CFRM 501 - Investment Science
2020 Midterm Test

- The duration of this test/exam is 150 minutes. This includes the time to scan and submit your answers on Canvas. Submissions made after this time will be flagged as late.
- If for any reason you are unable to upload your answers on Canvas, email them to `rdon@uw.edu` with an explanation of the problem.
- Answers must be justified to receive full credit (i.e., show your work).
- Use the definitions and notation from this course whenever possible.
- Questions are not allowed during the test/exam. If you find something on the test/exam unclear,
  - explain why you think it is unclear,
  - precisely state your interpretation of what is being asked,
  - provide an answer to your interpretation of the question.
- You may use notes and resources available on Canvas during the test/exam.
- You must work on this test/exam independently.
- The following statement must be written on the first page of your submission. Do not alter this statement in any way.

I did not give or receive any unauthorized help during this test.

Question 1: [10 marks]

A collection of risky assets has expected return vector $\vec{\mu}$ and covariance matrix of returns $\mathbf{\Sigma}$. Let $\vec{w}_b$ be the weights of a given benchmark portfolio, and let $\gamma > 0$ be the conversion factor between tracking error variance and mean active return, so that the manager's performance criteria is

$$\vec{w}_a^T \vec{\mu} - \frac{\gamma}{2} \vec{w}_a^T \mathbf{\Sigma} \vec{w}_a \ ,$$

where $\vec{w}_a = \vec{w}_p - \vec{w}_b$ are the active weights and $\vec{w}_p$ are the manager's selected portfolio weights. Compute the portfolio weights $\vec{w}_p$ (**which sum to** 1) that maximize the manager's performance criteria. You must derive these results, simply writing down the correct answer is not sufficient.

Question 2: [10 marks]

An investor has the opportunity to trade a single asset. Use the following notation to set up and solve an expected utility maximization problem:

- W_0 - the investor's initial wealth
- P_0 - the initial price of the asset
- θ - number of shares purchased of the asset
- P_1 - the end-of-period price of the asset

The quantity P_1 is equal to $P_0 + \Delta$ where $\Delta \sim \mathcal{N}(\mu, \sigma^2)$. Write down the end-of-period wealth W_1 in terms of θ and find the value of θ which maximizes expected exponential utility with parameter $\gamma > 0$. You must derive the optimal value of θ, simply writing down the correct answer is not sufficient.

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Question 3: [10 marks]

The risk-free asset has a return of r_f. A collection of risky assets has tangency portfolio with expected return μ_T and variance σ_T^2.

Part a) Compute the percentage of wealth invested in the tangency portfolio when maximizing mean-variance criteria:

$$\mathbb{E}[r] - \frac{\gamma}{2}\mathbb{V}[r]$$

Part b) Compute the value of the risk-aversion parameter γ which results in 100% of wealth invested in the tangency portfolio. Denote this value by γ^*.

Part c) Make a sketch which contains the following:

- The efficient frontier curve,
- The points representing the tangency portfolio and the risk-free portfolio,
- A line through the tangency portfolio which is tangent to the efficient frontier curve,
- The indifference curve of the tangency portfolio for γ^*. For this curve it is important to clearly label the y-intercept of the curve and to **give the value of this intercept**.

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Question 4: [10 marks] Short Answer

Part a) In Chapter 3 (Mean-Variance Investing) of Asset Management by Andrew Ang, the author gives several examples of traditional mean-variance investing being outperformed by other trading strategies. Even though mean-variance investing appears to underperform in these examples, he claims that mean-variance portfolio analysis still carries an important message which is repeated throughout the chapter. What is the important message? (the correct single word answer is sufficient, but if you don't know that word then one or two sentences to describe the message is enough)

Part b) Let $X \sim \mathcal{N}(\mu, \sigma^2)$. Compute $\mathbb{E}[e^X]$. (You do not need to derive the result, stating the correct formula is sufficient.)

Part c) In Chapter 1 (Asset Owners) of Asset Management by Andrew Ang, the author discusses several different classes of asset owners. Which of these classes is typically the largest?