CFRM 505: Monte Carlo Methods in Finance (Winter 2021)

CFRM Program, University of Washington

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Assignment 2

1. Use the inverse transform method to generate X specified as follows:

   (a) $X = \min(U_1, U_2)$ where U_1 and U_2 are IID $U(0,1)$

   (b) $X = \max(U_1, U_2)$ where U_1 and U_2 are IID $U(0,1)$

   (c) $X = \min(Y_1, Y_2, Y_3)$ where Y_1, Y_2, and Y_3 are IID exponential with parameter λ.

   Then estimate the mean, i.e. $E[X]$, in each of the three cases. Use a sample size (for X) no less than 10,000.

2. Write a MC simulation algorithm to generate 10000 samples of the Poisson random variable with $\lambda = 1$. Give the algorithm and report the mean and variance from your samples.

3. Generate a random variable X with the CDF:

$$\mathrm{F}(x) = \int_0^\infty x^y e^{-y}\, dy \quad \text{for } 0 \le x \le 1.$$

   Hint: Consider the composition method. In particular, let F denote the distribution function of X, and suppose that the conditional distribution of X given that $Y = y$ is

$$\mathrm{P}(X \le x | Y = y) = x^y, \quad \text{for } 0 \le x \le 1.$$

   Write the algorithm in R/Python to generate 10,000 samples of X with the CDF above. Report the mean and variance from your samples.

4. Give an acceptance-rejection algorithm to generate a random variable with pdf:

$$f(x) = 2(1-x) \quad \text{for } 0 \le x \le 1.$$

   Describe your choice of the density function $g(x)$ and constant a. Run your simulation code to generate $100,000$ samples, and compute the average number of iterations that were required until each sample was accepted.

5. Recall from the slides the acceptance-rejection algorithm for simulating $X \sim N(0,1)$ that utilizes

$$g(x) = 0.5e^x 1_{[-\infty,0)}(x) + 0.5e^{-x} 1_{[0,\infty)}(x).$$

   Write a simulation code to implement this algorithm to generate 10000 samples of X. Show the histogram of the samples. Estimate $E[X]$ and $Var(X)$. How many uniforms are used (on average) to generate one sample of X?