CFRM 501 - Investment Science
2020 Final Exam

- The duration of this test/exam is 150 minutes. This includes the time to scan and submit your answers on Canvas. Submissions made after this time will be flagged as late.
- If for any reason you are unable to upload your answers on Canvas, email them to `rdon@uw.edu` with an explanation of the problem.
- Answers must be justified to receive full credit (i.e., show your work).
- Use the definitions and notation from this course whenever possible.
- Questions are not allowed during the test/exam. If you find something on the test/exam unclear,
  - explain why you think it is unclear,
  - precisely state your interpretation of what is being asked,
  - provide an answer to your interpretation of the question.
- You may use notes and resources available on Canvas during the test/exam.
- You must work on this test/exam independently.
- The following statement must be written on the first page of your submission. Do not alter this statement in any way.

I did not give or receive any unauthorized help during this test.

Question 1: [10 marks]

Bond A and Bond B both have maturity T and face value F. Bond A pays coupons of c_A and Bond B pays coupons of c_B, where $c_A \neq c_B$ (these two coupon amounts are in dollars, they are not a fraction or percentage of F). The coupon payment dates for both bonds are the same. At time $t = 0$ the bonds trade at prices $p_0^{(A)}$ and $p_0^{(B)}$, where $p_0^{(A)} \neq p_0^{(B)}$. Compute $y_0(T)$ in terms of c_A, c_B, p_0^A, p_0^B, T, and F.

Hint: construct a portfolio of the two bonds which replicates a zero-coupon bond.

Question 2: [10 marks]

A market consists of three assets governed by a 2-factor model. The arithmetic returns of the three assets are given by:

$$
\begin{aligned}
r_1 &= 0.2 + 0.7\,F_1 + 0.8\,F_2 \\
r_2 &= 0.3 + 0.8\,F_1 + 0.9\,F_2 \\
r_3 &= 0.4 + 1.1\,F_1 + 1.2\,F_2
\end{aligned}
$$

where F_1 and F_2 are the common factors. All idiosyncratic error terms are zero. This model admits arbitrage opportunities. Construct a portfolio of three assets which is an arbitrage.

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Question 3: [10 marks]

Part a) [5 marks] The loss of a portfolio has exponential distribution with parameter λ, $L \sim Exp(\lambda)$. The probability density function for an exponentially distributed random variable is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Compute $VaR_\alpha(L)$ in terms of α and λ.

Part b) [5 marks] A collection of risky assets have arithmetic returns vector $\vec{r}$. The distribution of $\vec{r}$ is multivariate normal, $\vec{r} \sim \mathcal{N}(\vec{\mu}, \mathbf{\Sigma})$. Let $\vec{w}$ be the portfolio weights of a fully invested portfolio (the weights sum to 1), so the portfolio return is $r_p = \vec{w}^T \vec{r}$.

i) What is the distribution of r_p (including any associated parameters)?

ii) Write $VaR_\alpha(-r_p)$ in terms of $\vec{w}$, $\vec{\mu}$, and $\mathbf{\Sigma}$.

iii) Find the portfolio weights which minimize $VaR_\alpha^{mean}(-r_p)$.

Recall: $VaR_\alpha^{mean}(X) = VaR_\alpha(X) - \mu_X$.

Note: the following formula may or may not be useful. You may use it without deriving it. This is the portfolio weight vector for maximizing mean-variance performance with risk-aversion parameter γ.

$$\vec{w} = \frac{\mathbf{\Sigma}^{-1}\vec{1}}{\vec{1}^T\mathbf{\Sigma}^{-1}\vec{1}} + \frac{1}{\gamma}\left(\frac{\vec{1}^T\mathbf{\Sigma}^{-1}\vec{1}\mathbf{\Sigma}^{-1}\vec{\mu} - \vec{\mu}^T\mathbf{\Sigma}^{-1}\vec{1}\mathbf{\Sigma}^{-1}\vec{1}}{\vec{1}^T\mathbf{\Sigma}^{-1}\vec{1}}\right)$$

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Question 4: [10 marks] Short Answer

Part a) [3 marks] A very specific method for estimating a unique yield curve was presented in lectures. The resulting yield curve has a very undesirable feature. What is this undesirable feature?

Part b) [3 marks] The variance-covariance approach to computing $VaR_\alpha(L)$ for market risk assumes a specific distribution for the underlying risk factors changes. What is the full name of the underlying distribution?

Part c) [4 marks] In the book Asset Management by Andrew Ang, the author stresses several times that individual assets (or even classes of assets) should not be the main focus of investments as they themselves are not the main sources of risks. Instead of assets, what does the author claim should be the main focus of investing?