Engineering Science 331 (2023)
Eigen Problem Lab (Part 1)

Due 11:59 pm Sunday July 30th 2023

1. Compute all eigenvalues and eigenvectors for the following matrix:

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

   Give the dominant eigenpair of A?

2. Using `python`,

   a) Write a function called `power` which returns the dominant eigenpair of an arbitrary positive definite matrix $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ which has distinct eigenvalues *using the power method.*

      - The termination criteria should be of the form

$$\frac{|\lambda_1^{(\ell+1)} - \lambda_1^{(\ell)}|}{|\lambda_1^{(\ell+1)}|} \leq \Delta,$$

        with $\lambda_1^{(k)}$ denoting the k^{th} iteration estimate of the dominant eigenvalue using the power method, and Δ the user-specified tolerance.

   b) Check that your `power` function works correctly by comparing your output on a few small test matrices, such as that in Question 1.

   c) Write a function called `power_w_deflate` which returns all eigenpairs of an arbitrary positive definite matrix $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ which has distinct eigenvalues *using the power method with deflation.*

   d) Check that your `power_w_deflate` function works correctly by comparing your output on a few small test matrices, such as that in Question 1.

3. Consider the (simple) mass spring system shown in Figure 0.1; this can be used to model spring surging, a resonant phenomenon known to occur in the valve springs of high speed engines. This behaviour can be avoided by ensuring that the surging frequencies lie well above appropriate engine vibration frequencies. Applying Newton's second law to each of the masses (assuming each has a mass of 1) gives the following system of equations:

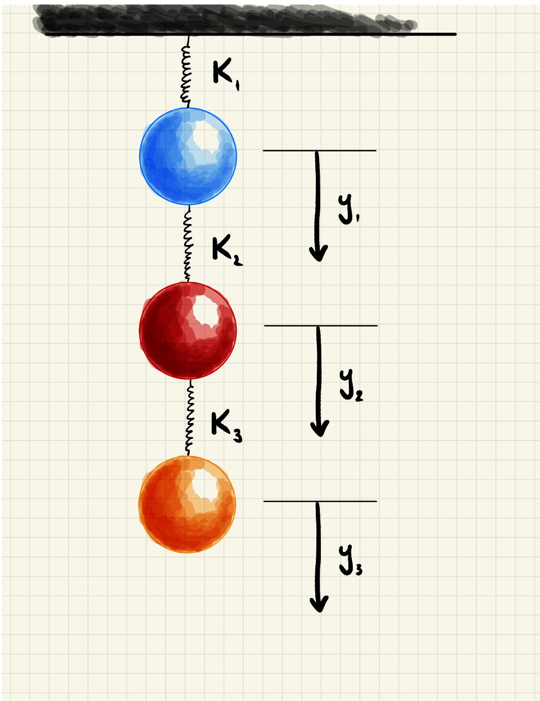

Figure 0.1: A (simple) mass spring system with three masses

$$
\begin{aligned}
\frac{d^2 y_1}{dt^2} &= -K_1 y_1 - K_2(y_1 - y_2) \\
\frac{d^2 y_2}{dt^2} &= -K_2(y_2 - y_1) - K_3(y_2 - y_3) \\
\frac{d^2 y_3}{dt^2} &= -K_3(y_3 - y_2).
\end{aligned}
$$

Assuming solutions of the form $y_i = x_i \exp(i\omega t)$, we arrive at the following system for $\boldsymbol{x} = [x_1, x_2, x_3]^T$;

$$
\begin{bmatrix} K_1 + K_2 & -K_2 & 0 \\ -K_2 & K_2 + K_3 & -K_3 \\ 0 & -K_3 & K_3 \end{bmatrix} \boldsymbol{x} = \omega^2 \boldsymbol{x}, \quad \text{or} \ \boldsymbol{A}\boldsymbol{x} = \lambda \boldsymbol{x},
$$

where $\lambda = \omega^2$.

This eigenproblem can be solved to give the modes of vibration for the mass-spring system. Here the mode shapes are given by the eigenvectors, x, and mode frequencies are given by the square root of the eigenvalues λ.

a) Write a function to construct the A matrix for a mass-spring system (like that shown in Figure 0.1) with N masses. The number of masses N and the spring constants K_i are user inputs.

b) *Using your functions from Question 2*, write a routine to display the natural frequency in Hz and the normalised eigenvector (modeshape) for each mode. (Note: the natural frequency in Hz (f)is the square root of the eigenvalue divided by 2π, that is, $f = \frac{\omega}{2\pi}$.