Aero 523, Fall 2020 **Midterm** 10/13/2020 2pm – 10/16/2020 11:55pm

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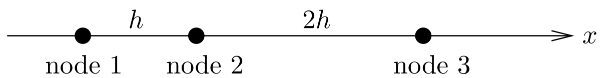
Take home, open-notes, computers allowed, no help from other people. Submit the exam online through Canvas. Upload one `.pdf` file containing your work (handwritten/scanned is acceptable). Printouts of any codes/calculations must be included in the `.pdf` file. Do not submit a `.zip` file. The submission limit is 10 sides, letter paper size, with legible fonts/writing – no exceptions. The late penalty is 1 point per minute.

1. Undetermined Coefficients [25%]

States u_i are given at three nodes in a non-uniform, one-dimensional grid, as shown below. Using the method of undetermined coefficients, derive the most accurate formula for du/dx at node 2, and give the order of accuracy, with respect to h, of your formula.



2. Gram Schmidt [25%]

Using the function inner product $(f, g) = \int_0^1 f g \, dx$, apply the Gram-Schmidt algorithm to orthonormalize the following two one-dimensional functions.

$$f_1 = x^4, \quad f_2 = 2x.$$

Then, project the function $g = x^2$ onto the space spanned by these two functions.

3. Gauss Seidel [25%]

Consider the successively over-relaxed, left-to-right Gauss-Seidel iterative smoother applied to a standard, second-order, central-difference discretization of the 1D Poisson equation ($-u_{xx} = f$) on $x \in [0, 1]$ with homogeneous Dirichlet boundary conditions. The grid is uniform and contains N intervals.

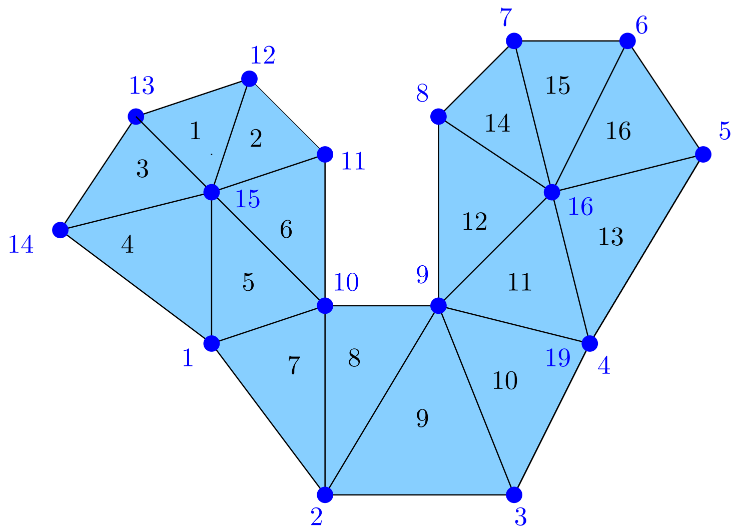
a) Write an expression for the iteration matrix, $\mathbf{S}$, for this smoother, using the decomposition $\mathbf{A} = \mathbf{L} + \mathbf{D} + \mathbf{U}$, where $\mathbf{A}\mathbf{u} = \mathbf{f}$ is the discretized system, as in the notes. Note that ω is a <u>successive</u> over-relaxation factor, applied immediately on each node update.

b) In the complex number plane, plot the eigenvalues of the iteration matrix, $\mathbf{S}$, for this smoother, using $N = 32$ and an over-relaxation factor of $\omega = 1.5$.

c) Make a plot of the magnitude of the largest magnitude eigenvalue of $\mathbf{S}$ versus ω, and identify the optimal over-relaxation factor. Use $N = 32$.

4. Mesh Connectivity [25%]

Section 1.7.5 of the notes introduces matrices $\mathbf{E}$ and $\mathbf{N}$ for describing connections between elements and nodes of a mesh. Assume that both matrices are made unique by using a counter-clockwise ordering of nodes/elements and by beginning each row with the smallest index. Element/node numbering starts at 1 and no numbers are skipped. Assume triangular elements.



a) For the mesh shown above, write down the matrices **E** and **N**.

b) Write a pseudo-code for <u>efficiently</u> determining **E** given **N**.

c) How many edges (total, interior and boundary) are in the above mesh? Write a pseudo-code for <u>efficiently</u> determining the number of edges in a mesh given **E**.