Exercises SIMPLEX METHOD

Exercise 1. Consider the problem

$$\begin{aligned} \text{minimize} \;\; & -2x_1 - x_2 \\ \text{subject to} \;\; & x_1 - x_2 < 2 \\ & x_1 + x_2 < 6 \\ & x_1,\; x_2 \geq 0. \end{aligned}$$

(a) Convert the problem into standard form and construct a basic feasible solution at which $(x_1, x_2) = (0, 0)$.
(b) Carry out the full tableau implementation of the simplex method, starting with the basic feasible solution of part (a).
(c) Draw a graphical representation of the problem in terms of the original variables x_1, x_2, and indicate the path taken by the simplex algorithm.

Exercise 2. Using the simplex procedure, solve minimize

$$\begin{aligned} \text{minimize} \; & 2x_1 + 4x_2 + x_3 + x_4 \\ \text{subject to} \; & x_1 + 3x_2 + x_4 \leq 4 \\ & 2x_1 + x_2 \leq 3 \\ & x_2 + 4x_3 + x_4 \leq 3 \\ & x_i \geq 0 \; i = 1, 2, 3, 4. \end{aligned}$$

(a) How much can the elements of $b = (4, 3, 3)$ be changed without changing the optimal basis?
(b) How much ca nth eelements of $c = (2, 4, 1, 1)$ be changed without changing the optimal basis?
(c) What happens to the optimal cost for small changes in b?
(d) What happens to the optimal cost for small changes in c?

Exercise 3. Consider the simplex method applied to a standard form problem and assume that the rows of the matrix A are linearly independent. For each of the statements that follow, give either a proof or a counterexample.

(a) An iteration of the simplex method may move the feasible solution by a positive distance while leaving the cost unchanged.
(b) A variable that has just left the basis cannot renter in the very next iteration.
(c) A variable that has just entered the basis cannot leave in the very next iteration.
(d) If there is a nondegenerate optimal basis, then there exists a unique optimal basis.
(e) If x is an optimal solution found by the simplex method, no more than m of its components can be positive, where m is the number of equality constraints.

Exercise 4. Using the two-phase simplex procedure solve

$$\begin{aligned} \text{minimize} \quad & -3x_1 + x_2 + 3x_3 - x_4 \\ \text{subject to} \quad & x_1 + 2x_2 - x_3 + x_4 = 0 \\ & 2x_1 - 2x_2 + 3x_3 + 3x_4 = 9 \\ & x_1 - x_2 + 2x_3 - x_4 = 6 \\ & x_i \geq 0 \; i = 1, 2, 3, 4. \end{aligned}$$

Exercise 5. The following tableau is an intermediate stage in the solution of a minimization problem:

	y_1	y_2	y_3	y_4	y_5	y_6	y_0
	1	2/3	0	0	4/3	0	4
	0	−7/3	3	1	−2/3	0	2
	0	−2/3	−2	0	2/3	1	2
r^T	0	8/3	−11	0	4/3	0	−8

(a) Determine the next pivot element.
(b) Given that the inverse of the current basis is

$$B^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 1 & -1 \\ 1 & -2 & 2 \\ -1 & 2 & 1 \end{pmatrix}$$

and the corresponding cost coefficients are

$$c_B^T = (c_1, c_4, c_6) = (-1, -3, 1),$$

find the original problem.

Exercise 6. Consider the following LPP

$$\begin{aligned} \text{minimize} \quad & -10x_1 - 12x_2 - 12x_3 \\ & x_1 + 2x_2 + 2x_3 \leq 20 \\ & 2x_1 + x_2 + 2x_3 \leq 20 \\ & 2x_1 + 2x_2 + x_3 \leq 20 \\ & x_i \geq 0 \; i = 1, 2, 3 \end{aligned}$$

(a) Verify that that the inverse of the matrix $B = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ is given by $B^{-1} = \begin{pmatrix} -0.6 & 0.4 & 0.4 \\ 0.4 & -0.6 & 0.4 \\ 0.4 & 0.4 & -0.6 \end{pmatrix}$
(b) Prove that the basic solution that corresponds to the basis B is optimal.
(c) Compute the optimal solution and the optimal objective value.