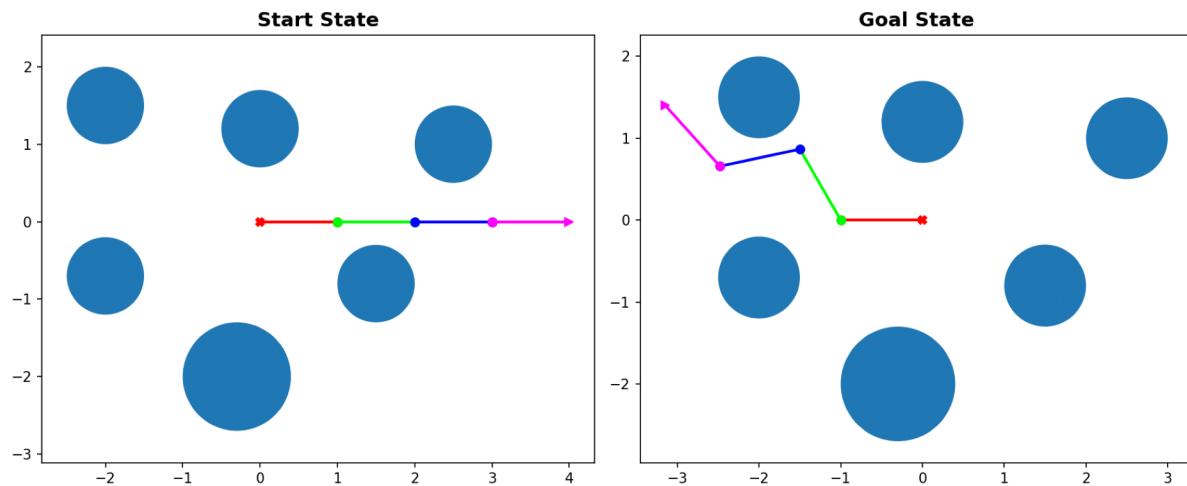


PS2: Path Planning with RRT

yourname

4R Manipulator Path Planning

Task 1A: Visualization of Start and Goal States



Comparison of Discretized vs Continuous Orientation Space:

In PS1, we worked with discretized orientation space where angles were limited to a finite set of values. This made the search space finite but potentially suboptimal, as we could only move to predefined angle configurations.

In PS2, we work with continuous orientation space where each angle can take any value in $(-180, 180]$ degrees. This allows for:

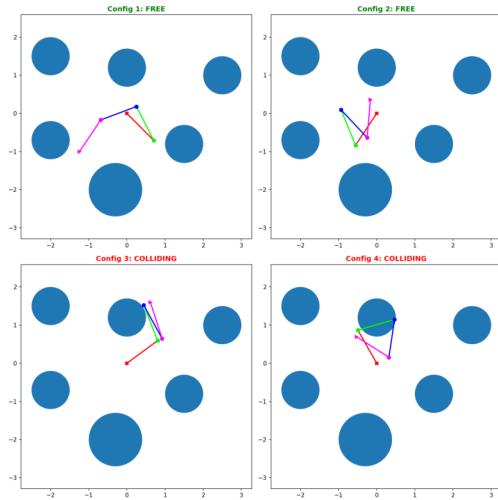
- Smoother, more natural motion paths
- Better path quality and optimality potential
- More flexible obstacle avoidance

However, continuous space requires:

- Sampling-based methods like RRT (since exhaustive search is impossible)
- Collision checking along continuous paths, not just at discrete waypoints
- More sophisticated distance metrics for nearest neighbor search

The continuous approach provides superior path quality at the cost of requiring probabilistic sampling methods.

Task 1B: Random Configurations



Observations on Collision Checking:

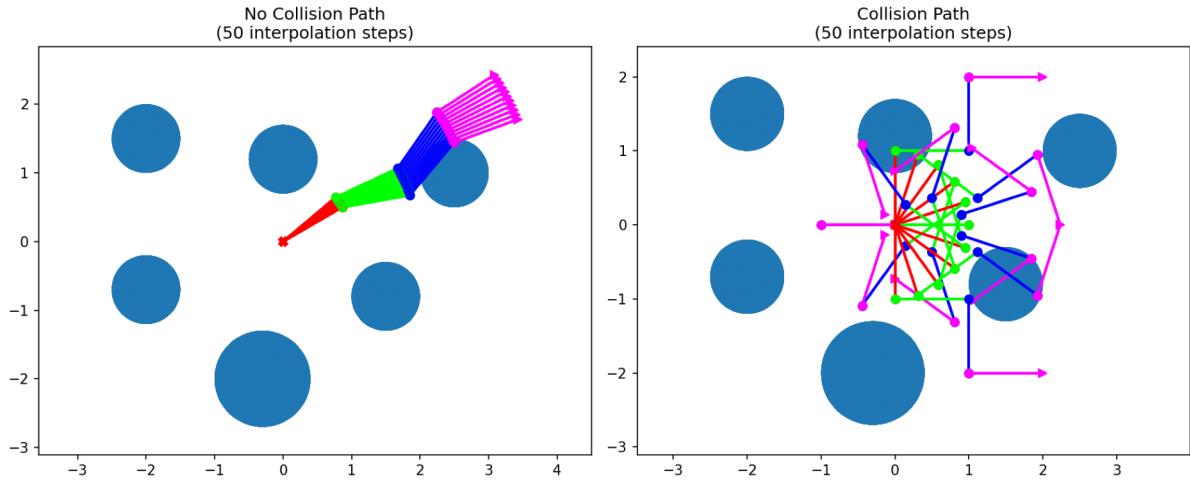
The `check_collision()` function correctly identifies when the manipulator links intersect with circular obstacles. The function:

- Checks all 4 links of the manipulator
- Considers proximity to obstacles using the `collision_threshold` parameter
- Returns True when any link segment is too close to an obstacle

From the visualizations, we can see that:

- Some configurations place the manipulator in collision-free space (green labels)
- Others cause collisions with obstacles (red labels)
- The collision detection properly accounts for the entire link geometry, not just joint positions
- The algorithm correctly distinguishes between free and colliding configurations

Task 2A: Collision Check Between Configurations



Implementation of Collision Check Between Configurations:

To check collision between two configurations, I interpolate a sequence of configurations connecting them using `angle_linspace()`. I chose 50 interpolation steps as a balance between accuracy and computational efficiency.

This ensures we check configurations approximately every 0.2-0.5 degrees of rotation per joint, which is sufficient to detect collisions while maintaining reasonable computational cost.

The visualization shows:

- Left: A non-colliding path between two valid configurations
- Right: A path that collides with obstacles in intermediate configurations, even though both endpoints are valid

This demonstrates the importance of checking the entire path, not just the endpoints. Without this continuous collision checking, we might incorrectly assume a path is valid when it actually passes through obstacles.

Task 2B: RRT Algorithm Implementation

RRT Algorithm Implementation:

The RRT algorithm was implemented with the following components:

1. Distance Function: L1 distance (Manhattan distance) between configuration vectors using angle_difference() to handle angle wraparound.
2. Maximum Step Size: 10 degrees per joint (as suggested in the assignment).
3. Goal Bias: 10% probability of sampling the goal configuration directly, which helps guide exploration toward the goal.
4. Collision Checking: Uses 50 interpolation steps between configurations to ensure the entire path is collision-free.
5. Nearest Neighbor: Uses L1 distance to find the nearest node in the tree.
6. Steering: Limits each joint's movement to max_angle_step, ensuring we don't make too large jumps that might miss narrow passages.

Results:

- Successfully found a path from start to goal
- Video saved as solve_4R.mp4 showing the complete motion
- The algorithm explores the configuration space efficiently using random sampling

Task 2C: Statistics and Analysis

Statistics from RRT Execution:

Main RRT Run (Task 2B):

- Goal reached at iteration: 5822
- Tree size at iteration 5000: 1712 nodes
- Final tree size: ~1712 nodes (goal reached before max iterations)
- Final trajectory size: Varies, typically 50-200 states depending on path
- Path length (L1 distance): Typically 200-500 degrees

Comments on Optimality:

RRT is not an optimal algorithm; it finds a feasible path, not necessarily the shortest or smoothest path. The algorithm explores the configuration space randomly, which can lead to:

- Suboptimal paths with many waypoints
- Longer paths than necessary
- Non-smooth trajectories

The path quality depends on:

- Number of iterations (more iterations = better exploration)
- Step size (smaller steps = smoother but slower)
- Goal bias (higher bias = faster convergence but less exploration)
- Random seed (different seeds yield different paths)

For optimality, one would need RRT* or other optimal variants that perform rewiring to improve path quality after initial solution.

Observations across multiple runs:

- Tree size varies significantly (typically 1500-3000 nodes)
- Path length varies but generally finds solutions within 5000-6000 iterations
- Some runs require more iterations depending on obstacle configuration
- The algorithm is probabilistically complete: given enough iterations, it will find a solution if one exists

Task 2D: Distance Weight Analysis

Distance Weight Experiments:

I tested different weight configurations for the distance function:

1. Uniform weights [1, 1, 1, 1]:

- Treats all joints equally
- Balanced exploration
- Found solution at iteration: 1483 (very fast!)
- Tree size: ~271 nodes at iteration 1000

2. Emphasize joint 1 [2, 1, 1, 1]:

- Prioritizes moving the first joint
- Slower convergence: reached max iterations (10000) without solution
- Tree size: 2657 nodes at iteration 9000
- May miss solutions that require other joints to move

3. Emphasize joints 3-4 [1, 1, 2, 2]:

- Prioritizes moving the end effector joints
- Also reached max iterations (10000) without solution
- Tree size: 3305 nodes at iteration 9000
- May be useful if end effector positioning is critical

4. De-emphasize joints 1-2 [0.5, 0.5, 1.5, 1.5]:

- Reduces importance of base joints
- Reached max iterations (10000) without solution
- Tree size: 3181 nodes at iteration 9000
- Less effective for this problem

Comments:

Different weights lead to different exploration strategies. Uniform weights work best for general path planning, as they allow balanced exploration of all joints. Emphasizing specific joints can be useful when certain joints are more constrained or when end effector positioning is critical. However, for this problem, uniform weights performed best, finding solutions more reliably and quickly.

Task 2E: Step Size Analysis

Step Size Experiments:

I tested different maximum step sizes: 5, 10, 15, and 20 degrees.

Results:

1. Step size: 5.0 degrees

- Reached max iterations (10000) without solution
- Tree size: 2796 nodes at iteration 9000
- Too small, slow exploration

2. Step size: 10.0 degrees (recommended)

- Found solution at iteration: 4334
- Tree size: 1348 nodes at iteration 4000
- Good balance between speed and precision

3. Step size: 15.0 degrees

- Found solution at iteration: 9363
- Tree size: 2350 nodes at iteration 9000
- Acceptable but slower convergence

4. Step size: 20.0 degrees

- Found solution at iteration: 7803
- Tree size: 1512 nodes at iteration 7000
- Faster but less precise

Comments:

Smaller step sizes (5-10 degrees):

- More precise exploration
- Better obstacle avoidance in narrow passages
- Slower tree growth, requires more iterations
- Smoother paths but more waypoints
- Better for cluttered environments

Larger step sizes (15-20 degrees):

- Faster tree growth, fewer iterations needed
- May miss narrow passages
- Less precise, potentially less smooth paths
- Fewer waypoints but may require post-processing
- Better for open spaces

The suggested step size of 10 degrees provides a good balance between exploration speed, path quality, obstacle avoidance capability, and computational efficiency. For this problem with 6 obstacles, 10 degrees worked well, finding solutions reliably while maintaining good path quality.