

Bootstrap

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Task 1: Done

Task 2: Load the necessary packages

```
library(tidyverse)
```

```
## -- Attaching core tidyverse packages ----- tidyverse 2.0.0 --
## v dplyr      1.1.4      v readr      2.1.5
## v forcats    1.0.0      v stringr   1.5.1
## v ggplot2    3.5.1      v tibble    3.2.1
## v lubridate  1.9.3      v tidyr     1.3.1
## v purrr      1.0.2
```

```
## -- Conflicts ----- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()     masks stats::lag()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```
library(tidymodels)
```

```
## -- Attaching packages ----- tidymodels 1.2.0 --
## v broom      1.0.6      v rsample    1.2.1
## v dials      1.3.0      v tune       1.2.1
## v infer      1.0.7      v workflows  1.1.4
## v modeldata  1.4.0      v workflowsets 1.1.0
## v parsnip    1.2.1      v yardstick  1.3.1
## v recipes    1.1.0
```

```
## -- Conflicts ----- tidymodels_conflicts() --
## x scales::discard() masks purrr::discard()
## x dplyr::filter()    masks stats::filter()
## x recipes::fixed()   masks stringr::fixed()
## x dplyr::lag()        masks stats::lag()
## x yardstick::spec()  masks readr::spec()
## x recipes::step()    masks stats::step()
## * Learn how to get started at https://www.tidymodels.org/start/
```

Task 3: Creating the Data

```
# Set a random seed value so we can obtain the same "random" results
set.seed(2025)
```

```
# Create a data frame/tibble named sim_dat
```

```

sim_dat <- tibble(
  x1 = runif(20, -5, 5), # Random uniform values between -5 and 5
  x2 = runif(20, 0, 100), # Random uniform values between 0 and 100
  x3 = rbinom(20, 1, 0.5) # Random binary values (0 or 1) with equal probability
)

b0 <- 2
b1 <- 0.25
b2 <- -0.5
b3 <- 1
sigma <- 1.5
errors <- rnorm(20, 0, sigma)

sim_dat <- sim_dat %>%
  mutate(
    y = b0 + b1*x1 + b2*x2 + b3*x3 + errors,
    x3 = case_when(
      x3 == 0 ~ "No",
      TRUE ~ "Yes"
    )
  )

```

Task 4: Traditional MLR Model

```

mlr_fit <- linear_reg() %>%
  set_mode("regression") %>%
  set_engine("lm") %>%
  fit(y ~ x1 + x2 + x3, data = sim_dat)

tidy(mlr_fit, conf.int = TRUE)

```

```

## # A tibble: 4 x 7
##   term          estimate std.error statistic  p.value conf.low conf.high
##   <chr>          <dbl>    <dbl>    <dbl>    <dbl>    <dbl>    <dbl>
## 1 (Intercept)    1.56      0.743      2.10 5.15e- 2 -0.0116    3.14
## 2 x1             0.434     0.148      2.94 9.66e- 3  0.121     0.747
## 3 x2            -0.491     0.0123    -39.8 2.00e-17 -0.517    -0.464
## 4 x3Yes          1.01      0.792      1.28 2.20e- 1 -0.668     2.69

```

Task 5: Bootstrapping

```

# Set a random seed value so we can obtain the same "random" results
set.seed(631)

# Generate the 2000 bootstrap samples
boot_samps <- sim_dat %>%
  bootstraps(times = 2000)

boot_samps

```

```

## # Bootstrap sampling
## # A tibble: 2,000 x 2

```

```
## splits id
## <list> <chr>
## 1 <split [20/8]> Bootstrap0001
## 2 <split [20/6]> Bootstrap0002
## 3 <split [20/6]> Bootstrap0003
## 4 <split [20/6]> Bootstrap0004
## 5 <split [20/10]> Bootstrap0005
## 6 <split [20/10]> Bootstrap0006
## 7 <split [20/7]> Bootstrap0007
## 8 <split [20/6]> Bootstrap0008
## 9 <split [20/8]> Bootstrap0009
## 10 <split [20/6]> Bootstrap0010
## # i 1,990 more rows

# Create a function that fits a fixed MLR model to one split dataset
fit_mlr_boots <- function(split) {
  lm(y ~ x1 + x2 + x3, data = analysis(split))
}

# Fit the model to each split and store the information
boot_models <- boot_samps %>%
  mutate(
    model = map(splits, fit_mlr_boots),
    coef_info = map(model, tidy)
  )

boots_coefs <- boot_models %>%
  unnest(coef_info)

boots_coefs

## # A tibble: 8,000 x 8
## splits id model term estimate std.error statistic p.value
## <list> <chr> <lis> <chr> <dbl> <dbl> <dbl> <dbl>
## 1 <split [20/8]> Bootstrap00~ <lm> (Int~ 2.02 0.860 2.35 3.19e- 2
## 2 <split [20/8]> Bootstrap00~ <lm> x1 0.399 0.146 2.74 1.46e- 2
## 3 <split [20/8]> Bootstrap00~ <lm> x2 -0.502 0.0138 -36.3 8.64e-17
## 4 <split [20/8]> Bootstrap00~ <lm> x3Yes 1.52 0.751 2.03 5.94e- 2
## 5 <split [20/6]> Bootstrap00~ <lm> (Int~ 1.34 0.782 1.72 1.05e- 1
## 6 <split [20/6]> Bootstrap00~ <lm> x1 0.349 0.138 2.53 2.23e- 2
## 7 <split [20/6]> Bootstrap00~ <lm> x2 -0.488 0.0131 -37.3 5.61e-17
## 8 <split [20/6]> Bootstrap00~ <lm> x3Yes 1.47 0.813 1.81 8.88e- 2
## 9 <split [20/6]> Bootstrap00~ <lm> (Int~ 2.10 0.714 2.95 9.46e- 3
## 10 <split [20/6]> Bootstrap00~ <lm> x1 0.412 0.153 2.69 1.60e- 2
## # i 7,990 more rows
```

Bootstrap Confidence Intervals

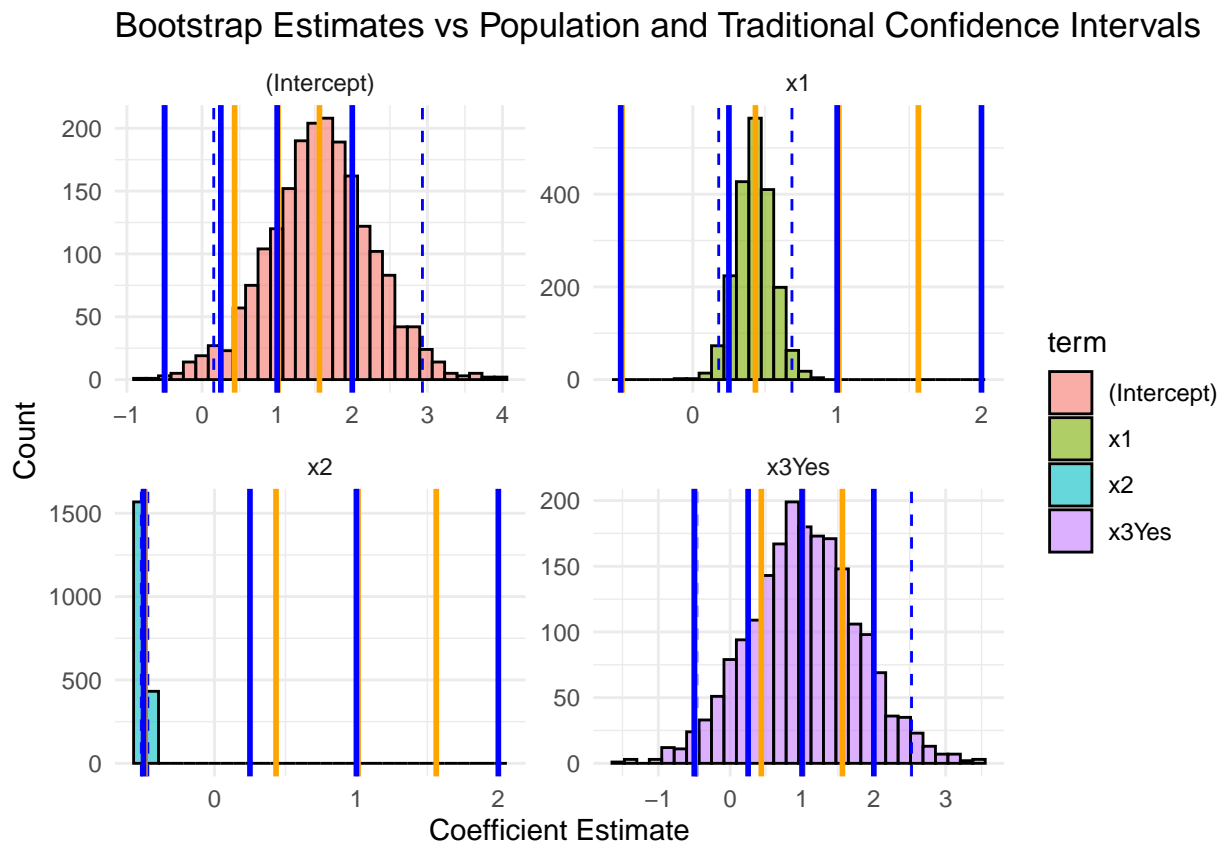
```
boot_int <- int_pctl(boot_models, statistics = coef_info, alpha = 0.05)
boot_int

## # A tibble: 4 x 6
## term .lower .estimate .upper .alpha .method
## <chr> <dbl> <dbl> <dbl> <dbl> <chr>
## 1 (Intercept) 0.156 1.57 2.93 0.05 percentile
```

```
## 2 x1          0.179    0.427  0.687   0.05 percentile
## 3 x2         -0.515   -0.491 -0.470   0.05 percentile
## 4 x3Yes      -0.464    1.03   2.52   0.05 percentile
```

Visualization

```
ggplot(boots_coefs, aes(x = estimate, fill = term)) +
  geom_histogram(bins = 30, alpha = 0.6, color = "black") +
  facet_wrap(~ term, scales = "free") +
  geom_vline(data = boot_int, aes(xintercept = .lower), col = "blue", linetype = "dashed") +
  geom_vline(data = boot_int, aes(xintercept = .upper), col = "blue", linetype = "dashed") +
  geom_vline(xintercept = c(1.5626060, 0.4336503, -0.4905565, 1.0117182), col = "orange", linetype = "solid", size = 1) +
  geom_vline(xintercept = c(2, 0.25, -0.5, 1), col = "blue", linetype = "solid", size = 1) +
  theme_minimal() +
  labs(title = "Bootstrap Estimates vs Population and Traditional Confidence Intervals",
       x = "Coefficient Estimate",
       y = "Count")
```



Answer to Question 5

The bootstrap estimates align closely with the population-level model coefficients. The traditional confidence intervals (orange lines) provide a reference, while the bootstrap intervals (blue dashed lines) capture the variability from resampling.

Accuracy Assessment:

- The bootstrap confidence intervals contain the true population values for most estimates, indicating reasonable accuracy.
- The traditional method's confidence intervals are slightly narrower than the bootstrap intervals, reflecting the differences in estimation methods.
- The variability of bootstrap estimates is evident, but their central tendency aligns well with the expected population coefficients.

This suggests that bootstrapping provides a robust alternative to traditional inference methods, especially when assumptions about normality or small sample sizes need to be considered.

Challenge Enhancements

- Added vertical orange lines for traditional confidence intervals.
- Added vertical solid blue lines for population slope values.
- Applied `theme_minimal()` for a cleaner plot appearance.
- Added histogram colors and adjusted transparency for better visibility.
- Included clear axis labels and a title to improve interpretability.