COMPUTATIONAL FINANCE & RISK MANAGEMENT

UNIVERSITY *of* WASHINGTON

Department of Applied Mathematics

Performance Ratios and Measures

CFRM 422/522 (009)

Introduction to Trading Systems

Lecture Reference

- Aldridge, Ch 6: Performance (reading assignment)

- Additional reading on performance measures, index models, and CAPM: Narang, Ch 9, *Research* (reading assignment)

- Additional References:
  - Clenow, Ch 4, for extra coverage of performance measures (will cover the remainder of the chapter in more detail later)
  - Bodie, Kane, Marcus
    - Ch 8, *Index Models*
    - Ch 9, *The Capital Asset Pricing Model*
  - David Luenberger, *Investment Science* (1E)

A good quantitative model…

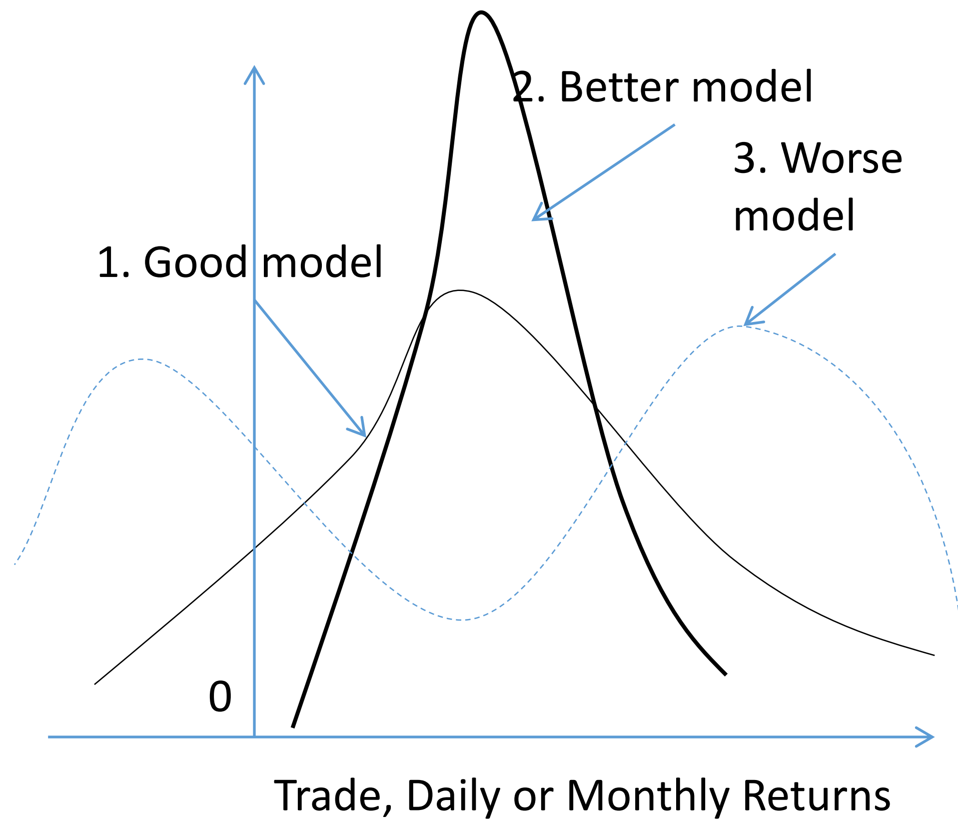
Produces high, precise returns

- Produces positive returns
- With little variation in returns
  - ie, few negative returns
- Results in a high "Sharpe ratio:"

$$Sharpe\ Ratio = \frac{E[r] - r_f}{\sigma[r]}$$

r_f = borrowing rate used to finance trading

Return distributions



Basic Performance Measures

1. Return
2. Volatility
3. Maximum drawdown
4. Win ratio
5. Avg gain/loss
6. Correlation
7. Alpha
8. Beta
9. Skewness
10. Kurtosis

Monthly performance	ABLE Gross Return	ABLE Return Net of fees and expenses*	S&P Return
Aug 23-31 2007	0.93%	0.34%	0.74%
Sep 2007	1.31%	0.91%	3.61%
Oct 2007	3.58%	2.73%	1.56%
Nov 2007	4.36%	3.35%	-4.22%
Dec 2007	0.57%	0.32%	-0.75%
Jan 2008	2.05		
Total:	**12.7**		

*Based on 2% annual manag
(performance fee is calculate

Daily Metrics Relative to S&P 500:	
Correlation:	-3.18%
Beta:	-0.0087
Alpha (excess risk-adjusted	0.09%

Daily Statistics	ABLE Return	S&P Return
Daily Avg	0.09%	-0.05%
Daily Stdev	0.35%	1.27%
Maximum	1.49%	2.92%
75% Quartile	0.20%	0.79%
Median	0.00%	0.00%
25% Quartile	-0.02%	-0.69%
Minimum	-0.89%	-3.20%

Performance Measures: Return

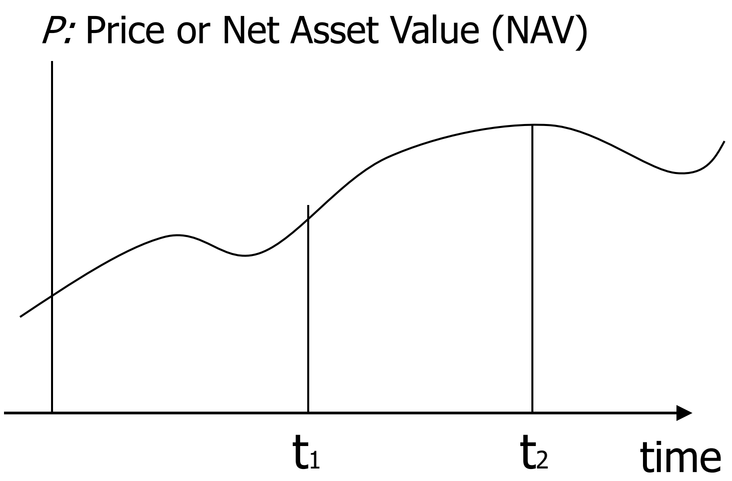
- Return can be expressed:
  - In dollar value
  - Most often, as a percentage
    - ➢ Allows easy cross-strategy and cross-asset comparison of performance, independent of the starting price

$$R_{t_2} = \frac{P_{t_2}}{P_{t_1}} - 1$$

  - Log returns are also common

$$R^*_{t_2} = \log \frac{P_{t_2}}{P_{t_1}}$$

- Illustration

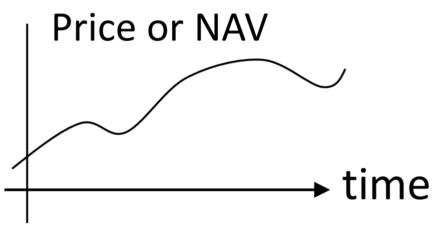


Why use log returns?

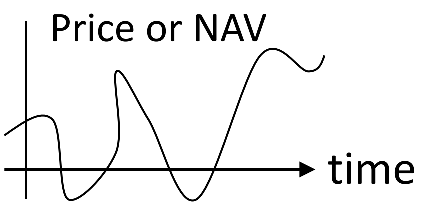
- Often want to know the cumulative return (over a period partitioned into *n* equal intervals, eg 1 day):
- $Let\ P_i = security\ price\ at\ time\ i$
- $Let\ r_i = return\ over [i-1,\ i),\ then\ 1 + \ r_i = \frac{P_i}{P_{i-1}}$
- $Then, the\ cumulative\ return\ r \quad is\ implicitly\ defined\ by$
- $1 + r \quad = (1 + \ r_1)(1 + \ r_2) \ldots (1 + \ r_n).$
- $But\ r \quad \cong \quad log(1 + r)\ , so$
- $r \quad \cong log \left\{ \left(\frac{P_1}{P_0}\right) \left(\frac{P_2}{P_1}\right) \ldots \left(\frac{P_n}{P_{n-1}}\right) \right\} = \log(P_n) - \log(P_0)$
- For r > 0.20, the approximation breaks down
- If assuming lognormal returns (eg for option pricing theory), the result is taken as the continuous cumulative rate of return over $[0, \mathrm{n})$; viz, $FV = \ e^{rt_n}$

Performance Measures: Volatility

- Measures how much the return moves up and down
- Is often taken to proxy risk
- Intuitively:
  - Low volatility



  - High volatility

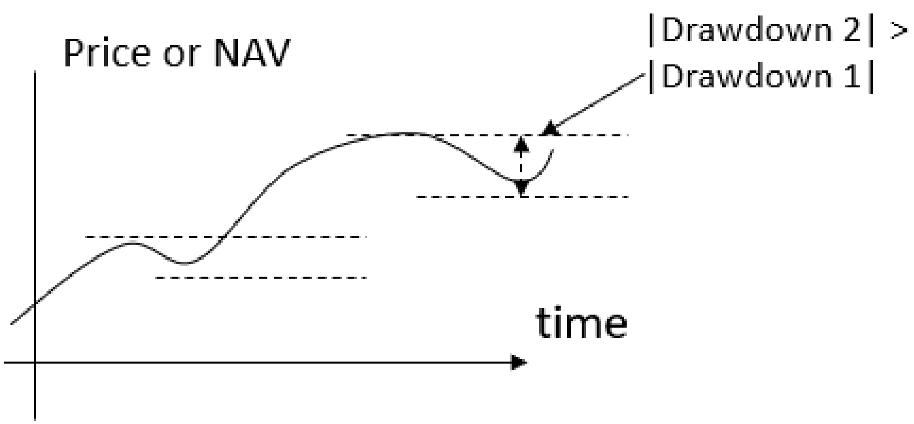


- At least a dozen measures of volatility exist, each measured per standardized period (round-trip trade, day, month)
- Standard deviation
  - Simple (most popular)
  - Weighted (later observations count more than earlier observations)

Drawdown

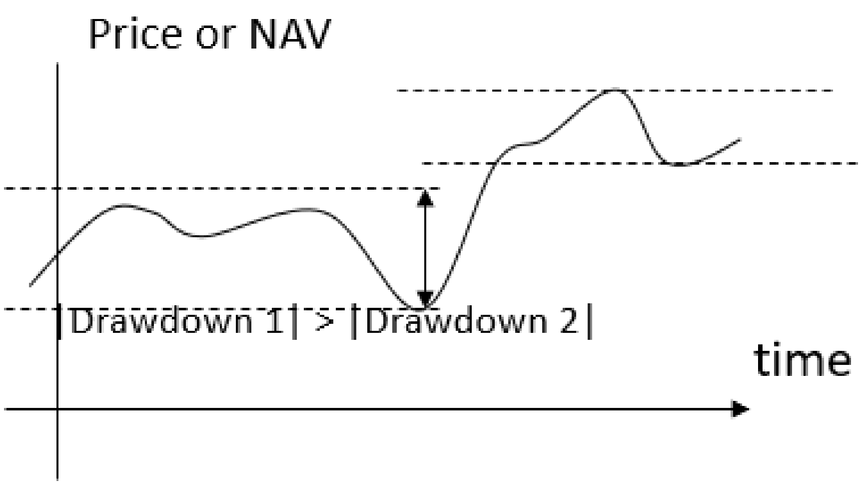
- Drawdown is a measure of historical and potential loss
- Maximum loss relative to the highest previous value or "watermark"
- Managers typically receive performance fees only after exceeding the highest watermark
- Maximum drawdown helps explain potential downside
- Key measure of portfolio or trading strategy performance

- Example 1

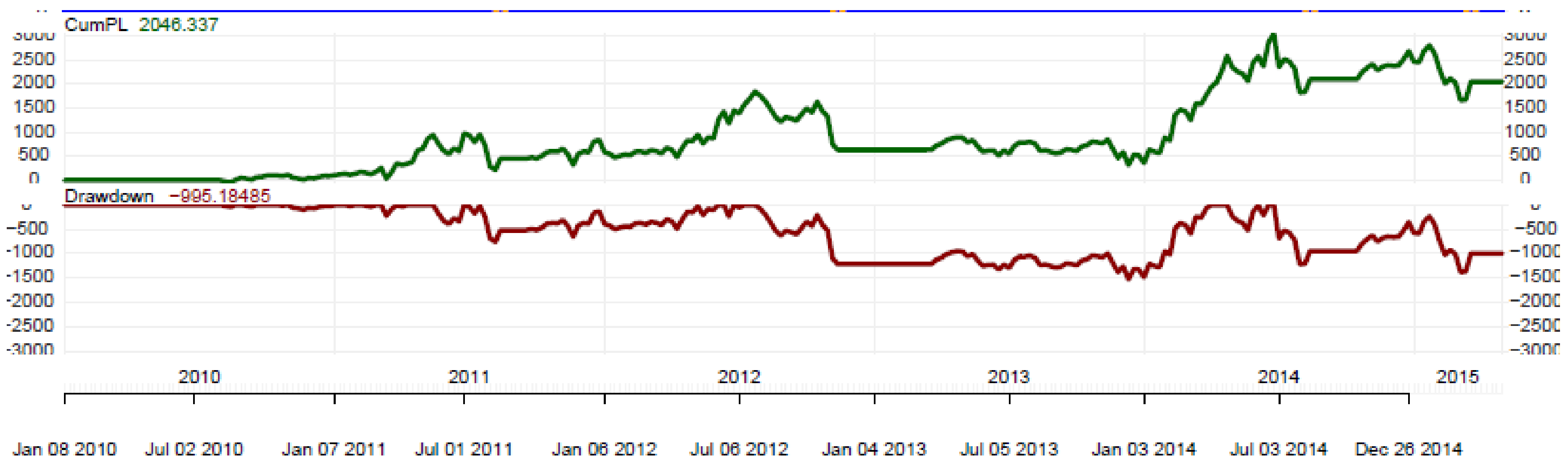


Irene Aldridge (Copyright © 2012)

Example 2



- Example using data from the XLU (utilities ETF)
- Using the quantstrat R package



Per-Trade Statistics (not in Alrdridge)

- Maximum adverse excursion (MAE) is the largest loss that a trade suffers while it is open
- Maximum favorable excursion (MFE) is the peak profit that a trade achieves while it is open
- These, and more, will be covered in Tomasini & Jaekle

Performance Measures: Win Ratio

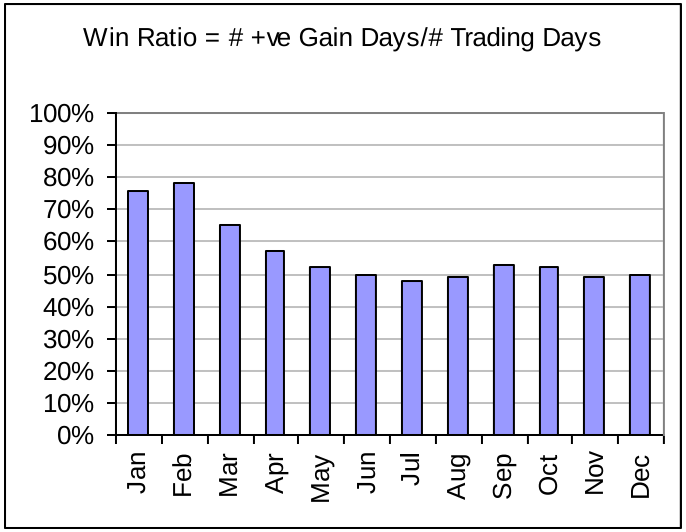
- What portion of the trades/days/months ended profitably?

$$WinRatio = \frac{\#Periods|_{Gain>0}}{Total\#Periods}$$

- Win ratio helps
  - compare “signals” of strategies
  - monitor run-time performance (is run-time win ratio consistent with prior performance?)

- Example:

Monthly day-by-day win ratio of a strategy



A decline in the win ratio may indicate that the strategy is reaching capacity

Performance Measures: Average Gain/Loss

- Measures related to drawdown:
  - When the strategy gains, what is the average gain?
  - When the strategy loses, what is the average loss?
- Best used together with win ratio:
  - High win ratio may tolerate lower avg gain
  - Lower win ratio requires higher avg gain

- Basic check of reported statistics:

$$E[R] \geq (WinRatio) * E[Gain] + (1 - WinRatio) * E[Loss]$$

	High *WinRatio*	Low *WinRatio*
High $\frac{E[Gain]}{E[\mid Loss \mid]}$	✓	High drawdowns
Low $\frac{E[Gain]}{E[\mid Loss \mid]}$	High volatility	✗

Performance Measures: Correlation

- Measures co-movement of strategy returns with those of another strategy or security
- Low correlation => high diversification
- Simple correlation:

$$\rho_{1,2} = \sum_t (R_{1,t} - E[R_1])(R_{2,t} - E[R_2])$$

- Asymmetric correlation can be more informative

$$\rho_{1,2}\,|_{R_1>0} = \sum_t (R_{1,t} - E[R_1])(R_{2,t} - E[R_2])\,|_{R_1>0}$$

$$\rho_{1,2}\,|_{R_1<0} = \sum_t (R_{1,t} - E[R_1])(R_{2,t} - E[R_2])\,|_{R_1<0}$$

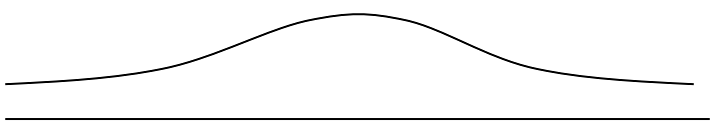
- Example:
  - Two ETFs: SPY (SPDR S&P 500) and GLD
  - Simple correlation (2010-2011 daily return data):
    - ➢ 5.6%
  - Asymmetric correlation:
    - ➢ On days when SPY > 0: Corr[SPY, GLD] = 4.3%
    - ➢ On days when SPY < 0:

      Corr[SPY, GLD] = -6.6%

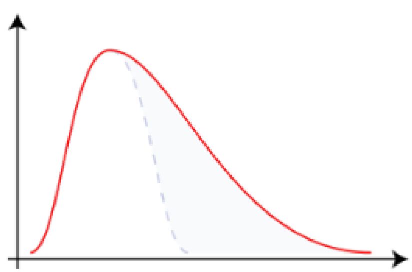
GLD has been a good diversifier of SPY: when SPY < 0, GLD can be > 0

Performance Measures: Skewness and Kurtosis

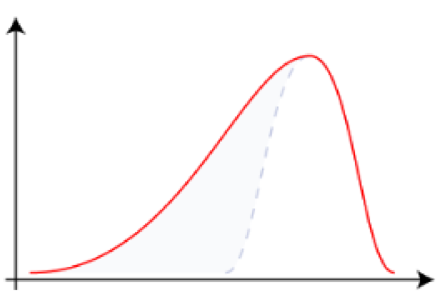
- Skewness describes tendency to skew to positive or negative side
  - Zero-skewness:



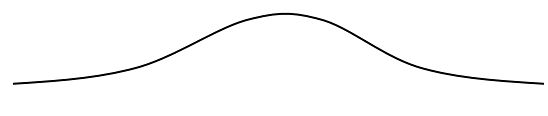
  - Positive skewness:



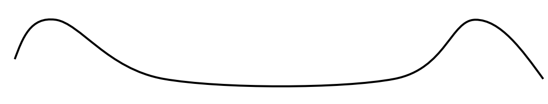
  - Negative skewness:



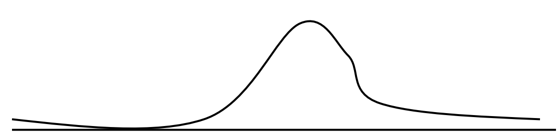
- Kurtosis measures the likelihood of extreme occurrences
  - "Normal" kurtosis



  - High kurtosis



  - Low kurtosis



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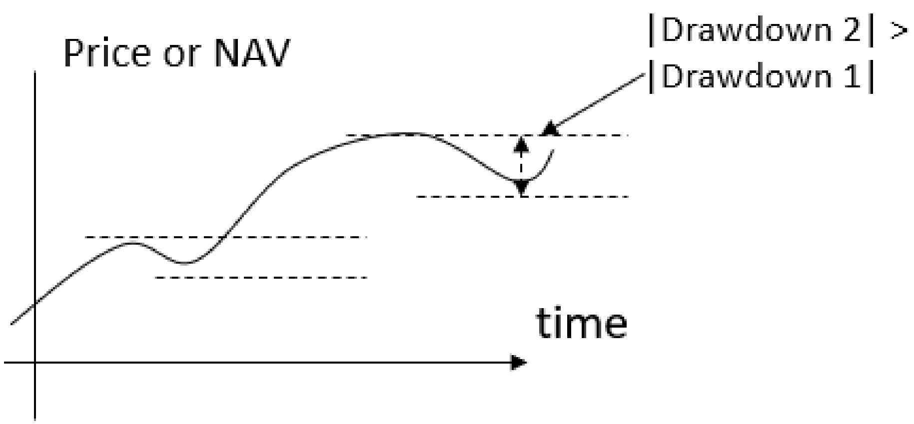
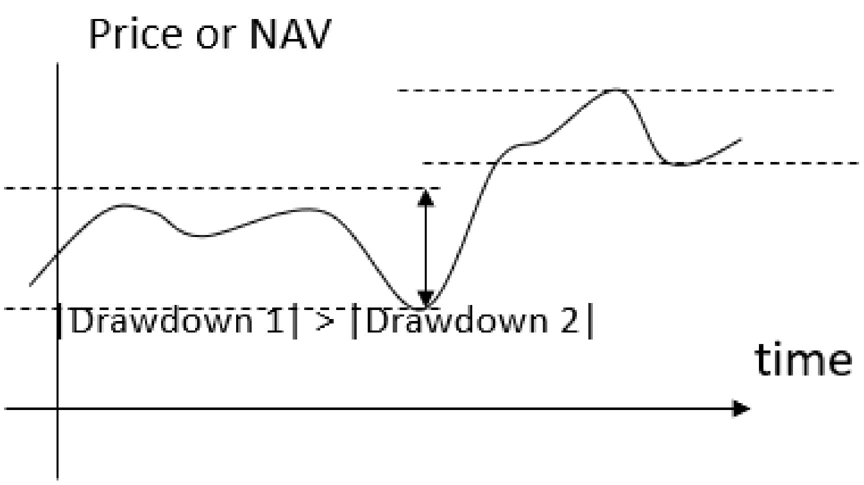


Example 2



Drawdown

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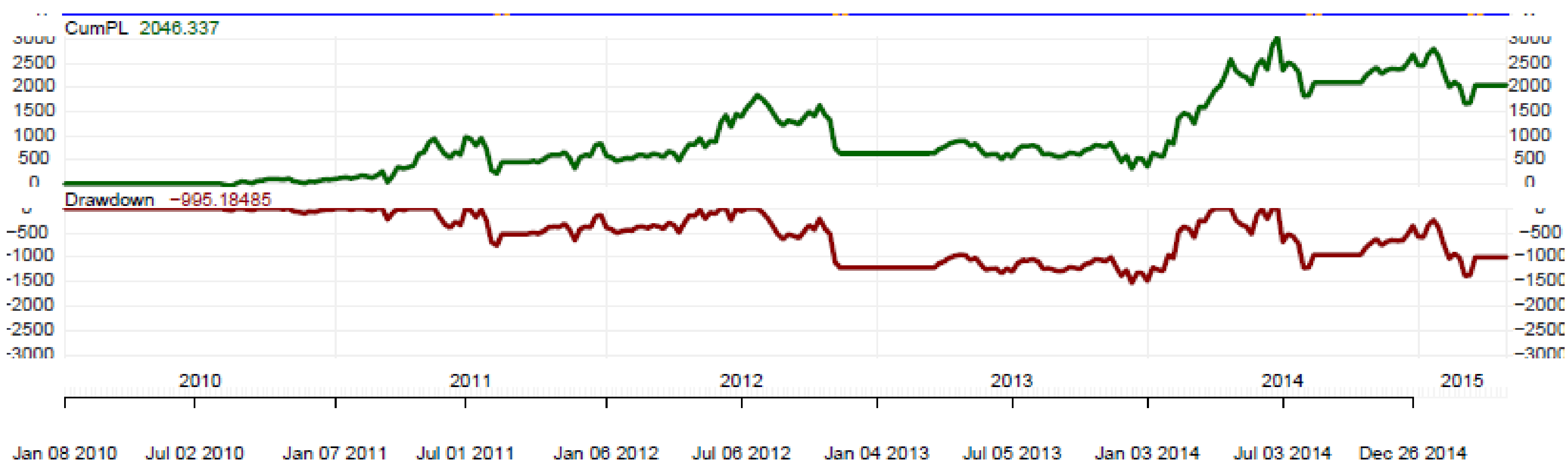

Ratios

- CAPM-based Ratios
  - Sharpe Ratio
  - Treynor Ratio
  - Jensen's Alpha

- VaR Ratios

Capital Asset Pricing Model (CAPM)

- The basic assumptions of the capital asset pricing model are as follows:

  - Market prices are in equilibrium
  - Everyone has the same forecasts of expected return and risks (everyone has access to the same information)
  - All investors choose an efficient portfolio according to their risk
  - Preferences so all portfolios are a combination of the *tangency portfolio* (to be discussed) and the risk-free asset
  - The risk premium for a single security is only a function of its contribution to the risk of the tangency portfolio

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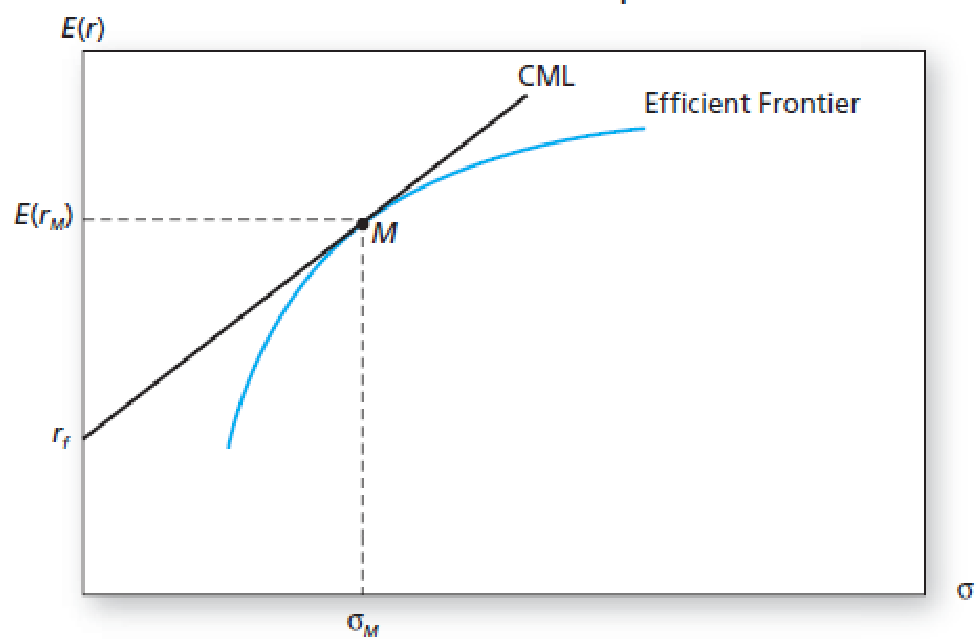
Guy Yollin (2016), Ruppert & Matteson

Capital Asset Pricing Model (CAPM)

- Markowitz => Everyone will optimally hold a weighted combination of a tangency portfolio and the risk-free asset
- CAPM assumptions => everyone has the same information
- Conclusions:
    - The one fund – the tangency portfolio – will be the same
    - This one fund must be the market portfolio
- The efficient frontier and its interior contain all possible portfolios in the entire market
- The market portfolio contains shares of every stock in the market in proportion to that stock's representation – ie capitalization – in the market
    - S&P 500
    - Wilshire 5000 ("Total Market Index")

B: The Efficient Frontier and the Capital Market Line



Luenberger (1E), p 174

Figure from Bodie/Kane/Marcus (10E)

Capital Asset Pricing Model (CAPM)

- Let p represent a portfolio comprised of securities in the overall market
  - Could contain multiple equities
  - Or, a single stock
- Its return is r_p, with weighted mean return μ_p and variance σ_p^2
- Let r_M, μ_M and σ_M^2 represent the same for the market tangency portfolio
- These are all scalar values
- The *central result* of the CAPM is that in equilibrium the riskiness of a portfolio in the market can be measured by a linear relationship between the expected return of the portfolio and the expected return of the market:

$$\mathbb{E}(r_p) = r_f + \beta(\mathbb{E}(r_M) - r_f)$$

where

$$\beta = \frac{Cov(r_p, r_M)}{\sigma_M^2}$$

Alpha

- "Empirical" CAPM =>

$$r_i = r_f + \beta_i(r_M - r_f) + \varepsilon_i$$

where ε_i is an error term with $\mathbb{E}(\varepsilon_i) = 0$ and variance σ_i^2 for security i in the market (M)

- Let R_i and R_M represent the excess returns above the risk-free rate; then,

$$R_i = \beta_i R_M + \varepsilon_i$$

- Empirical research has tested whether the CAPM is valid by testing regressions of the form

$$R_i = \alpha_i + \beta_i R_M + \varepsilon_i$$

- If the CAPM holds, then we can reject the hypothesis that $\alpha_i \neq 0$

- On the flip side, in the CAPM world, if $\alpha_i \neq 0$, then there is an arbitrage opportunity

Alpha

$$R_i = \alpha_i + \beta_i R_M + \varepsilon_i$$

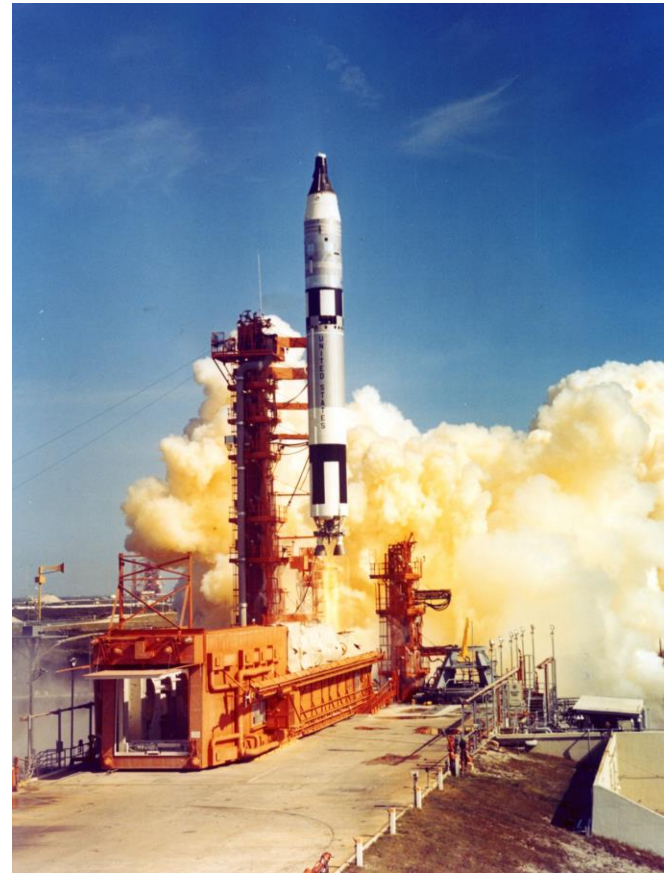
- In the CAPM world, if $\alpha_i \neq 0$, then there is an arbitrage opportunity
  - (Temporary) opportunity to make a profit risk-free
  - If $\alpha_i > 0$, buy the security; price will increase until R_i reverts lower, and α_i decreases back to 0
  - If $\alpha_i < 0$, sell the security; price will decrease until R_i reverts higher, and α_i increases back to 0

- Alpha is a performance metric in trading strategies

- Can apply outside of CAPM assumptions

- High and statistically significant Alpha is desirable

1966



Sharpe Ratio (1966)

- The Sharpe Ratio of a security i measures its excess return, adjusted for its risk:

$$\frac{\mathbb{E}(r_i) - r_f}{\sigma_i}$$

where $\mathbb{E}(r_M)$ and σ_i are substituted by their statistical estimators

- Compare, with r_f = 1%
  - Security 1 has an annual mean return of 46% (great!), but volatility 50%
  - Security 2 has an annual mean return of 11% (snore…), but volatility 5%
  - SR(1) = 0.9
  - SR(2) = 2.0
  - Risk-adjusted return of Security 2 is over twice that of Security 1

- Note that if $r_i = r_M$, the slope of the CML is the same as the Sharpe Ratio for the tangency portfolio in CAPM

Treynor Ratio

$$\frac{\mathbb{E}(r_i) - r_f}{\beta_i}$$

- Like the Sharpe Ratio, this gives excess return per unit of risk

- But it uses systematic risk instead of total risk

- Sometimes preferred to the Sharpe Ratio

Jensen's Alpha

$$\mathbb{E}(r_i) - r_f - \beta_i\left(r_M - r_f\right)$$

- The average return on the portfolio over and above that predicted by the CAPM, given the portfolio's beta and the average market return

- Rearrange the regression model from before

$$r_i - r_f = \beta_i\left(r_M - r_f\right) + \varepsilon_i$$

- And take expectations

- VaR Ratios
  - Excess return on value-at-risk
  - Modified Sharpe ratio

- Value at-Risk (VaR) describes the possible loss of an investment, which is not exceeded with a given probability of $1-\alpha$ in a certain period.

- For normally-distributed returns,

$$VaR_i = -(E[r_i] + z_\alpha \sigma_i)$$

where z_α is the α-quantile of the standard normal distribution.

Excess Return on VaR

- Dowd (2000)

$$\frac{\mathbb{E}(r_i) - r_f}{VaR_i}$$

- VaR_i = Value-at-Risk for a single security (or portfolio)

- Not suitable for parametric VaR when returns are not assumed to fit a normal distribution

Modified Sharpe ratio

- Gregoriou and Gueyie (2003)

$$Modified\ Sharpe = \frac{E[r] - r_f}{MVaR_i}$$

- Cornish-Fisher expansion is calculated as follows:

$$MVaR_i = -(E[r_i] + \sigma_i (z_\alpha + (z_\alpha{}^2 - 1)S_i / 6$$
$$+ (z_\alpha{}^3 - 3z_\alpha)EK_i / 24 - (2z_\alpha{}^3 - 5z_\alpha)S_i{}^2 / 36))$$

  - where S_i denotes skewness and EK_i the excess kurtosis for security *i* (Favre and Galeano, 2002).

- Suitable for non-normal returns.

Performance Attribution

- Consider Capital Asset Pricing Model (CAPM) again.

$$R_{i,t} = \alpha_i + \beta_i R_{M,t} + \varepsilon_{i,t}$$

- If Beta is high and Alpha is low, it may be cheaper and more effective to invest into R_M instead of R_i
  - Lower transaction costs
  - Often lower drawdown risk
  - Lower liquidation risk

- Example:
  - A trading strategy trades at least once a week
  - Relative to the SPDR S&P 500 ETF (SPY), the strategy has
    - ➢ Alpha of –0.01 (-1%)
    - ➢ Beta of 0.99 (99%)
  - It is cheaper and more effective to buy and hold SPDR S&P 500 ETF

- **[End]**