# Beamsplitters (Haus)

# 1. Reciprocity principle

Figure ?? shows all phases of HHO, which are described in the next subsections.

# 1.1. Exploration phase

In HHO, the Harris' hawks perch randomly on some locations and wait to detect a prey based on two strategies.

$$X(t+1) = \begin{cases} X_{rand}(t) - r_1 |X_{rand}(t) - 2r_2 X(t)| & q \ge 0.5\\ (X_{rabbit}(t) - X_m(t)) - r_3 (LB + r_4 (UB - LB)) & q < 0.5 \end{cases}$$
(1)

where X(t+1) is the position vector of hawks in the next iteration t,  $X_{rabbit}(t)$  is the position of rabbit, X(t) is the current position vector of hawks,  $r_1$ ,  $r_2$ ,  $r_3$ ,  $r_4$ , and q are random numbers inside (0,1), which are updated in each iteration, LB and UB show the upper and lower bounds of variables,  $X_{rand}(t)$  is a randomly selected hawk from the current population, and  $X_m$  is the average position of the current population of hawks. The average position of hawks is attained using Eq. (2):

$$X_m(t) = \frac{1}{N} \sum_{i=1}^{N} X_i(t)$$
 (2)

where  $X_i(t)$  indicates the location of each hawk in iteration t and N denotes the total number of hawks.

# 1.2. Transition from exploration to exploitation

To model this step, the energy of a rabbit is modeled as:

$$E = 2E_0(1 - \frac{t}{T})\tag{3}$$

where E indicates the escaping energy of the prey, T is the maximum number of iterations, and  $E_0$  is the initial state of its energy. The time-dependent behavior of E is also demonstrated in Fig. ??.

## 1.3. Exploitation phase

## 1.3.1. Soft besiege

This behavior is modeled by the following rules:

$$X(t+1) = \Delta X(t) - E \left| J X_{rabbit}(t) - X(t) \right| \tag{4}$$

$$\Delta X(t) = X_{rabbit}(t) - X(t) \tag{5}$$

where  $\Delta X(t)$  is the difference between the position vector of the rabbit and the current location in iteration t,  $r_5$  is a random number inside (0,1), and  $J=2(1-r_5)$  represents the random jump strength of the rabbit throughout the escaping procedure. The J value changes randomly in each iteration to simulate the nature of rabbit motions.

#### 1.3.2. Hard besiege

In this situation, the current positions are updated using Eq. (6):

$$X(t+1) = X_{rabbit}(t) - E\left|\Delta X(t)\right| \tag{6}$$

A simple example of this step with one hawk is depicted in Fig. ??.

## 1.3.3. Soft besiege with progressive rapid dives

To perform a soft besiege, we supposed that the hawks can evaluate (decide) their next move based on the following rule in Eq. (7):

$$Y = X_{rabbit}(t) - E \left| JX_{rabbit}(t) - X(t) \right| \tag{7}$$

We supposed that they will dive based on the LF-based patterns using the following rule:

$$Z = Y + S \times LF(D) \tag{8}$$

where D is the dimension of problem and S is a random vector by size  $1 \times D$  and LF is the levy flight function, which is calculated using Eq. (9):

$$LF(x) = 0.01 \times \frac{u \times \sigma}{|v|^{\frac{1}{\beta}}}, \sigma = \left(\frac{\Gamma(1+\beta) \times \sin(\frac{\pi\beta}{2})}{\Gamma(\frac{1+\beta}{2}) \times \beta \times 2^{(\frac{\beta-1}{2})})}\right)^{\frac{1}{\beta}}$$
(9)

where u, v are random values inside  $(0,1), \beta$  is a default constant set to 1.5.

Hence, the final strategy for updating the positions of hawks in the soft besiege phase can be performed by Eq. (10):

$$X(t+1) = \begin{cases} Y & if F(Y) < F(X(t)) \\ Z & if F(Z) < F(X(t)) \end{cases}$$

$$\tag{10}$$

where Y and Z are obtained using Eqs.(7) and (8).

A simple illustration of this step for one hawk is demonstrated in Fig. ??.

# 1.3.4. Hard besiege with progressive rapid dives

The following rule is performed in hard besiege condition:

$$X(t+1) = \begin{cases} Y & if F(Y) < F(X(t)) \\ Z & if F(Z) < F(X(t)) \end{cases}$$

$$\tag{11}$$

where Y and Z are obtained using new rules in Eqs. (12) and (13).

$$Y = X_{rabbit}(t) - E \left| J X_{rabbit}(t) - X_m(t) \right| \tag{12}$$

$$Z = Y + S \times LF(D) \tag{13}$$

where  $X_m(t)$  is obtained using Eq. (2).

A simple example of this step is demonstrated in Fig. ??.

## 1.4. Pseudocode of HHO

The pseudocode of the proposed HHO algorithm is reported in Algorithm 1.

## References

Harris Hawks Optimization: Algorithm and Applications, Ali Asghar Heidari and Seyedali Mirjalili and Hossam Faris and Ibrahim Aljarah and Majdi Mafarja and Huiling Chen, Future Generation Computer Systems, 2019.

```
Algorithm 1 Pseudo-code of HHO algorithm
```

Return  $X_{rabbit}$ 

```
Inputs: The population size N and maximum number of iterations T
Outputs: The location of rabbit and its fitness value
Initialize the random population X_i (i = 1, 2, ..., N)
while (stopping condition is not met) do
   Calculate the fitness values of hawks
   Set X_{rabbit} as the location of rabbit (best location)
   for (each hawk (X_i)) do
       Update the initial energy E_0 and jump strength J
                                                                        \triangleright E<sub>0</sub>=2rand()-1, J=2(1-rand())
       Update the E using Eq. (3)
       if (|E| > 1) then
                                                                                     ▷ Exploration phase
          Update the location vector using Eq. (1)
                                                                                    ▷ Exploitation phase
       if (|E| < 1) then
                                                                                              ▷ Soft besiege
          if (r \ge 0.5 \text{ and } |E| \ge 0.5) then
              Update the location vector using Eq. (4)
          else if (r \ge 0.5 \text{ and } |E| < 0.5) then
                                                                                             ▶ Hard besiege
              Update the location vector using Eq. (6)
          else if (r < 0.5 \text{ and } |E| \ge 0.5) then
                                                                ▷ Soft besiege with progressive rapid dives
              Update the location vector using Eq. (10)
          else if (r < 0.5 \text{ and } |E| < 0.5) then
                                                               ▶ Hard besiege with progressive rapid dives
              Update the location vector using Eq. (11)
```