Solution to exercises - Modelling 2

Electricity production management. Let us denote by

- $\overline{u}_{i,j}$ the maximal capacity of energy production of thermal plant j in region i per day;
- $\overline{v}_i$ the maximal hydro production in region i per day;
- $\overline{x}_i$ and $\underline{x}_i$ the maximal and minimal levels of reservoir i;
- $x_{t,i}$ the reservoir level at the beginning of day t;
- T the number of days of the planning horizon;
- $u_{t,i,j}$ the production of thermal plant j in region i for day t;
- $v_{t,i}$ the hydro production in region i for day t;
- $s_{t,i}$ the energy bought on the spot market for day t and region i;
- p_t the unit spot price for day t;
- $D_{t,i}$ the demand for day t;
- $A_{t,i}$ the inflows for day t and reservoir i;
- $c_{i,j}$ the unit production cost for thermal plant j in region i;
- $\mathcal{I}$ the set of regions and $\mathcal{T}_i$ the set of thermal units of region i;
- $E_{t,i,j}$ the energy exchanged from region i to region j for day t and G the corresponding graph;
- $\overline{E}_{i,j}$ is the maximal energy exchange from i to j.

We obtain the linear program:

$$
\begin{array}{l}
\min \displaystyle\sum_{t=1}^{T} \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{T}_i} c_{i,j} u_{t,i,j} + \sum_{t=1}^{T} \sum_{i\in\mathcal{I}} s_{t,i} p_t \\
x_{t,i} = x_{t-1,i} + 0.8A_{t,i} - v_{t,i},\ t = 2,\dots,T+1, i \in \mathcal{I}, \\
0.2A_{t,i} + \displaystyle\sum_{j\in\mathcal{T}_i} u_{t,i,j} + v_{t,i} + s_{t,i} + \sum_{j|(j,i)\in G} E_{t,j,i} - \sum_{j|(i,j)\in G} E_{t,i,j} \geq \mathcal{D}_{t,i}, \\
s_{t,i} \geq 0,\ t = 1,\dots,T, i \in \mathcal{I}, \\
0 \leq u_{t,i,j} \leq \overline{u}_{i,j},\ t = 1,\dots,T, i \in \mathcal{I}, \\
0 \leq v_{t,i} \leq \overline{v}_i,\ t = 1,\dots,T, i \in \mathcal{I}, \\
0 \leq E_{t,i,j} \leq \overline{E}_{i,j},\ t = 1,\dots,T, (i,j)|(i,j) \in G, \\
\underline{x}_i \leq x_{t,i} \leq \overline{x}_i,\ t = 1,\dots,T, i \in \mathcal{I},
\end{array}
$$

for $x_{1,i}$, $i \in \mathcal{I}$ given.

Asset management.

Denote by x_i the money invested in asset i $i = 1,\dots,n$. The model is

$$
\begin{array}{ll}
\max & \displaystyle\sum_{i=1}^{n} r_i x_i \\
\text{s.t} & \displaystyle\sum_{i=1}^{n} x_i \leq M \\
 & x_i \geq 0,\ i = 1,\dots,n.
\end{array}
$$

The optimal solution is given by $x_i = M$ where $r_i = \max\{r_j, j = 1,\dots,n\}$.

Exercise: production expansion.

Let us denote by:

(1) d_i: demand of product i with $i = 1, \ldots, n$;
(2) h_j and e_j: respectively, the number of regular and extra hours of usage of the machine j with $j = 1, \ldots, m$;
(3) s_i: quantity of product i unsold;
(4) $h_{i,j}$: number of hours machine j is used to produce product i;
(5) x_i: quantity of product i produced.

The linear program is

$$
\begin{aligned}
&\min \sum_{i=1}^{n}\sum_{j=1}^{m} g_{i,j}h_{i,j} + \sum_{j=1}^{m} c_j e_j + \sum_{i=1}^{n} p_i s_i \\
&h_{i,j}, e_j, s_i \geq 0, \forall\, i = 1, \ldots, n, j = 1, \ldots, m, \\
&\sum_{j=1}^{m} t_j(e_j + h_j) \leq T, \\
&x_i + s_i = d_i,\ i = 1, \ldots, n, \\
&x_i = \sum_{j=1}^{m} h_{i,j}a_{i,j},\ i = 1, \ldots, n, \\
&h_j + e_j \leq u_j,\ j = 1, \ldots, m, \\
&\sum_{i=1}^{n} h_{i,j} = h_j + e_j,\ j = 1, \ldots, m.
\end{aligned}
$$

LNG with cancellation options. Denote

- s_P^t the pipeline storage at the beginning of time t;
- S_t the spot price for month t;
- Q_t the quantity of contracted gas for t, one part of it, $u_P^{0,t}$, being injected in the pipeline and the rest $u_S^{1,t}$ being injected in the storage;
- u_n^t: quantity of gas from ship of contract n to storage at t;
- $u_n^{0,t}$: quantity of gas from ship of contract n to pipeline at t;
- u_P^t: quantity of gas transferred from the pipeline to clients at t;
- u_{NS}^t: quantity of unsatisfied demand at t;
- c_t: unit storage cost at t;
- p_t: cost of unsastisfied demand;
- s_n^t: stock in ship of contract n at $t \geq t_n - 1$;
- $\mathcal{D}_t$: demand at t,
- t_n: time ship of contract n will arrive;
- y_n^t is 1 if contract n is cancelled at t or before and 0 otherwise;
- Q_n: load of contract n;
- C_n^t: cancellation cost at t;
- F_n^t: ship load n cost at t;
- $f_{n,t}$: unit price for cancellation fee paid at t on contract n;

The model is

$$
\begin{aligned}
&\min \sum_{t=1}^{T}\Big(c_t(s^t+s_P^t)+p_t u_{NS}^t-1.3S_t u_P^t+\sum_{n=1}^{N}C_n^t+F_n^t\Big)\\
&Q_t=u_P^{0,t}+u_S^{1,t},\forall\, t=1.\dots,T,\\
&\underline{s}_t\le s^t\le \overline{s}_t,\forall\, t=1.\dots,T,\\
&s^t=s^{t-1}+\textstyle\sum_{n=1}^{N}u_n^t+u_s^{1,t}-u_s^{2,t},\forall\, t=1.\dots,T,\\
&\underline{s}_{P,t}\le s_P^t\le \overline{s}_{P,t},\forall\, t=1.\dots,T,\\
&s_P^t=s_P^{t-1}+\textstyle\sum_{n=1}^{N}u_n^{0,t}+u_s^{2,t}+u_P^{0,t}-u_P^t,\forall\, t=1.\dots,T,\\
&u_P^t+u_{NS}^t=\mathcal{D}_t,\ t=1.\dots,T,\\
&y_n^t\in\{0,1\},\\
&y_n^t\ge y_n^{t-1},2\le t\le t_n-1,\\
&s_n^t=(1-y_n^t)Q_n,t=t_n-1,\\
&s_n^t=s_n^{t-1}-u_n^t u_n^{0,t},t\ge t_n,\\
&C_1^t=y_n^1Q_nf_{n,1},\\
&C_n^t=(y_n^t-y_n^{t-1})Q_nf_{n,t},2\le t\le t_n-1\\
&F_n^t=(1-y_n^{t-1}S_tQ_n,\ t=t_n,\\
&F_n^t=c_ts_n^t,\ t\ge t_n+1.
\end{aligned}
$$

Exercise: Problema da mochila

Let us denote, for $i=1,\dots,n$:

- x_i a binary variable that indicates whether Mickey is taking object i (value 1) or not (value 0) $i=1,\dots,n$;
- u_i is the utility of object i;
- p_i the weight of object i;
- v_i volume of object i.

The LPP problem to be considered is

$$
\begin{aligned}
\max \sum_{i=1}^{n} u_i x_i&\\
\text{s.t}\qquad \sum_{i=1}^{n} p_i x_i &\le P\\
x_i\in\{0,1\}\ i&=1,\dots,n.
\end{aligned}
$$

If we add the condition on the volumes the LPP to be consider is

$$
\begin{aligned}
\min \sum_{i=1}^{n} u_i x_i&\\
\text{s.t}\qquad \sum_{i=1}^{n} p_i x_i &\le P\\
\sum_{i=1}^{n} v_i x_i &\le V\\
x_i\in\{0,1\}\ i&=1,\dots,n.
\end{aligned}
$$

“Unit commitment”

Let us denote

- $y_{t,i}$ is 1 if unit i is turned on at t and 0 otherwise;

- $u_{t,i}$: production of unit i at t;
- $\mathcal{D}_t$: demand at t;
- K_i: maximal production of unit i per step.

The model is

$$\begin{array}{l} \min \sum_{t,i} c_i u_{t,i} \\ y_{t,i} \geq y_{t-1,i}, t = 2, \ldots, T, \\ y_{t,i} \in \{0, 1\}, \\ 0 \leq u_{t,i} \leq y_{t,i} K_i, \\ \sum_i u_{t,i} = \mathcal{D}_t, \end{array}$$