CFRM 501 - Investment Science
Assignment 4

Due: November 9, 2020 - 11:59 pm

Late submissions will receive an automatic grade of zero.

Question 1: In a two asset economy, if $\rho = \pm 1$ it is possible to achieve a portfolio with zero variance. Show the following:

- If $\rho = 1$ then the portfolio which achieves zero variance has $w < 0$ or $w > 1$.

- If $\rho = -1$ then the portfolio which achieves zero variance has $0 < w < 1$.

Assume $\sigma_1 > 0$, $\sigma_2 > 0$, and $\sigma_1 \neq \sigma_2$.

Question 2: Download daily prices from November 1, 2015 to September 30, 2020 of the following equities: MSFT, AAPL, ORCL, EBAY, GOOG, INTC, BBBY, MMM, TEVA, GE.

Compute the sample mean vector and sample covariance matrix for the arithmetic returns. Convert these quantities to annualized values by the following (use $N = 252$ for the number of days in a year):

$$\mu_i^{(a)} = (\mu_i^{(d)} + 1)^N - 1\,,$$

$$(\sigma_i^{(a)})^2 = \left((\sigma_i^{(d)})^2 + (\mu_i^{(d)} + 1)^2 \right)^N - (\mu_i^{(d)} + 1)^{2N}\,,$$

$$\rho_{i,j}^{(a)} \sigma_i^{(a)} \sigma_j^{(a)} = \left(\rho_{i,j}^{(d)} \sigma_i^{(d)} \sigma_j^{(d)} + (\mu_i^{(d)} + 1)(\mu_j^{(d)} + 1) \right)^N - (\mu_i^{(d)} + 1)^N (\mu_j^{(d)} + 1)^N\,.$$

Using the annualized mean and covariance matrix, find the global minimum variance portfolio and report the values of the weights, standard deviation, and expected return. Also plot the following, all on the same figure:

1. the mean-variance boundary,

2. all points corresponding to individual assets,

3. the global minimum variance portfolio.

Which equity has the largest weight in the global minimum variance portfolio? When you consider the variance of each equity, does this result surprise you?

With ten assets, there are 45 pairwise correlations. Change all of the pairwise correlations by exactly ± 0.1 (choose the direction yourself, but move them all in the same direction). Using the new correlations redo the computations from above and comment in particular on the change the mean-variance boundary (you should plot the new frontier along with the original one).

Continued Reading: Chapters 1, 2, and 3 of Asset Management by Andrew Ang must be completed before the midterm (November 9, 2020).