This is just a text with beamer presentations because I want other theme colors

We want more colors

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Outline

1. Single tone case
   - Maximum likelihood estimators of frequency and amplitude
   - Computing probability density functions

Estimating frequency and amplitude of a single tone

Given a complex sinusoid defined by:

$$r(t) = A_1 e^{j2\pi f_1 t} + z(t)$$

Where $z(t)$ is a complex Gaussian white noise with zero mean and variance σ^2.
Givens a set of samples from $r(t)$, how could we estimate the values of A_1 and f_1?

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$$\lim_{n\to\infty} M_n = \mathrm{E}\{X\}$$

Estimating frequency and amplitude of a single tone

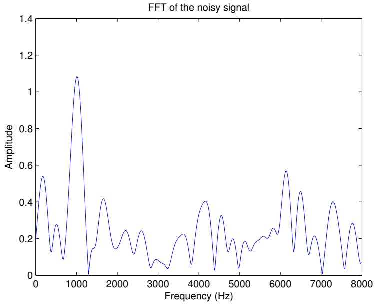

Figure: Discrete-time Fourier Transform and power spectral density of the signal with added noise.

Defining the PDF of the frequency

Being N the number of samples and T_s the sampling period $1/f_s$, we have.

$$\mathbf{r} = [r(0) \;\; r(T_s) \;\; r(2T_s) \;\; \dots \;\; r((N-1)T_s)]^T$$

$$\mathbf{e_1} = [e^{j2\pi f_1 0} \; e^{j2\pi f_1 T_s} \;\; \dots \;\; e^{j2\pi f_1 (N-1)T_s}]^T$$

$$\mathbf{r} = A_1\mathbf{e_1} + \mathbf{n}$$

$$\mathrm{p}(\mathbf{r}|f_1, A_1) = \frac{1}{\pi^N \sigma^{2N}} \exp\left[-\frac{(\mathbf{r} - A_1\mathbf{e_1})'(\mathbf{r} - A_1\mathbf{e_1})}{\sigma^2}\right]$$

$$\mathrm{p}(\mathbf{r}|A_1) = \int \mathrm{p}(\mathbf{r}|f_1, A_1)\mathrm{p}(f_1)df_1$$

$$\mathrm{p}(f_1|\mathbf{r}, A_1) = \frac{\mathrm{p}(\mathbf{r}|f_1, A_1)\mathrm{p}(f_1)}{\mathrm{p}(\mathbf{r}|A_1)}$$

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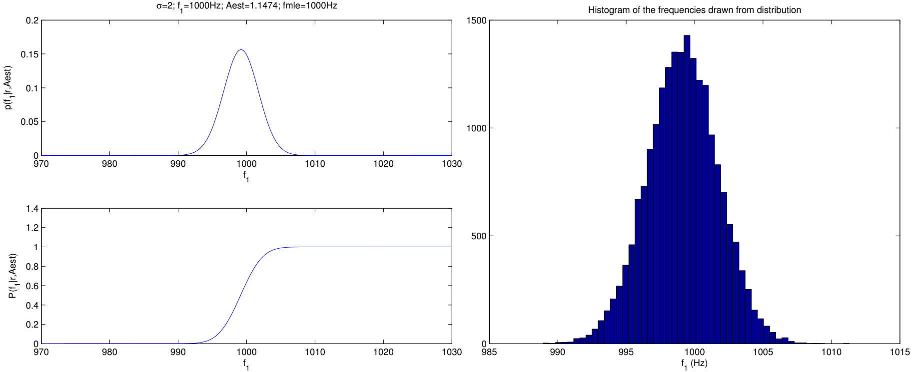

Figure: PDF of the frequency conditioned on the MLE of the amplitude.