COMPUTATIONAL FINANCE & RISK MANAGEMENT

UNIVERSITY of WASHINGTON

Department of Applied Mathematics

Statistical Arbitrage: An Introduction

CFRM 522

Introduction to Trading Systems

References

- Aldridge: Ch 8
- Vidyamurthy: Ch 5
- Kissell: Ch 13, pp 432-436

Arbitrage and Statistical Arbitrage

- Given an asset S_t and a replicating portfolio $R(S_t)$,
  - Arbitrage: $|S_t - R(S_t)| > TC$ (TC = Trading Costs)
  - Statistical Arbitrage: $E_t(|S_\tau - R(S_\tau)|) > TC + target$, for $\tau > t$
- Methods
  - Pairs trading
  - Cointegration (including pairs trading)
  - Risk Arbitrage

Simple Pairs Trading

- Simplest type of statistical arbitrage (aka “stat arb”) trading
- Signal is based on difference between the prices of two assets, i, j
- Look at movements over time, from, say $t = 0$ to $t = T$
- Write $\Delta S_t = S_{it} - S_{jt}$ (Drop the i, j subscripts on the left)
- For all assets i, j in the set being considered
  - Calculate the sum of squares of the differences over each time series:

$$\sum_{t=0}^{T} \Delta S_t^{\,2}$$

  - For each i, find j that yields the lowest sum of squares
  - Then, for each minimizing i, j, calculate the mean and variance of each series of ΔS_t

Simple Pairs Trading

- Now, apply the following trading rule:
- If for some time τ we have

$$S_{i\tau} - S_{j\tau} > E[\Delta S_{\tau}] - 2\sigma[\Delta S_{\tau}]$$

then, sell asset i and buy asset j.

- When the gap reverses and yields a desirable gain (covering trading costs as well), close out the positions

- Analyzing every possible pair of assets or equities can be a time-consuming process

- This leads into a refinement of the strategy, using *Cointegration*

Cointegration

- The idea is to find two assets v and m with the same trend; viz,

$$v_t = n_{t-1} + \varepsilon_t^n + \varepsilon_t^v$$

$$m_t = n_{t-1} + \varepsilon_t^n + \varepsilon_t^m$$

where ε_t^v and ε_t^m are stationary error processes (mean = 0), but v_t and m_t are nonstationary (due to trend term n_t)

- Although each asset price time series is nonstationary, the time series

$$v_t - m_t = \varepsilon_t^v - \varepsilon_t^m$$

is stationary, and is said to be *cointegrated*. This model can therefore be exploited when one asset is "out of balance" with the other, as the expectation is a reversion to a mean of zero.

- This topic involves a more in-depth discussion involving time series analysis. For further details, refer to the course bibliography, and the discussion in Kissell, Ch 13, and Vidyamurthy, Ch 5.

- Based on CAPM: All securities are influenced by the broad market returns (each return here is an excess return):

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_t$$

- Simplest example: trading pairs with the same response to the changes in the broader market conditions, but with different intrinsic returns, ie, α. (This is basically a specific example of cointegration).
- Run statistical tests (usual t-test – see text) to determine pairs of assets i, j such that

$$\beta_i - \beta_j = 0$$
$$\alpha_i - \alpha_j \neq 0$$

- Also require $|\alpha_i - \alpha_j| > TC$
- Once a pair is found satisfying these conditions, go long in the asset with the higher α and short the other
- As CAPM says that α will eventually tend to zero, reverse the trade as the difference converges, or hits a predetermined target

[END]