This is just a text with beamer presentations because I want other theme colors

We want more colors

David Anchieta

September 24, 2016

Outline

Estimating frequency and amplitude of a single tone

Given a complex sinusoid defined by:

$$r(t) = A_1 e^{j2\pi f_1 t} + z(t)$$

Where $z(t)$ is a complex Gaussian white noise with zero mean and variance σ^2.
Givens a set of samples from $r(t)$, how could we estimate the values of A_1 and f_1?

Note on Maximum Likelihood Estimation #1

- It's a method to estimate parameters of a statistical model.
- The value of the Maximum Likelihood Estimator of a parameter must make the observed data more likely.

Note on Maximum Likelihood Estimation #1

- It's a method to estimate parameters of a statistical model.
- The value of the Maximum Likelihood Estimator of a parameter must make the observed data more likely.

An example of maximum likelihood estimator: The sample mean of a set of n observations from a random variable X.

$$M_n = \frac{1}{n} \sum_{i=1}^{n} X_i$$

Note on Maximum Likelihood Estimation #1

- It's a method to estimate parameters of a statistical model.
- The value of the Maximum Likelihood Estimator of a parameter must make the observed data more likely.

An example of maximum likelihood estimator: The sample mean of a set of n observations from a random variable X.

$$M_n = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$\lim_{n \to \infty} M_n = \mathrm{E}\{X\}$$

Estimating frequency and amplitude of a single tone

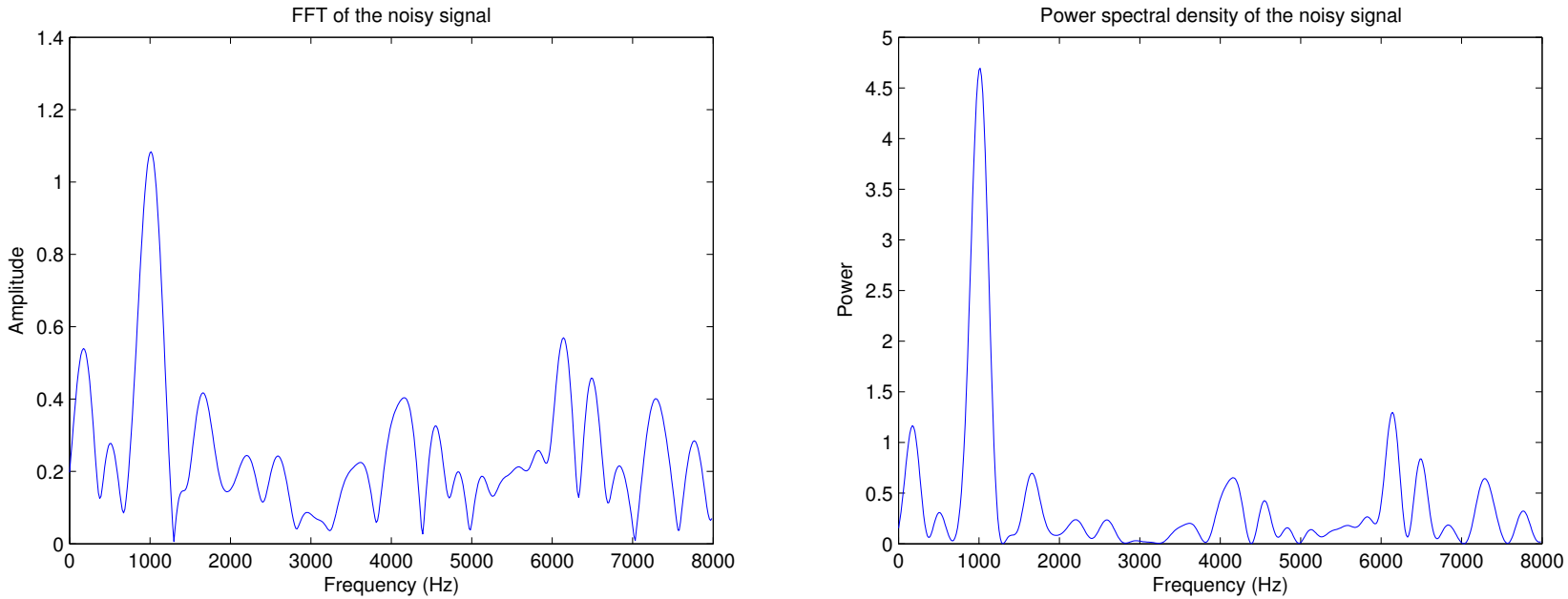

Figure: Discrete-time Fourier Transform and power spectral density of the signal with added noise.

Defining the PDF of the frequency

Being N the number of samples and T_s the sampling period $1/f_s$, we have.

$$\mathbf{r} = [r(0) \;\; r(T_s) \;\; r(2T_s) \;\; \dots \;\; r((N-1)T_s)]^T$$

$$\mathbf{e_1} = [e^{j2\pi f_1 0} \;\; e^{j2\pi f_1 T_s} \;\; \dots \;\; e^{j2\pi f_1 (N-1)T_s}]^T$$

$$\mathbf{r} = A_1\mathbf{e_1} + \mathbf{n}$$

$$\mathrm{p}(\mathbf{r}|f_1, A_1) = \frac{1}{\pi^N \sigma^{2N}} \exp\left[-\frac{(\mathbf{r} - A_1\mathbf{e_1})'(\mathbf{r} - A_1\mathbf{e_1})}{\sigma^2}\right]$$

$$\mathrm{p}(\mathbf{r}|A_1) = \int \mathrm{p}(\mathbf{r}|f_1, A_1)\mathrm{p}(f_1)df_1$$

$$\mathrm{p}(f_1|\mathbf{r}, A_1) = \frac{\mathrm{p}(\mathbf{r}|f_1, A_1)\mathrm{p}(f_1)}{\mathrm{p}(\mathbf{r}|A_1)}$$

Defining the PDF of the frequency

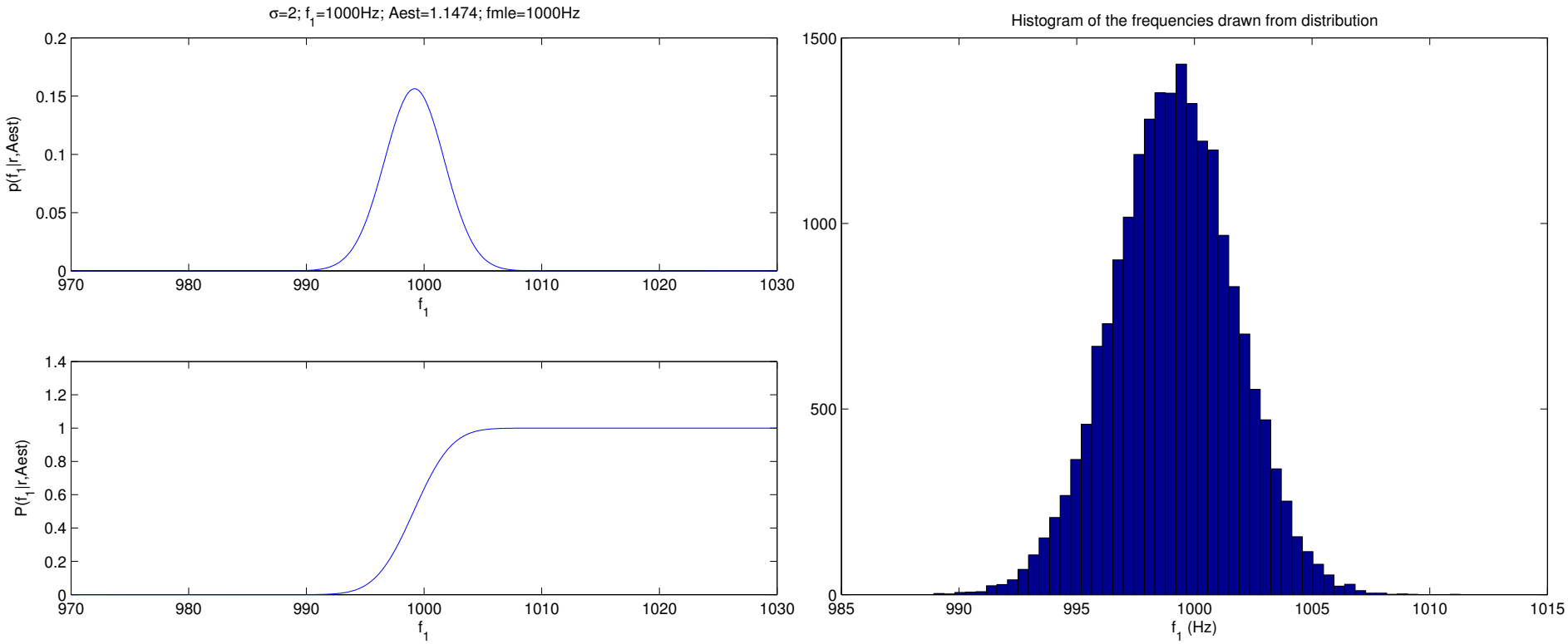

Figure: PDF of the frequency conditioned on the MLE of the amplitude.