The Mathematical Research Paper

HANCULUS

Hanan Basheer

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Abstract - According to Florian Cajori's '*History of Mathematical Notations*', volume 2, page 61, a capital sigma for sums was first used by Leonhard Euler (1707-1783) in 1755, but didn't catch on until the 1800s.

Introduction

Most Mathematicians are familiar with the summation symbol Σ which is used to find sum of n terms of any sequence. I being a great math enthusiast have been on constant research on multiple new topics like Han θ .etc. when I came to think about summation and its properties. This all happened while I was studying for the Prestigious Joint Entrance Exam (JEE Mains) of my country India in year 2019. But summation which is discrete form of integration hasn't been much explored and has no separate branch like calculus.

Hence I have made few laws for discrete form of calculus and named it under Math category *Hanculus* which also includes summation and many of its properties.

Important Symbols

$\frac{h}{hx}()$ - **Hanniferation** **[Counter-part of Differentiation]**

Σ - **Summation / Hintegration** **[Counter-part of Integration]**

Definitions

- **Hanniferation -** In simple words, opposite of summation. It can be understood by thinking of a function whose summation given to you.
  For e.g. given $\frac{h}{hx}(n^2)$

  Then the required answer is a function whose summation is n^2.
  We know sum of odd numbers is n^2.

  Hence $\frac{h}{hx}(n^2) = 2n-1$ [i.e. Odd numbers 1, 3, 5….]

- **Summation / Hintegration** – The adding of terms of a predefined function is called summation of a function.
  For e.g. given $\sum(n) = [n(n+1)]/2$

Hanniferation

Hanniferal of a function is a function whose summation is the initial function. I observed patterns to find the Hanniferal of a function.

I observed: $\frac{h}{hx}(n) = 1$ [For x^n]

$\frac{h}{hx}(n^2) = 2n-1$ and so on ….

Finally I noticed a superb thing:

Pascal's Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

The Coefficients where the lower half of Pascal Triangle but with alternating signs.

So the required formula for Hanniferation is:

$$\boxed{\frac{h}{hx}(f(x)) = f(x) - f(x-1)}$$

The above formula will generate desired Hanniferal of a function.

This formula is applicable for all functions as logic tells that on doing summation of RHS of above formula will finally give *f(x)* as terms will get cancelled.

$$\begin{aligned}
\Sigma\left(\frac{h}{hx}(f(x))\right) &= f(x) - \cancel{f(x-1)} && \text{[Terms cancel out]} \\
&\quad + \cancel{f(x-1)} - \cancel{f(x-2)} \\
&\quad + \cancel{f(x-2)} - \cancel{f(x-3)} \\
&\quad + \cancel{f(x-3)} - \cancel{f(x-4)} \\
&\qquad \ldots\ldots\ldots \\
&\quad + \cancel{f(1)} - f(0)
\end{aligned}$$

$$= f(x) - f(0) \qquad \text{[Mostly base case } f(0) = 0\text{, in case of } x^n \text{ it is 0]}$$

$$= f(x) \qquad \text{[But in other cases } \textbf{remember} \text{ to include the base case]}$$

Hence Proved.

Properties of Hanniferation

- **Cancellation property -** $\sum\left(\frac{h}{hx}(f(x))\right) = f(x) = \frac{h}{hx}\left(\sum(f(x))\right)$

  This property always holds True.

- **Constant's properties -** $\frac{h}{hx}(k.f(x)) = k.\frac{h}{hx}(f(x))$ and $\frac{h}{hx}(k) = 0$ [k = constant]

- **Addition/Subtraction –** Like differentiation it is also distributive over addition and subtraction.
  For e.g. $\frac{h}{hx}(f(x) \pm g(x)) = \frac{h}{hx}(f(x)) \pm \frac{h}{hx}(g(x))$
- **Multiplication** – The expansion of hanniferation is similar to differentiation. But slightly different. The formula is an empirical one as I have proved it by proof by patterns and analysis.

$$\boxed{\frac{h}{hx}(f(x).\ g(x)) = \frac{h}{hx}(f(x)).g(x) + \frac{h}{hx}(g(x)).f(x) - \frac{h}{hx}(f(x)).\frac{h}{hx}(g(x))}$$

For e.g. $\frac{h}{hx}(n^2) = \frac{h}{hx}(\text{n.n}) = \frac{h}{hx}(\text{n}).\text{n} + \frac{h}{hx}(\text{n}).\text{n} - \frac{h}{hx}(\text{n}).\frac{h}{hx}(\text{n})$

$= 1.\text{n} + 1.\text{n} - 1.1$ $\quad\quad [\ \frac{h}{hx}(\text{n}) = 1\ ]$

$= 2\text{n}-1$

Hence Proved

Hintegration/Summation

Hintegration also known as summation is a symbol to denote sum of n terms of a particular series or function.

Now apart from basic properties of Summation like distribution over addition or subtraction I have invented another new property which is **Summation by parts** which is like integration by parts which is shown below:

$$\boxed{\int udv = uv - \int vdu}$$

So Formula of **Summation by parts** is:

$$\boxed{\sum\big(f(x).g(x)\big) = f(x).\sum(g(x)) - \sum\Big(\frac{h}{hx}(f(x)).\sum(g(x))\Big) + \sum\Big(\frac{h}{hx}(f(x)).g(x)\Big)}$$

Note: Do NOT forget to put the limits after summation.

Proof of Summation by Parts

The empirical formula of product rule of Hanniferation shall be used to derive the formula for summation by parts.

So,

$$\frac{h}{hx}(r(x).\,g(x)) = \frac{h}{hx}(r(x)).g(x) + \frac{h}{hx}(g(x)).r(x) - \frac{h}{hx}(r(x)).\,\frac{h}{hx}(g(x))$$

$$\boxed{\text{Put } \frac{h}{hx}(r(x)) = f(x) \quad \text{or} \quad \sum(f(x)) = r(x)}$$

$$\Rightarrow \quad \frac{h}{hx}(\sum(f(x)).\,g(x)) = f(x).g(x) + \frac{h}{hx}(g(x)).\sum(f(x)) - f(x).\,\frac{h}{hx}(g(x))$$

Now take summation on both sides:

$$\Rightarrow \quad \sum\Big(\frac{h}{hx}(\sum(f(x)).\,g(x)\Big) = \sum(f(x).g(x)) + \sum\Big(\frac{h}{hx}(g(x)).\sum(f(x))\Big) - \sum\Big(f(x).\,\frac{h}{hx}(g(x))\Big)$$

$$\Rightarrow \quad \sum(f(x)).\,g(x) = \sum(f(x).g(x)) + \sum\Big(\frac{h}{hx}(g(x)).\sum(f(x))\Big) - \sum\Big(f(x).\,\frac{h}{hx}(g(x))\Big)$$

Now just rearrange the equation got and you will get:

$$\sum\left(f(x).g(x)\right) = f(x).\sum(g(x)) - \sum\left(\frac{h}{hx}(f(x)).\sum(g(x))\right) + \sum\left(\frac{h}{hx}(f(x)).g(x)\right)$$

Hence Proved.

Now an example to show summation by parts:

e.g. $\sum(n) = \sum(n.1) = n.\sum(1) - \sum\left(\frac{h}{hx}(n).\sum(1)\right) + \sum\left(\frac{h}{hx}(n).1\right)$

$$= n.n - \sum(1.n) + \sum(1.1)$$

$$\sum(n.1) = n^2 - \sum(n) + n$$

$$2(\sum(n.1)) = n^2 + n$$

$$\sum(n.1) = (n(n+1))/2$$

Hence Proved.

This formula works for all x^n as well as any other function.

Conclusion

There is no doubt that this latest concept of Hanculus is useful in many fields of mathematics especially in development of higher mathematics.

Hope this new theory proves to be beneficial for many.