UNIVERSITY
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IN4310 Probability and Random Variables

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Axioms for Events and Axioms of Probability [1, 2]

Sample space Ω the set of all sample points for a given experiment

Events are subsets of the sample space

A^c Complement of an event A is the set of points in Ω but not A

Axioms for events

Ω is an event

For every sequence of events $A_1, A_2, \cdots$, the $\bigcup_{i=1}^{\infty} A_i$ is an event

For every event A, the complement A^c is an event

Axiom of Probability: a probability rule $\Pr$ is a function mapping each event to a real number so that

$\Pr(\Omega) = 1$

For every event A, $\Pr(A) \geq 0$

For disjoint events $A_1, A_2, \cdots$, $\Pr\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{n=1}^{\infty} \Pr(A_i)$

Properties of Probability Rule $\Pr$

Range $0 \le \Pr(A) \le 1$ for any event A

Complement $\Pr(A^c) = 1 - \Pr(A)$

Events A and B are independent if $\Pr(A \cap B) = \Pr(A) \Pr(B)$

Events A and B are mutually exclusive if $\Pr(A \cap B) = 0$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

Conditional probability: probability of A given B under $\Pr(B) > 0$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\Pr(A) = \Pr(A \cap B) + \Pr(A \cap B^c) = \Pr(A|B)\Pr(B) + \Pr(A|B^c)\Pr(B^c)$$

Suppose $\bigcup_{i=1}^{\infty} B_i = \Omega$ where B_i's are mutually exclusive

$$\Pr(A) = \sum_{i=1}^{\infty} \Pr(A \cap B_i) = \sum_{i=1}^{\infty} \Pr(A|B_i)\Pr(B_i)$$

Bayes' rule: Given $\Pr(A|B_i)$, we obtain $\Pr(B_i|A)$

$$\Pr(B_i|A) = \frac{\Pr(A|B_i)\Pr(B_i)}{\Pr(A)} = \frac{\Pr(A|B_i)\Pr(B_i)}{\sum_{j=1}^{\infty} \Pr(A|B_j)\Pr(B_j)}$$