

Thermal compensation system comissioning for O3 and  
a study of the the Pockels effect of an AlGaAs coating

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# **Abstract**

Abstract goes here

# **Dedication**

# **Declaration**

I declare that

# Acknowledgements

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# Chapter 1

## Introduction

# Chapter 2

## TCS comissioning for O3

As shown in Chapter 1, increasing input power directly relates to an interferometer's sensitivity to detecting gravitational waves, one implication of this is the necessity for thermal compensation. As the interferometer increases input power, you directly couple more light into the Fabry-Pérot cavity arms. The input test masses, even with extra low absorption ( $\approx 400 \text{ ppb} \pm 150 \text{ ppb}$  [alog ? or point absorber paper](#)) still induce thermo-optic effects with the projected circulating arm power of 200 kW. A symptom of this is mode mismatching throughout the interferometer, a problem that contributes to loss of optical power at the anti-symmetric port which can reduce sensitivity two-fold: loss of power to your readout and reduced efficacy of implementing quantum squeezing.

### 2.1 The Mode Matching requirements

### 2.2 aLIGO Thermal Compensation System (TCS)

Mode Matching

- Annular actuation with RH actuators
- Central heating with central CO<sub>2</sub> laser

### 2.2.1 TCS preloading for O3

Reference to TVO's thesis for preloading settings based on his model [1]

### 2.2.2 RH input conditioning

Some notes about the analytical calculations of RH (There is a paper on this). 12 hours to sample single TCS setting

#### Optimizing RH thermo-optic response

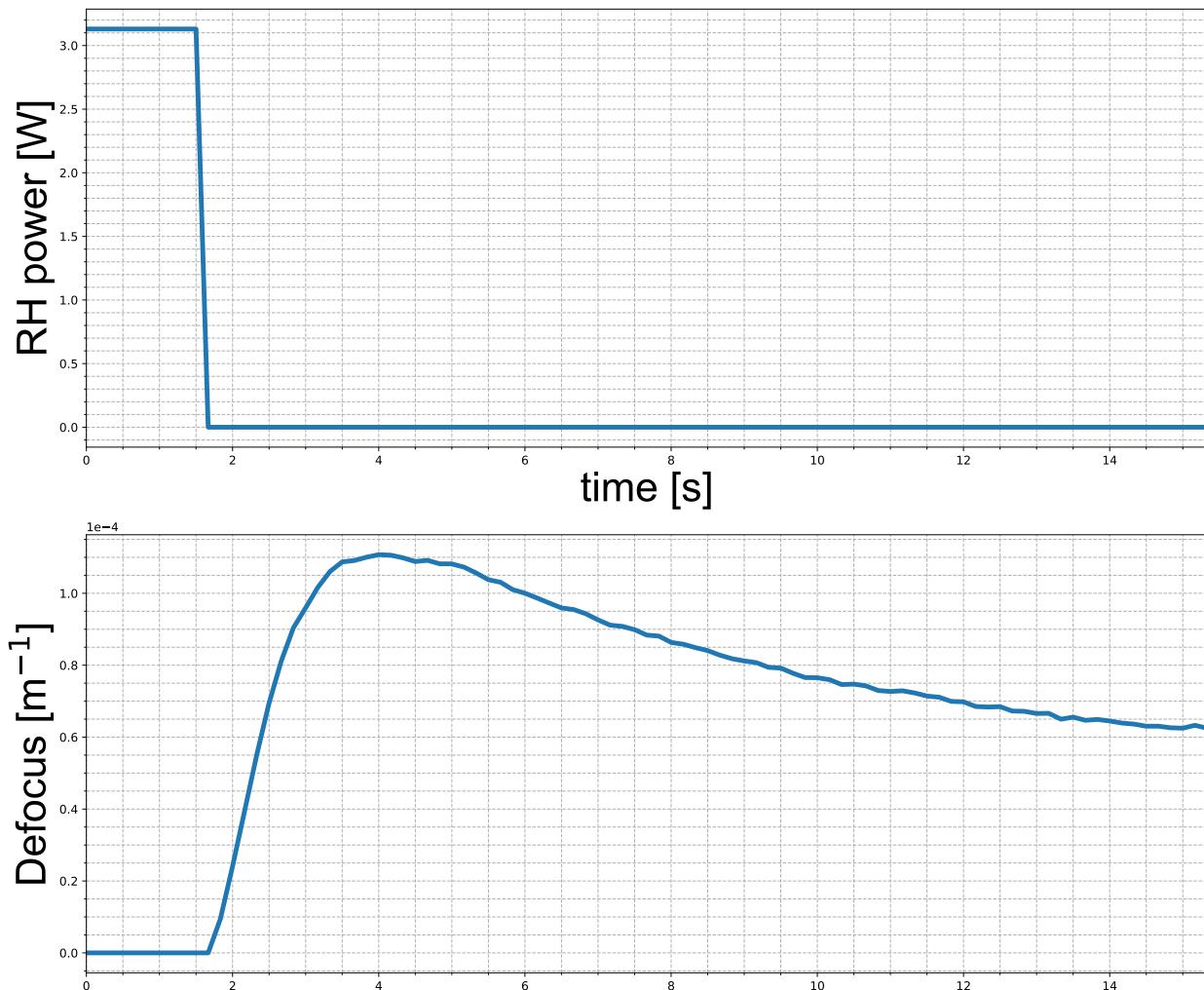


Figure 2.1: ITMY thermo-optic response to a 3.13 Watt power reduction to ring heaters. It's after  $\approx 12$  hours after the change was made do you start to see a small enough  $\frac{d\alpha_{sp}}{dt}$  when you can assume a steady thermal lens.

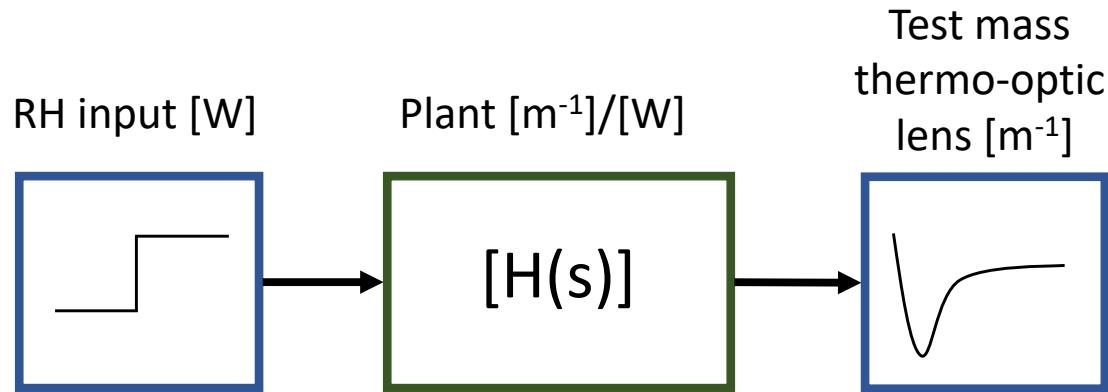


Figure 2.2: A pictograph showing how the plant transforms the signal. The example of this can be seen in Fig [2.1]

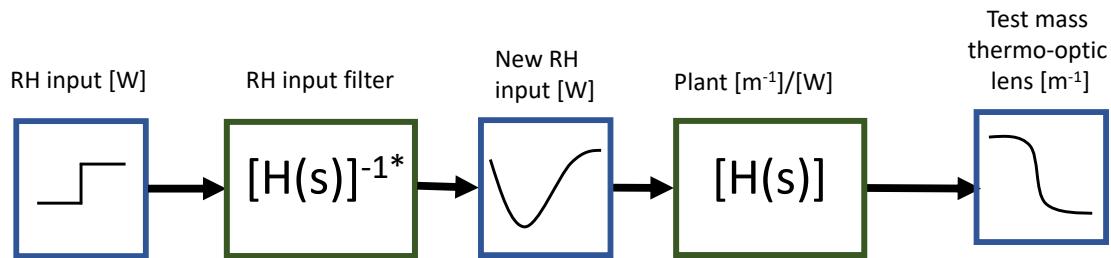


Figure 2.3: A pictograph showing the system with real time digital filtering for an improved thermo-optic response. The RH input filter is created by inverting the plant filter combine with a low pass and added poles to the zpk model to ensure stability.

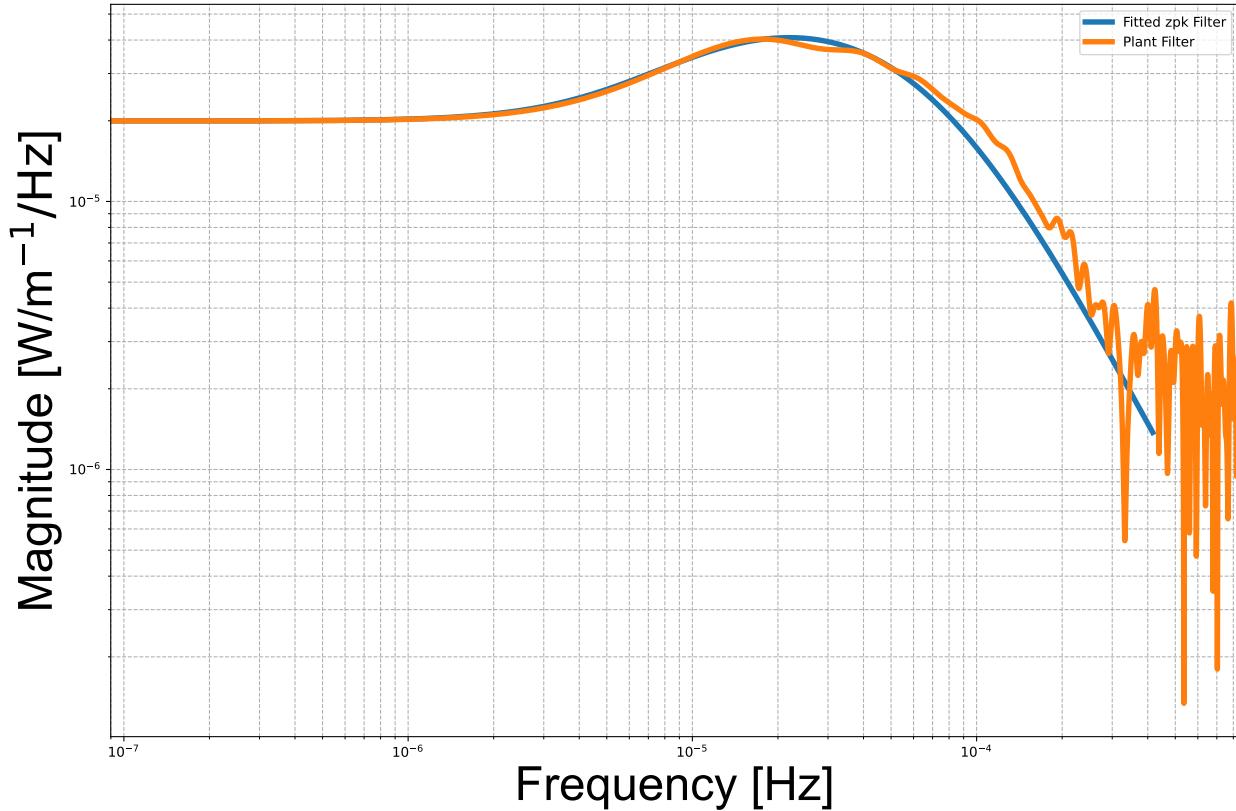


Figure 2.4: Showing the PSD of the RH response (normalized by the input RH power) over a an  $\approx 12.5$  hour period. The zpk model of the fitted filter ( $H(s)$ ) is  $9.2545e - 12 \frac{(s+3.14210e-5)}{(s+8.168e-5)(s+0.0003142)(s+0.0005969)}$

### Dynamic Thermal compensation

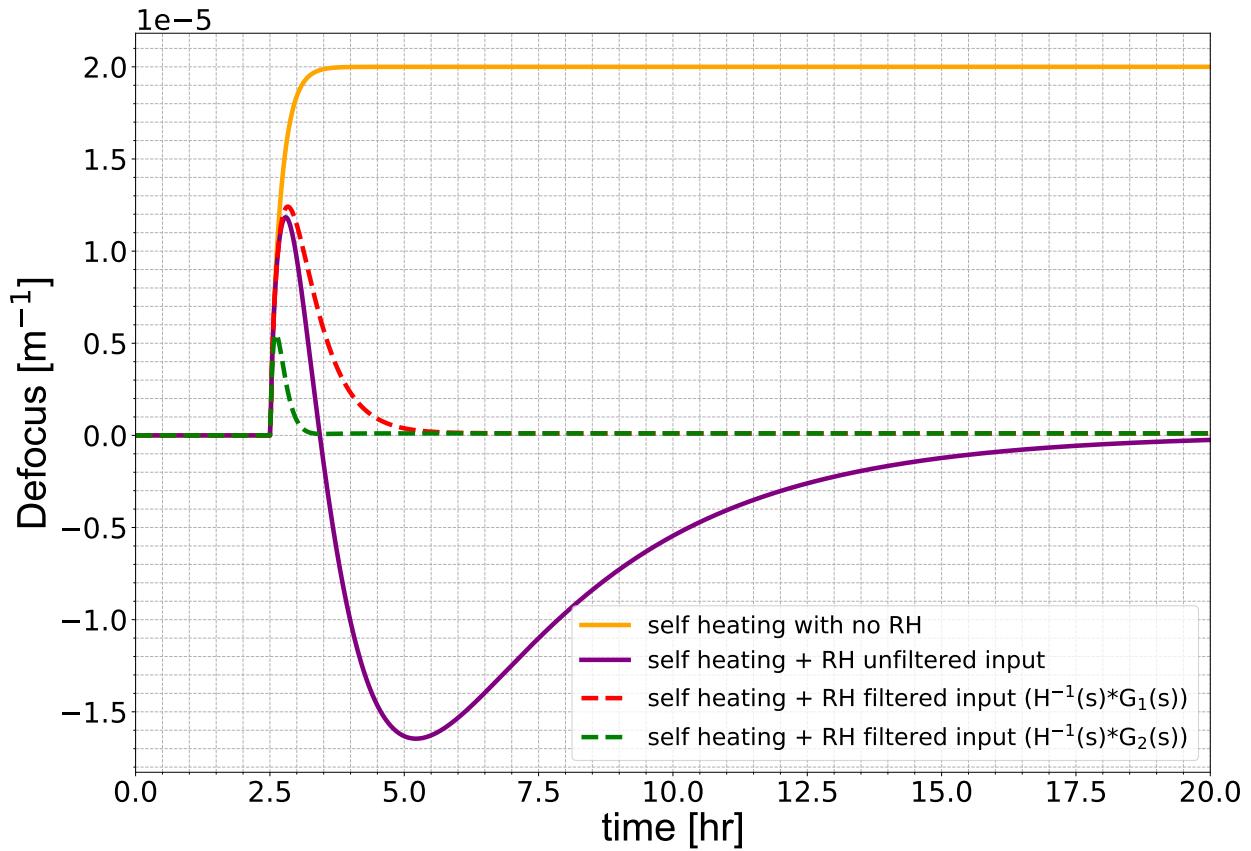


Figure 2.5: Comparison of the natural RH response and the response to the conditioned input. The above plot is simulated in Matlab by passing the RH input time series (top plot) through the  $[H(s)]^{-1}$  and  $H(s)$  to acquire with the result lensing behavior on the bottom plot.

## Limitations

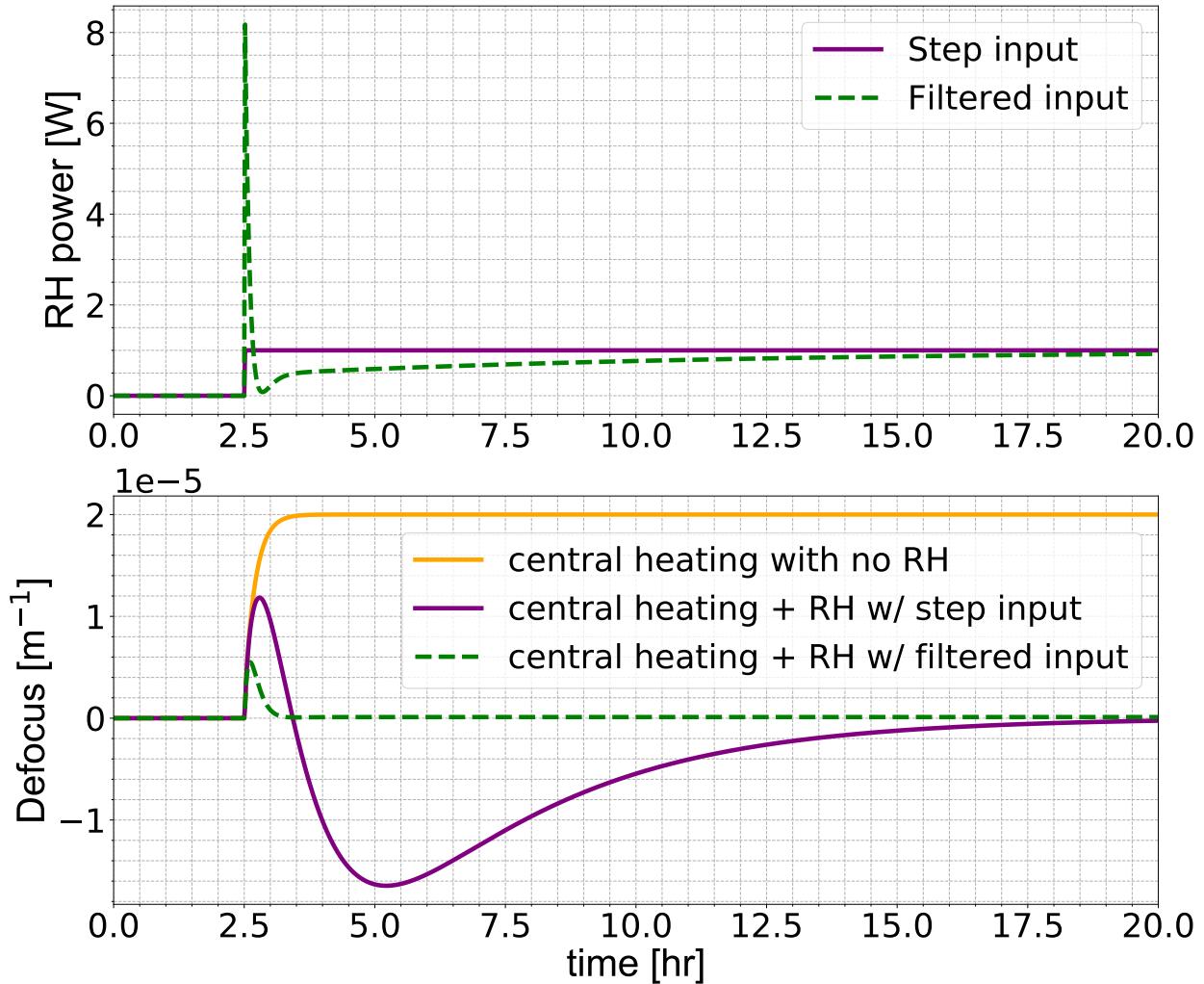


Figure 2.6: Comparison of the natural RH response and the response to the filtered input with RH power

Limitation on RH power is set at 8W **Double check source?**

## Implementation into CDS at LHO

### 2.3 Higher order TCS

#### 2.3.1 Point absorbers

- Impact on RF sidebands with interferometer thermalization

#### 2.3.2 Actuation using a CO<sub>2</sub> laser and mask

- Aiden's design
- Imaging
- Installation
- Metric of improvement? (Impact on sidebands after thermalization)

# Chapter 3

## Electro-optic study of a GaAs/Al<sub>0.92</sub>Ga<sub>0.08</sub>As coated mirror

As mentioned in Section (1?) one of the many LIGO fundamental noise sources is coating thermal noise from the SiO<sub>2</sub>/TiO<sub>2</sub> : Ta<sub>2</sub>O<sub>5</sub> aLIGO coatings. As aLIGO approaches its designed sensitivity various coating solutions are currently proposed to mitigate thermal noise coupling into the detector output. With the potential to reduce coating Brownian noise by a factor of 10 [2], Al<sub>0.92</sub>Ga<sub>0.08</sub>As shows much promise with next generation detectors with a corresponding strain reduction by a factor of (5?) in comparison to the aLIGO coatings, though with different material properties of these crystalline coatings introduce new coating noise couplings.

A notable source of noise is the linear electro-optic property of crystalline materials ( $dn/dE$ ), also known as the Pockels effect [3]. Characterizing currently proposed GaAs / Al<sub>0.92</sub>Ga<sub>0.08</sub>As coated “witness” samples through a calibrated measurement of the aforementioned property is worthwhile for the use of Al<sub>0.92</sub>Ga<sub>0.08</sub>As coatings in gravitational wave interferometers. The following section is dedicated to the discussion of brownian thermal noise and the fundamental limit it imposes on gravitational wave detection, the appeal of GaAs/Al<sub>0.92</sub>Ga<sub>0.08</sub>As coated mirrors for improved thermal noise, light propogation through

anisotropic materials, induced optical anisotropy of zincblende materials, estimates of the differential phase of light reflected from a GaAs/Al<sub>0.92</sub>Ga<sub>0.08</sub>As coating caused by electric field noise, and the measurements from a PDH locked single cavity experiment with a focus on a calibrated electro-optic response from a GaAs / Al<sub>0.92</sub>Ga<sub>0.08</sub>As sample from 1kHz to 1MHz.

### 3.0.1 Brownian Thermal Noise

In 1827 the Scottish botanist Robert Brown noticed a constant motion of pollen particulates on the surface of water; witnessing randomized collisions of the water molecules holding a kinetic energy proportional to the temperature ( $k_B T$ ) [4]. It is because of his documented observations we name the phenomena Brownian motion. And although the observations were on motion of particulates in liquids, molecules and atoms within gases and solids also exhibit Brownian motion. For high precision optical experiments operating at room temperature (and higher due to high power resonant beams), understanding how much differential phase noise is imparted on the interferometer light passing through and reflecting from core optics is crucial. This requires knowledge of the mean squared displacement from each degree of freedom of the system which can be realized through the Fluctuation Dissipation theorem. Derived by H.B. Callen and T.A. Welton, the theorem states that for a randomly fluctuating linear force [5]:

$$F_x^2(f) = 4k_B T \Re[Z] \quad (3.1)$$

Where  $\Re[Z]$  is the real part of the impedance of the system. This impedance directly relates to equations of motion:

$$Z = \frac{F}{\dot{x}} \quad (3.2)$$

Another useful form is the power spectrum of the fluctuating motion:

$$x^2(f) = \frac{4k_B T}{(2\pi f)^2} \Re[Y] \quad (3.3)$$

Where  $Y$  is the inverse of the impedance or admittance. With this power spectra, modelling and budgeting notable LIGO fundamental noise contributions attributed to the choice of the materials used for mirror substrates, and highly reflective mirror coatings becomes less daunting. Though adequate modelling of internal force couplings for the aforementioned components is required.

### Internal friction in Materials and Loss angle

Zener provides a model of the internal friction of materials incorporating anelasticity into the equations of motion [6]:

$$F = k(1 + i\phi)x + m\ddot{x} \quad (3.4)$$

Where  $m$  is mass attached to a spring with a spring constant  $k(1 + i\phi)$  incorporating the degree of anelasticity  $\phi$ . From equations 3.5 and 3.3 we perform a Laplace transform and acquire the following form of admittance:

$$Y(s) = \frac{\dot{x}(s)}{F(s)} = \frac{-s}{k(1 + i\phi) + ms^2} \quad (3.5)$$

Or more transparently the Fourier representation since we assume a linear time invariant system:

$$Y(\omega) = \frac{\dot{x}(\omega)}{F(\omega)} = \frac{-i\omega}{k(1 + i\phi) - m\omega^2} = \frac{k\omega\phi - i\omega(k - m\omega^2)}{(k - m\omega^2)^2 + k^2\phi^2} \quad (3.6)$$

Plugging equation 3.6 back into 3.3:

$$x^2(f) = \frac{2k_B T}{\pi} \frac{k\phi}{(k - 4\pi^2 m f^2)^2 + k^2 \phi^2} \quad (3.7)$$

Computing the admittance from a Gaussian beam impinging upon a HR mirror can require expansion of all individual mechanical degrees of freedom of the test mass system across a relevant frequency range, and with that approach convergence is not guaranteed. Saulson and Gonzalez provide an alternative method to computing the admittance coined the “direct approach” by Levin when computing the noise from a Gaussian beam on a LIGO HR test mass. The admittance can be acquired through:

$$\Re[Y] = \frac{W_{\text{diss}}}{F_o^2} \quad (3.8)$$

$W_{\text{diss}}$  is the dissipated power from the system due to an oscillating force  $F_o$ . This form of the admittance reveals an important result of the fluctuation dissipation theorem where an undriven system with a dissipative actor, imparts motion to the degrees of freedom via a driving force by virtue of that same actor at finite temperatures. This direct approach also allows the surface pressure applied by the Gaussian beam to interrogate which mechanical modes of the test mass impose a significant energy when 3.8 is plugged into 3.3. In the case of the gaussian beam / uncoated test mass studied by Levin [7]:

$$S_x(f) = \frac{4k_B T}{f} \frac{1 - \sigma^2}{\pi^3 E_o r_o} I \phi \left[ 1 - O\left(\frac{r_o}{R}\right) \right] \quad (3.9)$$

Refer to Levin appendix for more on how elasticity parameters are introduced? Where  $\phi$  and  $E_o$  are the Poisson ratio and Young's modulus respectively, and  $O(\frac{r_o}{R})$  contains a correction term contribution as a function of the small beam radius ( $r_o$ ) relative to the mirror radius ( $R$ ).

### Coating Brownian thermal noise

Further investigations into the beam/optic system utilizing this approach and elasticity theory led to a deeper understanding about Brownian thermal noise contributions from LIGO test masses (substrate, suspensions, HR coating). Levin mentions, with details from Harry, that the noise contributed by a lossy mirror coating is proven to be the most significant contributor of brownian thermal noise. Hong provides a power spectral density [8]:

$$S_j^X = \frac{4k_B T \lambda \phi_x^j (1 - \sigma_j - 2\sigma_j^2)}{3\pi^2 f Y_j (1 - \sigma_j)^2 \omega_o^2} \quad (3.10)$$

Where X represents bulk and shear with j = odd (material 1) and j = even (material 2) alternating layers representing high and low index materials j = odd (material 1) j = even (material 2) for an HR coating.

### SiO<sub>2</sub>/TiO<sub>2</sub> : Ta<sub>2</sub>O<sub>5</sub> coating parameters

Currently the LIGO interferometers deposit  $\lambda/4$  stacks of silica and titania doped tantalum on fused silica test mass substrates. Effective loss angle measurements [9]

### Current SiO<sub>2</sub>/TiO<sub>2</sub> : Ta<sub>2</sub>O<sub>5</sub> elasticity params, power spectra, and strain spectral density (order of magnitude estimate)

### GaAs/Al<sub>0.92</sub>Ga<sub>0.08</sub>As coating parameters

Specific coating parameters for most promising Al<sub>0.92</sub>Ga<sub>0.08</sub>As candidates? Chat with Steve. Or just mention parameters that are listed in Cole 2013 [2]

Insert computed curves of the most precise and recent (effective) loss angle measurements (Nick Demos measurements?). More instructive to plot strain spectral density or displacement power spectra

Currently thermal noise from the SiO<sub>2</sub>/TiO<sub>2</sub> : Ta<sub>2</sub>O<sub>5</sub> optical coatings is the largest contributor of Brownian noise in LIGO compared to estimated substrate and suspension thermal

noise [9]. As of the end of O3, Brownian thermal noise is estimated to be ? orders of magnitude below the current sensitivity and it will prove to be the limiting source of noise as that sensitivity is increased with various other upgrades mitigating fundamental and technical noise. (already mentioned in intro prior to this thermal noise section. Need to re-iterate in more detail?)

### 3.0.2 Anisotropic media

Unlike with isotropic media, we cannot assume that the index of refraction of anisotropic media is the same for all chosen wave vectors. This is a direct consequence of the birefringence of anisotropic media; characterized by the dielectric, permittivity, and polarization tensors.

#### The Dielectric tensor

Further elaborating on the nature of a generalized dielectric tensor for any wavevector is required to proceed:

$$D_i = \varepsilon_{ij} E_j \quad (3.11)$$

Where D is the displacement vector and E is the electric field vector and  $\varepsilon$  is the dielectric tensor. The displacement vector for isotropic media is retrieved when  $i = j$  and  $\varepsilon_i = \varepsilon$ . To further understand the nature of the dielectric tensor we assert Poynting's theorem providing an energy conservation requirement:

$$\nabla \cdot \vec{S} = \frac{dU}{dt} \quad (3.12)$$

Where  $\vec{S} = \vec{E} \times \vec{H}$  is the poynting vector and  $U = \frac{1}{8\pi}(\vec{E} \cdot \vec{D} + \vec{B} \cdot \vec{H})$  is the electromagnetic field density. The reader is left to perform the exercise and show that in order for 3.12 to hold true given 3.11

$$\varepsilon_{ij} = \varepsilon_{ji} \quad (3.13)$$

Demonstrating that the dielectric tensor is symmetric - exhibiting only six unique terms. Diagonalizing the tensor, the presence of two unique eigenvectors and eigenvalues indicates the existance of two eigenpolarizations with paired eigenindices.

### Monochromatic plane wave propogation

Revisiting Maxwell's equations for simple monochromatic plane wave solution gives provides further direction on how crystalline media may effect incident light. Further elaborating, the following assumptions are made:

$$\vec{E} = E_o e^{(i\omega(\frac{n}{c}\vec{r}\cdot\vec{s}-t))} \quad (3.14)$$

Where  $n$  is the index of refraction,  $c$  is the speed of light,  $\vec{r}$  is the position vector and  $\vec{s}$  is the unit wave normal.

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \quad (3.15)$$

Where  $\vec{H}$  is the magnetic field assuming the permeability  $\mu$ , and the generalized displacement vector  $\vec{D}$  and electric field vector  $\vec{E}$ .

$$\nabla \times \vec{E} = -\mu \vec{H} \quad (3.16)$$

Reducing to only the displacement and electric fields:

$$\vec{D} = \frac{n^2}{\mu} [\vec{E} - \vec{s}(\vec{s} \cdot \vec{E})] \quad (3.17)$$

Maxwell's equations show that the electric field is not necessarily parallel to the displacement field and in most materials with non-zero polarizability tensors and dielectric tensors, it is not. But as specified above, the displacement vector, Electric field and unit wave normal are co-planar while remaining orthogonal to  $\vec{H}$ . Assuming we are operating within a coordinate

system aligned with the principal dielectric axes, we substitute 3.11 into 3.17:

$$E_i = \frac{n^2 s_i (\vec{E} \cdot \vec{s})}{n^2 - \mu \varepsilon_i} \quad (3.18)$$

From here it can be shown that for a general plane wave there exist two unique refractive index solutions within the constructed dielectric. Though using this result to show this requires revisiting geometrical conditions that are best visualized using a method introduced in the next section. **For a more rigorous proof, see Appendix H in [10]**

### Indicatrix

Acquiring solutions of the two indices along with the corresponding directions of propagation in the crystal for a general plane wave with unit wave vector  $\vec{s}$  can be done via a convenient geometrical construction. The construction begins by considering a constant electric energy density ( $U_e$ ) surface in the  $\vec{D}$  space; an ellipsoid is formed:

$$\frac{D_x}{\varepsilon_x} + \frac{D_y}{\varepsilon_y} + \frac{D_z}{\varepsilon_z} = 2U_e \varepsilon_o \quad (3.19)$$

With redefined coordinates  $(\vec{D}/\sqrt{2U_e \varepsilon_o}) \rightarrow \vec{r}$  and setting  $\varepsilon_i = n_i^2$ :

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \quad (3.20)$$

This equation for the ellipsoid is known as the indicatrix. Given the co-planar solution demonstrated in the last section, we can impose the normal of the plane  $\vec{r} \cdot \vec{s} = 0$ :

$$\vec{r} \cdot \vec{s} = x s_x + y s_y + z s_z = 0 \quad (3.21)$$

Equations 3.19 and 3.21 both contribute constraints to the method of finding extrema using Lagrange multipliers for the function:

$$r^2 = x^2 + y^2 + z^2 \quad (3.22)$$

The Lagrangian ( $\mathcal{L}$ ) with the introduced multipliers ( $\lambda_1, \lambda_2$ ) then becomes:

$$\mathcal{L}(\vec{r}, \vec{s}, \lambda_1, \lambda_2) = x^2 + y^2 + z^2 + \lambda_1(xs_x + ys_y + zs_z) + \lambda_2\left(\frac{x^2}{\varepsilon_x} + \frac{y^2}{\varepsilon_y} + \frac{z^2}{\varepsilon_z} - 1\right) \quad (3.23)$$

With the generated system of equations from the Lagrange multiplier method ( $\partial F_i / \partial x_i = 0$ , and  $\partial F_j / \partial \lambda_j$ ) where index  $i = x, y, z$  and  $j = 1, 2$  we obtain a system of 3 equations:

$$i\left(1 - \frac{r^2}{\varepsilon_i}\right) + s_i\left(\frac{xs_x}{\varepsilon_x} + \frac{ys_y}{\varepsilon_y} + \frac{zs_z}{\varepsilon_z}\right) = 0 \quad (3.24)$$

The result is verified when substituting  $r \rightarrow \frac{\vec{D}}{\sqrt{\vec{E} \cdot \vec{D} \varepsilon_o}}$  back which recovers 3.18.

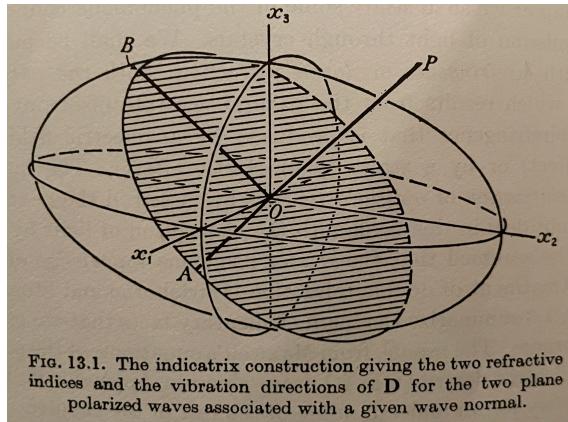


Figure 3.1: General ellipsoid indicatrix with a general propagation direction (Using Nye's figure as placeholder as of now)

### 3.0.3 GaAs and Al<sub>0.92</sub>Ga<sub>0.08</sub>As crystal classification

The space group of GaAs as well as Al<sub>0.92</sub>Ga<sub>0.08</sub>As are within the  $F\bar{4}3m$  space group. Crystals of this particular space group are commonly known as zincblende crystals; a common crystal configuration named after zinc sulfide (ZnS). Cubic crystals by their crystallographic structure display optically isotropic characteristics when stress free and no DC and/or slowly varying electric fields are present. Is this true? Yes. Though the birefringence seen from HR GaAs coatings is said to be due to an “intrinsic stress” in the high and low index layers. (What is breaking the symmetry to cause this? Heteroepitaxy? Annealing? Defects? Is this birefringence the same for all samples?) I think a dedicated high precision birefringence measurement on multiple samples would be cool.

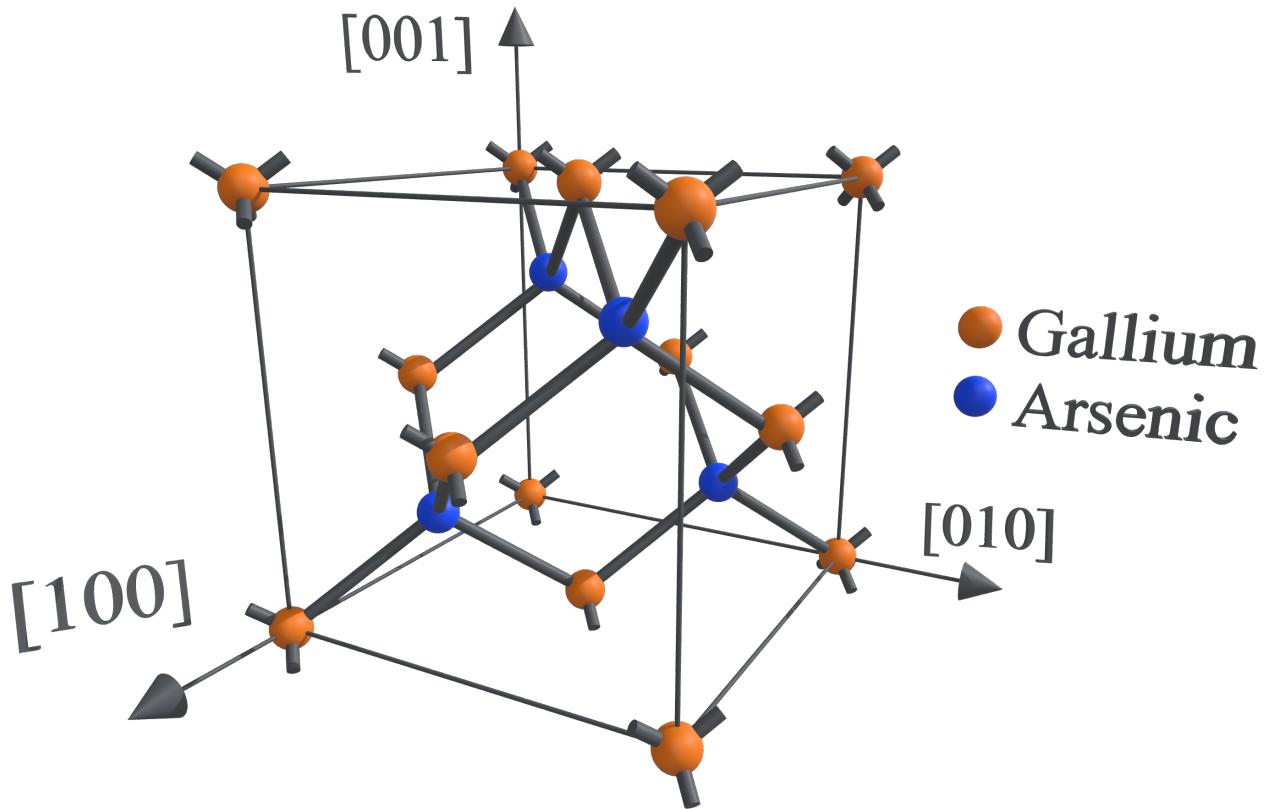


Figure 3.2: The unit cell of gallium arsenide (GaAs) with associated miller indices as coordinate axes

Mention the difference in lattice cell constant between GaAs and Al<sub>0.92</sub>Ga<sub>0.08</sub>As?

### 3.0.4 Induced anisotropy in zincblende crystals

Zincblende structures, like the crystalline materials in question can exhibit birefringent properties when under the influence of two factors: stress in the material, and present within DC electric fields. These two properties of crystalline materials are known as the photoelastic and electro-optic effects respectively.

#### The (linear) electro-optic effect

For non-centrosymmetric crystalline media there exists a non-zero rank 2,  $6 \times 3$  tensor ( $r_{ij}$ ) connecting a slowly varying electric field  $\vec{E}(f) = [E_x(f), E_y(f), E_z(f)]$  directly to the indicatrix [11, 10]:

$$\begin{bmatrix} (\frac{1}{\Delta n^2})_1 \\ (\frac{1}{\Delta n^2})_2 \\ (\frac{1}{\Delta n^2})_3 \\ (\frac{1}{\Delta n^2})_4 \\ (\frac{1}{\Delta n^2})_5 \\ (\frac{1}{\Delta n^2})_6 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{bmatrix} \begin{bmatrix} E_x(f) \\ E_y(f) \\ E_z(f) \end{bmatrix} \quad (3.25)$$

The  $i$  index runs over the terms in the indicatrix equation:

$$\left(\frac{1}{\Delta n_x^2}\right)x^2 + \left(\frac{1}{\Delta n_y^2}\right)y^2 + \left(\frac{1}{\Delta n_z^2}\right)z^2 + 2\left(\frac{1}{\Delta n_{xz}}\right)xz + 2\left(\frac{1}{\Delta n_{yz}}\right)yz + 2\left(\frac{1}{\Delta n_{xy}}\right)xy = 1 \quad (3.26)$$

Detail on some prior knowledge of  $f \leq f_{\max}$ ?

$r_{ij}$  for zincblende crystals ( $r_{43m,ij}$ )

The form of the electro-optic tensor for zincblende crystals (including GaAs and Al<sub>0.92</sub>Ga<sub>0.08</sub>As) reduces such that  $r_{ij} = r_{41} = r_{52} = r_{62} \neq 0$  with all other terms being zero:

$$r_{43m,ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{52} & 0 \\ 0 & 0 & r_{63} \end{bmatrix} \quad (3.27)$$

Where also  $r_{41} = r_{52} = r_{63}$

### New principal (electro-optic) dielectric axis for zincblende structures

In general the principle dielectric axes of the new ellipsoid do **not** coincide with the axes of the ellipsoid of the unperturbed crystal. The form of the index ellipsoid for a zincblende crystalline material accounting for the electro-optic tensor and some generalized DC electric field  $\vec{E}$  expressed in terms of the crystallographic axes is given by:

$$\left(\frac{1}{n_o^2}\right)x^2 + \left(\frac{1}{n_o^2}\right)y^2 + \left(\frac{1}{n_o^2}\right)z^2 + 2r_{41}E_{[100]}yz + 2r_{41}E_{[010]}xz + 2r_{41}E_{[001]}xy = 1 \quad (3.28)$$

Where we have set  $n_x = n_y = n_z = n_o$  for zincblende structures.

The two principal axes are given by the eigenvectors of the the matrix given from the equation above:

$$\begin{bmatrix} \left(\frac{1}{n_o^2}\right) & r_{41}E_{[001]} & r_{41}E_{[010]} \\ r_{41}E_{[001]} & \left(\frac{1}{n_o^2}\right) & E_{[100]} \\ r_{41}E_{[010]} & r_{41}E_{[100]} & \left(\frac{1}{n_o^2}\right) \end{bmatrix} \quad (3.29)$$

### The photoelastic effect?

When a general strains  $S_{kl}(r) = \frac{1}{2} \left[ \frac{\partial u_k(r)}{\partial x_i} + \frac{\partial u_i(r)}{\partial x_k} \right]$  are applied to a material, the photoelastic tensor  $p_{idkl}$  relates to the indicatrix by the following relation:

$$\left( \frac{1}{\Delta n^2} \right)_{id} = p_{idkl} S_{kl} \quad (3.30)$$

Supplementary comment to the measured birefringence from the mentioned intrinsic strain of the high and low index layers

### The generalized indicatrix

New principal dielectric axes for zincblende structures (zincblende photoelastic tensor, zincblende electro-optic tensor)

#### 3.0.5 Optical anisotropy of a HR GaAs / Al<sub>0.92</sub>Ga<sub>0.08</sub>As stack

The analysis of a highly reflective GaAs/Al<sub>0.92</sub>Ga<sub>0.08</sub>As stack is the sole purpose of the introduction of the geometric tools prepared in the prior section. This section implements said tools by: 1) making considerations of crystal coordinates when asserting an optical axis on a highly reflective crystalline stack manufactured by Thorlabs, 2) citing coating parameters and observed intrinsic birefringence from the highly reflective coating stack in question, 3) analyzing differential linear electro-optic effect on the phase of a reflected beam, and 4) estimating the differential phase noise in LIGO based on calibrated electric field measurements.

### Miller indices for highly reflective coatings GaAs/Al<sub>0.92</sub>Ga<sub>0.08</sub>As coatings

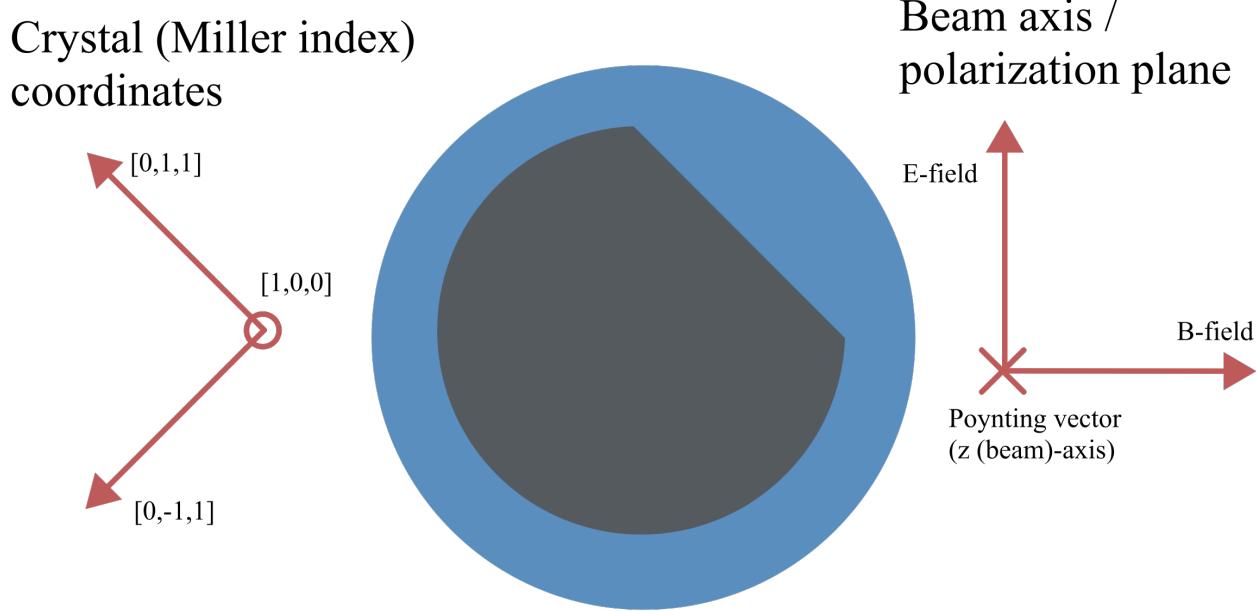


Figure 3.3: Figure of the beam propagation axis with respect to the AlGaAs/GaAs crystal axes (not final version). Within the [100] plane the AlGaAs coating is grown with a flat indicator that draws a line within the [0-11] plane where the bisecting vector of the plane normal points towards the sample center. The axis formed by the [100] coincides with the optical axis (z-axis). The polarizations of incident and reflected beam oscillate along vectors within the plane formed by the normal of that axis.

Up until this point we have discussed three different coordinate axes: the crystal axis (formally indicated by Miller indices), the principal dielectric axis (coordinates based in diagonalization of the indicatrix), and an optical axis (when considering a desired (laser) light propagation). We assert an optical axis with linearly polarized light with variable s and p contributions.

### Relevant coating parameters

What is the variability in this coating since [2]?

Cole 2013 coating

- 81 layers ( $6.83\mu m$  thick)

- GaAs ( $n_{\text{GaAs}} = 3.480$ )
- Al<sub>0.92</sub>Ga<sub>0.08</sub>As ( $n_{\text{Al}_{0.92}\text{Ga}_{0.08}\text{As}} = 2.977$ )
- Base coating layer of 270-nm-thick Al<sub>0.92</sub>Ga<sub>0.08</sub>As

### Our coating

The coating to be studied consists 36  $\lambda/4$  thick layers of GaAs interspersed with 35 layers of  $\lambda/4$  thick Al<sub>0.92</sub>Ga<sub>0.08</sub>As. GaAs forms the top and bottom layer to prevent oxygen absorption from the AlGaAs layer. The GaAs layers have an index of  $n_{\text{GaAs}} = 3.480$  and a thickness of  $\Delta d_{\text{GaAs}} = 76.43$  nm while the low index Al<sub>0.92</sub>Ga<sub>0.08</sub>As layers are  $n_{\text{Al}_{0.92}\text{Ga}_{0.08}\text{As}} = 2.977$  with thickness  $\Delta d_{\text{Al}_{0.92}\text{Ga}_{0.08}\text{As}} = 89.35$  nm.

Fejer and Bonilla take an analytical approximation approach when finding the impact of the electric field to the change in phase of the light through a crystalline anisotropic thin film ( $\lambda/4$ ) stack [12].

$$\hat{\phi}' = \frac{\pi n_1 z}{1 - z^2} (z^{2N} - 1) \frac{z^{2N} \frac{(n_f)^2}{n_2 n_3} (n_2 \kappa_{\gamma 2} + n_3 \kappa_{\gamma 3}) - (n_2 \kappa_{\gamma 3} + n_3 \kappa_{\gamma 2})}{(n_1)^2 - (n_f)^2 z^{4N}} \quad (3.31)$$

$$\text{with } z = \frac{n_2}{n_3} \text{ and } \kappa_{\gamma j} = \frac{d}{d\gamma} \log(n_j h_j) \Big|_{\gamma=\gamma_O} \left( \frac{\hat{n}'_j}{\hat{n}_j} + \frac{\hat{h}'_j}{\hat{h}_j} \right)$$

With  $\kappa$  being a scalar parameter.

Adding a schematic would be helpful.

Figure is in the works

### Electro-optic coupling to the reflected phase of a HR mirror coating

With our coordinate considerations and established beam axis, it is now worth considering the influence of a isotropic white noise field ( $E_n = [E_{nx}, E_{ny}, E_{nz}]$ ):

$$\begin{bmatrix} (\frac{1}{n_o^2}) & r_{41}E_{ny} & r_{41}E_{nx} \\ r_{41}E_{ny} & (\frac{1}{n_o^2}) & r_{41}E_{nz} \\ r_{41}E_{nx} & r_{41}E_{ny} & (\frac{1}{n_o^2}) \end{bmatrix} \quad (3.32)$$

Assuming  $E_n$  is small, the indicatrix change of  $E_{nx}$  and  $E_{ny}$  relative to  $E_{ny}$  (as seen by the beam polarization) will be small ( $r_{41}E_{n(x/y)} \ll r_{41}E_{nz}$ ). After diagonalizing with relevant terms in the tensor we are left with the following eigenindices:

$$n'_x = n_o - r_{41}E_{nz} \quad (3.33)$$

$$n'_y = n_o + r_{41}E_{nz} \quad (3.34)$$

How the modulation of the phase of the carrier field is dependent on the orientation of its wave vector with respect to the crystal structure, the modulating electric field direction and strength, (other items to discuss in terms of introducing the effect)

For GaAs @ 10.6 $\mu$   $r_{41} = 1.6 \times 10^{-12}$  [m/V]

Adachi estimate for Al<sub>x</sub>Ga<sub>1-x</sub>As?

Relevant eigenpolarizations, non-optical field  $E_y = E_z = 0$ ?

Figure: Transformed indicatrix (Before and after  $E_x$ )

Figure: Ellipse cross section. New eigenpolarizations and corresponding indices and their influence on incident field (Marty's result)

Assuming we are operating in a coordinate system suggested in Figure ?. Given this configuration, which plane is impacted by some  $E_{noise}$ ? Revisiting the indicatrix. We can see that for even non-zero z and y components that the only coupling to the input beam polarization is the index along the cross coupled zy axis through  $E_z$  is that of the  $E_x$  term. This gives us the ability to easily diagonalize the indicatrix tensor by setting the non-relevant field terms to zero.

### Numerically friendly estimate

In the appendix of [13] Ballmer explains a formalism that allows the coating transfer function given a differential change in phase due to a scalar parameter to be expressed as follows:

$$M = Q_N D_N \dots Q_k D_k \dots Q_1 D_1 Q_0 \quad (3.35)$$

For a single pass through the coating where the coating thickness and indices are left general.

Both approaches are essentially based on the same calculation method i.e., transfer matrices.

### 3.0.6 Measured birefringence from HR GaAs/Al<sub>0.92</sub>Ga<sub>0.08</sub>As mirrors

There seems to be different accounts of a measured birefringence from HR GaAs / Al<sub>0.92</sub>Ga<sub>0.08</sub>As (Satoshi, CTN, Aidan)

Is the measured birefringence static? (Layer bonding method might introduce something?)

Does it change from different mounting methods? (Photoelastic) (order of magnitude estimate)

Electro-optic ruled out based on field measurements and relative coupling factor.

Measurement precision of the coating birefringence? Cavity length, Polarization drifts, etc.

The measured birefringence is estimated to be caused by an intrinsic strain between the epitaxial layers of GaAs/Al<sub>0.92</sub>Ga<sub>0.08</sub>As. [2]

Marty's document about Birefringence in Crystalline mirror coatings V.8

## 3.1 Projected DARM coupling

To estimate how much DARM coupling can occur, we use use a measured field spectra acquired from installed electric field meters located within LIGO Hanford Observatory EX and EY vacuum chambers. Taking the upper and the lower EFM measurements in .3 [V/m/ $\sqrt{\text{Hz}}$ ] @ 60 Hz and  $4 \times 10^{-3}$  [V/m/ $\sqrt{\text{Hz}}$ ] @ 4kHz [14]. I don't think these values are calibrated. According to Martynov et al. 2016, the fluctuations in the electric filed is  $\sim 10^{-5}$  [(V/m)/ $\sqrt{\text{Hz}}$ ].

This along with computed estimate above allows us to create an upper limit for what this

noise might be assuming incoherent fields between the end stations and a flat frequency response within LIGO's bandwidth.

## 3.2 Experiment

To probe for this effect, we took the approach of an in-air Fabry-Pérot cavity with a GaAs/Al<sub>0.92</sub>Ga<sub>0.08</sub>As coated sample end mirror mounted in a custom designed MACOR mount with the ability to install electrodes maintaining a fixed distance from the sample surfaces for a controlled E-field injection. Details and specifications are discussed as well as relevant measurements. An improved experimental design is also detailed for a higher sensitivity measurement.

### 3.2.1 Design

This section details the Pound-Drever-Hall servo which is at the heart of this experiment, the on-table design, a sample / electrode assembly used for a controlled electric field injection, and electrode drive parameters

#### PDH servo

The Pound-Drever-Hall technique, originally imagined for laser frequency stabilization to an ultra-stable length reference, achieves constant resonance with said reference by tracking the phase of the cavity input light with respect to the light reflected from the cavity. Tracking the approximately linear phase response across resonance is at the heart of this technique and recognizes that physical property to be probed around a cavity's resonance cannot be the intensity of the transmitted or reflected light because of the quadratic symmetry. ([Derivative of the intensity could work but depends on us operating off resonance](#))

Insert figure of reflected intensity and phase response around resonance (cavity transfer function). Maybe also have a figure corresponding to what this could represent in a lab.

(sweeping a mirror of the laser cavity through resonance)

Another novelty and insight to the technique is the use of phase modulation onto the carrier field which is the mathematical and physical equivalent to imposing separate optical fields which in most cases do not resonate in the optical cavity.

$$E_{\text{inp}} = E_o e^{i\omega t + \beta \sin(\Omega t)} \quad (3.36)$$

Approximation with Bessel functions (assumptions about modulation depth)

$$E_{\text{inp}} \approx E_o E_0 [J_o J_0(\beta) e^{i\omega t} + J_1(\beta) e^{i(\omega+\Omega)t} - J_1(\beta) e^{i(\omega-\Omega)t}] \quad (3.37)$$

Discuss how the measurement of the beat power between the sideband and the carrier at the RFPD tracks the phase and how with demodulation and filtering, you can create an error signal.

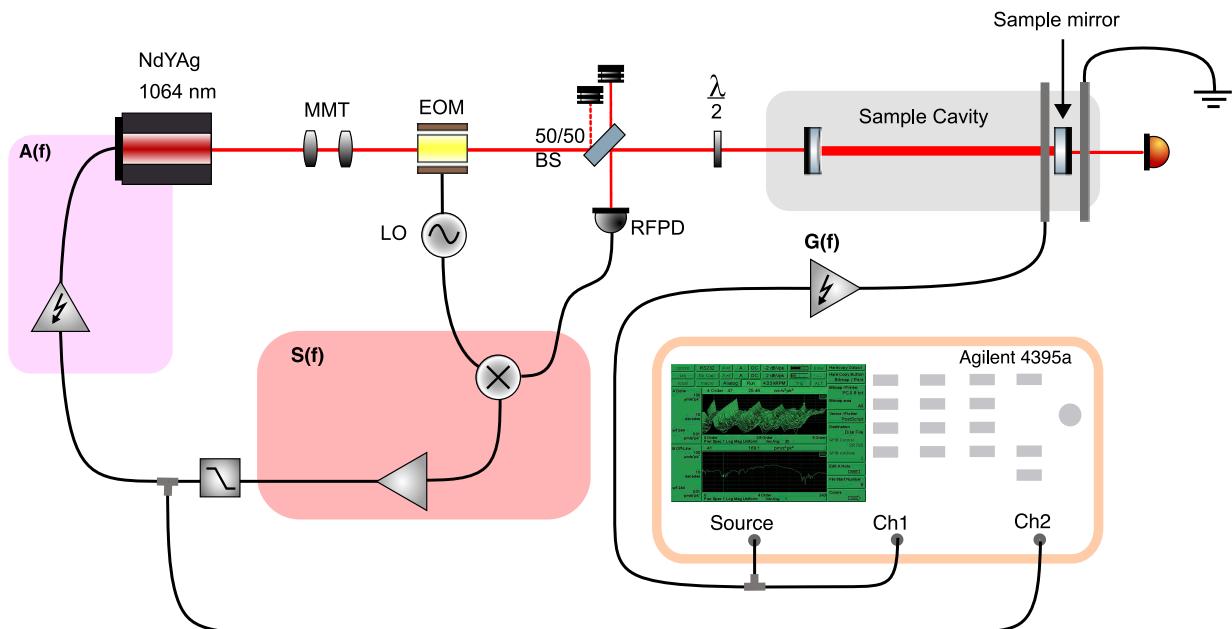


Figure 3.4: Simplified experiment schematic

### On-table schema

A figure that highlights path that is intended for locking onto the PMC, and a different color highlight for the path to the experiment

The setup presented in this section, branches off of a previously established optical path used to lock onto a separate optical cavity (PMC).

The laser light source is a Mephisto 2000 NE 1064nm laser.

25 MHz EOM is a New Focus Model 4003 IR resonant phase modulator.

The designed cavity is a 0.1651 m long cavity with a HR IBS coated sample (PL-CC, ROC =0.333 m) input coupler from CVI Melles-Griot and a GaAs/Al<sub>0.92</sub>Ga<sub>0.08</sub>As (PL-PL) fused silica substrate by the Crystalline Mirror Solutions (CMS) division of Thorlabs with the aforementioned parameters ([mentioned in coating parameters section](#)).

### Sensing S(f)

- 25 MHz RFPD
  - Transimpedance measurement (necessary? or should I just use the mixer out PDH to summarize PD/mixer response)
- Frequency Stabilization servo (modified MIT FSS (DCCD980536)) (LTspice model in appendix)

### Actuation A(f)

- Mephisto 2220 PZT response (capacitance estimated from HVA drive measurement with and without connection to PZT)
- Channel 3 of SVR 350-3 BIP High Voltage Amplifier from Piezomechanik GmbH with Pomona box (elog 412)
- [Figure of frequency response of A\(f\)](#)

### Low frequency servo (Thermal loop)

- Passed signal from FSS → integrators → Laser thermal actuator input

### OLG(f)

Isn't quite  $A(f) * S(f)$  as stated. Doesn't entirely account for the optical plant. How the measurement is taken (important to take between installations to account for the changes in the optical plant) (elog 831)

### Sample / Electrode assembly

Maximizing the electric field ( $|E_z|$ ) and within the coating while requiring a through beam to and through the HR coating lead us to imagine disk electrodes with a 3mm central aperture. The aperture size was chosen to be at least 5 times larger than the beam size at the plate locations. This was to avoid any beam clipping while still allowing to maximize the field strength at the region of interest.

Most commercial optical mounts are conductive which proved to be a problem when attempting to find a mounting solution while reducing the non-normal field gradients within the volume of interest around the sample. Because of this, we chose to construct an optical mount made of MACOR a machinable ceramic with high a high Young's modulus (66.9 GPa), and a moderate Poisson ratio (.29) [15]. An optical mount for the sample made with MACOR, along with glass bearings .48 ± .01 cm Ø and a McMaster-Carr 8-32, 1/2" ceramic screw were used to clamp and suspend the optical sample. A 1.24" Ø hole was bored into the MACOR with a (depth?) depth so that there is a ? mm clearance between the front and back surface of the sample to the electrode plates. [Figure with the sample in-situ](#)



Figure 3.5: Placeholder for more updated MACOR assembly

There are two relevant configurations of this experiment: 1) an all-in-one MACOR assembly where the electrodes are mechanically coupled to the optical mount, and 2) larger mechanically decoupled electrode plates.

A lot of time was dedicated towards preliminary mounts made of PLA and PETG. Do I want to do updated measurements and make statements about noise produced from these mounts?

### 3.2.2 |E<sub>z</sub>| strength estimate

To convey the problem at hand, it is useful to review the illustration seen [here](#) (figure showing the electrode plates, and sample with AlGaAs coating)

#### Math

To find the Electric field screened by the coating we begin with Gauss' Law:

$$\nabla \cdot D = \rho_{\text{free}} \quad (3.38)$$

For our problem we assume no free charge, but the fused silica substrate with the AlGaAs coating presents dielectric material between the plates. Our initial boundary conditions are also expressed in terms of plate potentials so it is natural to first solve for the potential ( $V$ ) for every point within our system. We can exploit the cylindrical symmetry with the optic and plate geometry in the  $r$  coordinate so we shall express the Laplacian accordingly:

$$(1 - \chi) \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial z^2} \right] V = 0 \quad (3.39)$$

Where  $\chi$  is a spatially dependent electric susceptibility. ([Establish coordinates for GaAs/Al<sub>0.92</sub>Ga<sub>0.08</sub>As, as well as the fused silica substrate so the computation is transparent](#))

Definition of  $\rho$  must be explained.  $\rho$  and  $\rho_{\text{free}}$  are confusing. Define  $\chi$  and  $V$ .

I will change  $\rho$  to  $r$  am going to modify figure text so it matches soon.

Utilizing this, we can proceed to a construction of a numerical Laplacian.

#### Numerical approximation

- Potential map computation in cylindrical
- Computing  $E_z$  from potential map
  - inside coating

- outside coating

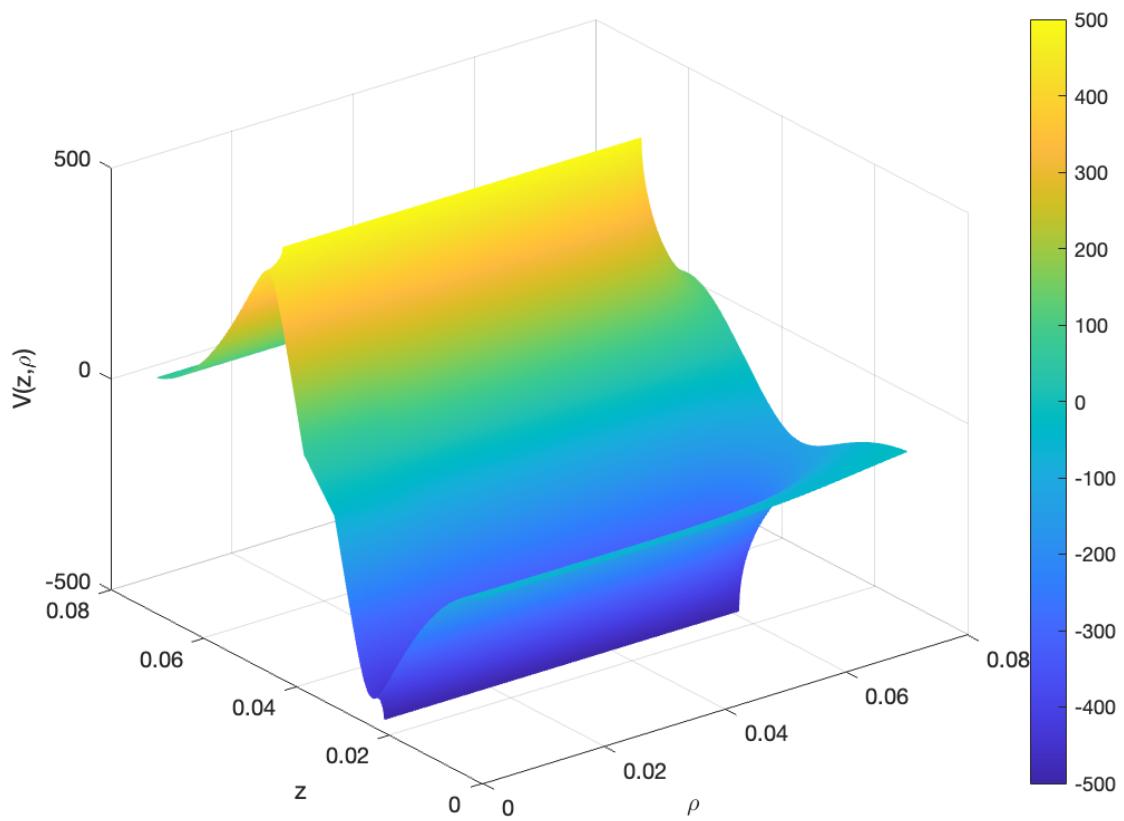


Figure 3.6: Poisson calculator output potential map ( $V(z, r)$ ) in cylindrical coordinates)

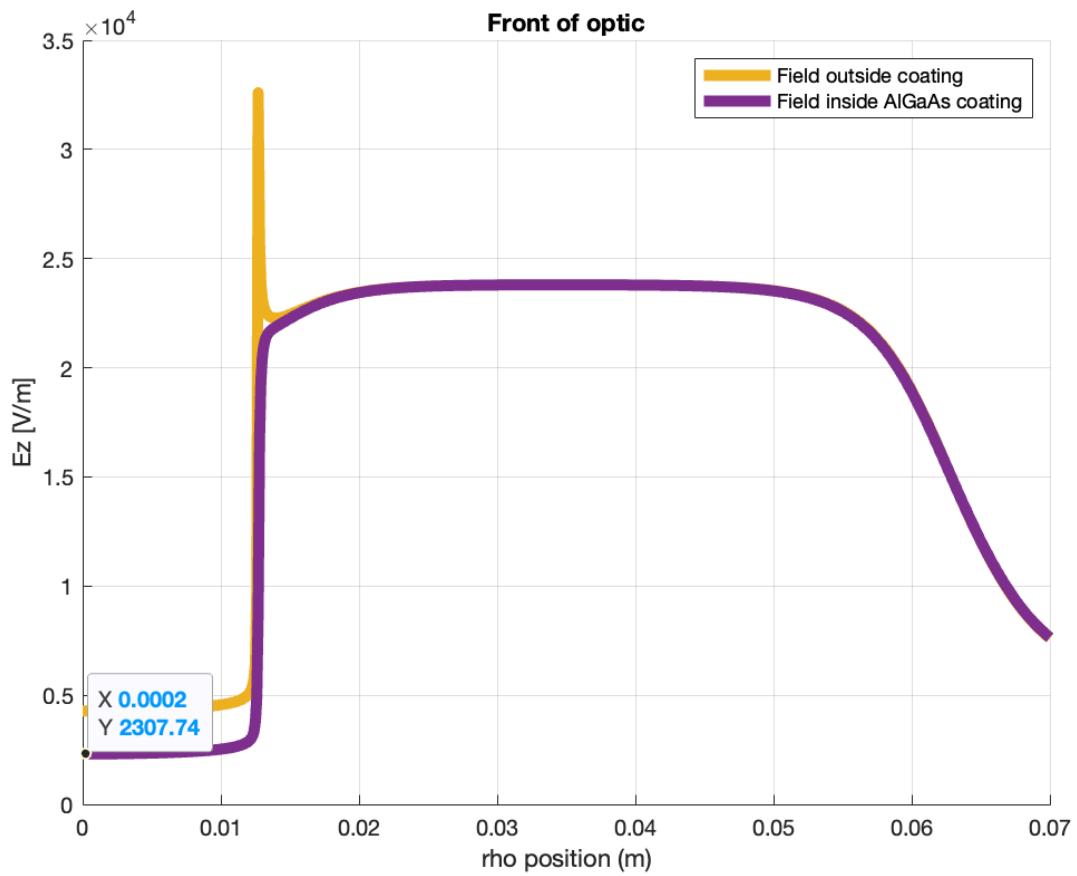


Figure 3.7:  $|E_z|$  screened by the scoating and immedately outside AlGaAs coating. Needs to be updated with more current settings  
How large applied voltage is assumed?

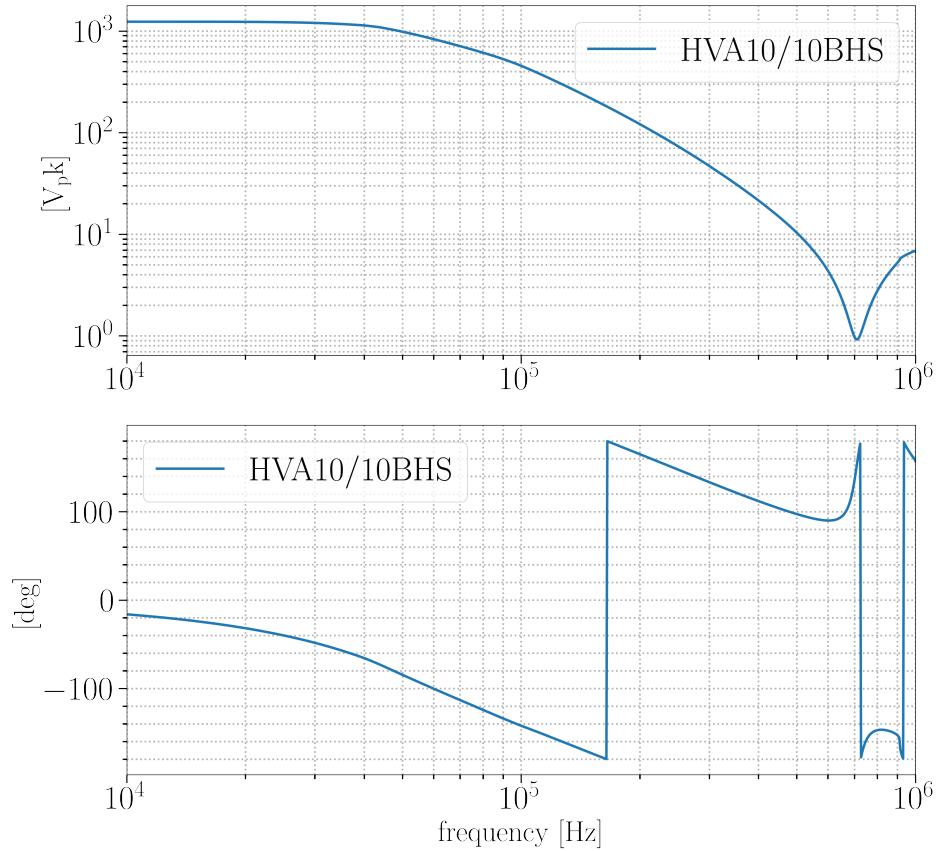


Figure 3.8: TREK 10/10B-HS HVA frequency dependent measurement. Using Poisson calculator to estimate field strength within coating. (Just HVA for now but will update. Also, assumes a flat response from coating within this studied region (is this a good assumption or could I do better? (dielectric frequency dependence))

### 3.2.3 Calibration

As discussed, we know that the error signal spectra provides us a voltage spectra that with the above information about the Plant/servo electronics, allows us to  $\text{VFSSOUT}_{\text{rms}}/\sqrt{\text{Hz}} \rightarrow m_{\text{rms}}/\sqrt{\text{Hz}}$

$$\Delta L = \text{source} * \alpha(f) A(f) * \frac{1 + \text{OLG}(f)}{\text{OLG}(f)} * \frac{L_{\text{cav}}}{f_{\text{laser}}}$$

### 3.2.4 Noise Floor

Various noise contributions that add up to measured noise floor

#### Shot noise

Look at the derived shot noise estimate in the appendix V in the Black paper [16]

#### Laser frequency noise

- Measure with initial LIGO PMC?
- There is also a spectra in the Mephisto laser spec sheets

Residual gas noise

Mount noise (3D printed mount mechanical noise)

### 3.2.5 Drive coupling

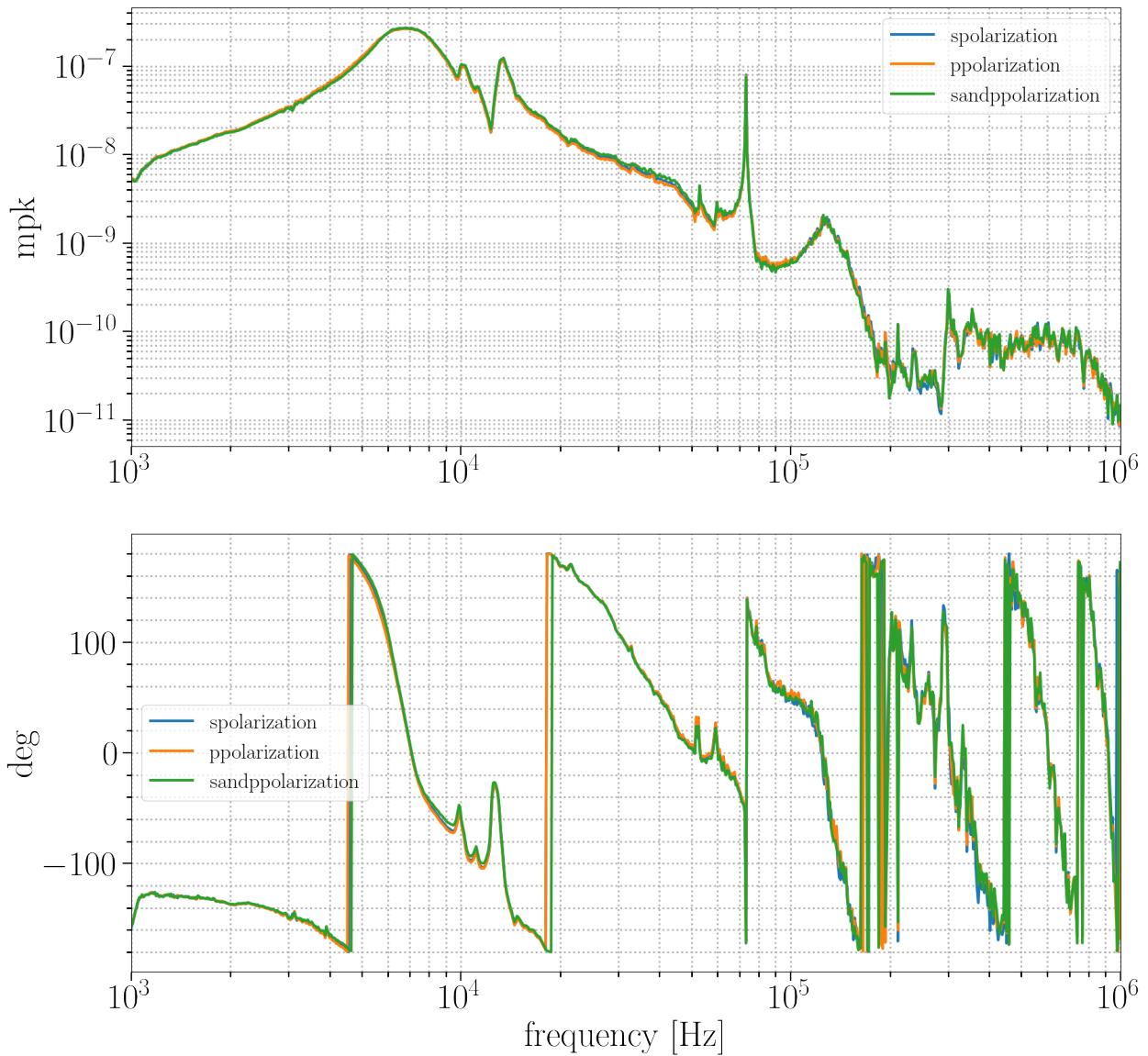


Figure 3.9: Figure that will include the displacement noise floor, (pockels estimate)\*(poisson calculator estimate)\*(HVA drive frequency dependence), and the drive coupled measurement figure size needs to be increased

### **Opto-mechanical coupling**

Sample and mount mechanical mode excitations. Seen with both AlGaAs and a HR coating from an AtFilm (IBS coating)

- **Vibration of plates (Leissa)** [17] Computing frequencies and order of magnitude
- Steve's COMSOL model results

### **Proposed alternative measurement schema**

# **Chapter 4**

## **Conclusion**

# **Chapter 5**

## **Appendix**

### **5.1 The Equipartition theorem and the Fluctuation dissipation theorem**



## 5.2 Crystal classification, point groups, and Bravais lattices

Crystal classification	Point groups		Bravais lattices
	Hermann-Maugin	Schoenflies	
Triclinic	1	$C_1$	P
	$\bar{1}$	$C_i$	
Monoclinic	2	$C_2$	P, C
	$m$	$C_s$	
	$2/m$	$C_{2h}$	
Orthorombic	222	$D_2$	P, C, I, F
	$mm2$	$C_{2v}$	
	$mmm$	$D_{2h}$	
Tetragonal	4	$C_4$	P, I
	$\bar{4}$	$S_4$	
	$4/m$	$C_{4h}$	
	422	$D_4$	
	$4mm$	$C_{4v}$	
	$\bar{4}2m$	$D_{2d}$	
	$4/mmm$	$D_{4h}$	
Trigonal	3	$C_3$	R
	$\bar{3}$	$C_{3i}$	
	32	$D_3$	
	$3m$	$C_{3v}$	
	$\bar{3}m$	$D_{3d}$	
Hexagonal	6	$C_6$	P
	$\bar{6}$	$C_{3h}$	
	$6/m$	$C_{6h}$	
	41 622	$D_6$	
	$6mm$	$C_{6v}$	

Crystal Class → geometric analogy

Point groups

Bravais lattices

Space groups

### 5.3 Calibration

The frequency response measurement shown in (?) records the following transfer function in dB of the following:

$$\alpha(f) = \frac{CH2(f)}{Source} \quad (5.1)$$

Channel

We also know that the error signal spectra of the loop is probed by  $CH2(f)$ :

$$CH2(f) = \frac{S(f) * signal_V}{(1 - OLG(f))} \quad (5.2)$$

Where  $signal_V$  is the uncalibrated voltage output from the mixer,  $S(f)$  is the FSS transfer function, and  $OLG(f)$  is the open loop gain of the PDH system.

And we know  $OLG(f) = S(f) * A(f)$  so :

$$signal_m = CH2(f) * A(f) \frac{1 - OLG(f)}{OLG(f)} \frac{L_{cav}}{f_{laser}} \quad (5.3)$$

Where  $A * (f)$  is the high voltage amplifier response with the Mephisto 2220 laser PZT response.  $L_{cav}$  is the sample cavity length,  $f_{laser}$  is the laser frequency. ( $signal_m$ ) is the effective cavity length change from the Pockels effect.

Substitute (1) into (4):

$$signal_m = Source * \alpha(f) * A(f) \frac{1 - OLG(f)}{OLG(f)} \frac{L_{cav}}{f_{laser}} \quad (5.4)$$

Measured : Source,  $\alpha(f)$ , OLG( $f$ ), A( $f$ ) and  $L_{\text{cav}}$

## 5.4 Laplace calculator / code

## 5.5 MACOR assembly

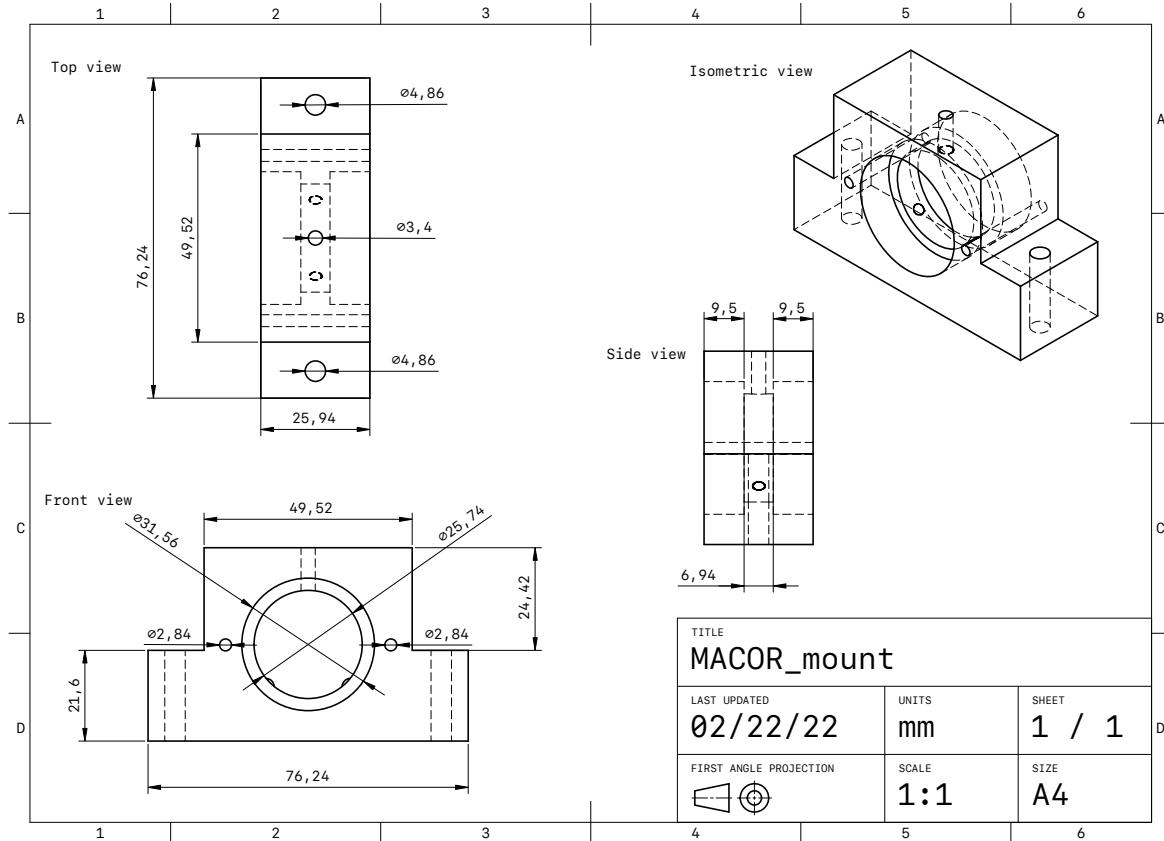


Figure 5.1: MACOR mount design constructed in Shapr3D  
First angle projection is used in Europe. In America, third angle projection is used, and I recommend to use it.

## 5.6 FSS LTSpice model

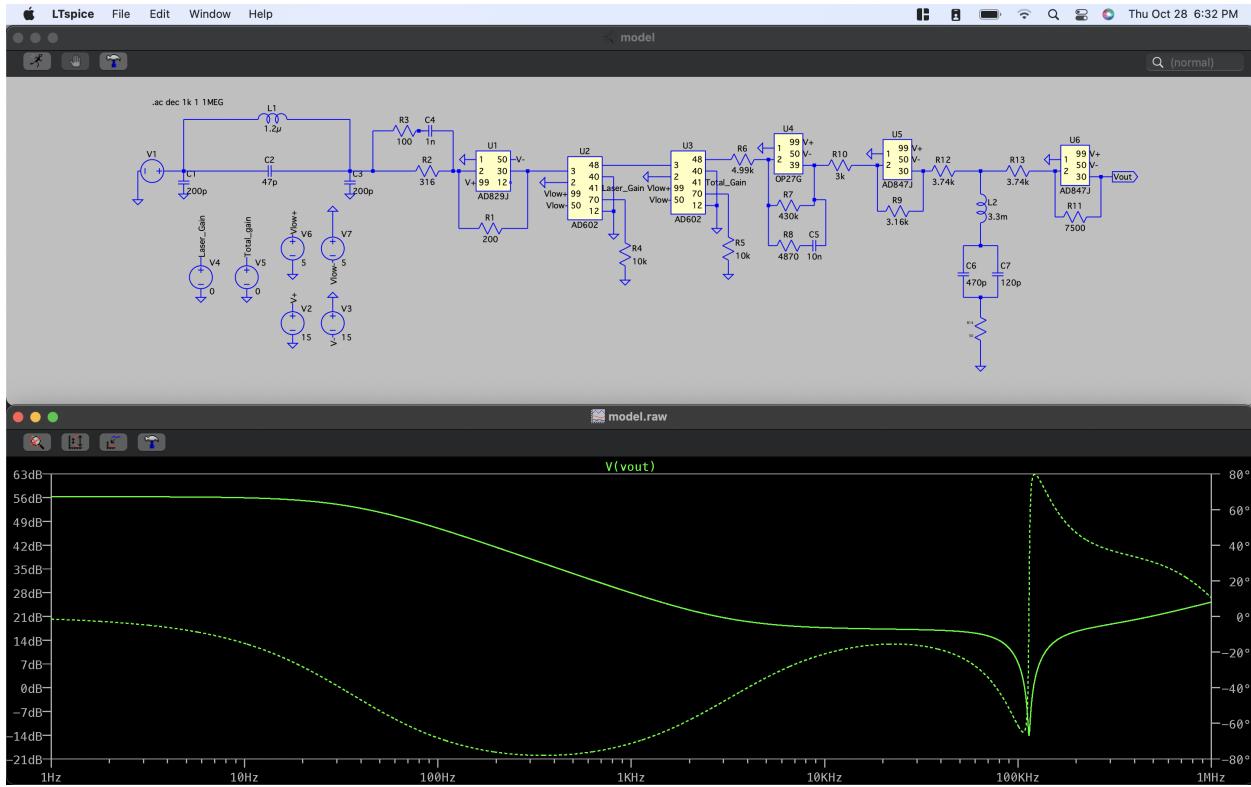


Figure 5.2: The FSS transfer function simulated in LTspice

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