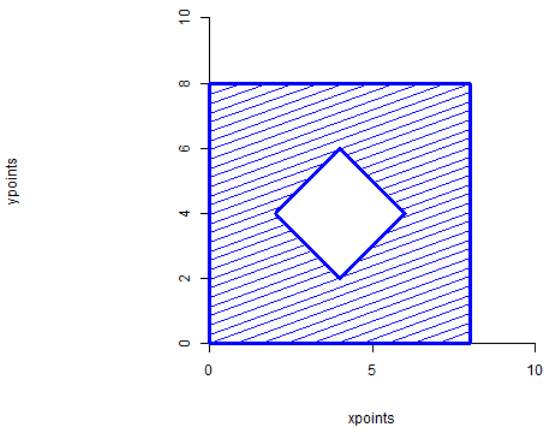
HOMEWORK 3, Due November 22, 2020

Problem statements that seem clear to one person may not be clear to another and it is difficult to fully specify all details and notes that could prevent you from interpreting these problem differently than intended. If you need clarification, please post to the Canvas Discussion Forum or contact the instructor or TA.

1. Consider the feasible region depicted by the shaded area in the following graph.



   (a) (10 points) Write down constraints that describe the feasible represented by the shaded part of the figure above.

2. An unnamed online retailer is looking to construct additional distribution centers to serve several cities. The distribution center must be located near rail and highway access and based on these requirements only 5 locations are being considered. The company would like to determine the number of distribution centers to construct and their locations in order to minimize the construction costs plus the one year cost to supply cities in this region. All orders for a given city are supplied from a single distribution center. The cost to supply each city from each potential distribution center is shown in the table below.

	City										
Dist. Center	A	B	C	D	E	F	G	H	I	J	Construction Cost
1	2	1	8	5	7	1	4	6	5	9	10
2	4	9	4	3	9	4	4	2	7	2	12
3	5	8	7	6	7	9	3	10	4	5	7
4	3	5	7	9	7	6	6	5	4	5	8
5	8	7	3	6	8	5	7	4	8	7	6

   (a) (10 points) Formulate the problem as an integer programming problem.

   (b) (10 points) Solve using AMPL or glpkAPI. Submit your input files along with the problem solution.

3. (20 points) You have an open position and seek to hire a qualified candidate. In your estimation, the Human Resources department is likely to find one candidate each day for each of the next 20 days before your deadline. Because HR cannot always fully appreciate the needs of hiring managers, they advise managers that the probability of a given candidate being exceptional is 2%. The chance that the candidate is merely acceptable is 20%. The chance of the candidate being completely unsuitable is 78%. You cannot hire an unsuitable candidate. If you don't fill the position, you won't be able to get all of the necessary work done. Your preferences can be summarized as:

   - Hiring an exceptional candidate is 5 times better than hiring a merely acceptable one.
   - Hiring an acceptable candidate is 5 times better than not hiring anyone.

   Use dynamic programming to solve this problem. Be sure to identify the stage, state, available decisions and the recursive relationship that could be used to solve this problem. Your solution should prescribe the best action to take at each stage and state as well as the expected outcome.

4. (20 points) You are a private equity fund manager screening investment opportunities. Over the next 3 years you anticipate finding one opportunity per month and seek to build a portfolio of between 14 and 16 investments with the highest expected return. As you screen the opportunities, you assign an anticipated return level. Of course, there is no telling whether the opportunities will actually provide the anticipated return, but you do your best to evaluate them nonetheless. The anticipated return levels and the probabilities that each month's project will be assessed at that level are shown in the table below.

Anticipated Return Level	Probability
20%	10%
15%	15%
10%	20%
less than 10%	55%

   You invest equal amounts in all opportunities that are funded and seek to build a portfolio with the highest anticipated return. Unlike the real world, in this homework problem you must decide whether to invest in an opportunity at the time it is proposed or those seeking capital will call on a different funder.
   You are not allowed to invest in projects anticipated to have less than a 10% return. To reflect your preference to have a sufficiently diversified portfolio, you apply a penalty for failing to invest in at least 14 projects. The penalty is a 4% decrement to your average return for each project less than 14. This penalty doesn't change the return levels of projects, it is simply a mechanism to express the trade off between attaining a high expected return level and not having enough investments in your portfolio.

You find yourself in month 33 committed to 12 of the 32 projects that you have already reviewed and with an average anticipated return of 14% and you anticipate that this 33^{rd} project will have a 10% return,

- What do you do? Do you commit to the project even though it will lower your average return or do you decline to participate and wait for the next one? Why?

Extra Credit

1. **Function Approximation**: Let $f(x)$ be a function with values given at $x_1, x_2, \ldots, x_n$ and functions $g_1(x)$, $g_2(x)$, ..., $g_m(x)$ be functions over the same range. We can approximate $f(x)$ at the given points with a linear combination of the functions $g_j(x)$. To determine the best approximation select t_1, t_2, ..., t_m such that the L-∞ norm, the largest deviation between $\sum_{j=1}^{m} t_j g_j(x)$ and $f(x)$ at the points x_1, x_2, ..., x_n, is as small as possible. That is, we want the maximum over $i = 1, 2, ..., n$ of $|\sum_{j=1}^{m} t_j g_j(x_i)$ - $f(x_i)|$ to be minimized.

   (a) (3 points) Formulate Function Approximation as a linear program

   (b) (3 points) Solve the linear programming approximation for $f(x) = e^{|x|} + 10cos(x+1)$ at $x = -3, -2, -1, 0, 1, 2, 3, 4$ using functions $g_j(x) = x^j$ for $j = 0, 1, 2, 3$ and again for $j = 0, 1, 2, 3, 4$. Assume x is in radians for the cosine function.

   (c) (3 points) An alternative objective is to minimize the L-1 norm, the sum of the absolute errors at each of the x_i. Formulate the linear program that uses this objective and solve for the example of part (b).