

**This is just a text with beamer presentations
because I want other theme colors**

We want more colors

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Outline

- 1 Single tone case
 - Maximum likelihood estimators of frequency and amplitude
 - Computing probability density functions

Estimating frequency and amplitude of a single tone

Given a complex sinusoid defined by:

$$r(t) = A_1 e^{j2\pi f_1 t} + z(t)$$

Where $z(t)$ is a complex Gaussian white noise with zero mean and variance σ^2 .

Given a set of samples from $r(t)$, how could we estimate the values of A_1 and f_1 ?

Note on Maximum Likelihood Estimation #1

- It's a method to estimate parameters of a statistical model.
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$$\lim_{n \rightarrow \infty} M_n = E\{X\}$$

Estimating frequency and amplitude of a single tone

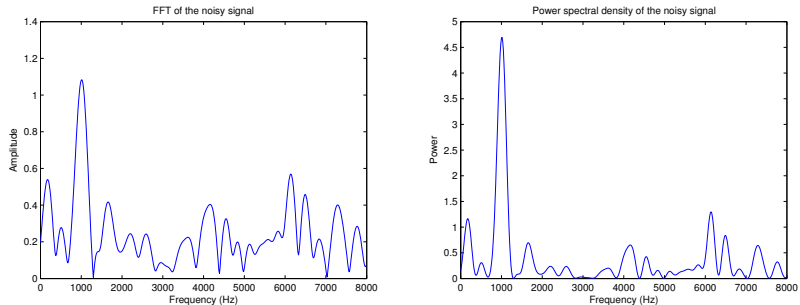


Figure: Discrete-time Fourier Transform and power spectral density of the signal with added noise.

Defining the PDF of the frequency

Being N the number of samples and T_s the sampling period $1/f_s$, we have.

$$\mathbf{r} = [r(0) \ r(T_s) \ r(2T_s) \ \dots \ r((N-1)T_s)]^T$$

$$\mathbf{e}_1 = [e^{j2\pi f_1 0} \ e^{j2\pi f_1 T_s} \ \dots \ e^{j2\pi f_1 (N-1)T_s}]^T$$

$$\mathbf{r} = A_1 \mathbf{e}_1 + \mathbf{n}$$

$$p(\mathbf{r}|f_1, A_1) = \frac{1}{\pi^N \sigma^{2N}} \exp \left[-\frac{(\mathbf{r} - A_1 \mathbf{e}_1)'(\mathbf{r} - A_1 \mathbf{e}_1)}{\sigma^2} \right]$$

$$p(\mathbf{r}|A_1) = \int p(\mathbf{r}|f_1, A_1) p(f_1) df_1$$

$$p(f_1|\mathbf{r}, A_1) = \frac{p(\mathbf{r}|f_1, A_1) p(f_1)}{p(\mathbf{r}|A_1)}$$

Defining the PDF of the frequency

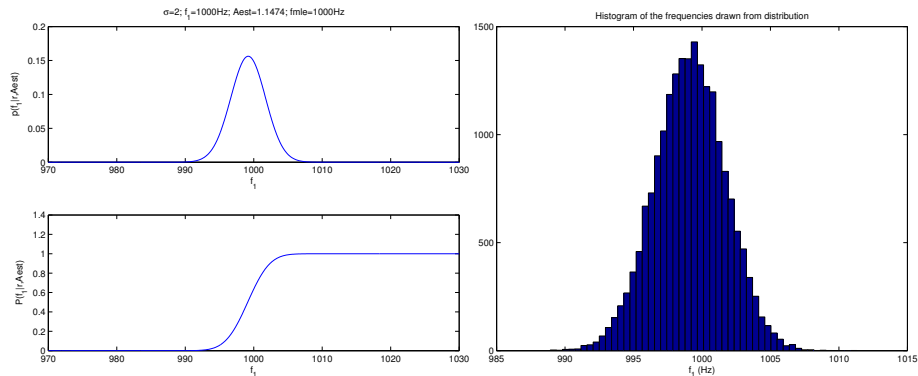


Figure: PDF of the frequency conditioned on the MLE of the amplitude.