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THE ROLE OF BINARIES IN GRAVITATIONAL WAVE PRODUCTION

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I. INTRODUCTION

Best estimates (Batten 1973) indicate that at least half the stars consist of binary or multiple systems. Since such systems have a time varying quadrupole moment they may be expected to generate gravitational waves (GW).

Peters and Mathews (1963) calculated the energy and angular momentum losses due to gravitational radiation in the weak field, slow motion limit of two point masses in a binary. The luminosity averaged over a period is

$$L_{GW} = 32\, G^4 m_1^2 m_2^2 (m_1 + m_2)\, f(e) \,/\, 5c^5 a^5 \tag{1a}$$

$$= (1.63\times10^{51} \mathrm{erg\,s^{-1}}) (m_1/m_\odot)^2 (m_2/m_\odot)^2 (m_1+m_2/m_\odot)(a/100\mathrm{km})^{-5} f(e) \tag{1b}$$

where a is the separation, m_1 and m_2 the masses of the two bodies, and $f(e)$ is a function of the orbital eccentricity, and equals one for a circular orbit.

The waves are emitted at a frequency equal to twice the orbital frequency

$$\nu_{GW} = G^{1/2} (m_1+m_2)^{1/2} / \pi a^{3/2} \tag{2a}$$

$$= (164\ \mathrm{Hz}) (m_1+m_2/2m_\odot)^{1/2} (a/100\ \mathrm{km})^{-3/2} \tag{2b}$$

The energy loss to gravitational waves causes the orbit to decay, and the components reach zero separation ("coalescence" or "collision") after a time (Peters 1964):

$$\tau_{GW} = 5c^5 a^4 / 256\, G^3 m_1 m_2 (m_1+m_2) \tag{3a}$$

$$= (2.0\mathrm{s})\ (m_1/m_\odot)^{-1} (m_2/m_\odot)^{-1} (m_1+m_2/m_\odot)^{-1} (a/100\ \mathrm{km})^4 \tag{3b}$$

Components of binaries, clearly have a real and very finite size, which restricts the separation of the components to be at least roughly as large as the Roche or tidal lobe of the more extended component, or a mass flow will ensue. Typically, for components of roughly equal mass, the Roche separation is approximately three times the radius of the extended component. A strong lower limit on the separation is that it should be larger than the sum of the radii of the components.

Table 1 shows the relative parameters for the closest possible binaries as a function of their most extended component, assuming two $\sim 1\ m_\odot$ objects.

Table 1: Parameters for Closest Possible Binaries

Most Extended Component	Typical radius (km)	L_{GW} (erg s^{-1})	ν_{GW} (Hz)	h @ 10 Kpc	Duration of binary
Main Sequence	10^6	$\sim 10^{29}$	3×10^{-5}	$\sim 3\times10^{-23}$	$\sim 10^6$ yr.
White dwarf	10^4	$\sim 10^{39}$	3×10^{-2}	$\sim 3\times10^{-21}$	$\sim 10^3$ yr.
Neutron Star	10	$\sim 10^{54}$	$\sim 1\times10^{3}$	$\sim 3\times10^{-18}$	~ 1 s.

The relative frequency of binaries in the galaxy, which will reach their Roche separation in less than 10^{10} years, however is approximately of the order of

$$N(\text{m.s.}) : N(\text{w.d.}) : N(\text{n.s.}) \approx 1 : 10^{-2} : 10^{-5}$$

II. BINARIES AS PERIODIC GRAVITATIONAL WAVE SOURCES

In this paper we shall only discuss close compact object binaries. A thorough review of binaries as GW sources is given by Douglass and Braginsky (1979).

To date it has been common to portray periodic sources individually on a plot of dimensionless amplitude versus frequency. We recommend (Ron Drever, Ray Weiss, private communication) that the density of these sources $d^2N/d(\log h)d(\log \nu)$ should be plotted as a function of log h and log ν. The sum of $d^2N/d(\log h)d(\log \nu)$ over all known and inferred sources will provide us with complete information on periodic sources. Since h and ν normally cover many orders of magnitude, they are plotted logarithmically.

Consider white dwarf-white dwarf binaries close enough that they reach their Roche separation in less than a Hubble time ($\sim 10^{10}$yr.). Then, assuming the density of systems to scale as r^2 from 100 pc to 10 Kpc from the sun (i.e. disk population), the white dwarfs to have masses of 1 $m_\odot$ each, and denoting the mean time between formation of such systems by τ_{wd}, we get:

$$d^2N/d(\log h)d(\log \nu) = 5\times10^{3}(\nu/10^{-3}\text{Hz})^{-4/3}(h/10^{-20})^{-2}(\tau_{wd}/1\text{yr})^{-1} \quad (4)$$

for:

$$9\times10^{-21}(\nu/1\ \text{Hz})^{2/3} \leq h \leq 9\times10^{-19}(\nu/1\text{Hz})^{2/3} \quad [100\text{pc} \leq r \leq 10^4\text{pc}] \quad (5a)$$

$$5\times10^{-5}\ \text{Hz} \leq \nu \leq 3\times10^{-2}\text{Hz} \quad [\tau_{GW} \lesssim 10^{10}\text{yr};\ a_{min} \gtrsim 3r_{wd}] \quad (5b)$$

The length of time between formation of such systems is probably of the order of decades to centuries. Figure 1 is a plot of equicontours of $d^2N/d(\log h)d(\log \nu)$, taking $\tau_{wd} = 10^2$yr.

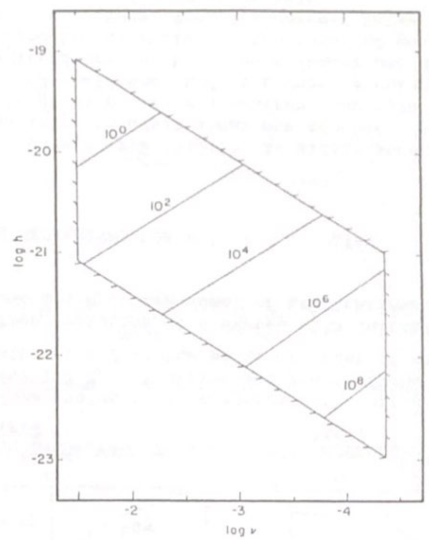

Fig. 1 - Equicontours of $d^2N/d(\log \nu)d(\log h)$ for close white dwarf-white dwarf binaries.

III. COMPACT OBJECT BINARIES

(i) Neutron Star - Neutron Star

Clark and Eardley (1977) studied the evolution of a binary system consisting of two neutron stars as their orbit decayed towards coalescence. In particular the loss of energy and angular momentum drove the components towards one another so that one of the neutron stars was within its tidal radius, whereupon tidal stripping ensued.

This scenario was envisioned as being the end product of either the orbital decay of a neutron star - neutron star binary whose initial period was less than about half a day, or the fissioning or fragmentation of a rapidly rotating core of a Type II supernova.

The calculation of the evolution of the components was carried out entirely using Newtonian dynamics, while allowing for gravitational wave energy and angular momentum losses. Mass loss from the less massive (more extended) neutron star as it spirals inside its tidal radius was found to stabilize the system against orbital decay. Depending on the masses of the components, mass and angular momentum losses from the system, and spin-angular momentum coupling, the system may be unstable to tidal breakup, otherwise prolonged mass transfer occurs. This substantially extends the duration of strong GW emission by the binary.

Extremely large neutrino and gravitational wave fluxes are generated for a few seconds until the components coalesce ($L_{\nu}^{peak} \sim 10^{56} erg.s^{-1}$; $L_{GW}^{peak} \sim 10^{54}$ erg.s^{-1}). Approximately 2% of the system rest mass is radiated as gravitational waves. Figures 2 and 3 show the time evolution of the neutrino and gravitational wave luminosities, and the GW frequency. Note that the GW signal for these events will be a "chirp" as the signal slides up the spectrum to $\sim$900 Hz and then back down again.

(ii) Black Hole - Neutron Star

Lattimer and Schramm (1976) considered the tidal disruption of a neutron star companion of a black hole as their orbit decayed towards coalescence. Unfortunately for our purposes they failed to estimate the resultant integrated GW flux during this event. However, to a first approximation, this problem is qualitatively the same as for n.s.-n.s. binaries, and thus probably has an efficiency of $\sim$2%, although some GW will be lost down the hole.

(iii) Black Hole - Black Hole

No detailed calculation of the final evolution of a binary

consisting of two black holes has yet been made. This is however a situation that will be rectified when researchers (cf. Eppley 1979 , Smarr 1979, Wilson 1979, these proceedings) in the field successfully complete a full 3-dimensional computer code.

Clark and Eardley (1977) crudely estimated the total efficiency in this case to be $\sim$2% or $\sim$3%, for nonrotating or corotating black holes, respectively. Similarly, Zel'dovich and Novikov (1971) estimated the efficiency to be $\sim$3% for two approximately equal mass black holes. See also Detweiler, this volume.

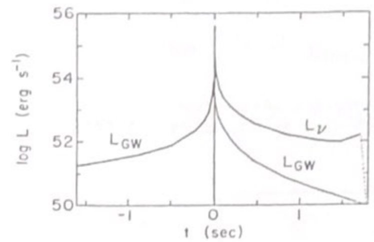

Fig.2

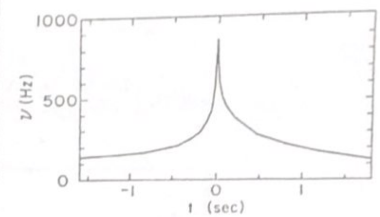

Fig.3

Fig.2 - Time evolution of a system with initial masses 0.8 and 1.3 $M_{\odot}$. Neutrino and gravitational wave luminosities. t=0 is the point of onset of mass stripping.

Fig.3 - Time evolution of a system with initial masses 0.8 and 1.3 $M_{\odot}$. Frequency of gravitational wave.

IV. FORMATION MECHANISMS FOR N.S. - N.S. PAIRS

(i) Binaries

The binary pulsar (PSR 1913+16) is widely assumed to be a pair of neutron stars in orbit about one another. Although the unseen companion is most likely a neutron star this has not yet been unequivocally confirmed, and it cannot be ruled out to be a helium star, white dwarf, or black hole (Smarr and Blandford 1976). The system is expected to evolve to the point of coalescence in $<10^9$ years due to GW losses, from its current separation of $\sim$1 $R_{\odot}$, and period of 7.75 hours (cf. Wagoner 1976).

Progenitors of n.s.-n.s. binaries (or other compact object binaries) are probably massive X-ray binaries (i.e. Cyg X-1, SMC X-1, Cen X-3), which originate from close binaries with primaries of mass $\gtrsim$ 15 $M_{\odot}$ (Flannery & van den Heuvel 1975, Smarr and Blandford 1976).

It is also possible that n.s.-n.s. binaries may form in dense globular clusters or galactic nuclei by exchange of a companion from a binary during the close passage of a third body.

(ii) Supernova Fission

O and B main sequence stars, which are the probable progenitors of Type II supernovae (SN) are observed to have $\sim 10^{51}$ erg.s of specific angular momentum. Since the collapsing core of such a SN will fission or fragment if its angular momentum is greater than $J_{fiss} \sim 10^{49}$ erg.s, it is clear that a n.s. - n.s. pair (or group) will form unless a great deal of angular momentum is lost prior to collapse (cf. Wiita & Press 1976). The subsequent evolution of the system in this case is extremely similar to the neutron star binary case, except that the stars will probably still be hot, and collapsing, whereas in the former case (Clark and Eardley) they were assumed to be cold, and static.

A considerable body of observational evidence indicates however that neutron stars may be born slowly rotating and thus that SN fission events are rare (Kazanas and Schramm 1977, Lamb, Lamb, and Arnett 1975, Greenstein et al. 1977). This matter is still, however, very far from a closed book.

V. N.S. - N.S. COALESCENCE EVENT RATES

(i) SN Fission

Only those SN whose angular momentum exceeds J_{fiss} will form n.s. - n.s. pairs. Denoting the fraction of SN that doesn't fission by β_1, we see that a fraction $\beta_2 = 1-\beta_1$ will fission. Taking the SN event rate given in Clark, van den Heuvel & Sutantyo (1978) [see Arnett (1979, these proceedings) for discussion; also see Figure 4], we see that

$$r_{SN}(J \gtrsim J_{Fiss}) \simeq 0.1\ \beta_2 \mathrm{yr}^{-1} \qquad d \lesssim 10 \text{ Kpc} \tag{6a}$$

$$\simeq \beta_2 \left(\frac{d}{10\mathrm{Mpc}}\right)^3 \left(\frac{H_o}{100\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}}\right)^3 \mathrm{yr}^{-1} \qquad d \gtrsim 10 \text{ Mpc} \tag{6b}$$

(ii) Binaries

Clark et al. estimated the event rate of original binary coalescence events in a number of independent ways. First, the existence of the binary pulsar as the only known binary out of over 300 pulsars indicates that the most probable value of the ratio of the binary pulsar formation rate (r_{bp}) to the pulsar formation rate, R_{prob} is $3\text{x}10^{-3}$. The true value of R is poorly known due to small number statistics (one object!), and $-3.5 \leq \log R \leq -1.9$ at the 90% confidence level.

Clark et al. estimate that massive X-ray binaries have a 10% probability of remaining bound after the final SN explosion if $\log R_{prob} = -2.5$. This value is consistent with the survival probability derived by Sutantyo (1978) on the assumption that SN explosions are slightly asymmetric.

Approximating the pulsar birthrate by the SN rate (we shall thus be counting binaries containing black holes as well as neutron stars) Clark et al. estimate the event rate of binary coalescences to be

$$r_{bp} \simeq 2.9\text{x}10^{-4}(315R)\ \mathrm{yr}^{-1} \qquad d \lesssim 10 \text{ kpc} \tag{7a}$$

$$\simeq 3.2\text{x}10^{-3}(315R)\left(\frac{H_o}{100\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}}\right)^3 \left(\frac{d}{10\mathrm{Mpc}}\right)^3 \mathrm{yr}^{-1} \quad d \gtrsim 10\mathrm{Mpc} \tag{7b}$$

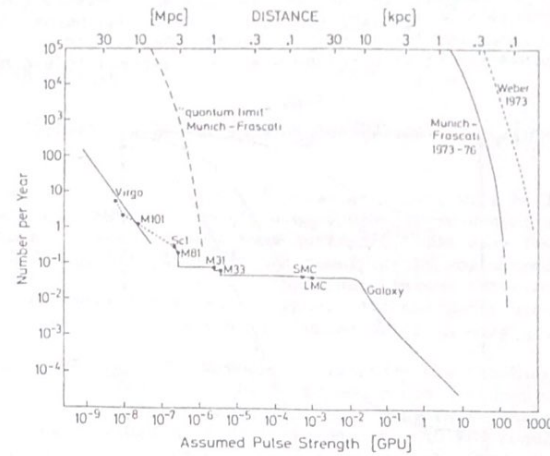

Fig.4 - Supernovae Event Rate (Kafka & Schnupp,1978)

VI. COMPARATIVE IMPORTANCE OF N.S. - N.S. COALESCENCES TO SN COLLAPSES

(i) Detectability

Recent calculations (Chia, Chau and Henriksen 1977, Shapiro 1977, Saenz and Shapiro 1978) indicate that peak gravitational wave efficiencies (η_{SN}) during core collapse probably do not exceed $\sim 0.1\%$.

Shapiro (1979, these proceedings) found recently that the effiency of homogeneous ellipsoidal SN cores was ~1%, independent of core angular momentum J, if the cores bounced ~5 times after the initial infall. The efficiency was high because the eccentricity grew on successive bounces. It is not yet possible to say if this important "eccentricity growth" effect occurs in real SN. If it does not, then the earlier calculations of Saenz and Shapiro (1978) for single-bounce collapse indicate that η_{SN} is a strong function of J. For collapse with a cold equation of state

$$\eta_{SN} \simeq 10^{-3}\ (J/J_{Fiss})^{3.5} \quad \text{for } J \lesssim J_{Fiss} \tag{8}$$

Consider two separate hypotheses. First, that the "eccentricity growth" effect occurs. And, second, that it does not. Then, taking γ_2 to be the fraction of SN with GW efficiencies of ~1%, and $\gamma_1 = 1 - \gamma_2$, we see that $\gamma_1 = 0$, $\gamma_2 = 1$ under the first hypothesis, and that $\gamma_1 = \beta_1$, $\gamma_2 = \beta_2$ under the second hypothesis.

Neutron star - neutron star coalescences have efficiencies (η_{bp}) of the order of 2% , the same order as SN efficiencies if the "growth" effect operates, but otherwise, substantially larger.

Now let us compare the detection rate of GW events due to SN, and binary coalescences, by a detector of sensitivity S_o GPU_{-10} (10^{-10}GPU). In addition let us require that the detector be sufficiently sensitive to detect more than one event a year - so our coverage will extend out well past our galaxy, to where the number of sources varies directly with the volume observed. Then, the ratio of detected events due to binary coalescences to all detected events is (cf. Clark et al. for details)

$$Q = \frac{dN_{bp}}{dt} \Big/ \frac{dN_{tot}}{dt}$$
$$= (315R)/[(315R)+0.366\ \gamma_1\ (J_o/J_{Fiss})^{5.25} + 129\ \gamma_2\ (\eta_{SN}^{Fiss}/10^{-2})^{1.50}] \tag{9}$$

where J_o is a weighted mean specific angular momentum of all SN rotating slower than fission, and η_{SN}^{Fiss} is the efficiency of SN which fission and then suffer n.s. - n.s. coalescence.

The total detected event rate dN_{tot}/dt is

$$(51\ \text{events.yr}^{-1})S_o^{-1.50}\ \left(\frac{H_o}{100\text{kms}^{-1}\text{Mpc}^{-1}}\right)\ [(315R) + 0.366\ \gamma_1\ (J_o/J_{Fiss})^{5.25} + 129\ \gamma_2(\eta_{SN}^{Fiss}/10^{-2})^{1.50}] \tag{10}$$

Consider two extreme cases: Case I, where all SN collapses occur near or above breakup angular momentum, i.e. $(J_o/J_{Fiss}) \sim 1$, and/or the "eccentricity growth" effect occurs. The $Q \sim 0.01$, i.e. only ~1% of all events will be due to binaries (R=1/315).

Case II, where no SN collapses occur near breakup, i.e. $(J_o/J_{Fiss}) << 1$, and the "eccentricity growth" effect is not present. Then $Q \sim 1.0$, i.e. 100% of all detected events are due to binaries.

Evidence pointing to the slow rotation of SN cores, thus indicates that the distinguishing criterion between the two cases above will be the existence or lack of "eccentricity growth".

Cases between these extremes may be anticipated if the "eccentricity growth" effect only occurs in some SN. A knowledge of the distribution of angular momenta of the other SN would be required in order to determine Q, whose dependence on γ_2 is shown in Figure 5.

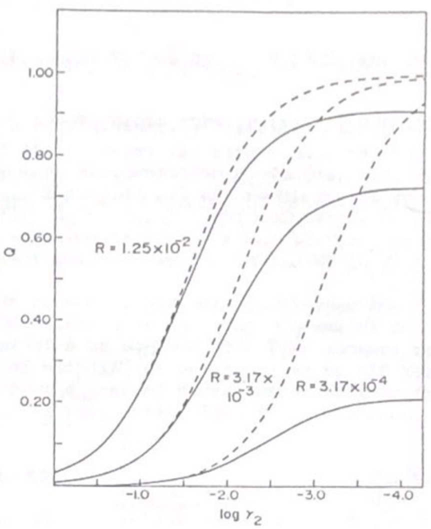

Fig. 5 - Q, the ratio of GW events due to binary pulsars to all GW events as a function of γ_2, the fraction of SN that have $\eta_{SN} \sim .01$, for (i) the most probable value of R (1/315), and (ii) 90% confidence limits for R (1/80; 1/3150). Solid lines represent log $(J_o/J_{Fiss})=0.0$, while dashed lines are for log $(J_o/J_{Fiss})=-1.0$.

A number of important conclusions can be drawn from this. In order to detect at least one event a year, a detector sensitivity of $\sim 10^{-7}$-10^{-8}GPU will be necessary, even if all SN have GW efficiencies of $\sim$1%. On the other hand, should SN be very inefficient generators of GW, one event a year will be detected at $\sim 10^{-9}$-10^{-10} GPU, but these bursts will be due to coalescence of binaries. The latter case corresponds to a dimensionless amplitude $h \sim 10^{-22}$ (cf. Figure 6).

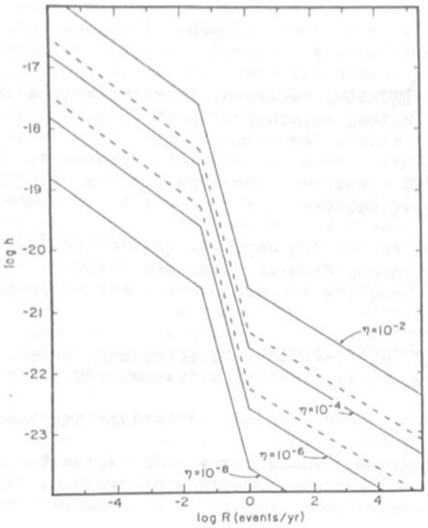

Fig.6 - Comparison of dimensionless amplitude h as a function of event rate. Solid lines are for SN collapses with efficiencies of 10^{-2}- 10^{-8}. Dashed lines are upper and lower estimates (99% confidence) for compact binary destruction events.

(ii) Frequency Spectrum

Clark and Eardley found that n.s. - n.s. coalescences generate waves with a peak frequency of $\lesssim$900 Hz.Clark et.al.showed that n.s.-n.s. pairs probably do not generate substantial amounts of gravitational waves above 1 kHz as mass transfer will tend to keep the bodies at least at the tidal limit of the body being stripped.

By contrast collapse of SN are expected to generate waves with a spectral peak of a few kHz (Thorne 1978). Saenz and Shapiro

however find that for $J \sim J_{Fiss}$ the bulk of the energy is emitted below 1 kHz, although for more slowly rotating SN they find that the bulk of the energy is emitted at high frequencies. Since their model does not adequately describe conditions as $J \rightarrow J_{Fiss}$, this result should be regarded as tentative.

The expectation that SN will be high frequency sources has led observers to concentrate on developing detectors whose optimum performance is above 1 kHz, such as silicon and sapphire crystal detectors which operate best around $\sim$10 kHz.

If we assume that non-fissioning SN generate high frequency waves ($\gtrsim$1 kHz), whereas fissioning SN, and neutron star binaries generate low frequency waves ($\lesssim$1 kHz) we shall if anything understate the case in favor of low frequency waves (recalling that Saenz and Shapiro found non-fissioning SN generating low frequency waves). Then the fraction of detected events which are high frequency out of all events will be

$$dN_{SN}(J \lesssim J_{Fiss})/dt / \frac{dN_{tot}}{dt} = 0.366\ \beta_1 (J_o/J_{Fiss})^{5.25}$$

$$/[(315R) = 0.366\ \gamma_1 (J_o/J_{Fiss})^{5.25} + 129\ \gamma_2 (\eta_{SN}/10^{-2})^{1.50}] \quad (11)$$

Clearly a vast majority of detected events will be low frequency ($\lesssim$1 kHz) unless: (i) binary neutron star systems are substantially less frequent than estimated, (ii) very few SN have $J \gtrsim J_{Fiss}$, but (iii) all the remaining SN are rotating close to fission (i.e. $J_o \sim J_{Fiss}$). The simulataneous satisfaction of these three requirements seems rather unlikely particularly since one would expect quite a few fissioning SN if $J_o \sim J_{Fiss}$.

The above reasoning strongly challenges the thinking of the last decade, and indicates that most GW events detected will have their peak emission below 1000 Hz. This appears to indicate that the strategy of building detectors tuned to $\sim$10 kHz requires rethinking unless the ease of construction at that frequency more than offsets the lower energy flux to be expected.

VII. CONCLUSIONS

Under the most optimistic assumptions a detector sensitive to $\sim 10^{-9}$ GPU or a dimensionless amplitude of $\sim 10^{-22}$ will be necessary to observe at least one compact binary event a year. This will, however, be detected in preference to SN events, if the SN efficiency is less than $\sim 10^{-5}$.

In contrast to our understanding of sources of gravitational radiation above $\sim$100 Hz as recently as two years ago (cf.Thorne

1978), current results indicate that compact binary objects may well be the most promising sources of gravitational waves, rather than SN collapses.

Furthermore, it appears that a majority of events will have peak frequencies below 1000 Hz, raising important questions about the design and implementation of the next generation of detectors. It is recommended that future discussions of periodic gravitational wave sources provide the density of such systems in dimensionless amplitude-frequency space, $d^2N/d(\log h)d(\log \nu)$.

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