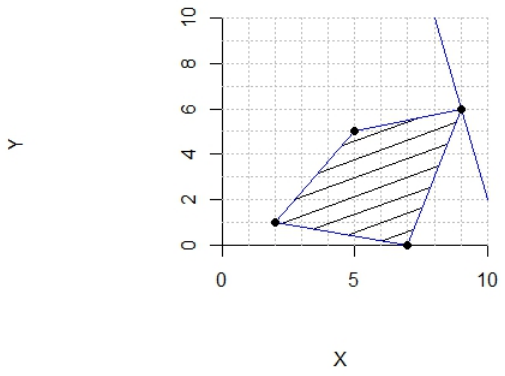
HOMEWORK 1, Due October 11, 2020

Problem statements that seem clear to one person may not be clear to another and it is difficult to fully specify all details and notes that could prevent you from interpreting these problem differently than intended. If you need clarification, please post to the Canvas Discussion Forum or contact the instructor or TA.

1. The feasible region of a linear program is shown in the graph below.



   (a) (4 points) List the basic solutions.

   (b) (4 points) Which constraint is redundant?

   (c) (4 points) Give an example of an objective function (identify the objective function, its value and the optimal solution) for which this LP would have a finite optimal solution.

   (d) (4 points) Give an example of an objective function for which the LP has multiple optimal solutions. Identify the objective function, its optimal value and two associated optimal solutions.

   (e) (4 points) Give an example of an objective function for which this LP would be unbounded.

   (f) (4 points) Give an example of an objective function for which this LP would be infeasible.

2. An insurance company issues life insurance policies as well as lifetime annuity products. Marketing, hedging and servicing these contracts requires time and resources, but is expected to generate a profit.
   The table below shows the time required for each product from various departments as well as the total time available from each department and the profit expected from each policy sold.

	Life Insurance	Life Annuity	Available
Underwriting	4	2	2400
Legal	1	1	1100
Administration	0	2	800
Claims	2	0	1400
Expected Profit	600	200	

The company would like to maximize expected profits.

(a) (5 points) Formulate a linear programming model for this problem in standard form.

(b) (5 points) Graphically represent the feasible region of this model.

(c) (5 points) Graphically identify the optimal solution.

(d) (5 points) Life insurance and annuities serve as natural hedges for each other in the sense that if annuity payouts go up due to increasing life expectancies, payouts from life insurance payouts will decrease (or at least be deferred). The insurance company would like to include an additional set of constraints to provide a balance between the exposure to life insurance liabilities and the exposure to annuity liabilities as follows:

$$\frac{Life\ Insurance\ sold}{Total\ products\ sold} \leq 60\%$$

$$\frac{Life\ Annuity\ sold}{Total\ products\ sold} \leq 60\%$$

Because these new constraints contain decision variables in both the numerator and denominator (afterall, the total products sold is simply equal to the sum of life insurance and annuities sold), these constraints are not linear. Rewrite the model from (a) as a linear model that includes these additional constraints and is in standard form.

(e) (5 points) Is the solution to the model in (d) the same as the solution from (c)?

(f) (5 points) Given an example of an objective function for which this LP would be unbounded.

(g) (5 points) Given an example of an objective function for which this LP would be infeasible.

3. Solve Problem 2 (including constraints from part (d)) using Excel. Highlight cells associated with decision variables in blue, cells associated with RHS in yellow, cells associated with objective function coefficients in green, and the cell holding the objective function value in red.

(a) (3 points) Please identify the optimal values of all variables.

(b) (3 points) Which variables are basic in the optimal solution?

(c) (3 points) Which constraints are active at the optimal solution?

(d) (3 points) Which constraints are inactive at the optimal solution?

(e) (3 points) What is the marginal value of an additional hour of services from each of the departments (Underwriting, Legal, Administration, and Claims)?

(f) (5 points) If a new policy type, Car Insurance, can be offer and requires 1.5, 2, 2.5, and 4 hours from the Underwriting, Legal, Administration and Claims departments, respectively, how much profit would each policy need to generate for this to be an attractive addition to the company's product line?
Support your answer using sensitivity information from the optimal solution rather than simply resolving.

4. (5 points) In the Furniture Maker problem, what if temporary labor can be hired for $25 per hour, but is less efficient and takes twice as long to finish tasks? Is it advisable to hire temporary labor? If so, at what level?
Don't simply formulate and solve a new problem, justify your solution using values in the optimal tableau that we discussed in class for the Furniture Maker problem.

5. Solve the Cash Flow Matching problem using Excel. Highlight cells associated with decision variables in blue, cells associated with RHS in yellow, cells associated with objective function coefficients in green, and the cell holding the objective function value in red.

   (a) (4 points) Please identify the optimal values of all variables.

   (b) (4 points) Which variables are basic in the optimal solution?

   (c) (4 points) Which constraints are active at the optimal solution?

   (d) (4 points) Which constraints are inactive at the optimal solution?

   (e) (4 points) What is the impact of a decrease of $1 in the size of the line of credit?
   Support your answer using sensitivity information from the optimal solution rather than simply resolving.

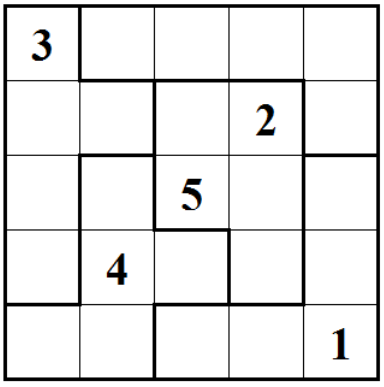
Formulate the following problems as mathematical programming models. There is no need to solve them. Be sure to define model variables (5 points), the objective function (5 points), and constraints (5 points). **Total 60 points**

2. A farm comprises 1250 acres of cropland that can be used to grow corn or oats. Each acre of corn provides $142 revenue and each acre of oats provides $128. Each acre of corn requires 2 hours of labor and each acre of oats requires 1.5 hours of labor. The total available labor hours is 350. The farmer seeks to maximize total revenue for the farm.

3. On January 1, you agree to lend $2000 to a friend with the promise to be paid back $350 each June 1 and each December 1 until the total payments meet or exceed the $2000 loan. What is the rate of return that you will receive?

4. Jigsaw Sudoku requires that the entries in each row and each column as well as in each highlighted region contain different values. Fill in the unspecified cells of the following grid with the numbers 1 through 5.

Irregular Sudoku



Rajesh Kumar @ www.FunWithPuzzles.com

Extra Credit

1. (5 points) Reformulate the following model into an equivalent model that is a linear program in standard form.

$$\begin{array}{rl} \text{Maximize} & -e^T|x| \\ \text{subject to} & Ax \geq b \\ & x \text{ unrestricted} \end{array}$$

where $e = (1, 1, ..., 1)$.

Hint: $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

2. (3 points) If A is skew-symmetric ($A^T = -A$), what is the optimal objective function value for the following linear program (assuming that it is feasible)?