ENGSCI 331 - Computational Techniques 2

Univariate Minimisation + Databases Lab (Part 1)

Department of Engineering Science

Due: Sunday 20 August 2023 (see Canvas)

Version 2.1

Introduction

Download `UMDBLab_Part1v2.zip` from `Canvas > Assignments > Databases Univariate Minimisation` and unzip in the folder that you've set aside for labs in this course.

Task 1 - Golden Section and Brent's Methods

Background and Aim: In this task we will explore the performance of the golden search method, and a simplified quadratic-only version of Brent's method for finding the minimum of the two one-dimensional functions, below:

$$\begin{aligned} f_0(x) &= x^4 - 8x^3 + 24x^2 - 32x + 17 \\ f_1(x) &= 2 - \exp(-2x^2 + 8x - 8). \end{aligned}$$

Methodology: In the extracted zip archive, you will find an incomplete function in `golden.py`. The definition of this function has already been written, specifying the inputs / outputs. Complete this function so that it converges towards the minimum of the input function, using the golden section search method.

The method should terminate when the maximum number of iterations is reached or when the interval of uncertainty is sufficiently small ($b-a \leq$ `tol`). Appropriate exit flags should be returned.

To help with the implementation, assume that we start iteration k with value $f(x'_k)$ for $a_k \leq x'_k \leq b_k$, where either $x'_k = \alpha_{k-1}$ or $x'_k = \beta_{k-1}$, depending on which of $f(\alpha_{k-1})$ or $f(\beta_{k-1})$ we are able to use from the previous iteration. Let x''_k denote the new x-value that we need to evaluate (being either α_k or β_k, which ever we have not evaluated yet).

1. Complete the following table to give a formula that determines x''_k in terms of a_k, b_k and x'_k. You need two different formulae for the two different cases shown.

$x'_k \leq (a_k + b_k)/2$	$x'_k > (a_k + b_k)/2$
$x''_k =?$	$x''_k =?$

Note that before we start iteration 1, we set $x'_1 = \alpha_1$. Then, in iteration 1, we compute x'' using the table above, which automatically gives us $x''_1 = \beta_1$. This means that no special code is needed for the first iteration.

2. Complete the following table to give expressions that specify a_{k+1}, x'_{k+1} and b_{k+1} to use in the next iteration in terms of a_k, x'_k, x''_k and b_k. You need two different formulae for the two different cases shown.

	$x'_k \leq (a_k + b_k)/2$	$x'_k > (a_k + b_k)/2$
$f(x''_k) \leq f(x'_k)$	$(a_{k+1}, x'_{k+1}, b_{k+1}) =$(?,?,?)	$(a_{k+1}, x'_{k+1}, b_{k+1}) =$(?,?,?)
$f(x''_k) > f(x'_k)$	$(a_{k+1}, x'_{k+1}, b_{k+1}) =$(?,?,?)	$(a_{k+1}, x'_{k+1}, b_{k+1}) =$(?,?,?)

3. Complete the `golden.py` code to implement the search method. You will need to submit this code. Your code should implement the logic specified by the tables above.

Ready-to-run code is provided in brent.py that implements just the quadratic approximation part of Brent's method. (Technically, this is 'succesive parabolic interpolation', also known as Jarrat's method.) You can use this code unchanged.

Verification: The script `task1.py` runs your code for the one-dimensional functions above, producing a number of plots. Check that your method is converging to the minimum of each function.

4. Using the plots and logging output from `task1.py`, comment on the performance of both the golden section method and the quadratic-only Brent's method (as provided in `brent.py`), and discuss how their performance is affected by the function being minimized. Briefly discuss reasons for the behaviours you observe.

Task 2 - A Bad Initial Interval of Uncertainty

The quadratic-only Brent's method, as currently coded up, does not have an interval of uncertainty, but instead just generates a sequence of quadratic approximations and uses these to give the next points. There is nothing in the code to stop the new point x_4 being outside the range $[\min(x_1, x_2, x_3), \max(x_1, x_2, x_3)]$ used to create the quadratic. Repeat Task 1 but now using a starting interval of uncertainty of [a,b]=[0,1] (but still plotting from $x = 0$ to $x = 3$).

5. Using these plots, and logging output from your code, carefully explain and contrast what has happened for each of the two functions for each method.

Task 3 - Challenging Polynomials

The file `functions.py` contains Python functions `f2(x)` to `f5(x)` that define functions $f_2(x) = x^2$, $f_3(x) = x^4$, ..., $f_5(x) = x^8$ that all have their minima at $x = 0$. The file `task3.py` runs Golden Section and the quadratic-only Brent on these functions using a starting interval of uncertainty of [-0.3,1].

6. Using the plots and output from `task3.py`, comment on and contrast the performance of the golden section method and quadratic-only Brent's method on these functions. Gve brief explanations for the key differences and trends you observe in the runs.

Separate instructions will be made available on how to submit your completed written lab answers and code.