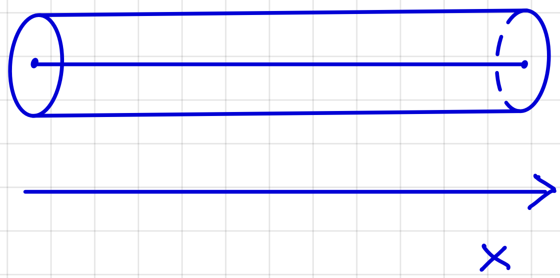
Coax-cable impedance

- Typical coax cables in labs (e.g. RG58) have impedance $Z = 50\,\Omega$,
- The analog TV standard in the US is (was) $Z = 75\,\Omega$.

What does that mean?



- Voltage between core and mantle at position x & time t $= V(x,t)$
- Current through position x of the core at time t $\equiv$ minus current through position x of the mantle at time t $= I(x,t)$

$$\Rightarrow \boxed{\text{Impedance } Z \text{ of Cable: } \quad V(x,t) = Z\, I(x,t)}$$

- for a signal traveling left to right or right to left
- we count $I(x,t)$ negative for a signal traveling right to left.

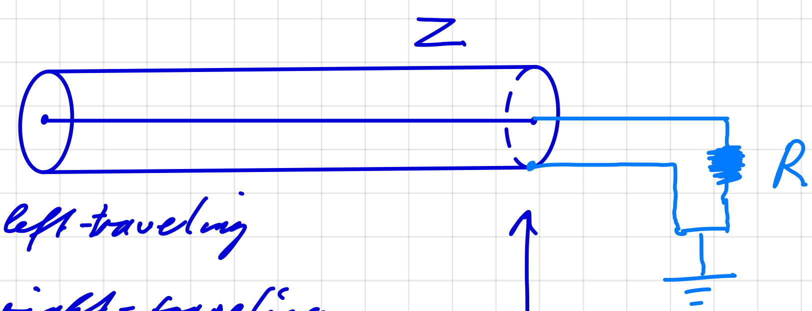
- Note: That V & I have same functional form is non-trivial, but follows from Maxwell equation

- $$Z = \frac{\ln \frac{R}{r}}{2\pi} \sqrt{\frac{\mu \mu_0}{\varepsilon \varepsilon_0}}$$

R: outer coax radius

r: inner coax radius

A) Cable termination & reflection:



$\vec{V}, \vec{I}$: left-traveling

$\overleftarrow{V}, \overleftarrow{I}$: right-traveling

- Total Voltage: $V = \vec{V} + \overleftarrow{V}$
- Total Current: $I = \vec{I} - \overleftarrow{I}$

$$V = \vec{V} + \overleftarrow{V} = V = R I = R\vec{I} - R\overleftarrow{I}$$

$$\Rightarrow \vec{V} + \overleftarrow{V} = \frac{R}{Z}\vec{V} - \frac{R}{Z}\overleftarrow{V}$$

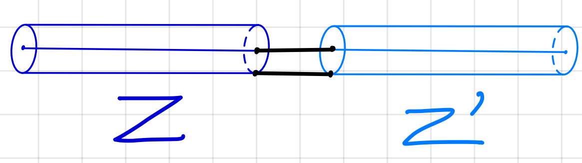
$$Z\vec{V} + Z\overleftarrow{V} = R\vec{V} - R\overleftarrow{V}$$

$$(R+Z)\overleftarrow{V} = (R-Z)\vec{V}$$

$$\boxed{\overleftarrow{V} = \frac{R-Z}{R+Z}\vec{V}}$$

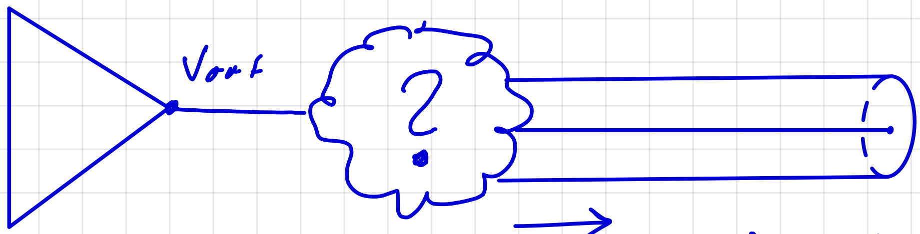
• Conclusions:

- The reflection coefficient is $\Gamma = \dfrac{R-Z}{R+Z}$
  - In particular: No reflection if $R = Z$!
- The voltage measured across R is $V = \vec{V} + \overleftarrow{V} = (1+\Gamma)\vec{V} = \dfrac{2R}{R+Z}\vec{V}$
  - In particular: $V = \vec{V}$ if $R = Z$!
- The same holds for cable impedance mismatch: $R \to Z'$



- Reflection $\overleftarrow{V} = \dfrac{Z'-Z}{Z'+Z}$
- Transmission $\vec{V}' = \dfrac{2Z'}{Z'+Z}$

B) Driving a coax cable



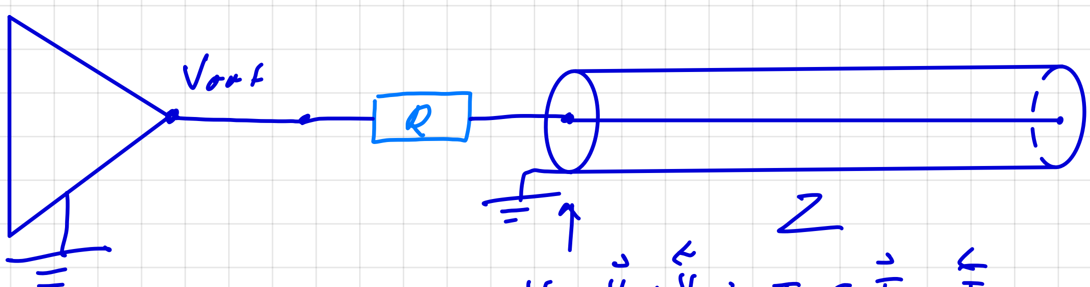
$\vec{V}$: right-traveling signal (output)

$\overleftarrow{V}$: left-traveling signal (reflection from the other end)

- We want:
  - 1) $\vec{V} = \alpha \cdot V_{out}$, independent of $\overleftarrow{V}$!
  - 2) ↳ this constant α as large as possible.

- **Solution**: • if $V_{out} \equiv 0$ (i.e. driving 0 V)

  $\Rightarrow$ {?} has to look termination: $\Rightarrow R = Z$



$V = \vec{V} + \overleftarrow{V}$; $I = \vec{I} - \overleftarrow{I}$

$$\Rightarrow V_{out} - (\overleftarrow{V} + \vec{V}) = R(\vec{I} - \overleftarrow{I}) = \frac{R}{Z}\vec{V} - \frac{R}{Z}\overleftarrow{V}$$

$$\Rightarrow V_{out} + \frac{R-Z}{Z}\overleftarrow{V} = \frac{R+Z}{Z}\vec{V}$$

$$\Rightarrow \boxed{\vec{V} = \frac{Z}{R+Z} V_{out} + \frac{R-Z}{R+Z}\overleftarrow{V}}$$

or, for $R = Z$: $\boxed{\vec{V} = \frac{1}{2} V_{out}}$, independent of left-traveling signal !

On RF Interference and Noise Rejection

Stefan Ballmer, Dec 14, 2020

This write-up summarizes a few simple principles that help when dealing with RF noise.

- RF interference can be thought of as an additional voltage signal picked up along cables. Particularly bad are any induction loops, that is large areas outlined by separated signal and ground wires. Faraday's law of Induction works against us in this situation.
- We therefore always route two wires (signal/ground, or signal & reference) together. Either in twisted pair cables or more commonly in our lab in coaxial cables.
- Note that we still pick up RF interference on the common signal of the pair (signal+reference), while we want to encode information of the differential signal (signal-reference).
- There are fundamentally two ways to try to reject signal contamination:
    - Balanced lines: Example: twisted-pair cable. The idea is to have two conductors with the same impedance, so any RF interference affects both cables exactly equal. For this to work, the receiver has to be differential and balanced (i.e. same impedance on both receiver inputs). The driver also has to have balanced impedance on both lines. Note though that the driver DOES NOT have to be differential for the noise cancellation to work.
      Wikipedia article:
      https://en.wikipedia.org/wiki/Balanced_line
    - Shielded (but unbalanced) cables: Example: coaxial cable. A coax cable has the advantage that the whole signal is contained between the inner conductor (or core) and the outer conductor (or shield). This helps to keep any unwanted signals out but requires as low as possible resistance on the shield. Thus, coax cables are typically not balanced. On long cables this can lead to some conversion of common to differential signal. There are simple strategies to kill this common signal, see e.g. the RF choke in the picture.
      Wikipedia article:
      https://en.wikipedia.org/wiki/Coaxial_cable
    - Twin-axial cables are sometimes used to get shielded and balanced signal transmission.

RF choke (a special kin of Balun): Wrapping a coax cable around an iron core increases the inductance for the common mode signal (but not the differential mode signal), thus killing any common-mode RF interference.

Another link on balanced vs. unbalanced: http://www.bluejeanscable.com/articles/balanced.htm

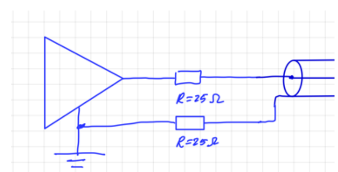
Strategies for mitigating RF interference:

- **Always use coaxial cables** (between electronics boxes) **or twisted pair** (short connections inside an electronics box).
- **Shield you RF electronics boxes** (metal box, or metal-covered plastic box).
- **Keep cables as short as possible**, pay attention to routing.
- **Use cables with heavy gauge shields**.
- **Maintain good connections**. (You won't believe how much time I have wasted due to bad connections.)
- **Do not add unnecessary grounds**. This might be surprising at first glance. But note that any additional ground connection forms an induction loop, putting additional signal on to your transmission line.
- **Use ground isolators in problem signal paths**.
- **Install RFI filters in signal path**. An example is the RF choke, or any balun.
- **Locate and treat the offending source.**

Here is an nice article on mitigating RF interference:
https://www.svconline.com/resources/understanding-and-controlling-rf-interference-364849

What about differential drive?

A common misconception is that a balanced line or a differential receiver require a differential drive (i.e. a. drive that puts the same signal with opposite sign on each of the two leads). This is wrong. A differential receiver obviously can receive single-sided signals. And the figure shows an example of a balanced, single-ended drive.



An example of a balanced, single-ended drive.

The only advantage of a differential drive is that the common-mode signal of a line is not driven. This reduces its effectiveness as antenna, thus reducing RF interference for *other* circuits.

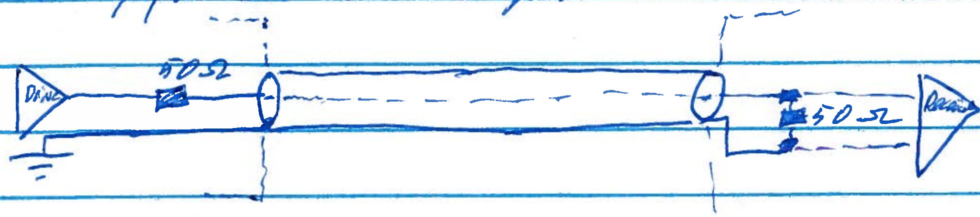
Wikipedia article: https://en.wikipedia.org/wiki/Differential_signaling

Simple rules to live by

- Make all drives single-sided. This also makes connecting an oscilloscope easy.
- Ground the cable shield at the driver end *only*.
- For RF signals put a 50 Ohm resistor in series between the driver opamp and the center conductor of the cable.
- The receiver should always be differential. Only exception:
    - Receiver in the same box or rack as driver, with a strong common ground.
- An RF receiver should have 50Ohm impedance.
- A low-frequency receiver (below ~100kHz) should have high impedance (>1kOhm, 10kOhm is typical).

· Practical tips

Thus, for an RF signal connection:



Driver side — Z=50Ω cable — Receiver side

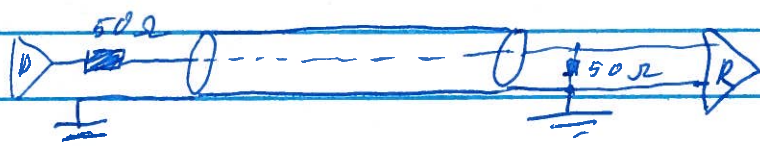
↓ 50Ω series resistor absorbs residual reflections

↓ if R≠50Ω (as seen by cable) ⇒ RF-reflections

· Grounding Rule of thumb:

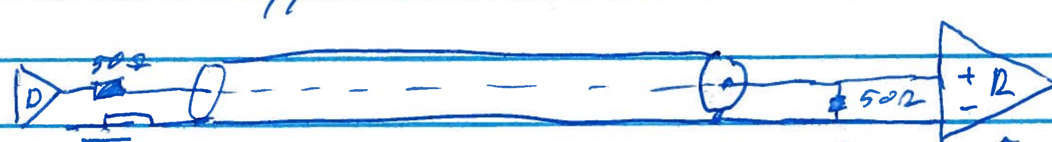
- if Driver & receiver are in same rack:

⇒ o.k. to ground receiver side of cable:



- if Driver & receiver are in separate racks:

⇒ need differential receiver



No grounding; true differentia receiver that rejects common mode signals

Signal propagation & Impedance-matching in coax-cables

Summary: Coax-cable inner radius r, outer radius R, vacuum

Impedance: $Z = \sqrt{\frac{\ell}{c}} = \ln\frac{R}{r}\,\frac{c\mu_0}{2\pi}$

Inductance per unit length: $\ell \equiv \frac{L}{d} = \ln\frac{R}{r}\,\frac{\mu_0}{2\pi}$

Capacitance per unit length: $c \equiv \frac{C}{d} = \frac{2\pi\varepsilon_0}{\ln\frac{R}{r}}$

Fourier space (Plane Wave)

$$\Rightarrow V = \mathrm{Re}\, V_0\, e^{\pm ikz}\, e^{-i\omega t}$$

$$I = \mathrm{Re} \pm I_0\, e^{\pm ikz}\, e^{-i\omega t} \qquad V_0 = Z\cdot I_0$$

$$E_\varrho(\varrho) = \mathrm{Re}\,\frac{V_0}{\ln\frac{R}{r}}\cdot\frac{1}{\varrho}\,e^{\pm ikz - i\omega t}$$

$$B_\varphi(\varrho) = \mathrm{Re} \pm \frac{\mu_0 I_0}{2\pi}\,\frac{1}{\varrho}\,e^{\pm ikz - i\omega t}$$

Time-domain or

$$\vec{V}(ct-x) = Z\,\vec{I}(ct-x) \quad \& \quad \overleftarrow{V}(ct+x) = Z\,\overleftarrow{I}(ct+x)$$

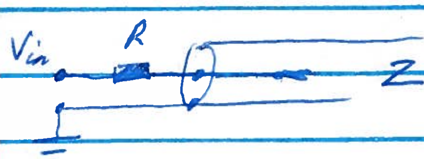
$$V = \vec{V} + \overleftarrow{V}\;;\quad I = \vec{I} - \overleftarrow{I}$$ Voltage & current

$$E_\varrho = \frac{V}{\varrho\ln\frac{R}{r}}\;;\quad B_\varphi = \frac{\mu_0 I}{2\pi\varrho}$$ fields

Reflection

$$\Rightarrow \overleftarrow{V} = \vec{V}\,\frac{R-Z}{R+Z}$$

Drive:



$$\Rightarrow \vec{V} = \underbrace{\frac{Z}{R+Z}V_{in}}_{\text{Drive}} + \underbrace{\frac{R-Z}{R+Z}}_{\text{Reflection}}\overleftarrow{V}$$