Mechanical Losses from Piezoelectric-Ohmic Loss Coupling

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1. Introduction

There exist more sources of thermodynamic noise in non-centrosymmetric semiconducting media than in conventional isotropic insulators. One example is the coupling of density fluctuations in charge carriers through the accompanying space-charge fields combined with the electro-optic effect into refractive index changes. In a bulk GaAs mirror substrate, this mechanism would cause $\sim 10^5$ larger linear spectral density of path length fluctuations than would the free-carrier dispersion alone (which is the only such coupling in centrosymmetric silicon).

Another mechanism is ohmic dissipation due to currents induced by the piezo-electric effect in an AlGaAs mirror. With an oscillatory Levin pressure field applied to the surface of an AlGaAs mirror, the piezo-electric effect generates an effective current through the volume of the mirror, which dissipates power due to the ohmic resistance of the mirror material. This process is analyzed here.

2. Piezoelectric effect in zincblende semiconductors

For our purposes, the pertinent part of the piezo-electric response in Voight notation is that a stress T_J generates a dipole moment density P_i described by the piezo-electric strain constants d_{iJ}. In materials like AlGaAs that are in the $\overline{4}3\mathrm{m}$ point group, the only non-zero tensor components are d_{14} = d_{25} = d_{36}, which are all conventionally referred to as d_{14}.. For a [001] oriented AlGaAs mirror, the pertinent relation is then

$$P_z = d_{14} T_6 \tag{1}$$

where we recall that in the Voight notation $T_6 = \sigma_{xy}$. To the extent that the stress is uniform through the thickness of the mirror layer, the polarization density is as well. Recalling that $\nabla \cdot \mathbf{P}$ enters into Maxwell's equations as an effective charge density, this charge will vanish except at the surfaces of the mirror (which for simplicity we consider for the present simply as a uniform layer of GaAs), where it creates an effective sheet charge density $\pm q = \pm P_z = \pm d_{14} T_6$ at the top and bottom interfaces, respectively.

3. Electrical model of piezo-electric layer

To model the electrical behavior of the piezo-electric layer, we can think of an area A of the mirror as a leaky capacitor, with the capacitance

$$C = \varepsilon_0 \varepsilon A / l \tag{2}$$

shunted by a resistance

$$R = l \,/\, \sigma A \tag{3}$$

where l is the thickness of the layer, σ is the conductivity of the mirror, and ε is the dielectric constant.

For a stress with a sinusoidal time dependence $\exp(i\omega t)$, the surface charge will also be oscillatory, so will enter as current driving the capacitor

$$I = A\frac{dq}{dt} = i\omega A d_{14} T_6 e^{i\omega t} \tag{4}$$

We now have an elementary circuit theory problem to solve. The real part of the impedance of the leaky capacitor is

$$Z_{\mathrm{Re}} = \frac{R}{1+\omega^2 C^2 R^2} = \frac{R}{1+\omega^2\tau^2} \tag{5}$$

where the RC time constant

$$\tau \equiv RC = \frac{\varepsilon\varepsilon_0}{\sigma} \tag{6}$$

is equivalent to Maxwell relaxation time for the mirror material.

The average power dissipated is then

$$\begin{aligned} P_{diss} &= \frac{1}{2} I_0^2 Z_{\mathrm{Re}} = \frac{1}{2}\left(\omega A d_{14} T_6\right)^2 \frac{R}{1+\omega^2\tau^2} \\ &= \frac{1}{2}\frac{\omega^2 A d_{14}^2 l}{1+\omega^2\tau^2}\frac{1}{\sigma} T_6^2 \\ \Rightarrow \frac{P_{diss}}{A\cdot l} &= \frac{\omega}{2}\frac{\omega\tau}{1+\omega^2\tau^2}\frac{d_{14}^2}{\varepsilon\varepsilon_0} T_6^2 \end{aligned} \tag{7}$$

where the second form follows with Eqs. (4) and (5), the third form with Eq. (3), and the final with Eq. (6). The final form will be recognized as the average power dissipated per unit volume.

4. Equivalent ϕ

Comparing Eq. (7) with the usual expression for the loss for shear field in a thin layer,

$$\begin{aligned} \frac{P_{diss}}{\mathrm{vol}} &= \frac{\omega}{2} c_{66}\phi_{66} S_{66}^2 \\ &= \frac{\omega}{2} c_{66}\phi_{66}\frac{T_6^2}{c_{66}^2} = \frac{\omega}{2}\phi_{66}\frac{T_6^2}{c_{66}} \end{aligned} \tag{8}$$

where c_{66} and ϕ_{66} are the shear stiffness and the loss angle, we see that the piezo-electric loss mechanism is equivalent to adding an additional effective loss angle of

$$\phi_{66,eq} = c_{66} \frac{d_{14}^2}{\varepsilon\varepsilon_0} \frac{\omega\tau}{1+\omega^2\tau^2} \tag{9}$$

With d_{14} = 2.6 x 10^{-12} C/nt, ε = 12.5, $c_{66} = c_{44}$ = 6 x 10^{10} nt/m^2, we find

$$\phi_{66,eq} = 4\times 10^{-3} \frac{\omega\tau}{1+\omega^2\tau^2} \tag{10}$$

5. Discussion

With the result in Eq. (10), the equivalent loss is clearly large enough to be a serious problem near the peak dissipation frequency $\omega \sim 1/\tau$. Measurements at room temperature on AlGaAs mirrors indicate a much lower value for the shear loss, <10^{-5}. This is not necessarily contradictory, since the relaxation time for GaAs, even at the low carrier densities in the AlGaAs mirrors should be sub-microsecond, putting the peak loss at frequencies well above the LIGO range. This explanation, though, is at odds with the recent Syracuse measurements of the electro-optic sensitivity, which was within a factor of ~2 of the result computed in the absence of carrier screening effects. Since both these calculations are relatively straightforward (other than carrier transport effects), and depend on no extrinsic parameters other than the conductivity (or equivalently the dielectric relaxation time), it is not clear yet how theory and experiment can be reconciled. It would seem likely that effects of the heterojunctions on the carrier transport normal to the layers are likely involved.

One possible explanation would be if the GaAs layers had a very short τ and the AlGaAs layers had a very long τ. In that the GaAs layers would have their loss peak at frequencies above the LIGO band while the loss in the AlGaAs layers would peak at frequencies below the LIGO band. Under those conditions, the applied electric field in the EO experiment would be screened out of the GaAs layers but would appear in the AlGaAs layers, so that the sensitivity would be reduced by a factor of ~2. This should be a computable question, if the doping in the two layers were known.

Another point to consider is that the conductivity will decrease exponentially with temperature as carriers freeze out in the cryogenic regime, and thus the conductivity will decrease (somewhat mitigated by an increase in mobility), the relaxation time will increase, and the peak frequency will move closer to the LIGO range. Since the other parameters in the piezoloss expression are weak functions of temperature, the piezo mechanism should be considered in understanding any unexpected increases in the observed loss at low temperatures.