Applied Machine Learning

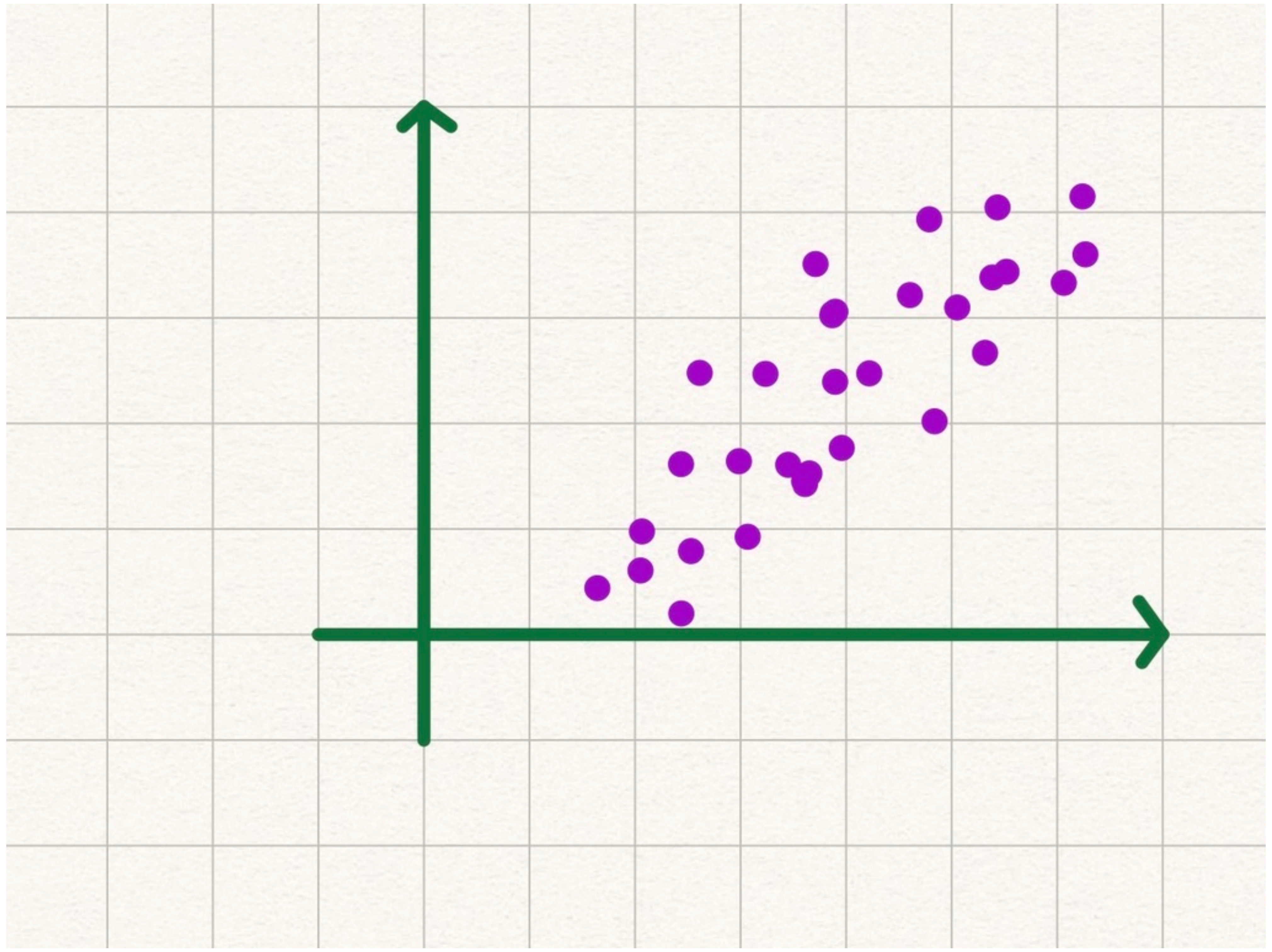
Data Transformations

Data Transformations

- Translating data
- Rotating data
- Eigenvectors and Eigenvalues

Transformations

- Blobs
  - Mean
  - Covariance



Transformations

- Translations and rotations
  - Source dataset: $\{\mathbf{x}\}$
  - Target dataset: $\{\mathbf{m}\}$
  - $\mathbf{m}_i = A\mathbf{x}_i + \mathbf{b}$
  - Rotation matrix: $A_{\{k\times d\}}$
  - Translation matrix: $\mathbf{b}_{\{d\times 1\}}$

- $$\begin{aligned} \text{mean}(\{\mathbf{m}\}) &= \text{mean}(\{A\mathbf{x}+\mathbf{b}\}) \\ &= A\cdot\text{mean}(\{\mathbf{x}\}) + \mathbf{b} \end{aligned}$$

- $$\begin{aligned} \text{Covmat}\{\mathbf{m}\} &= \frac{\sum_i (\mathbf{m}_i - mean(\{\mathbf{m}\}))(\mathbf{m}_i - mean(\{\mathbf{m}\}))^\top}{N} \\ &= \frac{\sum_i (A\cdot\mathbf{x}_i + \mathbf{b} - A\cdot\text{mean}(\{\mathbf{x}\}) - \mathbf{b})(A\cdot\mathbf{x}_i + \mathbf{b} - A\cdot\text{mean}(\{\mathbf{x}\}) - \mathbf{b})^\top}{N} \\ &= \frac{\sum_i A\cdot(\mathbf{x}_i - \text{mean}(\{\mathbf{x}\}))(A\cdot(\mathbf{x}_i - \text{mean}(\{\mathbf{x}\})))^\top}{N} \\ &= \frac{A\left[\sum_i (\mathbf{x}_i - \text{mean}(\{\mathbf{x}\}))(\mathbf{x}_i - \text{mean}(\{\mathbf{x}\}))^\top\right]A^\top}{N} \\ &= \; = A\text{Covmat}(\{\mathbf{x}\})A^\top \end{aligned}$$

Eigenvectors and Eigenvalues

- Eigenvector $\mathbf{u}$ and Eigenvalue λ of matrix S
  - $S\mathbf{u} = \lambda\mathbf{u}$
- Consider a symmetric $S_{\{d\times d\}} = S^{\top}_{\{d\times d\}}$ and $\mathbf{u}_{\{d\times 1\}}$
  - d distinct $(\mathbf{u}_i, \lambda_i)$
  - orthogonal: $\mathbf{u}_i \perp \mathbf{u}_j$ for $i \neq j$, can be with $\|\mathbf{u}\| = 1$
  - orthonormal matrix $U = \left[\mathbf{u}_1, \ldots, \mathbf{u}_d\right]$, thus $U^{\top}U = I$
- Construct diagonal matrix with eigenvalues $\Lambda_{\{d\times d\}}$
  - $\Lambda_{i,j} = \lambda_i$
  - $(\mathbf{u}_i, \lambda_i)$ in U and Λ: arrange λ_i's in Λ in descending order

- Eigenvectors U and eigenvalues Λ of S
  - $SU = U\Lambda$
- Diagonalization
  - $U^{\top}SU = \Lambda$

Transforming the dataset

- Source dataset: $\{\mathbf{x}\}$
- After translation: $\{\mathbf{m}\}$
  - $\mathbf{m}_i = \mathbf{x}_i - mean(\{\mathbf{x}\})$
  - $mean(\{\mathbf{m}\}) = 0$
- And, after rotation: $\{\mathbf{r}\}$
  - $\mathbf{r}_i = A\mathbf{m}_i = A(\mathbf{x}_i - \text{mean}(\{\mathbf{x}\}))$
  - $\text{Covmat}\{\mathbf{r}\} = A\text{Covmat}(\{\mathbf{x}\})A^\top = A\Sigma A^\top$
  - $\text{Covmat}\{\mathbf{r}\} = U^\top \Sigma U = \Lambda$
  - $\mathbf{r}_i = U^\top \mathbf{m}_i = U^\top(\mathbf{x}_i - \text{mean}(\{x\}))$

- Eigenvectors U and eigenvalues Λ of S
  - $SU = U\Lambda$
- Diagonalization
  - $U^\top SU = \Lambda$
- $A = U^\top$
- $\text{Covmat}\{\mathbf{x}\} = \Sigma = S$

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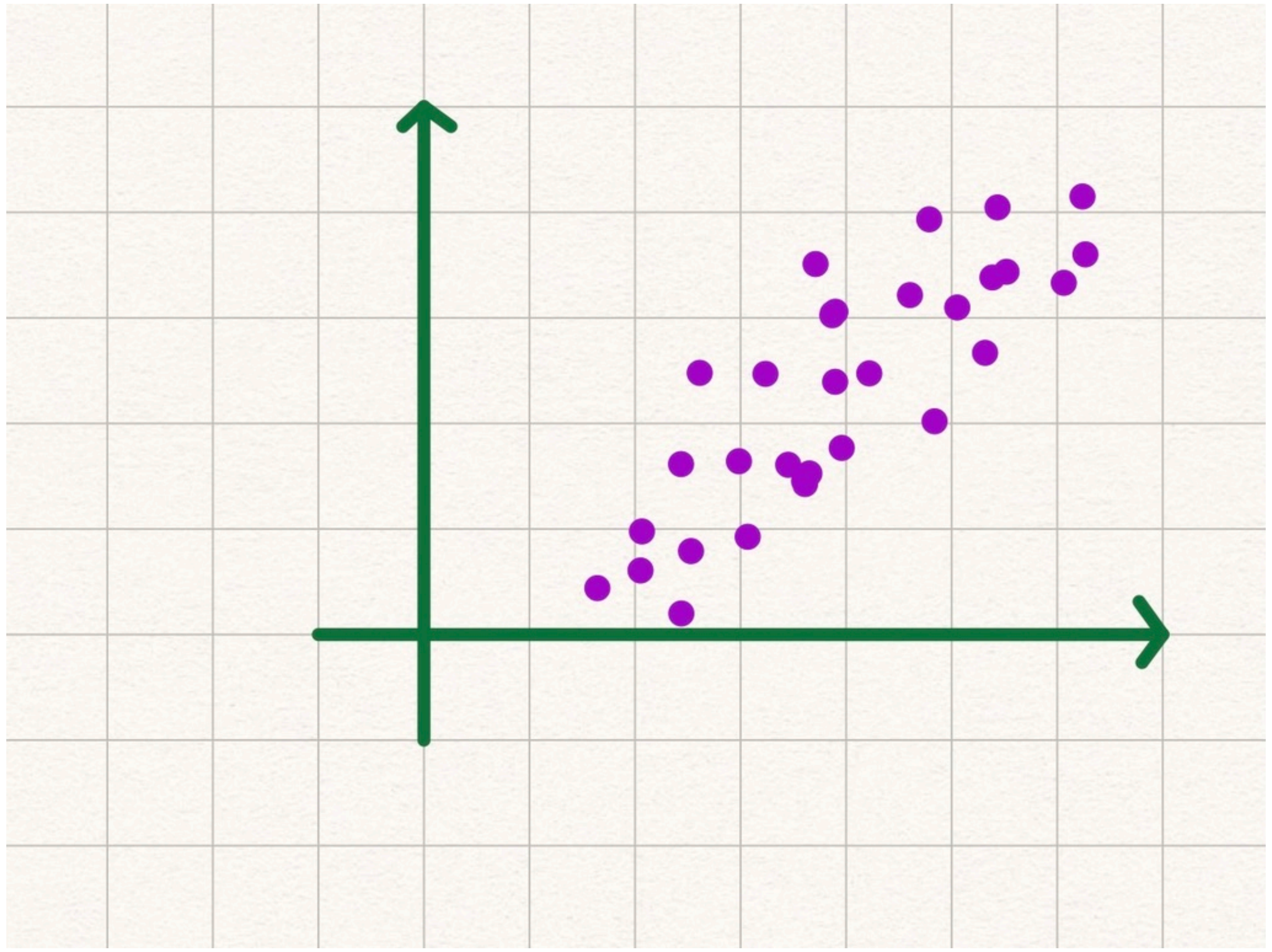

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