Algorithm for calculating imprecise signatures

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1 Notation

- number of components N
- number of types K
- number of components of type k in the system M_k
- state space $\Omega = \{0,1\}^N$
- Type State mapping : $L : \Omega \to \mathbb{N}^K$; maps the state to number of functioning components of each type
- Survivors $\mathbb{S} : \{0,1\} \times \mathbb{N}^K \to \mathbb{N}$, (number of unique states)

$$\mathbb{S}_i(l) := \#\{x \in \Omega : \varphi(x) = i \wedge L(x) = l\} \tag{1}$$

- The Survival signature

$$\Phi(l) := \frac{\mathbb{S}_1(l)}{\mathbb{S}_0(l) + \mathbb{S}_1(l)}, \tag{2}$$

2 the algorithm

2.1 Overall scheme

We will branch. At each step, the position in the branching scheme is tracked by two arrays:

- 'ones': a set which tracks which components are certainly functional,
- 'zeros': a set which tracks which components are certainly failed.

The accumulated 'survivors' (counts) will be held in structures 'sig0' and 'sig1', and converge to:

$$\text{sig0} \to \mathbb{S}_0, \quad \text{sig1} \to \mathbb{S}_1.$$

For each branch, we:

1. Find a minimal path, $\mathbb{MP}$, in the subgraph G, the original RBD, without the nodes in 'zeros'.
2. Account for all the states in Ω which will result in certain functionality of the system, those for which 'ones' $\cup\, \mathbb{MP}$ are functional. I.e. increase the counters held in 'sig1' (sec. 2.2).
3. Check, for each $p \in \mathbb{MP} -$ 'ones', whether $G -$ 'zeros' $- p$ can be functional (if the subgraph is connected). If not, increase the counter of 'sig0' (sec. 2.2).
4. Update the 'ones' with all the $p \in \mathbb{MP}$, which result into a cut set (so that we won't waste resources again in later branches).
5. For each $p \in \mathbb{MP}$, which did not result into a cut set and are not in 'ones', create a new branch, s.t. for arbitrary fixed indexing(ordering) i of (all) $p \in \mathbb{MP}$:
   - 'ones'$_i$ = 'ones' $\cup \bigcup_{j<i} p_j$,
   - 'zeros'$_i$ = 'zeros' $\cup\, p_i$.
6. Iterate for newly created branches.

Proposition 1 *Found minimal path (step 1) is an unique element of the RBD's minimal path set.*

Proposition 2 *If $G - zeros - p$ is not connected (step 3), then $zeros \cup p$ is a cut set (but not necessarily minimal). Additional computation would be needed if we would need the* **minimal** *cut set.*

Proposition 3 *The decomposition according to the found minimal path (step 5) is total and disjoint. It will decompose the space like '0xxxxx', '10xxxx', '110xxxx',..., where x denotes 'arbitrary'.*

Proposition 4 *Similar branching could be done (not tested) by finding the minimal cut instead of the minimal path. This would result into a direct construction of the minimal cut set.*

2.2 Increase counters

(This is the same routine that Sean Reed uses in his BDD paper.)

- We have the 'ones' and 'zeros' sets, which represent components for which the state is already certain.
- Let $M'(k) = M(k) -$ amount of components of type k in 'ones' and 'zeros' vectors.
  I.e. how many components of type k can still obtain arbitrary state at the current position in the branching.
- For each vector $0 \leq x \leq M'$:
  1. $y := L(\text{'ones'}) + x$ ... a bit cryptic, but this simply computes the l vector s.t. all components in 'ones' are functional + increased by x_k in each k.
  2. 'sig0/1'$(y)+ = \prod_k^K \binom{M'(k)}{x(k)}$. Which adds how many states there are for which 'ones' are functional, so are another x, and 'zeros' are not.

3 Output

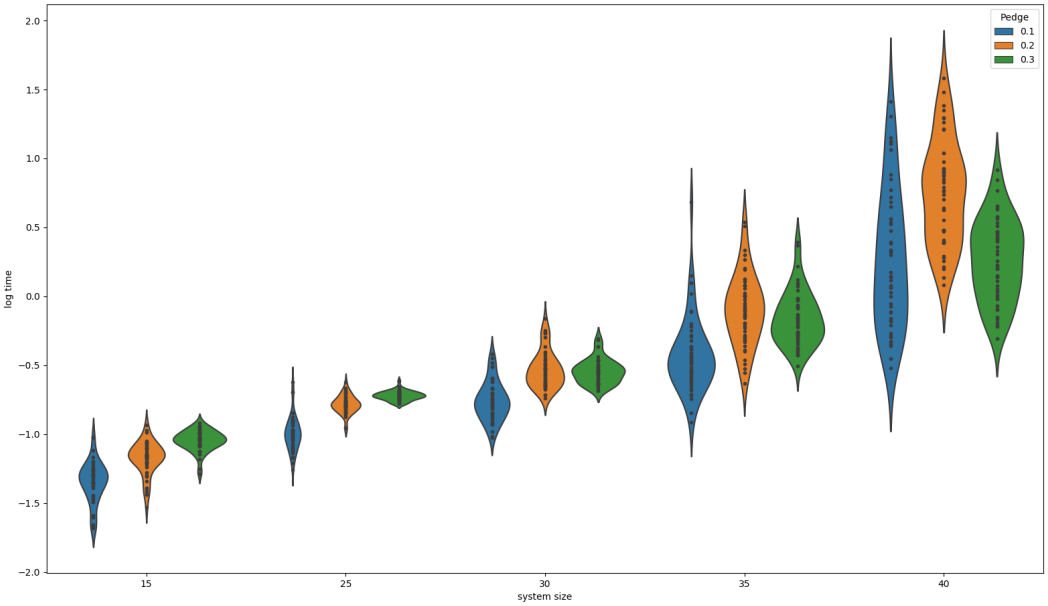
Whereever we stop in the branching process,

$$\frac{\text{`}sig1'(l)}{\prod_k \binom{M(k)}{l(k)}} \leq \Phi(l) \leq \frac{\prod_k \binom{M(k)}{l(k)} - \text{`}sig0'(l)}{\prod_k \binom{M(k)}{l(k)}},$$

with equalities, if we let the algorithm finish.

4 Test

Just to test, how fast it runs. Pedge corresponds to the random graph generator, where each edge is added with probability Pedge.


Pedge
0.1
0.2
0.3
log time
system size
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
15
25
30
35
40