

HOMEWORK 1, Due April 13, 2021, before class (100 points)

Problem statements that seem clear to one person may not be clear to another and it is difficult to fully specify all details and notes that could prevent you from interpreting this problem differently than intended. If you need clarification, please post to Piazza or contact the instructor or TA.

Please submit the assignment as a single PDF file – preferably not handwritten – with any code (not required) as appendix material.

1. You are considering portfolios that can be constructed from three investments, A, B and C. The expected returns, standard deviations and correlations among the investments are shown in the table below.

Expected Returns	Standard Deviations	Correlations		
		A	B	C
13%	20%	1.0	0.6	0.5
10%	18%	0.6	1.0	0.3
8%	12%	0.5	0.3	1.0

- (a) (10 points) Please write expressions for the portfolio expected return and portfolio standard deviation for a portfolio constructed using weights w_A, w_B and w_C , in the three investments, respectively.
 - (b) (15 points) Assuming that we require the portfolio weights to sum to 1.0, write an expression for the weights associated with the minimum uncertainty portfolio and evaluate it to obtain the minimum uncertainty portfolio weights. Please submit the expression, the weights, the associated uncertainty as a standard deviation value and the expected return of the minimum uncertainty portfolio.
 - (c) (20 points) Please solve for the efficient frontier for these three investments and submit as your solution a 100 x 6 matrix with risk tolerance of the frontier points in the first column, expected returns of frontier portfolios in the second column, standard deviations of the frontier portfolios in the third column and weights of the three investments in columns four through six. Make sure that your solution contains the entire frontier from the minimum uncertainty portfolio through the maximum expected return portfolio. Assume weights must sum to 1 and must be non-negative.
2. A portfolio is constructed from four investments with weights w_1, w_2, w_3 and w_4 and has expected return of

$$0.08w_1 + 0.07w_2 + 0.05w_3 + 0.04w_4$$

and portfolio variance

$$0.0289w_1^2 + 0.0306w_1w_2 + 0.01632w_1w_3 + 0.0153w_1w_4 + 0.0225w_2^2 + XXXw_2w_3 + \\ 0.0081w_2w_4 + 0.0144w_3^2 + 0.01512w_3w_4 + 0.0081w_4^2$$

- (a) (10 points) What are the standard deviations of the four investments?
 - (b) (15 points) What is the range of possible values for the coefficient of the term w_2w_3 currently shown as “XXX” ?
3. Investments A and B have expected returns and standard deviations as shown in the table for question 1. If the correlation between the returns of A and the returns of B is -1.0,
- (a) (15 points) What are the weights for A and B that can be used to create an uncertainty-free position?
 - (b) (15 points) What is the uncertainty-free rate of return?