

Electro-Optic Effects in AlGaAs Mirrors

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In addition to the effects that can alter the phase on reflection from amorphous mirrors, e.g. thermo-refractive and thermoelastic effects, there are additional phenomena present in crystalline mirrors that can also affect the reflection phase. Most obvious of these are the electro-optic and piezoelectric effects (electric-field dependence of the refractive index and the strain, respectively).

Result

As discussed in the following section, for an electric field E normal to the surface of a mirror grown along the [001] crystallographic direction, the electro-optic effect results in each layer of the mirror becoming birefringent, with principal axes along [110] and $[1\bar{1}0]$, and refractive indices

$$n_{x'} = n + \frac{1}{2}n^3 r_{41} E$$

$$n_{y'} = n - \frac{1}{2}n^3 r_{41} E$$

where r_{41} is the electro-optic coefficient. There is no piezoelectric strain induced in the surface normal direction for this geometry, due to the symmetry of the piezoelectric tensor, so it makes no contribution to the reflection phase shift.

As derived in the following section, with some simplifications the result for the slope of reflection phase vs electric field normal to the surface of an AlGaAs quarter-wave HR mirror operated at the center of its reflection band, for light polarized along a [110] direction can be given as

$$\left| \frac{d\phi}{dE} \right| = \pi \frac{r_{41}}{2} (n_2 n_3^2 + n_3 n_2^2) \frac{n_2 / n_3}{1 - (n_2 / n_3)^2}$$

where r_{41} is the electro-optic coefficient (assumed the same in GaAs and AlGaAs layers), n_2 and n_3 are the low and high refractive indexes, respectively, incidence is assumed from vacuum, and the number of layer pairs is large enough to neglect light reaching the substrate.

For GaAs around 1 μm wavelength, $r_{41} = -1.2 \times 10^{-12}$ m/V. If we guess the high and low index layers are 10% and 90% Al (the results do not depend strongly on the exact Al ratios), the refractive indices are $n_2 = 3.0087$ and $n_3 = 3.4426$. With these value we find

$$\left| \frac{d\phi}{dE} \right| = 4.5 \times 10^{-10} \text{ m/V}$$

Note that this means a 1 V/cm field would induce a 45 nrad phase shift, i.e. ~ 3 orders of magnitude larger than the displacements we hope to resolve.

As a physical interpretation, if we consider the phase shift as resulting from a double pass through a layer of an effective thickness l_{eff} with a refractive index perturbed by an amount Δn , i.e.

$$\Delta\phi = \frac{4\pi}{\lambda} \Delta n l_{\text{eff}}$$

we see that we can identify the effective length approximately as

$$l_{\text{eff}} \sim \frac{\lambda}{2} \frac{n_2 / n_3}{1 - (n_2 / n_3)^2}$$

For numerical values as in the previous calculation, we find $l_{\text{eff}} \sim 2 \mu\text{m}$, a plausible value.

There are several aspects of this result worth considering in evaluating its significance:

1. One implication is that environmental fields at frequencies within the LIGO signal band will have to be controlled to a level consistent with the sensitivity required of the interferometer. Naively applying the result computed above suggests that this requirement would be in the range of millivolts/cm. There are some mitigating factors:

a) the frequencies of interest are probably lower than the reciprocal of the dielectric relaxation time due to free carriers in the film. To the extent that that is true, the fields internal to the mirror will be partially screened and hence smaller than those in the surrounding environment. The residual carrier concentration (or equivalently the conductivity) of the layers should be obtained from CMS to estimate the actual screening time of typical mirrors.

b) The films are birefringent, with a positive index change for one axis $[110]$ and an equal and opposite change for the orthogonal axis $[1\bar{1}0]$. For polarizations 45° to these, i.e. $[100]$ and $[010]$ the index change vanishes, so the sensitivity of the reflection phase should also be reduced. Whether the fact that these are not principal polarization directions of the birefringent mirror, and hence are not true eigenpolarizations of the cavity, may matter and should be considered. It should also be noted that for orientations of the perturbing field other than the surface normal direction, the principal axes of the birefringence will be different than $[110]$ and $[1\bar{1}0]$, so the necessary polarization directions to avoid index perturbation may vary with time if the direction of the environmental field varies.

2. There are often interface fields in semiconductors on the order of the bandgap over some screening length, which can thus be on the order of $\text{V}/\mu\text{m}$ or equivalently kV/cm , which according to this calculation could cause phase shifts $\sim 10^6$ times larger than the phase shifts we hope to detect. The size and distribution of these fields will depend on properties like density of surface states, doping, heterojunctions, etc. While these are nominally DC changes and therefore not in the signal band, it would be worth considering the thermodynamic fluctuations in these fields, and perhaps the impact of other factors that influence the carrier concentration (e.g. laser power fluctuations, etc) and hence the magnitude of the interface fields.

It would seem that some experimental characterization and more careful analysis of the field sensitivity of the mirror phase is warranted.

Derivation

A description of the electro-optic effect in zincblende-structure crystals like AlGaAs can be found in section 14.5 of A. Yariv, *Quantum Electronics*, 3rd edition. For a crystal film with the crystallographic $[001]$ direction normal to the mirror surface (the orientation used in CMS mirrors) the change in refractive index for an electric field E normal to the surface of the mirror induces a birefringence in the film, with principal axes in the $x' = [110]$ and $y' = [1\bar{1}0]$ directions, and with refractive indices equal to

$$n_{x'} = n + \frac{1}{2}n^3 r_{41} E$$

$$n_{y'} = n - \frac{1}{2}n^3 r_{41} E$$

where r_{41} is the electro-optic coefficient. In GaAs around $1 \mu\text{m}$ wavelength, $r_{41} \sim -1.2 \times 10^{-12} \text{ m/V}$. In AlGaAs it is a probably a bit smaller, but I don't have data handy, and for simplicity will take it to be the same as in GaAs.

The constitutive relation for the piezoelectric strain can be written as

$$S_{ij} = d_{ijk}E_k + s_{ijkl}^E T_{kl} \\ \rightarrow d_{ijk}E_k$$

where S is the strain, T is the stress, s^E is the elastic compliance at constant electric field, and the components of d are the piezoelectric strain constants. The subscripts indicate Cartesian components, and the second form follows if the film is stress-free, as we assume here for simplicity rather than address the complete elastic problem. For the reflection phase, the pertinent component is the compressional strain in the surface normal direction; for our assumed geometry S_{zz} . For a surface normal field E_z , we then have

$$S_{zz} = d_{zzz}E_z = 0$$

where the second form follows from the fact that for point group $\bar{4}3m$ as for AlGaAs and other zincblende structure crystals, the only non-zero components of the piezoelectric tensor are d_{xyz} and its permutations (the electro-optic tensor has that same symmetry).

The effects of these perturbations to the refractive index and strain due to the presence of an electric field in a multilayer mirror are quite similar to those resulting from the perturbation of those quantities by a temperature change. We can thus use the analysis of the thermo-optic effect with a slight redefinition of pertinent quantities for the electric field case. The perturbation to the reflection phase of a quarter-wave stack operating at the center of reflectance band in the thermo-optic case, derived back in 2008 in the context of thermo-optic noise calculations in the document “Phase on Reflection from a Quarter-Wave Stack”, attached with this note, can be written as

$$\left. \frac{d\phi}{dT} \right|_0 = \pi n_1 \frac{n_l^2 (n_2^{-1} \kappa_3 + n_3^{-1} \kappa_2) \left(-\frac{n_2}{n_3} \right)^N - (n_2 \kappa_3 + n_3 \kappa_2) \left(-\frac{n_3}{n_2} \right)^N}{n_1^2 \left(-\frac{n_3}{n_2} \right)^{2N} - n_l^2 \left(-\frac{n_2}{n_3} \right)^{2N}} U_{N-1} \left[-\frac{1}{2} \left(\frac{n_2}{n_3} + \frac{n_3}{n_2} \right) \right]$$

where n_2 and n_3 are the refractive indices of the low and high index layers, respectively, N is the number of layer pairs, n_1 and n_l are the refractive indices of the semi-infinite input and output media, respectively. U_N is the Type II Chebyshev polynomial. The temperature coefficients κ_j in layers $j = (2, 3)$ are defined as

$$\kappa_j \equiv \frac{d \ln n_j}{dT} + \alpha_j$$

where α is the thermal expansion coefficient. For our electric-field problem, we can use the same form to calculate $d\phi / dE$ if we replace κ_j by

$$\kappa_j = \frac{d \ln n}{dE} + \frac{dS_{33}}{dE} \\ = \pm \frac{1}{2} n^2 r_{41} + d_{zzz} = \pm \frac{1}{2} n^2 r_{41}$$

where the \pm refer to light polarized along x', y' , respectively, and we again note that d_{zzz} vanishes in zincblende-structure crystals.

The case of interest here is for an incident refractive index $n_1 = 1$ and N large enough that negligible light reaches the substrate, in which case we have

$$\left. \frac{d\phi}{dE} \right|_0 = -\pi(n_2\kappa_3 + n_3\kappa_2) \left(-\frac{n_2}{n_3} \right)^N U_{N-1} \left[-\frac{1}{2} \left(\frac{n_2}{n_3} + \frac{n_3}{n_2} \right) \right]$$

where the simplification arises from neglecting terms going like $(n_2 / n_3)^N$ with respect to those going as $(n_3 / n_2)^N$. For large N we can approximate

$$f(N, x) \equiv x^N \left| U_{N-1} \left[-\frac{1}{2} \left(x + \frac{1}{x} \right) \right] \right| = \sum_{m=1, odd}^{2N-1} x^m = \frac{x(1-x^{2N})}{1-x^2} \approx \frac{x}{1-x^2}$$

where $x \equiv n_2 / n_3$. So in the limit of high reflectivity (large N) we have

$$\begin{aligned} \left. \frac{d\phi}{dE} \right|_0 &\approx -\pi(n_2\kappa_3 + n_3\kappa_2) \frac{n_2 / n_3}{1-(n_2 / n_3)^2} \\ &= -\pi \frac{r_{41}}{2} (n_2 n_3^2 + n_3 n_2^2) \frac{n_2 / n_3}{1-(n_2 / n_3)^2} \end{aligned}$$