CFRM 505: Monte Carlo Methods in Finance (Winter 2021)

CFRM Program, University of Washington

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Assignment 4

1. Let U_a and U_b be independent $U(-1,1)$ random variables. Suppose we want to estimate the probability

$$\mathbb{P}\{2U_a^2 + 2U_b^2 < 3\}.$$

   (a) First, estimate this probability by simulating both U_a and U_b. Implement your algorithm in R/python using 10000 samples for each random variable.

   (b) Describe a Monte Carlo simulation procedure that involves generating only samples of U_a (and not U_b). Write down the corresponding estimator. Then implement in R/python using a sample size of 10000.

2. Let X and Y be independent exponentials with X having mean 2 and Y having mean 3, and suppose we want to use simulation to estimate $\mathbb{P}(X + Y \geq 4)$. Let's use conditional expectation for variance reduction. Would you condition on X or Y? Explain your reasoning, state what you will simulate. Then, implement your conditional MC algorithm in R/python.

3. Consider the random variable:

$$V = \max\{X_1, X_2\},$$

   where X_i's are independent and $X_i \sim \exp(i)$, $i = 1, 2$. The objective is to estimate the probability $\mathbb{P}\{V > 20\}$.

   In R/Python, implement a MC simulation algorithm that estimates this probability using importance sampling on *both* X_1 and X_2. Write down the likelihood ratio used in your algoritm. Use a sample size of at least 100000 for each random variable.

4. Consider the random variable:

$$V = \min\{X_1, X_2\},$$

   where $X_1 \sim N(0,1)$ and $X_2 \sim N(0,2)$, and they're independent. The objective is to estimate the probability $\mathbb{P}\{V > 5\}$.

   In R/Python, implement a MC simulation algorithm that estimates this probability using importance sampling on *both* X_1 and X_2. Write down the likelihood ratio used in your algoritm. Use a sample size of 100000 or more for each random variable.