

**This is just a text with beamer presentations  
because I want other theme colors**

We want more colors

David Anchieta

January 18, 2017

# Outline

1

## Single tone case

- Maximum likelihood estimators of frequency and amplitude
- Computing probability density functions

# Estimating frequency and amplitude of a single tone

## Theorem

*How you have fallen from heaven,  
morning star, son of the dawn!  
You have been cast down to the earth,  
you who once laid low the nations!*

*Isaiah 14:12*

# Estimating frequency and amplitude of a single tone

Given a complex sinusoid defined by:

$$r(t) = A_1 e^{j2\pi f_1 t} + z(t)$$

Where  $z(t)$  is a complex Gaussian white noise with zero mean and variance  $\sigma^2$ .

Given a set of samples from  $r(t)$ , how could we estimate the values of  $A_1$  and  $f_1$ ?

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$$\lim_{n \rightarrow \infty} M_n = E\{X\}$$

# Defining the PDF of the frequency

Being  $N$  the number of samples and  $T_s$  the sampling period  $1/f_s$ , we have.

$$\mathbf{r} = [r(0) \ r(T_s) \ r(2T_s) \ \dots \ r((N-1)T_s)]^T$$

$$\mathbf{e}_1 = [e^{j2\pi f_1 0} \ e^{j2\pi f_1 T_s} \ \dots \ e^{j2\pi f_1 (N-1)T_s}]^T$$

$$\mathbf{r} = A_1 \mathbf{e}_1 + \mathbf{n}$$

$$p(\mathbf{r}|f_1, A_1) = \frac{1}{\pi^N \sigma^{2N}} \exp \left[ -\frac{(\mathbf{r} - A_1 \mathbf{e}_1)'(\mathbf{r} - A_1 \mathbf{e}_1)}{\sigma^2} \right]$$

$$p(\mathbf{r}|A_1) = \int p(\mathbf{r}|f_1, A_1) p(f_1) df_1$$

$$p(f_1|\mathbf{r}, A_1) = \frac{p(\mathbf{r}|f_1, A_1) p(f_1)}{p(\mathbf{r}|A_1)}$$



# Defining the PDF of the frequency

## Just a list

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# Defining the PDF of the frequency

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