```
In [1]:
import numpy as np
import pandas as pd
import seaborn as sns
from matplotlib import pyplot as plt
import statsmodels.tools.eval measures as
                                              em
                                      import mean squared error
     sklearn.metrics
     statsmodels.tsa.api
from
                                      import ExponentialSmoothing, SimpleExpSmoothing,
Holt
from
     IPython.display
                                       import display
                                       import rcParams
from
     pylab
```

Q1. Read the data as an appropriate Time Series data and plot the data.

```
In [2]:
df 1 = pd.read csv("Sparkling.csv",parse dates=True,squeeze=True,index col=0)
In [3]:
print(df 1.head())
print(df_1.tail())
YearMonth
1980-01-01
              1686
1980-02-01
              1591
1980-03-01
              2304
1980-04-01
             1712
1980-05-01
             1471
Name: Sparkling, dtype: int64
YearMonth
1995-03-01
             1897
1995-04-01
             1862
1995-05-01
             1670
1995-06-01
             1688
1995-07-01
            2031
Name: Sparkling, dtype: int64
In [4]:
df 2 = pd.read csv("Sparkling.csv")
In [5]:
date = pd.date range(start='1/1/1980', end='1/7/1995', freq='M')
Out[5]:
DatetimeIndex(['1980-01-31', '1980-02-29', '1980-03-31', '1980-04-30',
               '1980-05-31', '1980-06-30', '1980-07-31', '1980-08-31',
               '1980-09-30', '1980-10-31',
               '1994-03-31', '1994-04-30', '1994-05-31', '1994-06-30',
               '1994-07-31', '1994-08-31', '1994-09-30', '1994-10-31',
               '1994-11-30', '1994-12-31'],
              dtype='datetime64[ns]', length=180, freq='M')
In [6]:
df 2['Time Stamp'] = pd.DataFrame(date,columns=['YearMonth'])
df 2.head()
Out[6]:
```

	YearMonth	Sparkling	Time_Stamp
0	1980-01	1686	1980-01-31
1	1980-02	1591	1980-02-29
2	1980-03	2304	1980-03-31
3	1980-04	1712	1980-04-30
4	1980-05	1471	1980-05-31

In [7]:

```
df_2['Time_Stamp'] = pd.to_datetime(df_2['Time_Stamp'])
```

In [8]:

```
df = df_2.set_index('Time_Stamp')
df.drop(['YearMonth'], axis=1, inplace=True)
df.head()
```

Out[8]:

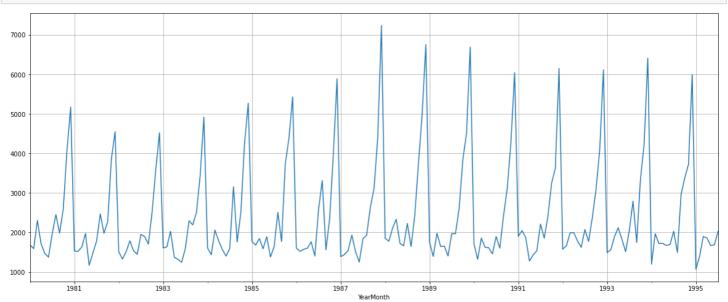
Sparkling

Time_Stamp

1980-01-31	1686
1980-02-29	1591
1980-03-31	2304
1980-04-30	1712
1980-05-31	1471

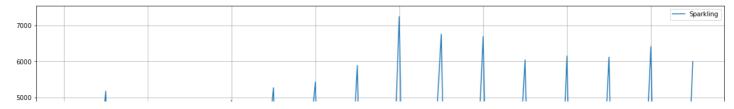
In [9]:

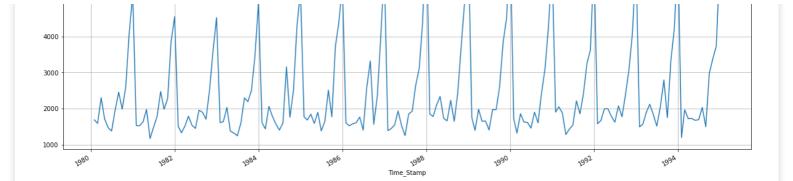
```
df_1.plot(figsize=(20,8))
plt.grid();
```



In [10]:

```
df.plot(figsize=(20,8))
plt.grid()
```





Q2. Perform appropriate Exploratory Data Analysis to understand the data and also perform decomposition.

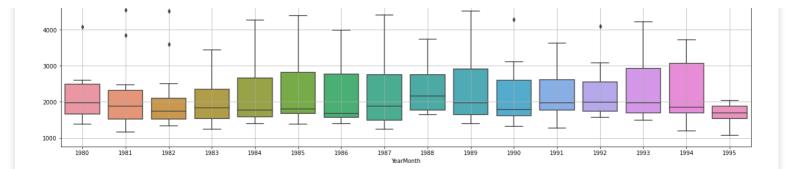
```
In [11]:
round(df.describe(),3)
Out[11]:
      Sparkling
        187.000
count
       2402.417
 mean
       1295.112
  std
       1070.000
  min
 25%
       1605.000
 50%
       1874.000
 75%
       2549.000
 max 7242.000
In [12]:
df.isna().sum()
Out[12]:
Sparkling
dtype: int64
In [13]:
df.shape
Out[13]:
(187, 1)
```

YEARLY BOXPLOT

```
In [14]:
```

```
_, ax = plt.subplots(figsize=(22,8))
sns.boxplot(x = df_1.index.year,y = df.values[:,0],ax=ax)
plt.grid();
```

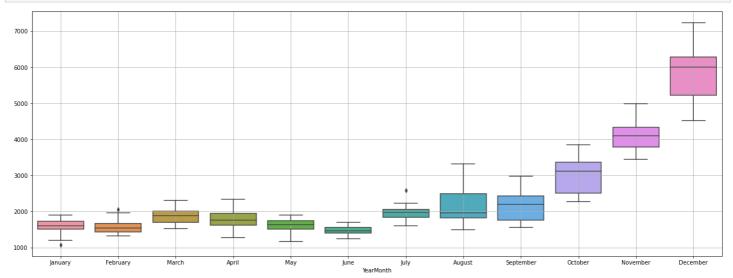




MONTHLY BOXPLOT

In [15]:

```
_, ax = plt.subplots(figsize=(22,8))
sns.boxplot(x = df_1.index.month_name(),y = df.values[:,0],ax=ax)
plt.grid();
```



In [16]:

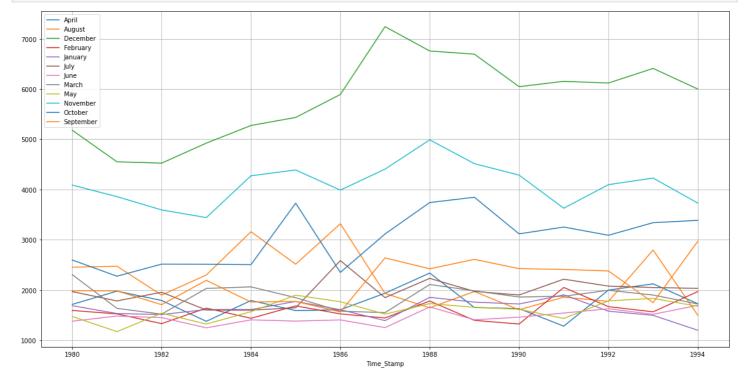
monthly_sales_across_years = pd.pivot_table(df, values = 'Sparkling', columns = df.index
.month_name(), index = df.index.year)
monthly_sales_across_years

Out[16]:

Time_Stamp	April	August	December	February	January	July	June	March	May	November	October	September
Time_Stamp												
1980.0	1712	2453	5179	1591	1686	1966	1377	2304	1471	4087	2596	1984
1981.0	1976	2472	4551	1523	1530	1781	1480	1633	1170	3857	2273	1981
1982.0	1790	1897	4524	1329	1510	1954	1449	1518	1537	3593	2514	1706
1983.0	1375	2298	4923	1638	1609	1600	1245	2030	1320	3440	2511	2191
1984.0	1789	3159	5274	1435	1609	1597	1404	2061	1567	4273	2504	1759
1985.0	1589	2512	5434	1682	1771	1645	1379	1846	1896	4388	3727	1771
1986.0	1605	3318	5891	1523	1606	2584	1403	1577	1765	3987	2349	1562
1987.0	1935	1930	7242	1442	1389	1847	1250	1548	1518	4405	3114	2638
1988.0	2336	1645	6757	1779	1853	2230	1661	2108	1728	4988	3740	2421
1989.0	1650	1968	6694	1394	1757	1971	1406	1982	1654	4514	3845	2608
1990.0	1628	1605	6047	1321	1720	1899	1457	1859	1615	4286	3116	2424
1991.0	1279	1857	6153	2049	1902	2214	1540	1874	1432	3627	3252	2408
1992.0	1997	1773	6119	1667	1577	2076	1625	1993	1783	4096	3088	2377

```
In [17]:
```

```
monthly_sales_across_years.plot(figsize=(20,10))
plt.grid()
plt.legend(loc='best');
```

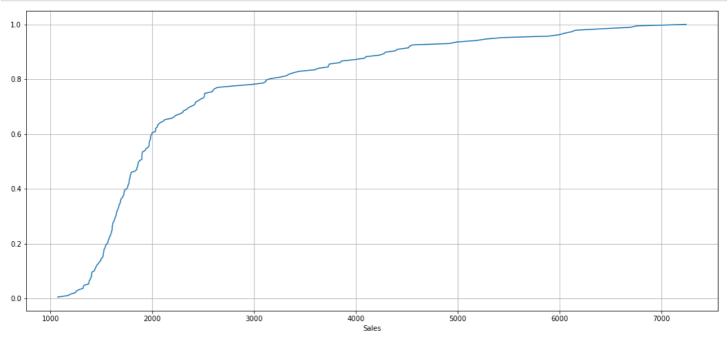


Plot the Empirical Cumulative Distribution.

In [18]:

```
# statistics
from statsmodels.distributions.empirical_distribution import ECDF

plt.figure(figsize = (18, 8))
cdf = ECDF(df['Sparkling'])
plt.plot(cdf.x, cdf.y, label = "statmodels");
plt.grid()
plt.xlabel('Sales');
```



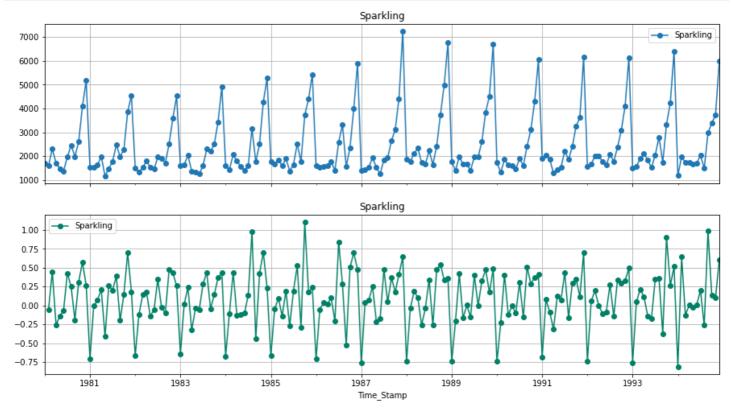
Plot the average RetailSales per month and the month on month percentage change of RetailSales.

```
In [19]:
```

```
# group by date and get average RetailSales, and precent change
average = df.groupby(df.index)["Sparkling"].mean()
pct_change = df.groupby(df.index)["Sparkling"].sum().pct_change()

fig, (axis1,axis2) = plt.subplots(2,1,sharex=True,figsize=(15,8))

# plot average Sparkling over time(year-month)
ax1 = average.plot(legend=True,ax=axis1,marker='o',title="Sparkling",grid=True)
ax1.set_xticks(range(len(average)))
ax1.set_xticklabels(average.index.tolist())
# plot precent change for Sparkling over time(year-month)
ax2 = pct_change.plot(legend=True,ax=axis2,marker='o',colormap="summer",title="Sparkling",grid=True)
```



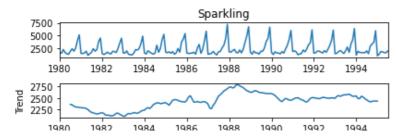
Decompose the Time Series and plot the different components.

```
In [20]:
from statsmodels.tsa.seasonal import seasonal_decompose
In [21]:
```

```
from statsmodels.tsa.seasonal import seasonal_decompose
```

```
In [22]:
```

```
decomposition = seasonal_decompose(df_1, model='additive')
decomposition.plot();
```



```
....
                                                                   ....
Seasonal
  2500 -
      0
      1980
               1982
                        1984
                                1986
                                         1988
                                                  1990
                                                           1992
                                                                   1994
  1000
 -1000
                       1984
                                1986
                                                  1990
                                                           1992
      1980
               1982
                                         1988
                                                                   1994
```

In [23]:

```
trend = decomposition.trend
seasonality = decomposition.seasonal
residual = decomposition.resid

print('Trend','\n',trend.head(12),'\n')
print('Seasonality','\n',seasonality.head(12),'\n')
print('Residual','\n',residual.head(12),'\n')
```

```
Trend
 YearMonth
1980-01-01
                       NaN
1980-02-01
                       NaN
1980-03-01
                       NaN
1980-04-01
                       NaN
1980-05-01
                       NaN
1980-06-01
                       NaN
              2360.666667
1980-07-01
1980-08-01
              2351.333333
1980-09-01
              2320.541667
1980-10-01
              2303.583333
1980-11-01
              2302.041667
1980-12-01
              2293.791667
Name: trend, dtype: float64
```

Seasonality YearMonth 1980-01-01 -854.260599 1980-02-01 -830.350678 1980-03-01 -592.356630 1980-04-01 -658.490559 1980-05-01 -824.416154 -967.434011 1980-06-01 -465.502265 1980-07-01 1980-08-01 -214.332821 1980-09-01 -254.677265 1980-10-01 599.769957 1980-11-01 1675.067179 1980-12-01 3386.983846 Name: seasonal, dtype: float64

Residual YearMonth 1980-01-01 NaN 1980-02-01 NaN 1980-03-01 NaN 1980-04-01 NaN 1980-05-01 NaN 1980-06-01 NaN 1980-07-01 70.835599 1980-08-01 315.999487 1980-09-01 -81.864401 -307.353290 1980-10-01 1980-11-01 109.891154

Name: resid, dtype: float64

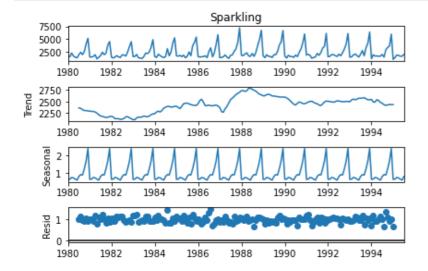
-501.775513

In [24]:

1980-12-01

decomposition = seasonal decompose(df 1, model='multipicative')

```
decomposition.plot();
```



In [25]:

```
trend = decomposition.trend
seasonality = decomposition.seasonal
residual = decomposition.resid

print('Trend','\n',trend.head(12),'\n')
print('Seasonality','\n',seasonality.head(12),'\n')
print('Residual','\n',residual.head(12),'\n')
```

Trend YearMonth 1980-01-01 NaN 1980-02-01 NaN 1980-03-01 NaN 1980-04-01 NaN 1980-05-01 NaN 1980-06-01 NaN 1980-07-01 2360.666667 2351.333333 1980-08-01 1980-09-01 2320.541667 1980-10-01 2303.583333 2302.041667 1980-11-01 2293.791667 1980-12-01 Name: trend, dtype: float64

Seasonality YearMonth 0.649843 1980-01-01 1980-02-01 0.659214 1980-03-01 0.757440 0.730351 1980-04-01 1980-05-01 0.660609 0.603468 1980-06-01 1980-07-01 0.809164 1980-08-01 0.918822 1980-09-01 0.894367 1980-10-01 1.241789 1980-11-01 1.690158 1980-12-01 2.384776

Name: seasonal, dtype: float64

Residual	
YearMonth	
1980-01-01	NaN
1980-02-01	NaN
1980-03-01	NaN
1980-04-01	NaN
1980-05-01	NaN
1980-06-01	NaN
1980-07-01	1.029230
1980-08-01	1.135407
1 0 0 0 - 0 0 - 0 1	n a55a5/

```
1980-10-01 0.907513
1980-11-01 1.050423
1980-12-01 0.946770
Name: resid, dtype: float64
```

In [30]:

(132, 1)

print(train.shape)
print(test.shape)

Q3. Split the data into training and test. The test data should start in 1991.

```
In [26]:
df.index.year.unique()
Out[26]:
Float64Index([1980.0, 1981.0, 1982.0, 1983.0, 1984.0, 1985.0, 1986.0, 1987.0,
               1988.0, 1989.0, 1990.0, 1991.0, 1992.0, 1993.0, 1994.0, nan],
              dtype='float64', name='Time Stamp')
In [27]:
# Drop the last year rows as data is incomplete ( only for 7 months data is provided)
df = df.head(-7)
# Printing dataframe
df.tail()
Out[27]:
           Sparkling
Time_Stamp
 1994-08-31
               1495
 1994-09-30
               2968
 1994-10-31
               3385
               3729
 1994-11-30
 1994-12-31
               5999
In [28]:
train = df[df.index<'1991']</pre>
test = df[df.index>='1991']
In [29]:
test.tail(5)
Out[29]:
           Sparkling
Time_Stamp
 1994-08-31
               1495
 1994-09-30
               2968
 1994-10-31
               3385
 1994-11-30
               3729
 1994-12-31
               5999
```

```
(48, 1)
In [31]:
print('First\ few\ rows\ of\ Training\ Data','\n',train.head(),'\n')
print('Last few rows of Training Data','\n', train.tail(),'\n')
print('First few rows of Test Data', '\n', test.head(), '\n')
print('Last few rows of Test Data', '\n', test.tail(), '\n')
First few rows of Training Data
             Sparkling
Time_Stamp
1980-01-31
                  1686
1980-02-29
                  1591
1980-03-31
                  2304
1980-04-30
                  1712
1980-05-31
                  1471
Last few rows of Training Data
             Sparkling
```

Time_Stamp 1990-08-31 1605 1990-09-30 2424 1990-10-31 3116 1990-11-30 4286 1990-12-31 6047

First few rows of Test Data Sparkling

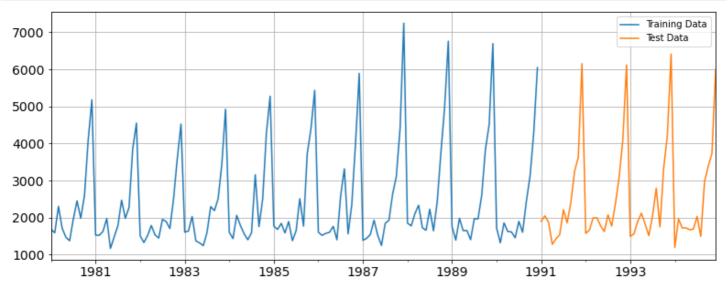
Time_Stamp 1991-01-31 1902 1991-02-28 2049 1991-03-31 1874 1991-04-30 1279 1991-05-31 1432

Last few rows of Test Data Sparkling

Time_Stamp 1994-08-31 1495 1994-09-30 2968 1994-10-31 3385 1994-11-30 3729 1994-12-31 5999

In [32]:

```
train['Sparkling'].plot(figsize=(13,5), fontsize=14)
test['Sparkling'].plot(figsize=(13,5), fontsize=14)
plt.grid()
plt.legend(['Training Data','Test Data'])
plt.show()
```



Q4. Build various exponential smoothing models on the training data and evaluate the model using RMSE on the test data. Other models such as regression,naïve forecast models, simple average models etc. should also be built on the training data and check the performance on the test data using RMSE.

Model 1: Linear Regression

For this particular linear regression, we are going to regress the 'Sparkling' variable against the order of the occurrence. For this we need to modify our training data before fitting it into a linear regression.

```
In [33]:
```

```
train time = [i+1 for i in range(len(train))]
test time = [i+133 for i in range(len(test))]
print('Training Time instance','\n',train time)
print('Test Time instance','\n',test time)
Training Time instance
 [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 2
5, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47,
48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70
 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92,
93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 1
12, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129,
130, 131, 132]
Test Time instance
 [133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 15
0, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 1
68, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180]
```

We see that we have successfully the generated the numerical time instance order for both the training and test set. Now we will add these values in the training and test set.

```
In [34]:
```

```
LinearRegression_train = train.copy()
LinearRegression_test = test.copy()
```

In [35]:

```
LinearRegression_train['time'] = train_time
LinearRegression_test['time'] = test_time

print('First few rows of Training Data', '\n', LinearRegression_train.head(), '\n')
print('Last few rows of Training Data', '\n', LinearRegression_train.tail(), '\n')
print('First few rows of Test Data', '\n', LinearRegression_test.head(), '\n')
print('Last few rows of Test Data', '\n', LinearRegression_test.tail(), '\n')
```

```
First few rows of Training Data
            Sparkling time
Time Stamp
1980-01-31
                1686
                         1
1980-02-29
                1591
                         2
                         3
1980-03-31
                2304
1980-04-30
                1712
                         4
1980-05-31
                1471
```

```
Last few rows of Training Data
Sparkling time
Time_Stamp
```

1990-08-31 1605 128 1990-09-30 2424 129 1000-10-31 3116 130

```
100 IU 01
                  \cup \perp \perp \cup
                          エンし
1990-11-30
                  4286
                          131
                          132
1990-12-31
                  6047
First few rows of Test Data
              Sparkling time
Time Stamp
1991-01-31
                  1902
                          133
                          134
1991-02-28
                  2049
1991-03-31
                  1874
                          135
1991-04-30
                  1279
                          136
1991-05-31
                  1432
                          137
Last few rows of Test Data
              Sparkling time
Time Stamp
                  1495
                          176
1994-08-31
1994-09-30
                  2968
                          177
1994-10-31
                  3385
                          178
1994-11-30
                  3729
                          179
1994-12-31
                  5999
                          180
```

LinearRegression

Now that our training and test data has been modified, let us go ahead use to build the model on the training data and test the model on the test data.

```
In [36]:
```

```
from sklearn.linear_model import LinearRegression
```

In [37]:

```
lr = LinearRegression()
```

In [38]:

```
lr.fit(LinearRegression_train[['time']],LinearRegression_train['Sparkling'].values)
```

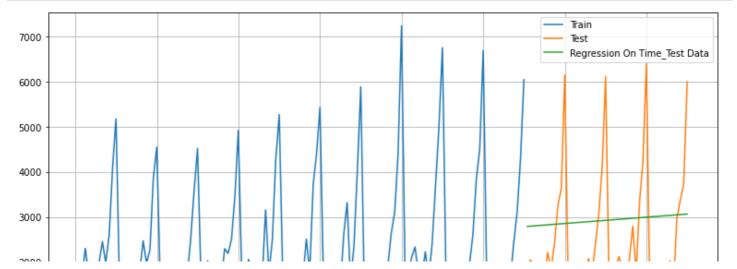
Out[38]:

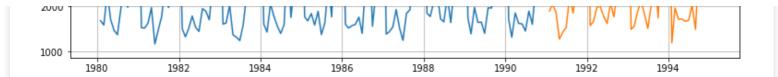
LinearRegression()

In [39]:

```
test_predictions_model1 = lr.predict(LinearRegression_test[['time']])
LinearRegression_test['RegOnTime'] = test_predictions_model1

plt.figure(figsize=(13,6))
plt.plot( train['Sparkling'], label='Train')
plt.plot(test['Sparkling'], label='Test')
plt.plot(LinearRegression_test['RegOnTime'], label='Regression On Time_Test Data')
plt.legend(loc='best')
plt.grid();
```





Defining the accuracy metrics.

```
In [40]:
```

```
from sklearn import metrics
```

Model Evaluation

```
In [41]:
```

```
## Test Data - RMSE

rmse_modell_test = metrics.mean_squared_error(test['Sparkling'], test_predictions_model1,s
quared=False)
print("For RegressionOnTime forecast on the Test Data, RMSE is %3.3f" %(rmse_model1_test
))
```

For RegressionOnTime forecast on the Test Data, RMSE is 1378.580

```
In [42]:
```

```
resultsDf_1 = pd.DataFrame({'RMSE': [rmse_model1_test]},index=['RegressionOnTime'])
resultsDf = resultsDf_1
resultsDf
```

Out[42]:

RMSE

RegressionOnTime 1378.580256

Model 2: Naive Approach: $\hat{Y}_{t+1} = y_t$

For this particular naive model, we say that the prediction for tomorrow is the same as today and the prediction for day after tomorrow is tomorrow and since the prediction of tomorrow is same as today, therefore the prediction for day after tomorrow is also today.

```
In [43]:
```

```
NaiveModel_train = train.copy()
NaiveModel_test = test.copy()
```

```
In [44]:
```

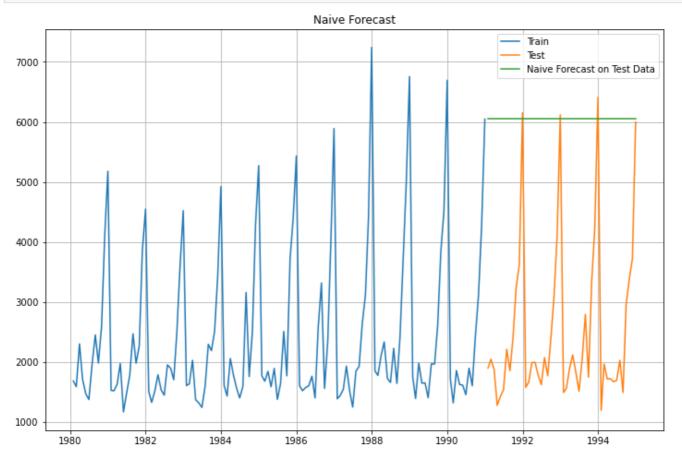
```
NaiveModel_test['naive'] = np.asarray(train['Sparkling'])[len(np.asarray(train['Sparklin
g']))-1]
NaiveModel_test['naive'].head()
```

Out[44]:

```
Time_Stamp
1991-01-31 6047
1991-02-28 6047
1991-03-31 6047
1991-04-30 6047
1991-05-31 6047
Name: naive, dtype: int64
```

In [45]:

plt.figure(figsize=(12,8))
plt.plot(NaiveModel_train['Sparkling'], label='Train')
plt.plot(test['Sparkling'], label='Test')
plt.plot(NaiveModel_test['naive'], label='Naive Forecast on Test Data')
plt.legend(loc='best')
plt.title("Naive Forecast")
plt.grid();



Model Evaluation

```
In [46]:
```

```
## Test Data - RMSE

rmse_model2_test = metrics.mean_squared_error(test['Sparkling'], NaiveModel_test['naive']
, squared=False)
print("For RegressionOnTime forecast on the Test Data, RMSE is %3.3f" %(rmse_model2_test
))
```

For RegressionOnTime forecast on the Test Data, RMSE is 3780.227

In [47]:

```
resultsDf_2 = pd.DataFrame({'RMSE': [rmse_model2_test]},index=['NaiveModel'])
resultsDf = pd.concat([resultsDf, resultsDf_2])
resultsDf
```

Out[47]:

RMSE

RegressionOnTime 1378.580256 **NaiveModel** 3780.226721

Method 3: Simple Average

Ear this norticular simple sucress method we will forecast by using the sucress of the training values

ror uns parucular simple average meulou, we will forecast by using the average of the training values.

```
In [48]:
```

```
SimpleAverage_train = train.copy()
SimpleAverage_test = test.copy()
```

In [49]:

```
SimpleAverage_test['mean_forecast'] = train['Sparkling'].mean()
SimpleAverage_test.head()
```

Out[49]:

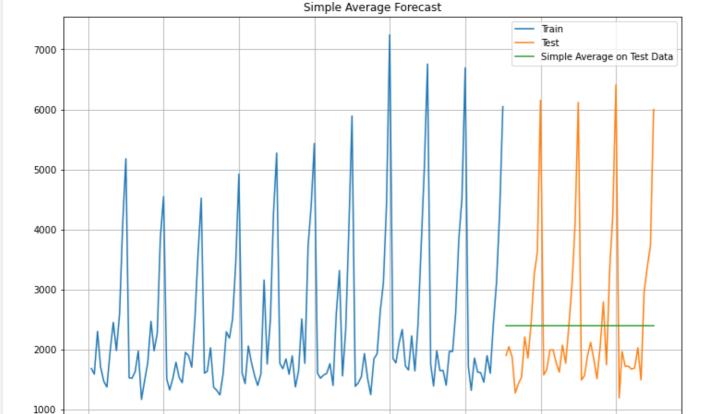
Sparkling mean_forecast

Time_Stamp

1991-01-31	1902	2403.780303
1991-02-28	2049	2403.780303
1991-03-31	1874	2403.780303
1991-04-30	1279	2403.780303
1991-05-31	1432	2403.780303

In [50]:

```
plt.figure(figsize=(12,8))
plt.plot(SimpleAverage_train['Sparkling'], label='Train')
plt.plot(SimpleAverage_test['Sparkling'], label='Test')
plt.plot(SimpleAverage_test['mean_forecast'], label='Simple Average on Test Data')
plt.legend(loc='best')
plt.title("Simple Average Forecast")
plt.grid();
```



Model Evaluation

1980

```
In [51]:
```

```
## Test Data - RMSE
```

1984

1986

1988

1990

1992

1994

1982

```
rmse_model3_test = metrics.mean_squared_error(test['Sparkling'],SimpleAverage_test['mean_forecast'],squared=False)
print("For Simple Average forecast on the Test Data, RMSE is %3.3f" %(rmse_model3_test))
```

For Simple Average forecast on the Test Data, RMSE is 1329.930

In [52]:

```
resultsDf_3 = pd.DataFrame({'RMSE': [rmse_model3_test]},index=['SimpleAverageModel'])
resultsDf = pd.concat([resultsDf, resultsDf_3])
resultsDf
```

Out[52]:

RMSE

 RegressionOnTime
 1378.580256

 NaiveModel
 3780.226721

 SimpleAverageModel
 1329.929685

In [53]:

df 2.tail(10)

Out[53]:

	YearMonth	Sparkling	Time_Stamp
177	1994-10	3385	1994-10-31
178	1994-11	3729	1994-11-30
179	1994-12	5999	1994-12-31
180	1995-01	1070	NaT
181	1995-02	1402	NaT
182	1995-03	1897	NaT
183	1995-04	1862	NaT
184	1995-05	1670	NaT
185	1995-06	1688	NaT
186	1995-07	2031	NaT

METHOD 4: MOVING AVERAGE

In [54]:

MovingAverage = df.copy()
MovingAverage.tail()

Out[54]:

Sparkling

Time_Stamp				
1994-08-31	1495			
1994-09-30	2968			
1994-10-31	3385			
1994-11-30	3729			
1994-12-31	5999			

In [55]:

```
MovingAverage['Trailing_2'] = MovingAverage['Sparkling'].rolling(2).mean()
MovingAverage['Trailing_4'] = MovingAverage['Sparkling'].rolling(4).mean()
MovingAverage['Trailing_6'] = MovingAverage['Sparkling'].rolling(6).mean()
MovingAverage['Trailing_9'] = MovingAverage['Sparkling'].rolling(9).mean()
MovingAverage.head()
```

Out[55]:

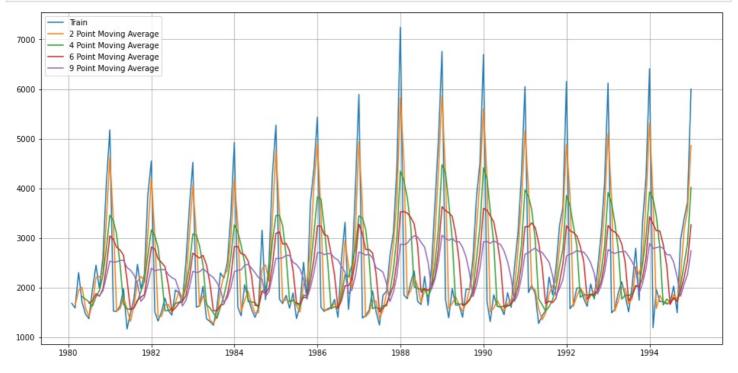
Sparkling Trailing_2 Trailing_4 Trailing_6 Trailing_9

Time_Stamp

1980-01-31	1686	NaN	NaN	NaN	NaN
1980-02-29	1591	1638.5	NaN	NaN	NaN
1980-03-31	2304	1947.5	NaN	NaN	NaN
1980-04-30	1712	2008.0	1823.25	NaN	NaN
1980-05-31	1471	1591.5	1769.50	NaN	NaN

In [56]:

```
plt.figure(figsize=(16,8))
plt.plot(MovingAverage['Sparkling'], label='Train')
plt.plot(MovingAverage['Trailing_2'], label='2 Point Moving Average')
plt.plot(MovingAverage['Trailing_4'], label='4 Point Moving Average')
plt.plot(MovingAverage['Trailing_6'], label = '6 Point Moving Average')
plt.plot(MovingAverage['Trailing_9'], label = '9 Point Moving Average')
plt.legend(loc = 'best')
plt.grid();
```



In [57]:

```
trailing_MovingAverage_train=MovingAverage[0:int(len(MovingAverage)*0.734)]
trailing_MovingAverage_test=MovingAverage[int(len(MovingAverage)*0.734):]
```

In [58]:

```
trailing_MovingAverage_train.tail(5)
```

Out[58]:

Time_Stamp	Sparkling	Trailing_2	Trailing_4	Trailing_6	Trailing_9
Tit00 <u>0</u> 838+6¢	1605	1752.0	1644.00	1677.166667	2199.777778
1990-09-30	2424	2014.5	1846.25	1771.333333	1725.333333
1990-10-31	3116	2770.0	2261.00	2019.333333	1880.44444
1990-11-30	4286	3701.0	2857.75	2464.500000	2209.888889
1990-12-31	6047	5166.5	3968.25	3229.500000	2675.222222

In [59]:

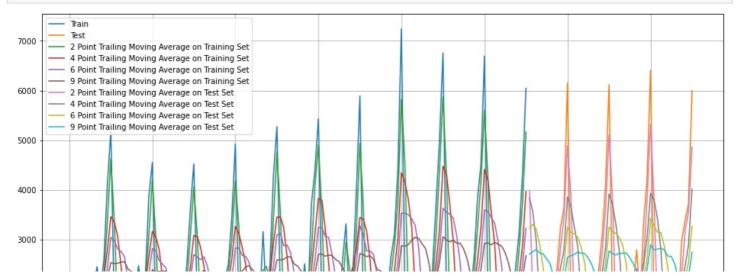
```
trailing_MovingAverage_test.head(5)
```

Out[59]:

	Sparkling	Trailing_2	Trailing_4	Trailing_6	Trailing_9
Time_Stamp					
1991-01-31	1902	3974.5	3837.75	3230.000000	2705.666667
1991-02-28	2049	1975.5	3571.00	3304.000000	2753.888889
1991-03-31	1874	1961.5	2968.00	3212.333333	2800.222222
1991-04-30	1279	1576.5	1776.00	2906.166667	2731.333333
1991-05-31	1432	1355.5	1658.50	2430.500000	2712.111111

In [60]:

```
plt.figure(figsize=(16,8))
plt.plot(trailing_MovingAverage_train['Sparkling'], label='Train')
plt.plot(trailing MovingAverage test['Sparkling'], label='Test')
plt.plot(trailing MovingAverage train['Trailing 2'], label='2 Point Trailing Moving Avera
ge on Training Set')
plt.plot(trailing MovingAverage train['Trailing 4'], label='4 Point Trailing Moving Avera
ge on Training Set')
plt.plot(trailing MovingAverage train['Trailing 6'], label = '6 Point Trailing Moving Aver
age on Training Set')
plt.plot(trailing MovingAverage train['Trailing 9'], label = '9 Point Trailing Moving Aver
age on Training Set')
plt.plot(trailing MovingAverage test['Trailing 2'], label='2 Point Trailing Moving Avera
ge on Test Set')
plt.plot(trailing MovingAverage test['Trailing 4'], label='4 Point Trailing Moving Avera
ge on Test Set')
plt.plot(trailing MovingAverage test['Trailing 6'], label = '6 Point Trailing Moving Aver
age on Test Set')
plt.plot(trailing MovingAverage test['Trailing 9'], label = '9 Point Trailing Moving Aver
age on Test Set')
plt.legend(loc = 'best')
plt.grid();
```



```
1000
```

In [61]:

```
# Test Data - RMSE --> 2 point Trailing MA
rmse model4 test 2 = metrics.mean squared error(test['Sparkling'], trailing MovingAverage
test['Trailing_2'], squared=False)
print("For 2 point Moving Average Model forecast on the Test Data, RMSE is %3.3f" %(rmse
model4 test 2))
## Test Data - RMSE --> 4 point Trailing MA
rmse model4 test 4 = metrics.mean squared error(test['Sparkling'],trailing MovingAverage
test['Trailing 4'], squared=False)
print("For 4 point Moving Average Model forecast on the Test Data, RMSE is %3.3f" %(rmse
_model4 test 4))
## Test Data - RMSE --> 6 point Trailing MA
rmse model4 test 6 = metrics.mean squared error(test['Sparkling'],trailing MovingAverage
test['Trailing 6'], squared=False)
print("For 6 point Moving Average Model forecast on the Test Data, RMSE is %3.3f" %(rmse
model4 test 6))
## Test Data - RMSE --> 9 point Trailing MA
rmse_model4_test_9 = metrics.mean_squared_error(test['Sparkling'],trailing_MovingAverage_
test['Trailing 9'], squared=False)
print("For 9 point Moving Average Model forecast on the Test Data, RMSE is %3.3f " %(rms
e model4 test 9))
For 2 point Moving Average Model forecast on the Test Data,
                                                             RMSE is 793.037
For 4 point Moving Average Model forecast on the Test Data,
                                                             RMSE is 1155.560
For 6 point Moving Average Model forecast on the Test Data,
                                                             RMSE is 1302.932
For 9 point Moving Average Model forecast on the Test Data, RMSE is 1388.802
In [62]:
```

Out[62]:

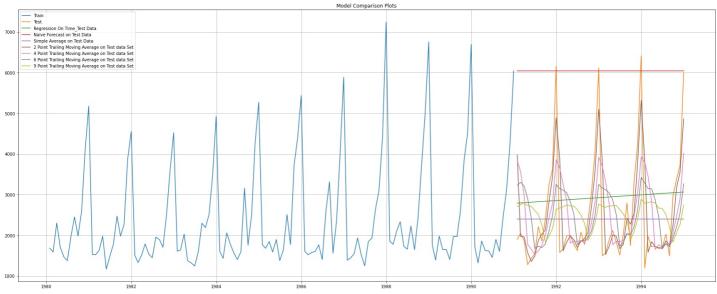
RMSE

RegressionOnTime	1378.580256
NaiveModel	3780.226721
SimpleAverageModel	1329.929685
2pointTrailingMovingAverage	793.036740
4pointTrailingMovingAverage	1155.559612
6pointTrailingMovingAverage	1302.932089
9pointTrailingMovingAverage	1388.801927

and compare the Time Series plots.

```
In [63]:
```

```
plt.figure(figsize=(30,12))
plt.plot(train['Sparkling'], label='Train')
plt.plot(test['Sparkling'], label='Test')
plt.plot(LinearRegression test['RegOnTime'], label='Regression On Time Test Data')
plt.plot(NaiveModel test['naive'], label='Naive Forecast on Test Data')
plt.plot(SimpleAverage test['mean forecast'], label='Simple Average on Test Data')
plt.plot(trailing MovingAverage test['Trailing 2'], label='2 Point Trailing Moving Avera
ge on Test data Set')
plt.plot(trailing MovingAverage test['Trailing 4'], label='4 Point Trailing Moving Avera
ge on Test data Set')
plt.plot(trailing MovingAverage test['Trailing 6'], label='6 Point Trailing Moving Avera
plt.plot(trailing MovingAverage test['Trailing 9'], label='9 Point Trailing Moving Avera
ge on Test data Set')
plt.legend(loc='best')
plt.title("Model Comparison Plots")
plt.grid();
```



Method 5: Simple Exponential Smoothing

```
In [64]:
```

```
from statsmodels.tsa.api import ExponentialSmoothing, SimpleExpSmoothing, Holt
```

```
In [65]:
```

```
SES_train = train.copy()
SES_test = test.copy()
```

```
In [66]:
```

```
model_SES = SimpleExpSmoothing(SES_train['Sparkling'])
```

C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py: 524: ValueWarning: No frequency information was provided, so inferred frequency M will be used.

```
warnings.warn('No frequency information was'
```

```
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\holtwinters\model.
py:427: FutureWarning: After 0.13 initialization must be handled at model creation
  warnings.warn(
In [67]:
model SES autofit = model SES.fit(optimized=True)
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\holtwinters\model.
py:920: ConvergenceWarning: Optimization failed to converge. Check mle_retvals.
  warnings.warn(
In [68]:
model SES autofit.params
Out[68]:
{'smoothing level': 0.049607360581862936,
 'smoothing_trend': nan,
 'smoothing seasonal': nan,
 'damping trend': nan,
 'initial level': 1818.535750008871,
 'initial trend': nan,
 'initial seasons': array([], dtype=float64),
 'use boxcox': False,
 'lamda': None,
 'remove bias': False}
In [69]:
SES test['predict'] = model SES autofit.forecast(steps=len(test))
SES test.head(5)
Out[69]:
                      predict
          Sparkling
Time Stamp
 1991-01-31
              1902 2724.932624
 1991-02-28
              2049 2724.932624
 1991-03-31
              1874 2724.932624
              1279 2724.932624
 1991-04-30
```

In [70]:

1991-05-31

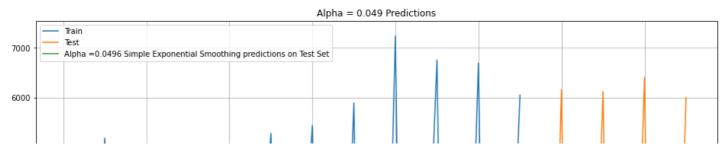
1432 2724.932624

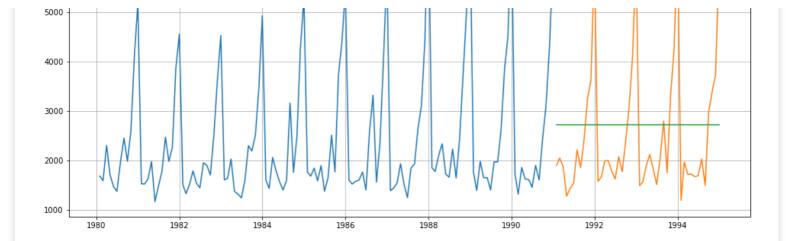
```
## Plotting on both the Training and Test data

plt.figure(figsize=(16,8))
plt.plot(SES_train['Sparkling'], label='Train')
plt.plot(SES_test['Sparkling'], label='Test')

plt.plot(SES_test['predict'], label='Alpha =0.0496 Simple Exponential Smoothing predictions on Test Set')

plt.legend(loc='best')
plt.grid()
plt.title('Alpha = 0.049 Predictions');
```





Model Evaluation for α = 0.0496 : Simple Exponential Smoothing

```
In [71]:
```

```
## Test Data

rmse_model5_test_1 = metrics.mean_squared_error(SES_test['Sparkling'], SES_test['predict']
, squared=False)
print("For Alpha =0.0496 Simple Exponential Smoothing Model forecast on the Test Data, RM
SE is %3.3f" % (rmse_model5_test_1))
```

For Alpha =0.0496 Simple Exponential Smoothing Model forecast on the Test Data, RMSE is 1343.724

In [72]:

```
resultsDf_5 = pd.DataFrame({'RMSE': [rmse_model5_test_1]},index=['Alpha=0.0496,SimpleExpo
nentialSmoothing'])
resultsDf = pd.concat([resultsDf, resultsDf_5])
resultsDf
```

Out[72]:

	RMSE
RegressionOnTime	1378.580256
NaiveModel	3780.226721
SimpleAverageModel	1329.929685
2pointTrailingMovingAverage	793.036740
4pointTrailingMovingAverage	1155.559612
6pointTrailingMovingAverage	1302.932089
9pointTrailingMovingAverage	1388.801927
Alpha=0.0496,SimpleExponentialSmoothing	1343.723719

SES - ETS(A, N, N) - Simple Exponential Smoothing with additive errors

The simplest of the exponentially smoothing methods is naturally called simple exponential smoothing (SES).

This method is suitable for forecasting data with no clear trend or seasonal pattern.

In Single ES, the forecast at time (t + 1) is given by Winters, 1960

$$\bullet \quad F_{t+1} = \alpha Y_t + (1 - \alpha) F_t$$

Parameter n

.

is called the smoothing constant and its value lies between 0 and 1. Since the model uses only one smoothing constant, it is called Single Exponential Smoothing.

Note: Here, there is both trend and seasonality in the data. So, we should have directly gone for the Triple Exponential Smoothing but Simple Exponential Smoothing and the Double Exponential Smoothing models are built over here to get an idea of how the three types of models compare in this case.

SimpleExpSmoothing class must be instantiated and passed the training data.

The fit() function is then called providing the fit configuration, the alpha value, smoothing_level. If this is omitted or set to None, the model will automatically optimize the value.

```
In [73]:
# create class
model SES = SimpleExpSmoothing(train,initialization method='estimated')
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa model.py:
524: ValueWarning: No frequency information was provided, so inferred frequency M will be
used.
 warnings.warn('No frequency information was'
In [74]:
# Fitting the Simple Exponential Smoothing model and asking python to choose the optimal
parameters
model SES autofit = model SES.fit(optimized=True)
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\holtwinters\model.
py:920: ConvergenceWarning: Optimization failed to converge. Check mle retvals.
 warnings.warn(
In [75]:
## Let us check the parameters
model SES autofit.params
Out [75]:
{'smoothing level': 0.07029120765764557,
 'smoothing trend': nan,
 'smoothing seasonal': nan,
 'damping trend': nan,
 'initial level': 1764.0137060346985,
 'initial trend': nan,
 'initial seasons': array([], dtype=float64),
 'use boxcox': False,
 'lamda': None,
 'remove bias': False}
```

Here, Python has optimized the smoothing level to be almost 1.

```
In [76]:
```

```
# Using the fitted model on the training set to forecast on the test set
SES predict = model SES autofit.forecast(steps=len(test))
SES predict
Out [76]:
1991-01-31
            2804.675124
1991-02-28
             2804.675124
1991-03-31
             2804.675124
1991-04-30
             2804.675124
1991-05-31
             2804.675124
1991-06-30
             2804.675124
1991-07-31
             2804.675124
             2804.675124
1991-08-31
             2804.675124
1991-09-30
```

```
1991-10-31
               2804.675124
1991-11-30
               2804.675124
1991-12-31
               2804.675124
1992-01-31
               2804.675124
1992-02-29
               2804.675124
1992-03-31
               2804.675124
1992-04-30
               2804.675124
1992-05-31
               2804.675124
               2804.675124
1992-06-30
1992-07-31
               2804.675124
1992-08-31
               2804.675124
1992-09-30
               2804.675124
1992-10-31
               2804.675124
1992-11-30
               2804.675124
1992-12-31
               2804.675124
1993-01-31
               2804.675124
1993-02-28
               2804.675124
               2804.675124
1993-03-31
1993-04-30
               2804.675124
1993-05-31
               2804.675124
1993-06-30
               2804.675124
               2804.675124
1993-07-31
1993-08-31
               2804.675124
1993-09-30
               2804.675124
1993-10-31
               2804.675124
1993-11-30
               2804.675124
1993-12-31
               2804.675124
1994-01-31
               2804.675124
1994-02-28
               2804.675124
1994-03-31
               2804.675124
1994-04-30
               2804.675124
1994-05-31
               2804.675124
1994-06-30
               2804.675124
               2804.675124
1994-07-31
1994-08-31
               2804.675124
1994-09-30
               2804.675124
1994-10-31
               2804.675124
1994-11-30
               2804.675124
1994-12-31
               2804.675124
Freq: M, dtype: float64
```

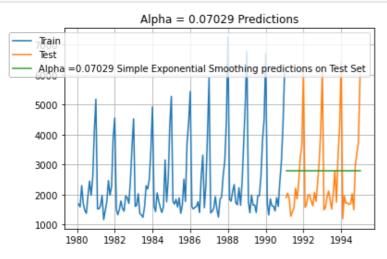
In [77]:

```
## Plotting the Training data, Test data and the forecasted values

plt.plot(train, label='Train')
plt.plot(test, label='Test')

plt.plot(SES_predict, label='Alpha =0.07029 Simple Exponential Smoothing predictions on T
    est Set')

plt.legend(loc='best')
plt.grid()
plt.title('Alpha = 0.07029 Predictions');
```



```
## Mean Absolute Percentage Error (MAPE) - Function Definition
def MAPE(y true, y pred):
    return np.mean((np.abs(y true-y pred))/(y true))*100
In [79]:
print('SES RMSE:', mean squared error(test.values, SES predict.values, squared=False))
#different way to calculate RMSE
print('SES RMSE (calculated using statsmodels):',em.rmse(test.values,SES predict.values)
[0])
SES RMSE: 1358.9404884833777
SES RMSE (calculated using statsmodels): 1358.9404884833775
In [80]:
resultsDf 6= pd.DataFrame({'RMSE': [em.rmse(test.values, SES predict.values)[0]]},index=[
'Alpha=0.0702, SES'])
resultsDf = pd.concat([resultsDf, resultsDf 6])
resultsDf
Out[80]:
```

RMSE

RegressionOnTime	1378.580256
NaiveModel	3780.226721
SimpleAverageModel	1329.929685
2pointTrailingMovingAverage	793.036740
4pointTrailingMovingAverage	1155.559612
6pointTrailingMovingAverage	1302.932089
9pointTrailingMovingAverage	1388.801927
Alpha=0.0496,SimpleExponentialSmoothing	1343.723719
Alpha=0.0702,SES	1358.940488

Holt - ETS(A, A, N) - Holt's linear method with additive errors

Double Exponential Smoothing

- One of the drawbacks of the simple exponential smoothing is that the model does not do well in the
 presence of the trend.
- This model is an extension of SES known as Double Exponential model which estimates two smoothing parameters.
- Applicable when data has Trend but no seasonality.
- Two separate components are considered: Level and Trend.
- · Level is the local mean.
- One smoothing parameter α corresponds to the level series
- $\bullet\,$ A second smoothing parameter β corresponds to the trend series.

Double Exponential Smoothing uses two equations to forecast future values of the time series, one for forecating the short term avarage value or level and the other for capturing the trend.

- Intercept or Level equation, L_t is given by: $L_t = \alpha Y_t + (1 \alpha)F_t$
- Trend equation is given by $T_t = \beta(L_t L_{t-1}) + (1 \beta)T_{t-1}$

In [78]:

are the smoothing constants for level and trend, respectively,

```
0 < α</li>1 and 0 < β</li>1.
```

The forecast at time t + 1 is given by

```
• F_{t+1} = L_t + T_t
• F_{t+n} = L_t + nT_t
```

```
In [81]:
```

```
# Initializing the Double Exponential Smoothing Model
model_DES = Holt(train,initialization_method='estimated')
# Fitting the model
model_DES = model_DES.fit()

print('')
print('=Holt model Exponential Smoothing Estimated Parameters ==')
print('')
print(model_DES.params)
```

==Holt model Exponential Smoothing Estimated Parameters ==

{'smoothing_level': 0.6649999999999999999, 'smoothing_trend': 0.0001, 'smoothing_seasonal':
nan, 'damping_trend': nan, 'initial_level': 1502.199999999991, 'initial_trend': 74.87272
727272739, 'initial_seasons': array([], dtype=float64), 'use_boxcox': False, 'lamda': Non
e, 'remove bias': False}

C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py: 524: ValueWarning: No frequency information was provided, so inferred frequency M will be used.

warnings.warn('No frequency information was'
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\holtwinters\model.
py:920: ConvergenceWarning: Optimization failed to converge. Check mle_retvals.
 warnings.warn(

In [82]:

```
# Forecasting using this model for the duration of the test set
DES_predict = model_DES.forecast(len(test))
DES_predict
```

Out[82]:

```
5401.733026
1991-01-31
            5476.005230
1991-02-28
1991-03-31
            5550.277433
1991-04-30
            5624.549637
1991-05-31
             5698.821840
             5773.094044
1991-06-30
1991-07-31
             5847.366248
1991-08-31
             5921.638451
1991-09-30 5995.910655
1991-10-31
           6070.182858
1991-11-30 6144.455062
1991-12-31 6218.727266
1992-01-31
           6292.999469
1992-02-29 6367.271673
1992-03-31 6441.543876
1992-04-30 6515.816080
1992-05-31
           6590.088284
1992-06-30 6664.360487
            6738.632691
1992-07-31
           6812.904894
1992-08-31
           6887.177098
1992-09-30
1992-10-31
             6961.449302
1992-11-30
             7035.721505
1992-12-31
             7109.993709
1002-01-21
             7101 265012
```

```
エフプンーしエーンエ
              1104.200712
1993-02-28
              7258.538116
1993-03-31
              7332.810320
1993-04-30
              7407.082523
1993-05-31
              7481.354727
1993-06-30
              7555.626930
1993-07-31
              7629.899134
1993-08-31
              7704.171338
              7778.443541
1993-09-30
1993-10-31
              7852.715745
1993-11-30
              7926.987948
1993-12-31
              8001.260152
              8075.532356
1994-01-31
1994-02-28
              8149.804559
1994-03-31
              8224.076763
1994-04-30
              8298.348967
1994-05-31
              8372.621170
1994-06-30
              8446.893374
1994-07-31
              8521.165577
1994-08-31
              8595.437781
1994-09-30
              8669.709985
1994-10-31
              8743.982188
1994-11-30
              8818.254392
1994-12-31
              8892.526595
Freq: M, dtype: float64
```

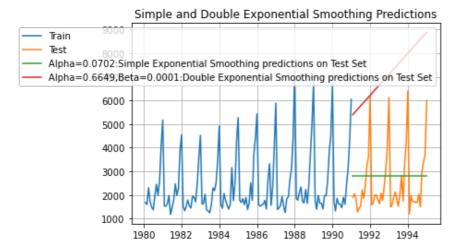
In [83]:

```
## Plotting the Training data, Test data and the forecasted values

plt.plot(train, label='Train')
plt.plot(test, label='Test')

plt.plot(SES_predict, label='Alpha=0.0702:Simple Exponential Smoothing predictions on Test Set')
plt.plot(DES_predict, label='Alpha=0.6649,Beta=0.0001:Double Exponential Smoothing predictions on Test Set')

plt.legend(loc='best')
plt.grid()
plt.title('Simple and Double Exponential Smoothing Predictions');
```



We see that the double exponential smoothing is picking up the trend component along with the level component as well.

```
In [84]:
print('DES RMSE:', mean_squared_error(test.values, DES_predict.values, squared=False))

DES RMSE: 4879.86657314999

In [85]:
resultsDf 7 = pd.DataFrame({'RMSE': [mean squared error(test.values, DES predict.values, s]
```

DMCE

Out[85]:

	RIVISE
RegressionOnTime	1378.580256
NaiveModel	3780.226721
SimpleAverageModel	1329.929685
2pointTrailingMovingAverage	793.036740
4pointTrailingMovingAverage	1155.559612
6pointTrailingMovingAverage	1302.932089
9pointTrailingMovingAverage	1388.801927
Alpha=0.0496,SimpleExponentialSmoothing	1343.723719
Alpha=0.0702,SES	1358.940488
Alpha=0.66499,Beta=0.0001:DES	4879.866573

Inference

Here, we see that the Single Exponential Smoothing has performed well when compared to the Double Exponential Smoothing. This is because of low values in the trend component.

Holt-Winters - ETS(A, A, A) - Holt Winter's linear method with additive errors

```
In [86]:
# Initializing the Double Exponential Smoothing Model
model TES = ExponentialSmoothing(train, trend='additive', seasonal='additive', initializatio
n method='estimated')
# Fitting the model
model TES = model TES.fit()
print('')
print('==Holt Winters model Exponential Smoothing Estimated Parameters ==')
print('')
print(model_TES.params)
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa model.py:
524: ValueWarning: No frequency information was provided, so inferred frequency M will be
 warnings.warn('No frequency information was'
==Holt Winters model Exponential Smoothing Estimated Parameters ==
{'smoothing level': 0.11127217746964548, 'smoothing_trend': 0.012360783178126748, 'smooth
ing seasonal': 0.46071776538894016, 'damping trend': nan, 'initial level': 2356.578364436
942, 'initial trend': -0.018474239476475418, 'initial seasons': array([-636.23349217, -72
```

```
# Forecasting using this model for the duration of the test set
TES_predict = model_TES.forecast(len(test))
TES_predict
```

```
Out[87]:
```

```
1991-01-31
              1490.574551
1991-02-28
              1204.714462
1991-03-31
              1688.941256
1991-04-30
              1551.450972
              1461.440602
1991-05-31
1991-06-30
              1278.907350
1991-07-31
              1805.164222
1991-08-31
              1679.251604
1991-09-30
              2315.687875
1991-10-31
              3225.309145
1991-11-30
              4299.652613
1991-12-31
              6411.161656
1992-01-31
              1483.296915
1992-02-29
              1197.436826
1992-03-31
              1681.663620
1992-04-30
              1544.173336
1992-05-31
              1454.162966
1992-06-30
              1271.629714
1992-07-31
              1797.886586
1992-08-31
              1671.973968
              2308.410239
1992-09-30
1992-10-31
              3218.031509
              4292.374977
1992-11-30
1992-12-31
              6403.884020
1993-01-31
              1476.019279
1993-02-28
              1190.159190
1993-03-31
              1674.385984
1993-04-30
              1536.895700
1993-05-31
              1446.885330
1993-06-30
              1264.352078
1993-07-31
              1790.608950
1993-08-31
              1664.696332
1993-09-30
              2301.132603
1993-10-31
              3210.753873
1993-11-30
              4285.097341
1993-12-31
              6396.606384
1994-01-31
              1468.741643
1994-02-28
              1182.881554
1994-03-31
              1667.108348
1994-04-30
              1529.618064
1994-05-31
              1439.607694
1994-06-30
              1257.074442
1994-07-31
              1783.331314
1994-08-31
              1657.418696
1994-09-30
              2293.854967
1994-10-31
              3203.476237
1994-11-30
              4277.819705
1994-12-31
              6389.328748
Freq: M, dtype: float64
```

In [88]:

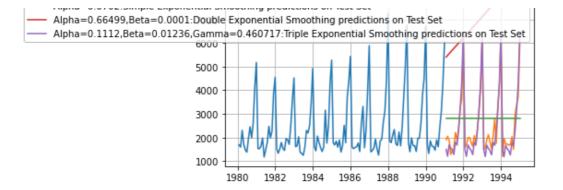
```
## Plotting the Training data, Test data and the forecasted values

plt.plot(train, label='Train')
plt.plot(test, label='Test')

plt.plot(SES_predict, label='Alpha=0.0702:Simple Exponential Smoothing predictions on Test Set')
plt.plot(DES_predict, label='Alpha=0.66499, Beta=0.0001:Double Exponential Smoothing predictions on Test Set')
plt.plot(TES_predict, label='Alpha=0.1112, Beta=0.01236, Gamma=0.460717:Triple Exponential Smoothing predictions on Test Set')

plt.legend(loc='best')
plt.legend(loc='best')
plt.grid()
plt.title('Simple, Double and Triple Exponential Smoothing Predictions');
```

Train
Test
8000
Alpha=0 0702-Simple Exponential Smoothing predictions on Test Set



We see that the Triple Exponential Smoothing is picking up the seasonal component as well.

	RMSE
RegressionOnTime	1378.580256
NaiveModel	3780.226721
SimpleAverageModel	1329.929685
2pointTrailingMovingAverage	793.036740
4pointTrailingMovingAverage	1155.559612
6pointTrailingMovingAverage	1302.932089
9pointTrailingMovingAverage	1388.801927
Alpha=0.0496,SimpleExponentialSmoothing	1343.723719
Alpha=0.0702,SES	1358.940488
Alpha=0.66499,Beta=0.0001:DES	4879.866573
Alpha=0.1112,Beta=0.01235,Gamma=0.460717:TES	387.165102

Inference

In [89]:

Out[90]:

Triple Exponential Smoothing has performed the best on the test as expected since the data had high component of seasonality.

But we see that our triple exponential smoothing is under forecasting. Let us try to tweak some of the parameters in order to get a better forecast on the test set.

Holt-Winters - ETS(A, A, M) - Holt Winter's linear method (Multiplicative seasonality)

ETS(A, A, M) model

In [91]:

Initializing the Double Exponential Smoothing Model

```
model TES am = ExponentialSmoothing(train, trend='add', seasonal='multiplicative', initializ
ation method='estimated')
# Fitting the model
model TES am = model TES am.fit()
print('')
print('==Holt Winters model Exponential Smoothing Estimated Parameters ==')
print('')
print(model TES am.params)
==Holt Winters model Exponential Smoothing Estimated Parameters ==
{'smoothing level': 0.11101523064586813, 'smoothing trend': 0.0493150519338607, 'smoothin
g seasonal': 0.3624464792152574, 'damping trend': nan, 'initial level': 2356.498115747180
7, 'initial trend': -9.804740061967985, 'initial seasons': array([0.71351261, 0.68262962,
0.90447098, 0.80523467, 0.65565848,
       0.65382021, 0.88609683, 1.13334681, 0.91877227, 1.21174825,
       1.87063097, 2.37421776]), 'use boxcox': False, 'lamda': None, 'remove bias': False
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa model.py:
524: ValueWarning: No frequency information was provided, so inferred frequency M will be
  warnings.warn('No frequency information was'
In [92]:
# Forecasting using this model for the duration of the test set
TES predict am = model TES am.forecast(len(test))
TES predict am
Out[92]:
             1587.921914
1991-01-31
1991-02-28
             1356.650344
1991-03-31
             1763.350219
1991-04-30
             1656.523868
1991-05-31
             1542.386219
1991-06-30
             1355.480857
             1854.708560
1991-07-31
             1820.665822
1991-08-31
1991-09-30
             2277.675614
             3123.118775
1991-10-31
1991-11-30
             4130.016056
1991-12-31
             5893.139956
1992-01-31
             1539.025473
1992-02-29
             1314.767912
1992-03-31
             1708.771760
             1605.119264
1992-04-30
1992-05-31
             1494.399399
1992-06-30
             1313.199426
1992-07-31
             1796.703973
1992-08-31
             1763.577111
1992-09-30
            2206.069820
             3024.675950
1992-10-31
1992-11-30
             3999.492295
1992-12-31
             5706.403171
1993-01-31
             1490.129033
1993-02-28
             1272.885480
1993-03-31
             1654.193301
1993-04-30
             1553.714661
1993-05-31
             1446.412578
1993-06-30
             1270.917996
1993-07-31
             1738.699385
1993-08-31
             1706.488400
1993-09-30
              2134.464026
1993-10-31
             2926.233125
1993-11-30
             3868.968534
```

1993-12-31

1994-01-31

1994-03-31

1994-04-30

1994-02-28

5519.666385

1441.232592

1231.003048

1599.614842

1502.310057

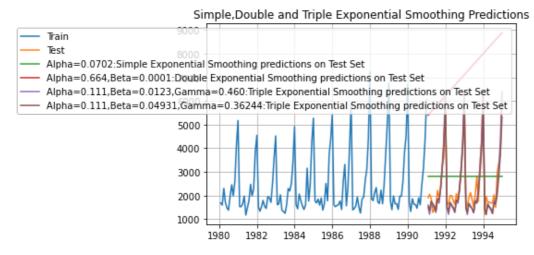
```
1994-05-31
              1398.425757
1994-06-30
              1228.636565
1994-07-31
             1680.694798
1994-08-31
             1649.399690
1994-09-30
              2062.858232
1994-10-31
              2827.790300
1994-11-30
              3738.444773
1994-12-31
              5332.929599
Freq: M, dtype: float64
```

In [93]:

```
## Plotting the Training data, Test data and the forecasted values

plt.plot(train, label='Train')
plt.plot(test, label='Test')

plt.plot(SES_predict, label='Alpha=0.0702:Simple Exponential Smoothing predictions on Test Set')
plt.plot(DES_predict, label='Alpha=0.664, Beta=0.0001:Double Exponential Smoothing predictions on Test Set')
plt.plot(TES_predict, label='Alpha=0.111, Beta=0.0123, Gamma=0.460:Triple Exponential Smoothing predictions on Test Set')
plt.plot(TES_predict_am, label='Alpha=0.111, Beta=0.04931, Gamma=0.36244:Triple Exponential Smoothing predictions on Test Set')
plt.legend(loc='best')
plt.legend(loc='best')
plt.grid()
plt.title('Simple, Double and Triple Exponential Smoothing Predictions');
```



Report model accuracy

```
In [94]:
```

```
print('TES_am RMSE:', mean_squared_error(test.values, TES_predict_am.values, squared=False)
)
```

TES_am RMSE: 407.41029599793075

In [95]:

Out[95]:

NaiveModel	3780.2 4673£
SimpleAverageModel	1329.929685
2pointTrailingMovingAverage	793.036740
4pointTrailingMovingAverage	1155.559612
6pointTrailingMovingAverage	1302.932089
9pointTrailingMovingAverage	1388.801927
Alpha=0.0496,SimpleExponentialSmoothing	1343.723719
Alpha=0.0702,SES	1358.940488
Alpha=0.66499,Beta=0.0001:DES	4879.866573
Alpha=0.1112,Beta=0.01235,Gamma=0.460717:TES	387.165102
Alpha=0.111,Beta=0.04931,Gamma=0.36244:TES	407.410296

We see that the multiplicative seasonality model has not done that well when compared to the additive seasonality Triple Exponential Smoothing model.

There are various other parameters in the models. Please do feel free to play around with those in the hope of getting a better forecast on the test set.

Q5. Check for the stationarity of the data on which the model is being built on using appropriate statistical tests and also mention the hypothesis for the statistical test. If the data is found to be non-stationary, take appropriate steps to make it stationary. Check the new data for stationarity and comment.

Note: Stationarity should be checked at alpha = 0.05.

Check for stationarity of the whole Time Series data.

The Augmented Dickey-Fuller test is an unit root test which determines whether there is a unit root and subsequently whether the series is non-stationary.

The hypothesis in a simple form for the ADF test is:

- *H*₀
 - : The Time Series has a unit root and is thus non-stationary.
- H₁
 - : The Time Series does not have a unit root and is thus stationary.

We would want the series to be stationary for building ARIMA models and thus we would want the p-value of this test to be less than the α value.

```
In [96]:
```

```
from statsmodels.tsa.stattools import adfuller
def test_stationarity(timeseries):

    #Determing rolling statistics
    rolmean = timeseries.rolling(window=7).mean() #determining the rolling mean
    rolstd = timeseries.rolling(window=7).std() #determining the rolling standard devi
ation

#Plot rolling statistics:
    orig = plt.plot(timeseries, color='blue', label='Original')
    mean = plt.plot(rolmean, color='red', label='Rolling Mean')
    std = plt.plot(rolstd, color='black', label = 'Rolling Std')
    plt.legend(loc='best')
```

```
plt.title('Rolling Mean & Standard Deviation')
plt.show(block=False)

#Perform Dickey-Fuller test:
print ('Results of Dickey-Fuller Test:')
dftest = adfuller(timeseries, autolag='AIC')
dfoutput = pd.Series(dftest[0:4], index=['Test Statistic','p-value','#Lags Used','Nu
mber of Observations Used'])
for key,value in dftest[4].items():
    dfoutput['Critical Value (%s)'%key] = value
print (dfoutput,'\n')
```

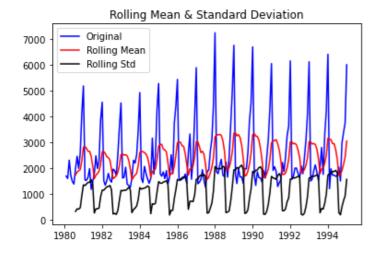
In [97]:

```
dftest = adfuller(df,regression='ct')
print('DF test statistic is %3.3f' %dftest[0])
print('DF test p-value is' ,dftest[1])
print('Number of lags used' ,dftest[2])
```

DF test statistic is -1.832 DF test p-value is 0.6888980541551387 Number of lags used 11

In [98]:

```
test_stationarity(df['Sparkling'].dropna())
```



```
Results of Dickey-Fuller Test:
Test Statistic
                                -1.311362
                                 0.623958
p-value
                                11.000000
#Lags Used
Number of Observations Used
                              168.000000
Critical Value (1%)
                                -3.469886
Critical Value (5%)
                                -2.878903
Critical Value (10%)
                                -2.576027
dtype: float64
```

We see that at 5% significant level the Time Series is non-stationary.

There are various ways that Python allows us to select the appropriate number of lags at which we check whether the Time Series is stationary. To know more about the how to select the various ways, please refer to the link over *here*

Let us take one level of differencing to see whether the series becomes stationary.

In [99]:

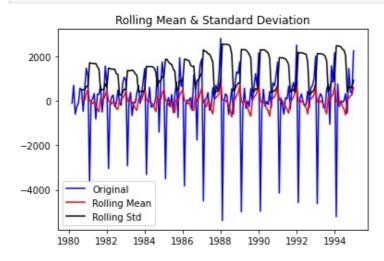
```
dftest = adfuller(df.diff().dropna(),regression='ct')
print('DF test statistic is %3.3f' %dftest[0])
print('DF test p-value is' ,dftest[1])
```

print('Number of lags used' ,dftest[2])

DF test statistic is -42.700 DF test p-value is 0.0 Number of lags used 10

In [100]:

test stationarity(df['Sparkling'].diff().dropna())



Results of Dickey-Fuller Test:

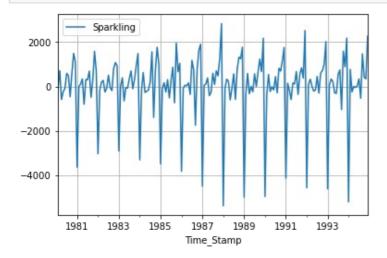
Test Statistic -42.902281
p-value 0.000000
#Lags Used 10.000000

Number of Observations Used 168.000000
Critical Value (1%) -3.469886
Critical Value (5%) -2.878903
Critical Value (10%) -2.576027
dtype: float64

Now, let us go ahead and plot the stationary series.

In [101]:

df.diff().dropna().plot(grid=True);



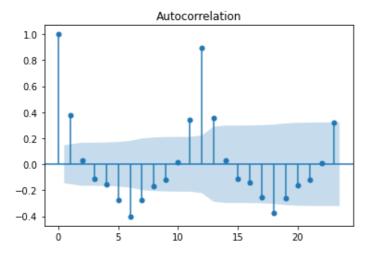
Plot the Autocorrelation and the Partial Autocorrelation function plots on the whole data.

In [102]:

from statsmodels.graphics.tsaplots import plot_acf, plot_pacf

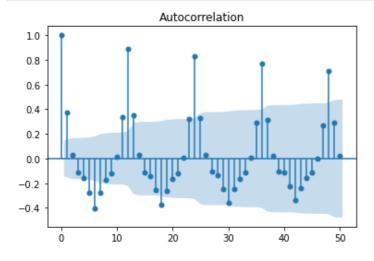
In [103]:

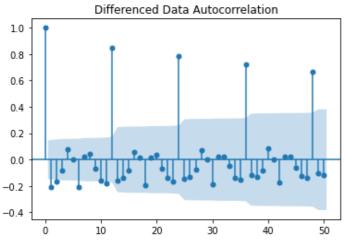
plot acf(df,alpha=0.05);



In [104]:

```
plot_acf(df['Sparkling'],lags=50)
plot_acf(df['Sparkling'].diff().dropna(),lags=50,title='Differenced Data Autocorrelation
')
plt.show() #q=2 (from 2nd plot) and S=6,12 for seasonal arima (from 2nd plot)
```

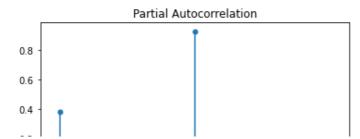




From the above plots, we can say that there seems to be a seasonality in the data.

In [105]:

```
plot_pacf(df,zero=False,alpha=0.05);
```



```
0.2

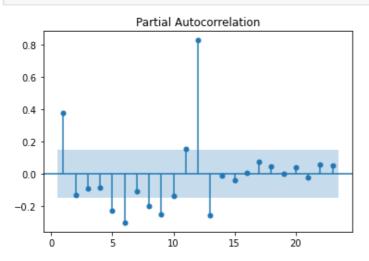
0.0

-0.2

0 5 10 15 20
```

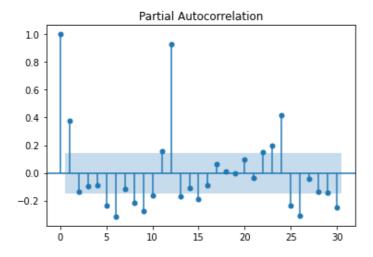
In [106]:

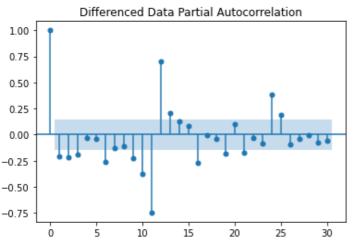
```
plot_pacf(df,zero=False,alpha=0.05,method='ywmle'); # p=1
```



In [107]:

```
plot_pacf(df['Sparkling'],lags=30)
plot_pacf(df['Sparkling'].diff().dropna(),lags=30,title='Differenced Data Partial Autocor
relation')
plt.show() #p=3 (from 2nd plot)
```





Split the data into train and test and plot the training and test data.

Training Data is till the end of 1990. Test Data is from the beginning of 1991 to the last time stamp provided.

```
In [108]:
```

In [109]:

```
train = df[df.index<'1991']
test = df[df.index>='1991']
```

In [110]:

```
## This is to display multiple data frames from one cell from IPython.display import display
```

In [111]:

```
print('First few rows of Training Data')
display(train.head())
print('Last few rows of Training Data')
display(train.tail())
print('First few rows of Test Data')
display(test.head())
print('Last few rows of Test Data')
display(test.tail())
```

First few rows of Training Data

Sparkling

Time_Stamp

1980-01-31	1686
1980-02-29	1591
1980-03-31	2304
1980-04-30	1712
1980-05-31	1471

Last few rows of Training Data

Sparkling

Time_Stamp

1990-08-31	1605
1990-09-30	2424
1990-10-31	3116
1990-11-30	4286
1990-12-31	6047

First few rows of Test Data

Sparkling

Time_Stamp

1991-01-31	1902
1991-02-28	2049
1991-03-31	1874

```
1991-04-30 Sparkling
1279
Time Stamp
1432
```

Last few rows of Test Data

Sparkling

Time_Stamp	
1994-08-31	1495
1994-09-30	2968
1994-10-31	3385
1994-11-30	3729
1994-12-31	5999

In [112]:

```
print(train.shape)
print(test.shape)
(132, 1)
```

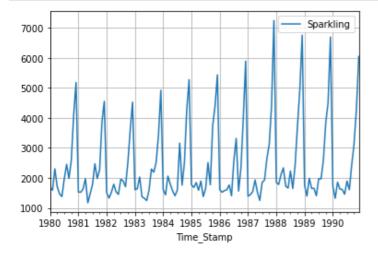
(48, 1)

Check for stationarity of the Training Data Time Series.

Let us plot the training data once.

In [113]:

```
train.plot(grid=True);
```



In [114]:

```
dftest = adfuller(train, regression='ct')
print('DF test statistic is %3.3f' %dftest[0])
print('DF test p-value is' ,dftest[1])
print('Number of lags used' ,dftest[2])
```

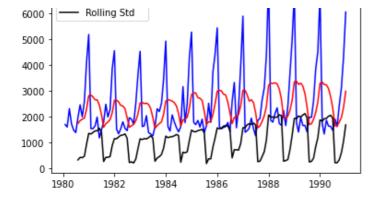
DF test statistic is -2.062 DF test p-value is 0.5674110388593686 Number of lags used 12

In [115]:

```
test_stationarity(train['Sparkling'])
```

Rolling Mean & Standard Deviation

```
7000 - Original Rolling Mean
```



```
Results of Dickey-Fuller Test:
                                 -1.208926
Test Statistic
p-value
                                  0.669744
#Lags Used
                                 12.000000
Number of Observations Used
                                119.000000
Critical Value (1%)
                                -3.486535
Critical Value (5%)
                                 -2.886151
Critical Value (10%)
                                 -2.579896
dtype: float64
```

The training data is non-stationary at 95% confidence level. Let us take a first level of differencing to stationarize the Time Series.

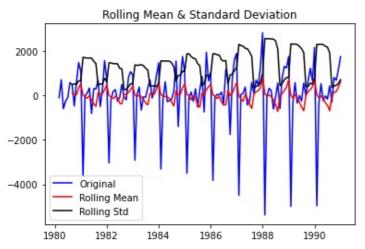
In [116]:

```
dftest = adfuller(train.diff().dropna(),regression='ct')
print('DF test statistic is %3.3f' %dftest[0])
print('DF test p-value is' ,dftest[1])
print('Number of lags used' ,dftest[2])
```

DF test statistic is -7.968 DF test p-value is 8.479210655514366e-11 Number of lags used 11

In [117]:

```
test stationarity(train['Sparkling'].diff().dropna())
```

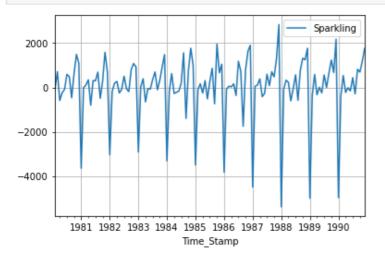


```
Results of Dickey-Fuller Test:
                               -8.005007e+00
Test Statistic
                                2.280104e-12
p-value
                               1.100000e+01
#Lags Used
Number of Observations Used
                               1.190000e+02
Critical Value (1%)
                               -3.486535e+00
Critical Value (5%)
                               -2.886151e+00
                               -2.579896e+00
Critical Value (10%)
dtype: float64
```

Now, let us go ahead and plot the differenced training data.

```
In [118]:
```

```
train.diff().dropna().plot(grid=True);
```



Note: If the series is non-stationary, stationarize the Time Series by taking a difference of the Time Series. Then we can use this particular differenced series to train the ARIMA models. We do not need to worry about stationarity for the Test Data because we are not building any models on the Test Data, we are evaluating our models over there. You can look at other kinds of transformations as part of making the time series stationary like taking logarithms.

```
In [119]:
```

Q6. Build an automated version of the ARIMA/SARIMA model in which the parameters are selected using the lowest Akaike Information Criteria (AIC) on the training data and evaluate this model on the test data using RMSE.

Building an automated version of ARIMA model by using lowest AIC

```
In [120]:
```

```
import itertools
p = q = range(0, 4)
d= range(1,2)
pdq = list(itertools.product(p, d, q))
print('Examples of the parameter combinations for the Model')
for i in range(0,len(pdq)):
    print('Model: {}'.format(pdq[i]))
Examples of the parameter combinations for the Model
```

```
Examples of the parameter combinations for the Mode Model: (0, 1, 0)
Model: (0, 1, 1)
Model: (0, 1, 2)
Model: (0, 1, 3)
Model: (1, 1, 0)
Model: (1, 1, 1)
Model: (1, 1, 2)
Model: (1, 1, 3)
Model: (2, 1, 0)
```

```
Model: (2, 1, 2)
Model: (2, 1, 3)
Model: (3, 1, 0)
Model: (3, 1, 1)
Model: (3, 1, 2)
Model: (3, 1, 3)
In [121]:
ARIMA AIC = pd.DataFrame(columns=['param', 'AIC'])
ARIMA AIC
Out[121]:
  param AIC
In [122]:
from statsmodels.tsa.arima.model import ARIMA
for param in pdq: # running a loop within the pdq parameters defined by itertools
    ARIMA model = ARIMA(train['Sparkling'].values, order=param).fit() #fitting the ARIMA m
    #using the parameters from the loop
   print('ARIMA{} - AIC:{}'.format(param, ARIMA model.aic)) #printing the parameters and
the AIC
    #from the fitted models
    ARIMA AIC = ARIMA AIC.append({'param':param, 'AIC': ARIMA model.aic}, ignore index=T
rue)
    #appending the AIC values and the model parameters to the previously created data fra
me
    #for easier understanding and sorting of the AIC values
ARIMA(0, 1, 0) - AIC:2267.6630357855465
ARIMA(0, 1, 1) - AIC:2263.060015591336
ARIMA(0, 1, 2) - AIC:2234.4083231283275
ARIMA(0, 1, 3) - AIC:2233.994857753515
ARIMA(1, 1, 0) - AIC:2266.6085393190087
ARIMA(1, 1, 1) - AIC:2235.755094673383
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\statespace\sarimax
.py:977: UserWarning: Non-invertible starting MA parameters found. Using zeros as startin
g parameters.
  warn('Non-invertible starting MA parameters found.'
ARIMA(1, 1, 2) - AIC:2234.527200452466
ARIMA(1, 1, 3) - AIC:2235.607816390617
ARIMA(2, 1, 0) - AIC:2260.365743968086
ARIMA(2, 1, 1) - AIC:2233.777626239922
ARIMA(2, 1, 2) - AIC:2213.509212306332
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\base\model.py:566: Con
vergenceWarning: Maximum Likelihood optimization failed to converge. Check mle retvals
  warnings.warn("Maximum Likelihood optimization failed to "
ARIMA(2, 1, 3) - AIC:2232.921136688177
ARIMA(3, 1, 0) - AIC:2257.72337899794
ARIMA(3, 1, 1) - AIC:2235.498878057432
ARIMA(3, 1, 2) - AIC:2230.759636959836
ARIMA(3, 1, 3) - AIC:2221.4566102276085
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\base\model.py:566: Con
vergenceWarning: Maximum Likelihood optimization failed to converge. Check mle retvals
  warnings.warn("Maximum Likelihood optimization failed to "
In [123]:
ARIMA AIC.sort values(by='AIC', ascending=True).head()
Out[123]:
```

Model: (2, 1, 1)

 param
 AIC

 10
 (2, 1, 2)
 2213.509212

 15
 (3, 1, 3)
 2221.456610

 14
 (3, 1, 2)
 2230.759637

 11
 (2, 1, 3)
 2232.921137

 9
 (2, 1, 1)
 2233.777626

In [124]:

```
auto_ARIMA = ARIMA(train, order=(2,1,2), freq='M')
results_auto_ARIMA = auto_ARIMA.fit()
print(results_auto_ARIMA.summary())
```

C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py: 524: ValueWarning: No frequency information was provided, so inferred frequency M will be used.

warnings.warn('No frequency information was'

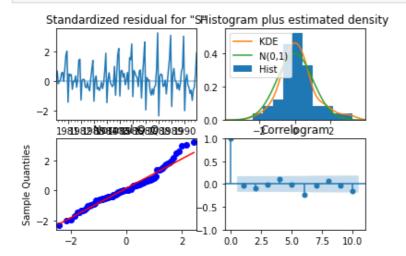
		-10.01.07				
		SAI	RIMAX Resul	ts		
Dep. Varia Model: Date: Time: Sample:	Sı		, 2) Log 2021 AIC 3:58 BIC 1980 HQIC			132 -1101.755 2213.509 2227.885 2219.351
Covariance	e Type: 		opg			
	coef	std err	Z	P> z	[0.025	0.975]
ar.L2 ma.L1 ma.L2	1.3121 -0.5593 -1.9917 0.9999 1.099e+06	0.072 0.109 0.110	-7.741 -18.217 9.109	0.000 0.000 0.000	-0.701 -2.206 0.785	-0.418 -1.777 1.215
Prob(Q): Heteroske	(L1) (Q): dasticity (H): two-sided):		2.43	Jarque-Bera Prob(JB): Skew: Kurtosis:	(JB):	14.4 0.0 0.6 4.0

Warnings:

- [1] Covariance matrix calculated using the outer product of gradients (complex-step).
- [2] Covariance matrix is singular or near-singular, with condition number 3.24e+27. Stand ard errors may be unstable.

In [125]:

results auto ARIMA.plot diagnostics();



Predict on the Test Set using automated ARIMA model and evaluate the model

```
In [126]:
predicted auto ARIMA = results auto ARIMA.forecast(steps=len(test))
In [127]:
def mean absolute_percentage_error(y_true, y_pred):
    return np.mean((np.abs(y true-y pred))/(y true))*100
## Importing the mean squared error function from sklearn to calculate the RMSE
from sklearn.metrics import mean_squared_error
In [128]:
rmse = mean squared error(test['Sparkling'], predicted auto ARIMA, squared=False)
mape = mean absolute percentage error(test['Sparkling'], predicted auto ARIMA)
print('RMSE:',rmse,'\nMAPE:',mape)
RMSE: 1329.406019119613
MAPE: 43.893226520603946
In [129]:
resultsDf 10 = pd.DataFrame({'Test RMSE': rmse,'MAPE':mape}
                           ,index=['ARIMA(2,1,2)'])
resultsDf = pd.concat([resultsDf, resultsDf 10])
resultsDf
Out[129]:
```

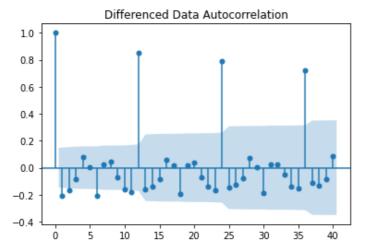
	RMSE	Test RMSE	MAPE
RegressionOnTime	1378.580256	NaN	NaN
NaiveModel	3780.226721	NaN	NaN
SimpleAverageModel	1329.929685	NaN	NaN
2pointTrailingMovingAverage	793.036740	NaN	NaN
4pointTrailingMovingAverage	1155.559612	NaN	NaN
6pointTrailingMovingAverage	1302.932089	NaN	NaN
9pointTrailingMovingAverage	1388.801927	NaN	NaN
Alpha=0.0496,SimpleExponentialSmoothing	1343.723719	NaN	NaN
Alpha=0.0702,SES	1358.940488	NaN	NaN
Alpha=0.66499,Beta=0.0001:DES	4879.866573	NaN	NaN
Alpha=0.1112,Beta=0.01235,Gamma=0.460717:TES	387.165102	NaN	NaN
Alpha=0.111,Beta=0.04931,Gamma=0.36244:TES	407.410296	NaN	NaN
ARIMA(2,1,2)	NaN	1329.406019	43.893227

Build an Automated version of a SARIMA model for which the best parameters are selected in accordance with the lowest Akaike Information Criteria (AIC).

Let us look at the ACF plot once more to understand the seasonal parameter for the SARIMA model.

```
In [130]:
plot_acf(df['Sparkling'].diff().dropna(),lags=40,title='Differenced Data Autocorrelation
```





We see that there can be a seasonality of 6 as well as 12. We will run our auto SARIMA models by setting seasonality both as 6 and 12.

Setting the seasonality as 6 for the first iteration of the auto SARIMA model.

```
In [131]:
```

```
import itertools
p = q = range(0, 3)
d= range(1,2)
D = range(0,1)
pdq = list(itertools.product(p, d, q))
model_pdq = [(x[0], x[1], x[2], 6) for x in list(itertools.product(p, D, q))]
print('Examples of some parameter combinations for Model...')
for i in range(1,len(pdq)):
    print('Model: {}{}'.format(pdq[i], model_pdq[i]))
Examples of some parameter combinations for Model...
Model: (0, 1, 1) (0, 0, 1, 6)
Model: (0, 1, 2) (0, 0, 2, 6)
```

Model: (0, 1, 2) (0, 0, 2, 6)

Model: (1, 1, 0) (1, 0, 0, 6)

Model: (1, 1, 1) (1, 0, 1, 6)

Model: (1, 1, 2) (1, 0, 2, 6)

Model: (2, 1, 0) (2, 0, 0, 6)

Model: (2, 1, 1) (2, 0, 1, 6)

Model: (2, 1, 2) (2, 0, 2, 6)

In [132]:

```
SARIMA_AIC = pd.DataFrame(columns=['param', 'seasonal', 'AIC'])
SARIMA_AIC
```

Out[132]:

param seasonal AIC

In [133]:

```
results SARIMA = SARIMA model.fit(maxiter=1000)
        print('SARIMA{}x{} - AIC:{}'.format(param, param_seasonal, results_SARIMA.aic))
        SARIMA AIC = SARIMA AIC.append({'param':param,'seasonal':param seasonal ,'AIC':
results SARIMA.aic}, ignore index=True)
SARIMA(0, 1, 0) \times (0, 0, 0, 6) - AIC:2251.3597196862966
SARIMA(0, 1, 0)x(0, 0, 1, 6) - AIC:2152.37807617163
SARIMA(0, 1, 0)x(0, 0, 2, 6) - AIC:1955.6355536885285
SARIMA(0, 1, 0) \times (1, 0, 0, 6) - AIC:2164.4097581959904
SARIMA(0, 1, 0) \times (1, 0, 1, 6) - AIC:2079.5599844427693
SARIMA(0, 1, 0)x(1, 0, 2, 6) - AIC:1926.9360123815902
SARIMA(0, 1, 0)x(2, 0, 0, 6) - AIC:1839.4012986872267
SARIMA(0, 1, 0)x(2, 0, 1, 6) - AIC:1841.1993617510798
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\base\model.py:566: Con
vergenceWarning: Maximum Likelihood optimization failed to converge. Check mle retvals
 warnings.warn("Maximum Likelihood optimization failed to "
SARIMA(0, 1, 0)\times(2, 0, 2, 6) - AIC:1810.917780565727
SARIMA(0, 1, 1)\times(0, 0, 0, 6) - AIC:2230.1629078505825
SARIMA(0, 1, 1)x(0, 0, 1, 6) - AIC:2130.5652859082825
SARIMA(0, 1, 1)x(0, 0, 2, 6) - AIC:1918.1876339541468
SARIMA(0, 1, 1)x(1, 0, 0, 6) - AIC:2139.5732428785805
SARIMA(0, 1, 1)x(1, 0, 1, 6) - AIC:2006.5174298135562
SARIMA(0, 1, 1)x(1, 0, 2, 6) - AIC:1855.7093274257234
SARIMA(0, 1, 1)x(2, 0, 0, 6) - AIC:1798.7885103818865
SARIMA(0, 1, 1)x(2, 0, 1, 6) - AIC:1800.7717933228848
SARIMA(0, 1, 1)x(2, 0, 2, 6) - AIC:1741.7036718705149
SARIMA(0, 1, 2)x(0, 0, 0, 6) - AIC:2187.441010168729
SARIMA(0, 1, 2)\times(0, 0, 1, 6) - AIC:2087.6843840215843
SARIMA(0, 1, 2)\times(0, 0, 2, 6) - AIC:1886.1151460686604
SARIMA(0, 1, 2)x(1, 0, 0, 6) - AIC:2129.7395689235263
SARIMA(0, 1, 2)x(1, 0, 1, 6) - AIC:1988.4111271789998
SARIMA(0, 1, 2)x(1, 0, 2, 6) - AIC:1839.6938117659683
SARIMA(0, 1, 2)\times(2, 0, 0, 6) - AIC:1791.653707904919
SARIMA(0, 1, 2)x(2, 0, 1, 6) - AIC:1793.6190985640728
SARIMA(0, 1, 2)x(2, 0, 2, 6) - AIC:1727.8888051077456
SARIMA(1, 1, 0) \times (0, 0, 0, 6) - AIC:2250.3181267386713
SARIMA(1, 1, 0)x(0, 0, 1, 6) - AIC:2151.078268308422
SARIMA(1, 1, 0)x(0, 0, 2, 6) - AIC:1953.3652245477156
SARIMA(1, 1, 0)x(1, 0, 0, 6) - AIC:2146.1836648562185
SARIMA(1, 1, 0)x(1, 0, 1, 6) - AIC:2073.981368525434
SARIMA(1, 1, 0) \times (1, 0, 2, 6) - AIC:1917.5889468332687
SARIMA(1, 1, 0)x(2, 0, 0, 6) - AIC:1813.2423977988667
SARIMA(1, 1, 0) \times (2, 0, 1, 6) - AIC:1814.8301602827999
SARIMA(1, 1, 0) \times (2, 0, 2, 6) - AIC:1791.3715257067154
SARIMA(1, 1, 1)\times(0, 0, 0, 6) - AIC:2204.934049154573
SARIMA(1, 1, 1)\times(0, 0, 1, 6) - AIC:2103.2471520743843
SARIMA(1, 1, 1)x(0, 0, 2, 6) - AIC:1906.3976381404113
SARIMA(1, 1, 1)x(1, 0, 0, 6) - AIC:2109.6671209734727
SARIMA(1, 1, 1)\times(1, 0, 1, 6) - AIC:2005.6125660487137
SARIMA(1, 1, 1)x(1, 0, 2, 6) - AIC:1856.0775237169466
SARIMA(1, 1, 1) \times (2, 0, 0, 6) - AIC:1776.9417670616047
SARIMA(1, 1, 1) \times (2, 0, 1, 6) - AIC:1778.8222557972151
SARIMA(1, 1, 1)x(2, 0, 2, 6) - AIC:1743.3797786882963
SARIMA(1, 1, 2)x(0, 0, 0, 6) - AIC:2188.463345050375
SARIMA(1, 1, 2)x(0, 0, 1, 6) - AIC:2089.132092446035
SARIMA(1, 1, 2)x(0, 0, 2, 6) - AIC:1888.1170920288248
SARIMA(1, 1, 2)\times(1, 0, 0, 6) - AIC:2108.5645510269565
SARIMA(1, 1, 2)x(1, 0, 1, 6) - AIC:1987.1476985820211
SARIMA(1, 1, 2)x(1, 0, 2, 6) - AIC:1838.975595322297
SARIMA(1, 1, 2) \times (2, 0, 0, 6) - AIC:1773.422939038036
SARIMA(1, 1, 2)x(2, 0, 1, 6) - AIC:1775.2584006213904
SARIMA(1, 1, 2) \times (2, 0, 2, 6) - AIC:1729.938537373231
SARIMA(2, 1, 0)\times(0, 0, 0, 6) - AIC:2227.302761872421
SARIMA(2, 1, 0)\times(0, 0, 1, 6) - AIC:2145.357699120112
SARIMA(2, 1, 0)x(0, 0, 2, 6) - AIC:1945.156142608436
SARIMA(2, 1, 0)\times(1, 0, 0, 6) - AIC:2124.907178631821
```

SARIMA(2, 1, 0) \times (1, 0, 1, 6) - AIC:2054.170071229263 SARIMA(2, 1, 0) \times (1, 0, 2, 6) - AIC:1915.6336922498174 SARIMA(2, 1, 0) \times (2, 0, 0, 6) - AIC:1782.7357821344322 SARIMA(2, 1, 0) \times (2, 0, 1, 6) - AIC:1782.359816911855 SARIMA(2, 1, 0) \times (2, 0, 2, 6) - AIC:1760.3426708650063

```
SARIMA(2, 1, 1)\times(0, 0, 0, 6) - AIC:2199.858613145845
SARIMA(2, 1, 1) \times (0, 0, 1, 6) - AIC:2103.0859058222977
SARIMA(2, 1, 1) \times (0, 0, 2, 6) - AIC:1903.0416542541525
SARIMA(2, 1, 1) \times (1, 0, 0, 6) - AIC:2088.1336363678556
SARIMA(2, 1, 1)x(1, 0, 1, 6) - AIC:1997.3692882240744
SARIMA(2, 1, 1)x(1, 0, 2, 6) - AIC:1852.7863807284743
SARIMA(2, 1, 1)x(2, 0, 0, 6) - AIC:1761.267512870908
SARIMA(2, 1, 1)x(2, 0, 1, 6) - AIC:1763.191458211221
SARIMA(2, 1, 1)x(2, 0, 2, 6) - AIC:1744.0407495463405
SARIMA(2, 1, 2)\times(0, 0, 0, 6) - AIC:2176.8681147684483
SARIMA(2, 1, 2)\times(0, 0, 1, 6) - AIC:2068.7782222388523
SARIMA(2, 1, 2)x(0, 0, 2, 6) - AIC:1889.7875404630274
SARIMA(2, 1, 2)x(1, 0, 0, 6) - AIC:2074.1102217625075
SARIMA(2, 1, 2)x(1, 0, 1, 6) - AIC:1955.605896996941
SARIMA(2, 1, 2)x(1, 0, 2, 6) - AIC:1825.8956954615257
SARIMA(2, 1, 2)x(2, 0, 0, 6) - AIC:1763.272840917471
SARIMA(2, 1, 2)x(2, 0, 1, 6) - AIC:1760.8267435833768
SARIMA(2, 1, 2)x(2, 0, 2, 6) - AIC:1782.3426569473315
In [134]:
SARIMA AIC.sort values(by=['AIC']).head()
Out[134]:
   param seasonal
26 (0, 1, 2) (2, 0, 2, 6) 1727.888805
53 (1, 1, 2) (2, 0, 2, 6) 1729.938537
17 (0, 1, 1) (2, 0, 2, 6) 1741.703672
44 (1, 1, 1) (2, 0, 2, 6) 1743.379779
71 (2, 1, 1) (2, 0, 2, 6) 1744.040750
In [135]:
import statsmodels.api as sm
auto SARIMA 6 = sm.tsa.statespace.SARIMAX(train['Sparkling'].values,
                                  order=(0, 1, 2),
                                  seasonal order=(2, 0, 2, 6),
                                  enforce stationarity=False,
                                  enforce invertibility=False)
results auto SARIMA 6 = auto SARIMA 6.fit(maxiter=1000)
print(results auto SARIMA 6.summary())
                                       SARIMAX Results
______
Dep. Variable:
                                                 y No. Observations:
2
                   SARIMAX(0, 1, 2)\times(2, 0, 2, 6)
                                                     Log Likelihood
                                                                                     -856.94
Model:
Date:
                                 Sun, 23 May 2021
                                                     AIC
                                                                                     1727.88
                                          23:04:48
Time:
                                                     BIC
                                                                                     1747.16
                                                 0
                                                    HQIC
                                                                                     1735.7
Sample:
13
                                             - 132
Covariance Type:
                                               opg
______
                coef std err z P>|z| [0.025 0.975]
______

    ma.L1
    -0.7851
    0.103
    -7.655
    0.000
    -0.986
    -0.584

    ma.L2
    -0.0976
    0.112
    -0.871
    0.384
    -0.317
    0.122

    ar.S.L6
    0.0022
    0.026
    0.084
    0.933
    -0.049
    0.053

    ar.S.L12
    1.0396
    0.018
    58.254
    0.000
    1.005
    1.075
```

0111(1111(2) 1) 0/2(2) 0/ 2/ 0/ 1110.1/00.5/20/000000

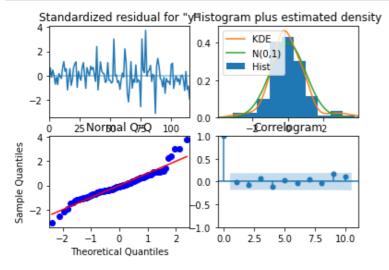
```
ma.S.L6
         0.0427
                 0.143
                        0.298
                               0.766
                                      -0.238
                                               0.324
       -0.6202
                                      -0.797
                 0.090
                        -6.878
                               0.000
                                               -0.443
ma.S.L12
sigma2 1.475e+05 1.42e+04
                       10.372
                               0.000
                                      1.2e+05 1.75e+05
______
                        0.00
Ljung-Box (L1) (Q):
                             Jarque-Bera (JB):
                                                   38.96
                            Prob(JB):
Prob(Q):
                        0.97
                                                   0.00
Heteroskedasticity (H):
                        2.85
                             Skew:
                                                   0.58
Prob(H) (two-sided):
                        0.00
                                                   5.59
                             Kurtosis:
______
```

Warnings:

[1] Covariance matrix calculated using the outer product of gradients (complex-step).

In [136]:

```
results_auto_SARIMA_6.plot_diagnostics()
plt.show()
```



Predict on the Test Set using automated SARIMA model (seasonality=6) and evaluate the model.

In [137]:

```
predicted_auto_SARIMA_6 = results_auto_SARIMA_6.get_forecast(steps=len(test))
```

In [138]:

```
predicted_auto_SARIMA_6.summary_frame(alpha=0.05).head()
```

Out[138]:

у	mean	mean_se	mean_ci_lower	mean_ci_upper
0	1375.723266	384.067675	622.964456	2128.482077
1	1116.867397	392.833844	346.927212	1886.807582
2	1667.668042	395.405759	892.686995	2442.649089
3	1528.423107	397.964142	748.427721	2308.418492
4	1372.367185	400.506237	587.389385	2157.344984

In [139]:

RMSE: 572.0193624004195 MAPE: 22.966429718234306

Tn [1/10] •

	RMSE	Test RMSE	MAPE
RegressionOnTime	1378.580256	NaN	NaN
NaiveModel	3780.226721	NaN	NaN
SimpleAverageModel	1329.929685	NaN	NaN
2pointTrailingMovingAverage	793.036740	NaN	NaN
4pointTrailingMovingAverage	1155.559612	NaN	NaN
6pointTrailingMovingAverage	1302.932089	NaN	NaN
9pointTrailingMovingAverage	1388.801927	NaN	NaN
Alpha=0.0496,SimpleExponentialSmoothing	1343.723719	NaN	NaN
Alpha=0.0702,SES	1358.940488	NaN	NaN
Alpha=0.66499,Beta=0.0001:DES	4879.866573	NaN	NaN
Alpha=0.1112,Beta=0.01235,Gamma=0.460717:TES	387.165102	NaN	NaN
Alpha=0.111,Beta=0.04931,Gamma=0.36244:TES	407.410296	NaN	NaN
ARIMA(2,1,2)	NaN	1329.406019	43.893227
SARIMA(0,1,2)(2,0,2,6)	NaN	572.019362	22.966430

Setting the seasonality as 12 for the first iteration of the auto SARIMA model.

```
In [141]:
import itertools
p = q = range(0, 3)
d= range(1,2)
D = range(0,1)
pdq = list(itertools.product(p, d, q))
model_pdq = [(x[0], x[1], x[2], 12)  for x in list(itertools.product(p, D, q))]
print('Examples of some parameter combinations for Model...')
for i in range(1,len(pdq)):
    print('Model: {}{}'.format(pdq[i], model_pdq[i]))
Examples of some parameter combinations for Model...
Model: (0, 1, 1)(0, 0, 1, 12)
Model: (0, 1, 2)(0, 0, 2, 12)
Model: (1, 1, 0)(1, 0, 0, 12)
Model: (1, 1, 1)(1, 0, 1, 12)
Model: (1, 1, 2)(1, 0, 2, 12)
Model: (2, 1, 0)(2, 0, 0, 12)
Model: (2, 1, 1)(2, 0, 1, 12)
Model: (2, 1, 2)(2, 0, 2, 12)
In [142]:
SARIMA AIC = pd.DataFrame(columns=['param', 'seasonal', 'AIC'])
SARIMA AIC
Out[142]:
 param seasonal AIC
In [143]:
```

```
import statsmoders.api as sm
for param in pdq:
    for param seasonal in model pdg:
        SARIMA model = sm.tsa.statespace.SARIMAX(train['Sparkling'].values,
                                               order=param,
                                               seasonal order=param_seasonal,
                                               enforce stationarity=False,
                                               enforce invertibility=False)
        results SARIMA = SARIMA model.fit(maxiter=1000)
        print('SARIMA{}x{} - AIC:{}'.format(param, param seasonal, results SARIMA.aic))
        SARIMA AIC = SARIMA AIC.append({'param':param,'seasonal':param seasonal ,'AIC':
results SARIMA.aic}, ignore index=True)
SARIMA(0, 1, 0)\times(0, 0, 0, 12) - AIC:2251.3597196862966
SARIMA(0, 1, 0)x(0, 0, 1, 12) - AIC:1956.2614616846013
SARIMA(0, 1, 0)x(0, 0, 2, 12) - AIC:1723.1533640236748
SARIMA(0, 1, 0)x(1, 0, 0, 12) - AIC:1837.4366022456675
SARIMA(0, 1, 0)x(1, 0, 1, 12) - AIC:1806.9905301388796
SARIMA(0, 1, 0)x(1, 0, 2, 12) - AIC:1633.2108735791871
SARIMA(0, 1, 0)\times(2, 0, 0, 12) - AIC:1648.3776153470856
SARIMA(0, 1, 0)x(2, 0, 1, 12) - AIC:1647.2054158610893
SARIMA(0, 1, 0)x(2, 0, 2, 12) - AIC:1630.9898053920836
SARIMA(0, 1, 1)x(0, 0, 0, 12) - AIC:2230.1629078505825
SARIMA(0, 1, 1) \times (0, 0, 1, 12) - AIC:1923.7688649566526
SARIMA(0, 1, 1) \times (0, 0, 2, 12) - AIC:1692.7089573264204
SARIMA(0, 1, 1)x(1, 0, 0, 12) - AIC:1797.1795881838495
SARIMA(0, 1, 1)x(1, 0, 1, 12) - AIC:1738.090319377595
SARIMA(0, 1, 1)x(1, 0, 2, 12) - AIC:1570.1509144030938
SARIMA(0, 1, 1)x(2, 0, 0, 12) - AIC:1605.6751954177198
SARIMA(0, 1, 1)x(2, 0, 1, 12) - AIC:1599.2245085400543
SARIMA(0, 1, 1)\times(2, 0, 2, 12) - AIC:1570.4018826079407
SARIMA(0, 1, 2)x(0, 0, 0, 12) - AIC:2187.441010168729
SARIMA(0, 1, 2)x(0, 0, 1, 12) - AIC:1887.9128007202514
SARIMA(0, 1, 2)x(0, 0, 2, 12) - AIC:1659.878989082306
SARIMA(0, 1, 2)x(1, 0, 0, 12) - AIC:1790.032633215157
SARIMA(0, 1, 2)x(1, 0, 1, 12) - AIC:1724.1675070727338
SARIMA(0, 1, 2)x(1, 0, 2, 12) - AIC:1557.1605068181761
SARIMA(0, 1, 2)\times(2, 0, 0, 12) - AIC:1603.9654774397775 SARIMA(0, 1, 2)\times(2, 0, 1, 12) - AIC:1600.5438855184834
\mathtt{SARIMA}\,(0,\ 1,\ 2)\,\mathtt{x}\,(2,\ 0,\ 2,\ 12)\ -\ \mathtt{AIC:}1557.121578530881
SARIMA(1, 1, 0) \times (0, 0, 12) - AIC:2250.3181267386713
SARIMA(1, 1, 0) \times (0, 0, 1, 12) - AIC:1954.3938339904391
SARIMA(1, 1, 0)x(0, 0, 2, 12) - AIC:1721.2688476354874
SARIMA(1, 1, 0)\times(1, 0, 0, 12) - AIC:1811.2440279330933
SARIMA(1, 1, 0)x(1, 0, 1, 12) - AIC:1788.5343594768467
SARIMA(1, 1, 0)\times(1, 0, 2, 12) - AIC:1616.4894405860919
SARIMA(1, 1, 0)\times(2, 0, 0, 12) - AIC:1621.6355080129133
SARIMA(1, 1, 0)x(2, 0, 1, 12) - AIC:1617.1356133219958
SARIMA(1, 1, 0)x(2, 0, 2, 12) - AIC:1616.5412067483685
SARIMA(1, 1, 1) \times (0, 0, 0, 12) - AIC:2204.934049154573
SARIMA(1, 1, 1)\times(0, 0, 1, 12) - AIC:1907.3558974127466
SARIMA(1, 1, 1)\times(0, 0, 2, 12) - AIC:1678.098135263109
SARIMA(1, 1, 1)x(1, 0, 0, 12) - AIC:1775.1424466870326
SARIMA(1, 1, 1)x(1, 0, 1, 12) - AIC:1739.7120332657666
SARIMA(1, 1, 1)x(1, 0, 2, 12) - AIC:1571.3248863599367
SARIMA(1, 1, 1)x(2, 0, 0, 12) - AIC:1590.6161606879166
SARIMA(1, 1, 1)x(2, 0, 1, 12) - AIC:1586.3142234344425
SARIMA(1, 1, 1) \times (2, 0, 2, 12) - AIC:1571.806997673572
SARIMA(1, 1, 2)x(0, 0, 12) - AIC:2188.463345050375
SARIMA(1, 1, 2) \times (0, 0, 1, 12) - AIC:1889.7708307500739
SARIMA(1, 1, 2)\times(0, 0, 2, 12) - AIC:1659.6291421455642
SARIMA(1, 1, 2)x(1, 0, 0, 12) - AIC:1771.8259799349605
SARIMA(1, 1, 2)x(1, 0, 1, 12) - AIC:1723.99521831177
```

SARIMA(1, 1, 2)×(1, 0, 2, 12) - AIC:1555.5842478283373 SARIMA(1, 1, 2)×(2, 0, 0, 12) - AIC:1588.4216931771698 SARIMA(1, 1, 2)×(2, 0, 1, 12) - AIC:1585.5152893733252 SARIMA(1, 1, 2)×(2, 0, 2, 12) - AIC:1556.076790161543 SARIMA(2, 1, 0)×(0, 0, 0, 12) - AIC:2227.302761872421 SARIMA(2, 1, 0)×(0, 0, 1, 12) - AIC:1946.4383435413138 SARIMA(2, 1, 0)×(0, 0, 2, 12) - AIC:1711.4123039860508 SARIMA(2, 1, 0)×(1, 0, 12) - AIC:1780.7646066056914

```
1110.1100.1010000000
SARIMA(2, 1, 0)\times(1, 0, 1, 12) - AIC:1756.9357349691973
SARIMA(2, 1, 0)x(1, 0, 2, 12) - AIC:1600.9702206733857
SARIMA(2, 1, 0)x(2, 0, 0, 12) - AIC:1592.240346481407
SARIMA(2, 1, 0)x(2, 0, 1, 12) - AIC:1587.6344987869593
SARIMA(2, 1, 0)x(2, 0, 2, 12) - AIC:1585.9191733762755
SARIMA(2, 1, 1) \times (0, 0, 12) - AIC:2199.858613145845
SARIMA(2, 1, 1)\times(0, 0, 1, 12) - AIC:1905.0209495390427
SARIMA(2, 1, 1)x(0, 0, 2, 12) - AIC:1675.423408033302
SARIMA(2, 1, 1)x(1, 0, 0, 12) - AIC:1792.8234290502423
SARIMA(2, 1, 1)x(1, 0, 1, 12) - AIC:1740.0911246863095
SARIMA(2, 1, 1)x(1, 0, 2, 12) - AIC:1571.9886809388709
SARIMA(2, 1, 1)x(2, 0, 0, 12) - AIC:1577.123506089788
SARIMA(2, 1, 1)\times(2, 0, 1, 12) - AIC:1573.1595853781935
SARIMA(2, 1, 1)x(2, 0, 2, 12) - AIC:1572.3428647349142
SARIMA(2, 1, 2)x(0, 0, 0, 12) - AIC:2176.8681147684483
SARIMA(2, 1, 2)x(0, 0, 1, 12) - AIC:1892.2372631948035
SARIMA(2, 1, 2)x(0, 0, 2, 12) - AIC:1661.5523433366088
SARIMA(2, 1, 2)x(1, 0, 0, 12) - AIC:1757.214097728816
SARIMA(2, 1, 2)x(1, 0, 1, 12) - AIC:1725.6086050460815
SARIMA(2, 1, 2)x(1, 0, 2, 12) - AIC:1557.340402131229
SARIMA(2, 1, 2)x(2, 0, 0, 12) - AIC:1625.2275288931048
SARIMA(2, 1, 2)x(2, 0, 1, 12) - AIC:1573.5435589066155
SARIMA(2, 1, 2)x(2, 0, 2, 12) - AIC:1557.8401270149407
```

In [144]:

```
SARIMA_AIC.sort_values(by=['AIC']).head()
```

Out[144]:

	param	seasonal	AIC
50	(1, 1, 2)	(1, 0, 2, 12)	1555.584248
53	(1, 1, 2)	(2, 0, 2, 12)	1556.076790
26	(0, 1, 2)	(2, 0, 2, 12)	1557.121579
23	(0, 1, 2)	(1, 0, 2, 12)	1557.160507
77	(2, 1, 2)	(1, 0, 2, 12)	1557.340402

In [145]:

SARIMAX Results

```
Dep. Variable:
                                                       No. Observations:
                                                                                               1
32
Model:
                    SARIMAX(1, 1, 2)\times(1, 0, 2, 12)
                                                        Log Likelihood
                                                                                          -770.7
92
Date:
                                    Sun, 23 May 2021
                                                        AIC
                                                                                          1555.5
84
Time:
                                             23:05:54
                                                        BIC
                                                                                          1574.
095
                                                    0
                                                        HQIC
                                                                                          1563.
Sample:
083
```

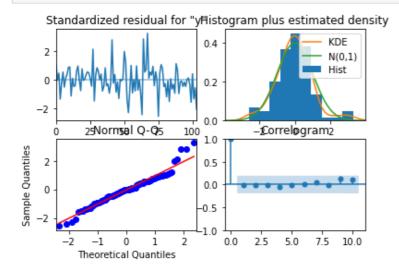
========	==========	=========		:=========	=========	========
	coef	std err	Z	P> z	[0.025	0.975]
ar.L1	-0.6283	0.255	-2.464	0.014	-1.128	-0.128
ma.L1 ma.L2	-0.1040 -0.7277	0.225 0.154	-0.463 -4.736	0.644	-0.545 -1.029	0.337 -0.427
ar.S.L12	1.0439	0.014	72.834	0.000	1.016	1.072
ma.S.L12 ma.S.L24	-0.5550 -0.1354	0.098 0.120	-5.663 -1.133	0.000 0.257	-0.747 -0.370	-0.363 0.099
sigma2	1.506e+05	2.03e+04	7.401	0.000	1.11e+05	1.9e+05
Ljung-Box (L1) (Q): Prob(Q):			0.04	Jarque-Bera Prob(JB):	(JB):	11.
	dasticity (H): two-sided):	:	1.47 0.26	Skew: Kurtosis:		0. 4.

Warnings:

[1] Covariance matrix calculated using the outer product of gradients (complex-step).

In [146]:

```
results_auto_SARIMA_12.plot_diagnostics()
plt.show()
```



Predict on the Test Set using automated SARIMA model (seasonality=12) and evaluate the model.

In [147]:

```
predicted_auto_SARIMA_12 = results_auto_SARIMA_12.get_forecast(steps=len(test))
```

In [148]:

```
predicted_auto_SARIMA_12.summary_frame(alpha=0.05).head()
```

Out[148]:

У	mean	mean_se	mean_ci_lower	mean_ci_upper
0	1327.420690	388.340435	566.287424	2088.553956
1	1315.131011	401.998581	527.228271	2103.033752
2	1621.592088	401.992199	833.701856	2409.482321
3	1598.894150	407.229343	800.739304	2397.048996
4	1392.700709	407.958777	593.116199	2192.285219

In [149]:

Out[150]:

	RMSE	Test RMSE	MAPE
RegressionOnTime	1378.580256	NaN	NaN
NaiveModel	3780.226721	NaN	NaN
SimpleAverageModel	1329.929685	NaN	NaN
2pointTrailingMovingAverage	793.036740	NaN	NaN
4pointTrailingMovingAverage	1155.559612	NaN	NaN
6pointTrailingMovingAverage	1302.932089	NaN	NaN
9pointTrailingMovingAverage	1388.801927	NaN	NaN
Alpha=0.0496,SimpleExponentialSmoothing	1343.723719	NaN	NaN
Alpha=0.0702,SES	1358.940488	NaN	NaN
Alpha=0.66499,Beta=0.0001:DES	4879.866573	NaN	NaN
Alpha=0.1112,Beta=0.01235,Gamma=0.460717:TES	387.165102	NaN	NaN
Alpha=0.111,Beta=0.04931,Gamma=0.36244:TES	407.410296	NaN	NaN
ARIMA(2,1,2)	NaN	1329.406019	43.893227
SARIMA(0,1,2)(2,0,2,6)	NaN	572.019362	22.966430
SARIMA(1,1,2)(1,0,2,12)	NaN	522.981539	19.659414

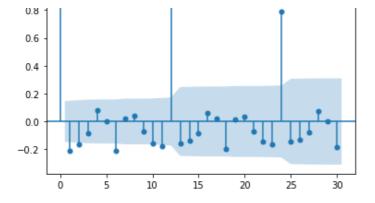
Q7. Build ARIMA/SARIMA models based on the cut-off points of ACF and PACF on the training data and evaluate this model on the test data using RMSE.

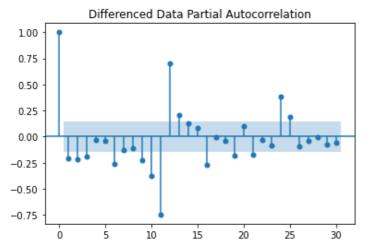
Building a version of the ARIMA model for which the best parameters are selected by looking at the ACF and the PACF plots.

Let us look at the ACF and the PACF plots once more.

```
In [151]:
plot_acf(df['Sparkling'].diff().dropna(),lags=30,title='Differenced Data Autocorrelation
')
plot_pacf(df['Sparkling'].diff().dropna(),lags=30,title='Differenced Data Partial Autocor
relation')
plt.show()
```

Differenced Data Autocorrelation





p=3, q=2, d= 1, SA= 6,12

In [152]:

```
manual_ARIMA = ARIMA(train['Sparkling'], order=(3,1,2), freq='M')
results_manual_ARIMA = manual_ARIMA.fit()
print(results_manual_ARIMA.summary())
```

C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py: 524: ValueWarning: No frequency information was provided, so inferred frequency M will be used.

warnings.warn('No frequency information was'

SARIMAX Results

Dep. Vari Model: Date: Time: Sample:		ARIMA(3, 1, 23 May 2 23:05	, 2) Log 2021 AIC 5:55 BIC 1980 HQIC		132 -1109.380 2230.760 2248.011 2237.770		
Covarianc	e Type:		opg				
=======	coef	std err	z	P> z	[0.025	0.975]	
ar.L1	-0.4318	0.044	-9.897	0.000	-0.517	-0.346	
ar.L2	0.3297	0.109	3.022	0.003	0.116	0.544	
ar.L3	-0.2384	0.066	-3.640	0.000	-0.367	-0.110	
ma.L1	0.0170	0.128	0.132	0.895	-0.235	0.269	
ma.L2	-0.9828	0.136	-7.224	0.000	-1.250	-0.716	
sigma2	1.276e+06	1.95e-07	6.55e+12	0.000	1.28e+06	1.28e+06	
2 2	(L1) (Q):		0.02	Jarque-Bera	(JB):		4.66
<pre>Prob(Q): Heteroskedasticity (H):</pre>		0.88 2.72	Prob(JB): Skew:			0.10	
	two-sided):	•	0.00				3.55
LION(U) (two-sided):		0.00	NullOSIS:			5.55

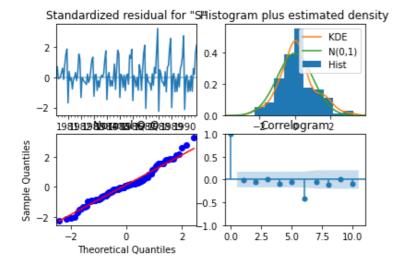
Warnings:

Il Comprished matrix calculated using the outer product of gradients (complex-ster

[2] Covariance matrix is singular or near-singular, with condition number 6.6e+27. Standard errors may be unstable.

In [153]:

results_manual_ARIMA.plot_diagnostics();



Predict on the Test Set using manual ARIMA model and evaluate the model.

In [154]:

```
predicted_manual_ARIMA = results_manual_ARIMA.forecast(steps=len(test))
```

In [155]:

```
rmse = mean_squared_error(test['Sparkling'], predicted_manual_ARIMA, squared=False)
mape = mean_absolute_percentage_error(test['Sparkling'], predicted_manual_ARIMA)
print('RMSE:', rmse, '\nMAPE:', mape)
```

RMSE: 1325.920593997978 MAPE: 41.71428902796663

In [156]:

Out[156]:

	RMSE	Test RMSE	MAPE
RegressionOnTime	1378.580256	NaN	NaN
NaiveModel	3780.226721	NaN	NaN
SimpleAverageModel	1329.929685	NaN	NaN
2pointTrailingMovingAverage	793.036740	NaN	NaN
4pointTrailingMovingAverage	1155.559612	NaN	NaN
6pointTrailingMovingAverage	1302.932089	NaN	NaN
9pointTrailingMovingAverage	1388.801927	NaN	NaN
Alpha=0.0496,SimpleExponentialSmoothing	1343.723719	NaN	NaN
Alpha=0.0702,SES	1358.940488	NaN	NaN
Alpha=0.66499,Beta=0.0001:DES	4879.866573	NaN	NaN
ALL 0.4440 D. 0.4400F.0 0.400747 TEO	007 105100		

Alpna=U.1112,Beta=U.U1235,Gamma=U.46U/17:1ES	387.165102 RMSE	nan Test RMSE	nan MAPE
Alpha=0.111,Beta=0.04931,Gamma=0.36244:TES	407.410296	NaN	NaN
ARIMA(2,1,2)	NaN	1329.406019	43.893227
SARIMA(0,1,2)(2,0,2,6)	NaN	572.019362	22.966430
SARIMA(1,1,2)(1,0,2,12)	NaN	522.981539	19.659414
ARIMA(3,1,2)	NaN	1325.920594	41.714289

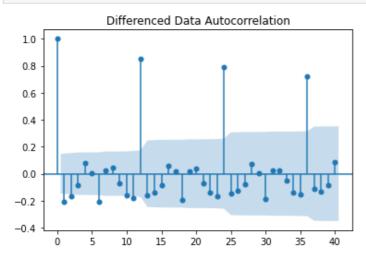
In []:

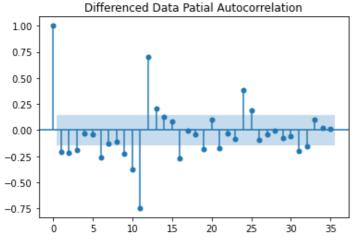
Building a version of the SARIMA model for which the best parameters are selected by looking at the ACF and the PACF plots. - Seasonality at 6.

Let us look at the ACF and the PACF plots once more.

In [157]:

```
plot_acf(df['Sparkling'].diff().dropna(),lags=40,title='Differenced Data Autocorrelation
')
plot_pacf(df['Sparkling'].diff().dropna(),lags=35,title='Differenced Data Patial Autocor
relation')
plt.show()
```

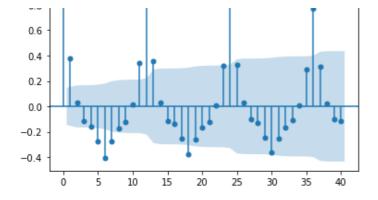


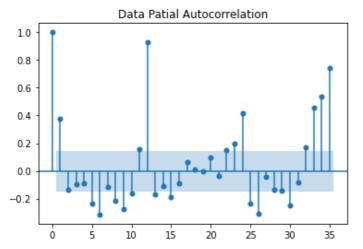


```
In [158]:
```

```
plot_acf(df['Sparkling'],lags=40,title='Data Autocorrelation')
plot_pacf(df['Sparkling'],lags=35,title='Data Patial Autocorrelation')
plt.show() #To find P and Q P=1, Q=1
```

```
Data Autocorrelation
```

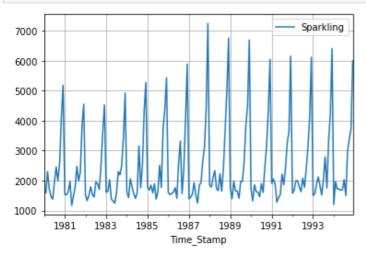




We see that our ACF plot at the seasonal interval (6) does not taper off. So, we go ahead and take a seasonal differencing of the original series. Before that let us look at the original series.

In [159]:

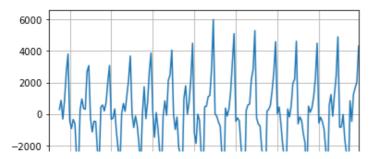
```
df.plot()
plt.grid();
```



We see that there is a trend and a seasonality. So, now we take a seasonal differencing and check the series.

In [160]:

```
(df['Sparkling'].diff(6)).plot()
plt.grid();
```



```
-4000

-6000

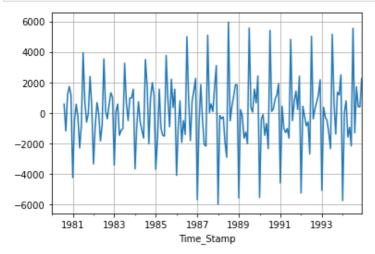
1981 1983 1985 1987 1989 1991 1993

Time_Stamp
```

We see that there might be a slight trend which can be noticed in the data. So we take a differencing of first order on the seasonally differenced series.

In [161]:

```
(df['Sparkling'].diff(6)).diff().plot()
plt.grid();
```



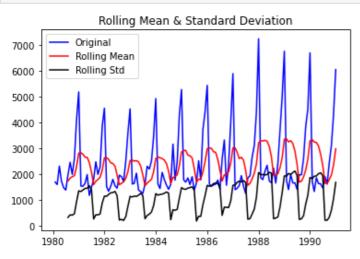
Now we see that there is almost no trend present in the data. Seasonality is only present in the data.

CHECKINGT HE STATIONARITY OF TEST DATA

Let us go ahead and check the stationarity of the above series before fitting the SARIMA model.

In [162]:

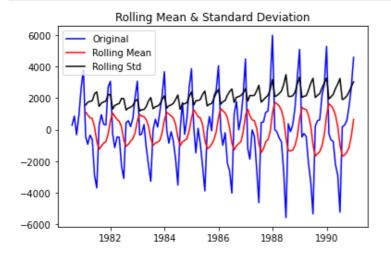
```
test_stationarity((train['Sparkling'].dropna()))
```



```
Results of Dickey-Fuller Test:
Test Statistic
                                 -1.208926
p-value
                                  0.669744
                                 12.000000
#Lags Used
Number of Observations Used
                                119.000000
Critical Value (1%)
                                 -3.486535
Critical Value (5%)
                                 -2.886151
Critical Value (10%)
                                 -2.579896
dtype: float64
```

In [163]:

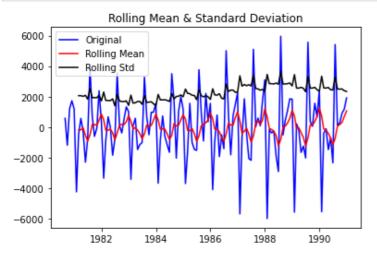
```
test_stationarity((train['Sparkling'].diff(6).dropna()))
```



Results of Dickey-Fuller Test: Test Statistic -8.181919e+00 p-value 8.088278e-13 #Lags Used 6.000000e+00 Number of Observations Used 1.190000e+02 Critical Value (1%) -3.486535e+00 Critical Value (5%) -2.886151e+00 Critical Value (10%) -2.579896e+00 dtype: float64

In [164]:

```
test stationarity((train['Sparkling'].diff(6).dropna()).diff(1).dropna())
```

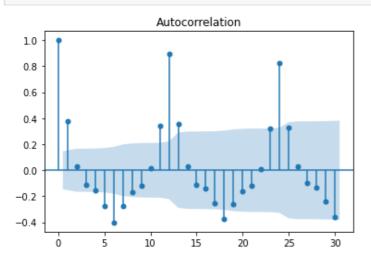


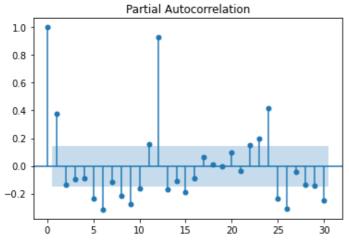
```
Results of Dickey-Fuller Test:
                              -7.017242e+00
Test Statistic
p-value
                               6.683657e-10
                              1.300000e+01
#Lags Used
Number of Observations Used
                              1.110000e+02
Critical Value (1%)
                             -3.490683e+00
Critical Value (5%)
                             -2.887952e+00
Critical Value (10%)
                             -2.580857e+00
dtype: float64
```

Checking the ACF and the PACF plots for the new modified Time Series.

In [165]:

```
plot_acf(df['Sparkling'], lags=30)
plot_pacf(df['Sparkling'], lags=30)
plt.show()
```





Here, we have taken alpha=0.05.

We are going to take the seasonal period as 6. We will keep the p(3) and q(2) parameters same as the ARIMA model.

- The Auto-Regressive parameter in an SARIMA model is 'P' which comes from the significant lag after which the PACF plot cuts-off to 0.
- The Moving-Average parameter in an SARIMA model is 'q' which comes from the significant lag after which the ACF plot cuts-off to 0. Remember to check the ACF and the PACF plots only at multiples of 6 (since 6 is the seasonal period).

By looking at the plots we see that the ACF and the PACF do not directly cut-off to 0.

This is a common problem while building models by looking at the ACF and the PACF plots. But we are able to explain the model.

Please do play around with the data and try out different kinds of transformations and different levels of differencing on this data. We have not taken the logarithm of the series and then trying it out.

Please do refer to this link

to read more about Seasonal Auto-Regressive Integrtaed Moving Average Models.

In [166]:

SARIMAX Results

== Dep. Variable: No. Observations: 132 Model: SARIMAX(3, 1, 2) \times (1, 0, [1], 6) Log Likelihood -965. 436 1946. Sun, 23 May 2021 Date: AIC 873 23:06:01 BIC 1969 Time: .305

- 132

Covariance Type:

Sample:

.984

opg

0

HQIC

1955

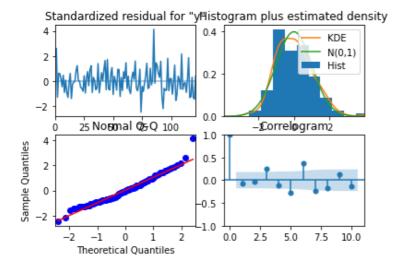
=========				=========	-=======	========
	coef	std err	Z	P> z	[0.025	0.975]
ar.L1	0.4404	0.112	3.931	0.000	0.221	0.660
ar.L2	-0.3676	0.097	-3.809	0.000	-0.557	-0.178
ar.L3	-0.3569	0.104	-3.427	0.001	-0.561	-0.153
ma.L1	-1.4683	0.119	-12.314	0.000	-1.702	-1.235
ma.L2	0.6168	0.112	5.494	0.000	0.397	0.837
ar.S.L6	-1.0302	0.003	-306.824	0.000	-1.037	-1.024
ma.S.L6	0.9938	0.118	8.435	0.000	0.763	1.225
sigma2	3.69e+05	3.2e-07	1.15e+12	0.000	3.69e+05	3.69e+05
Ljung-Box	 (L1) (Q):		0.51	Jarque-Bera	(JB):	16.3
Prob(Q):			0.47	Prob(JB):		0.00
Heteroskeda	asticity (H):		1.88	Skew:		0.65
Prob(H) (tv	wo-sided):		0.05	Kurtosis:		4.23

Warnings:

- [1] Covariance matrix calculated using the outer product of gradients (complex-step).
- [2] Covariance matrix is singular or near-singular, with condition number 1.92e+27. Stand ard errors may be unstable.

In [167]:

results_manual_SARIMA_6.plot_diagnostics()
plt.show()



The model diagnostics plot looks okay.

Predict on the Test Set using manual SARIMA model (Seasonality=6) and evaluate the model.

```
τιι [τρω]:
predicted manual SARIMA 6 = results manual SARIMA 6.get forecast(steps=len(test))
In [169]:
predicted manual SARIMA 6.summary frame(alpha=0.05).head()
Out[169]:
               mean_se mean_ci_lower mean_ci_upper
0 2439.636701 619.486052
                          1225.466350
                                        3653.807051
                           373.888707
1 1588.517937 619.720178
                                        2803.147167
2 1612.447839 635.476356
                           366.937069
                                        2857.958610
3 1334.062046 662.033342
                            36.500538
                                        2631.623554
4 805.327578 665.550788
                           -499.127996
                                        2109.783151
In [170]:
```

```
rmse = mean_squared_error(test['Sparkling'],predicted_manual_SARIMA_6.predicted_mean,squa
red=False)
mape = mean_absolute_percentage_error(test['Sparkling'],predicted_manual_SARIMA_6.predict
ed_mean)
print('RMSE:',rmse,'\nMAPE:',mape)
```

RMSE: 911.0149075155055 MAPE: 35.67575890169932

In [171]:

Out[171]:

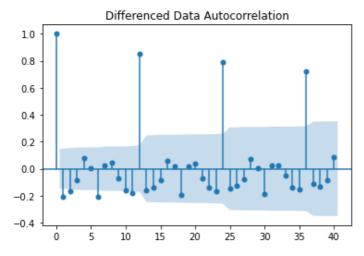
	RMSE	Test RMSE	MAPE
RegressionOnTime	1378.580256	NaN	NaN
NaiveModel	3780.226721	NaN	NaN
SimpleAverageModel	1329.929685	NaN	NaN
2pointTrailingMovingAverage	793.036740	NaN	NaN
4pointTrailingMovingAverage	1155.559612	NaN	NaN
6pointTrailingMovingAverage	1302.932089	NaN	NaN
9pointTrailingMovingAverage	1388.801927	NaN	NaN
Alpha=0.0496,SimpleExponentialSmoothing	1343.723719	NaN	NaN
Alpha=0.0702,SES	1358.940488	NaN	NaN
Alpha=0.66499,Beta=0.0001:DES	4879.866573	NaN	NaN
Alpha=0.1112,Beta=0.01235,Gamma=0.460717:TES	387.165102	NaN	NaN
Alpha=0.111,Beta=0.04931,Gamma=0.36244:TES	407.410296	NaN	NaN
ARIMA(2,1,2)	NaN	1329.406019	43.893227
SARIMA(0,1,2)(2,0,2,6)	NaN	572.019362	22.966430
SARIMA(1,1,2)(1,0,2,12)	NaN	522.981539	19.659414
ARIMA(3,1,2)	NaN	1325.920594	41.714289
SARIMA(3,1,2)(1,0,1,6)	NaN	911.014908	35.675759

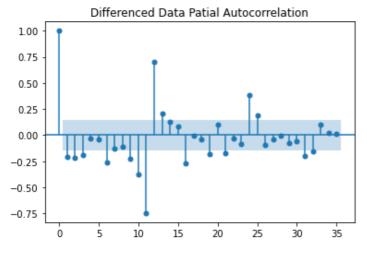
Building a version of the SARIMA model for which the best parameters are selected by looking at the ACF and the PACF plots. - Seasonality at 12.

Let us look at the ACF and the PACF plots once more.

```
In [172]:
```

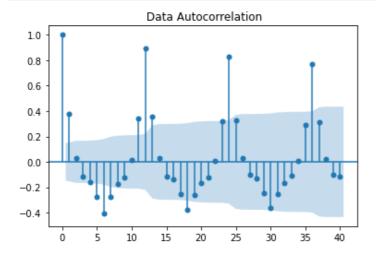
```
plot_acf(df['Sparkling'].diff().dropna(),lags=40,title='Differenced Data Autocorrelation ') plot_pacf(df['Sparkling'].diff().dropna(),lags=35,title='Differenced Data Patial Autocorrelation') plt.show() # p = 2, q = 3
```

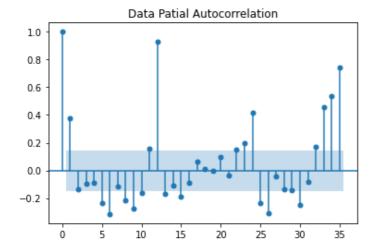




In [173]:

```
plot_acf(df['Sparkling'],lags=40,title=' Data Autocorrelation')
plot_pacf(df['Sparkling'],lags=35,title='Data Patial Autocorrelation')
plt.show()
# P=1, Q=1
```

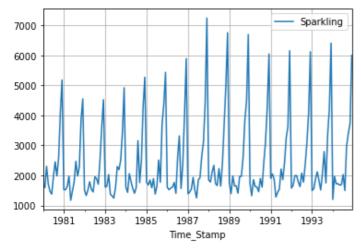




We see that our ACF plot at the seasonal interval (6) does not taper off. So, we go ahead and take a seasonal differencing of the original series. Before that let us look at the original series.

In [174]:

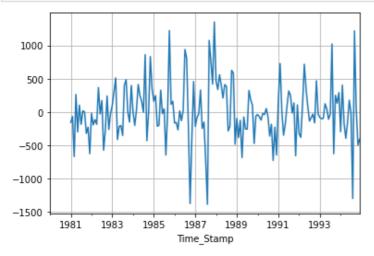
```
df.plot()
plt.grid();
```



We see that there is a trend and a seasonality. So, now we take a seasonal differencing and check the series.

In [175]:

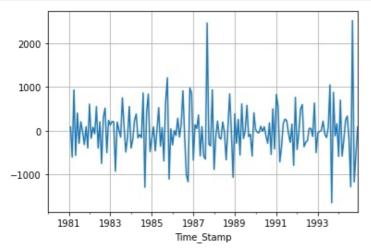
```
(df['Sparkling'].diff(12)).plot()
plt.grid();
```



We see that there might be a slight trend which can be noticed in the data. So we take a differencing of first order on the seasonally differenced series.

+ (17/7)

in [1/6]: (df['Sparkling'].diff(12)).diff().plot() plt.grid();



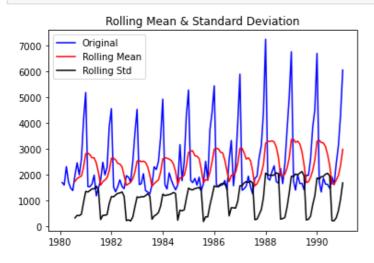
Now we see that there is almost no trend present in the data. Seasonality is only present in the data.

CHECKINGT HE STATIONARITY OF TEST DATA

Let us go ahead and check the stationarity of the above series before fitting the SARIMA model.

In [177]:

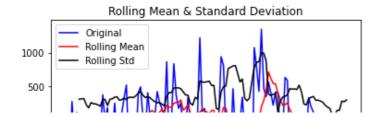
```
test stationarity((train['Sparkling'].dropna()))
```



```
Results of Dickey-Fuller Test:
                                 -1.208926
Test Statistic
                                  0.669744
p-value
#Lags Used
                                 12.000000
Number of Observations Used
                                119.000000
Critical Value (1%)
                                 -3.486535
Critical Value (5%)
                                 -2.886151
Critical Value (10%)
                                 -2.579896
dtype: float64
```

In [178]:

```
test stationarity((train['Sparkling'].diff(12).dropna()))
```

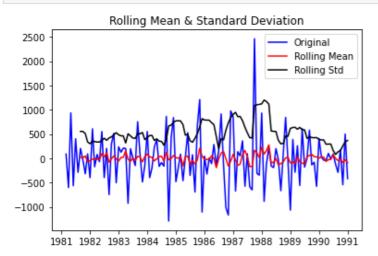


```
0
-500
-1000
-1500
1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991
```

```
Results of Dickey-Fuller Test:
Test Statistic
                                 -3.136812
                                  0.023946
p-value
#Lags Used
                                 11.000000
Number of Observations Used
                                108.000000
Critical Value (1%)
                                 -3.492401
Critical Value (5%)
                                 -2.888697
                                 -2.581255
Critical Value (10%)
dtype: float64
```

In [179]:

test stationarity((train['Sparkling'].diff(12).dropna()).diff(1).dropna())

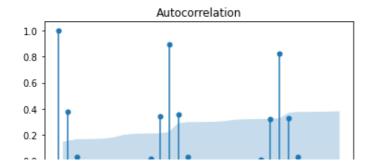


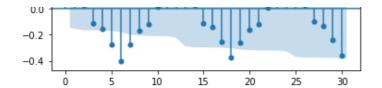
```
Results of Dickey-Fuller Test:
Test Statistic
                                 -3.342905
                                  0.013066
p-value
                                 10.000000
#Lags Used
Number of Observations Used
                                108.000000
Critical Value (1%)
                                 -3.492401
Critical Value (5%)
                                 -2.888697
Critical Value (10%)
                                 -2.581255
dtype: float64
```

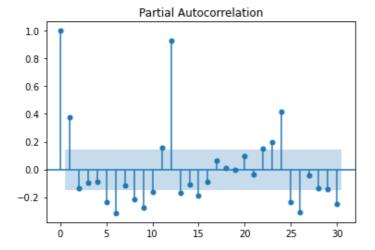
Checking the ACF and the PACF plots for the new modified Time Series.

In [180]:

```
plot_acf(df['Sparkling'],lags=30)
plot_pacf(df['Sparkling'],lags=30)
plt.show()
     # To find P and Q P=1, Q=1
```







Here, we have taken alpha=0.05.

We are going to take the seasonal period as 6. We will keep the p(3) and q(2) as same parameters of the ARIMA model.

- The Auto-Regressive parameter in an SARIMA model is 'P' which comes from the significant lag after which the PACF plot cuts-off to 0.
- The Moving-Average parameter in an SARIMA model is 'q' which comes from the significant lag after which the ACF plot cuts-off to 0. Remember to check the ACF and the PACF plots only at multiples of 6 (since 6 is the seasonal period).

By looking at the plots we see that the ACF and the PACF do not directly cut-off to 0.

This is a common problem while building models by looking at the ACF and the PACF plots. But we are able to explain the model.

Please do play around with the data and try out different kinds of transformations and different levels of differencing on this data. We have not taken the logarithm of the series and then trying it out.

References for Seasonal Auto-Regressive Integrtaed Moving Average Models: link

In [181]:

SARIMAX Results

```
Dep. Variable:
                                                          No. Observations:
132
Model:
                    SARIMAX(3, 1, 2)x(1, 0, [1], 12)
                                                          Log Likelihood
                                                                                           -855
.683
                                      Sun, 23 May 2021
Date:
                                                           AIC
                                                                                           1727
.365
                                               23:06:06
                                                                                           174
Time:
                                                          BIC
9.394
                                                      0
                                                          HQIC
                                                                                           173
Sample:
```

Covariance Type:

opg

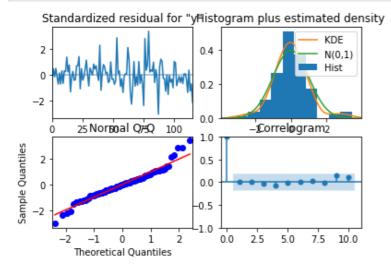
========	=========			=========	========	=======
	coef	std err	Z	P> z	[0.025	0.975]
ar.L1	-0.6468	0.291	-2.220	0.026	-1.218	-0.076
ar.L2	-0.0562	0.165	-0.340	0.734	-0.380	0.268
ar.L3	0.0161	0.106	0.153	0.879	-0.191	0.223
ma.L1	-0.1333	0.293	-0.455	0.649	-0.707	0.440
ma.L2	-0.6440	0.248	-2.594	0.009	-1.131	-0.157
ar.S.L12	1.0403	0.012	85.002	0.000	1.016	1.064
ma.S.L12	-1.5661	0.209	-7.496	0.000	-1.976	-1.157
sigma2	5.894e+04	1.61e+04	3.652	0.000	2.73e+04	9.06e+04
Ljung-Box	(L1) (Q):		0.00	Jarque-Bera	========= (JB):	19.0
Prob(Q):			0.95	Prob(JB):		0.0
Heterosked	lasticity (H):	:	2.76	Skew:		0.3
	wo-sided):		0.00	Kurtosis:		4.8
========	=========	-=========		==========		=========

Warnings:

[1] Covariance matrix calculated using the outer product of gradients (complex-step).

In [182]:

```
results_manual_SARIMA_12.plot_diagnostics()
plt.show()
```



The model diagnostics plot looks okay.

Predict on the Test Set using manual SARIMA model (Seasonality=12) and evaluate the model.

In [183]:

```
predicted_manual_SARIMA_12 = results_manual_SARIMA_12.get_forecast(steps=len(test))
```

In [184]:

```
predicted_manual_SARIMA_12.summary_frame(alpha=0.05).head()
```

Out[184]:

У	mean	mean_se	mean_ci_lower	mean_ci_upper
0	1317.584843	380.204587	572.397546	2062.772140
1	1234.489559	389.287489	471.500100	1997.479017
2	1543.632035	389.396455	780.429008	2306.835062
3	1521.868664	397.554386	742.676387	2301.060942

y mean mean se mean ci lower mean ci upper 4 1324.861569 398.977562 542.879917 2106.843921

Out[186]:

In [187]:

	RMSE	Test RMSE	MAPE
RegressionOnTime	1378.580256	NaN	NaN
NaiveModel	3780.226721	NaN	NaN
SimpleAverageModel	1329.929685	NaN	NaN
2pointTrailingMovingAverage	793.036740	NaN	NaN
4pointTrailingMovingAverage	1155.559612	NaN	NaN
6pointTrailingMovingAverage	1302.932089	NaN	NaN
9pointTrailingMovingAverage	1388.801927	NaN	NaN
Alpha=0.0496,SimpleExponentialSmoothing	1343.723719	NaN	NaN
Alpha=0.0702,SES	1358.940488	NaN	NaN
Alpha=0.66499,Beta=0.0001:DES	4879.866573	NaN	NaN
Alpha=0.1112,Beta=0.01235,Gamma=0.460717:TES	387.165102	NaN	NaN
Alpha=0.111,Beta=0.04931,Gamma=0.36244:TES	407.410296	NaN	NaN
ARIMA(2,1,2)	NaN	1329.406019	43.893227
SARIMA(0,1,2)(2,0,2,6)	NaN	572.019362	22.966430
SARIMA(1,1,2)(1,0,2,12)	NaN	522.981539	19.659414
ARIMA(3,1,2)	NaN	1325.920594	41.714289
SARIMA(3,1,2)(1,0,1,6)	NaN	911.014908	35.675759
SARIMA(3,1,2)(1,0,1,12)	NaN	581.269173	23.827279

The model building ends here. The least RMSE model is considered the best for predicting the forecast. Now, we will take our best model and forecast 12 months into the future with appropriate confidence intervals to see how the predictions look. We have to build our model on the full data for this.

Q8. Build a table with all the models built along with their corresponding parameters and the respective RMSE values on the test data.

In [188]:

results table s

Out[188]:

	RMSE	Test RMSE	MAPE
RegressionOnTime	1378.580256	NaN	NaN
NaiveModel	3780.226721	NaN	NaN
SimpleAverageModel	1329.929685	NaN	NaN
2pointTrailingMovingAverage	793.036740	NaN	NaN
4pointTrailingMovingAverage	1155.559612	NaN	NaN
6pointTrailingMovingAverage	1302.932089	NaN	NaN
9pointTrailingMovingAverage	1388.801927	NaN	NaN
Alpha=0.0496,SimpleExponentialSmoothing	1343.723719	NaN	NaN
Alpha=0.0702,SES	1358.940488	NaN	NaN
Alpha=0.66499,Beta=0.0001:DES	4879.866573	NaN	NaN
Alpha=0.1112,Beta=0.01235,Gamma=0.460717:TES	387.165102	NaN	NaN
Alpha=0.111,Beta=0.04931,Gamma=0.36244:TES	407.410296	NaN	NaN
ARIMA(2,1,2)	NaN	1329.406019	43.893227
SARIMA(0,1,2)(2,0,2,6)	NaN	572.019362	22.966430
SARIMA(1,1,2)(1,0,2,12)	NaN	522.981539	19.659414
ARIMA(3,1,2)	NaN	1325.920594	41.714289
SARIMA(3,1,2)(1,0,1,6)	NaN	911.014908	35.675759
SARIMA(3,1,2)(1,0,1,12)	NaN	581.269173	23.827279

Q9. Based on the model-building exercise, build the most optimum model(s) on the complete data and predict 12 months into the future with appropriate confidence intervals/bands

In [189]:

results_table_s

Out[189]:

	RMSE	Test RMSE	MAPE
RegressionOnTime	1378.580256	NaN	NaN
NaiveModel	3780.226721	NaN	NaN
SimpleAverageModel	1329.929685	NaN	NaN
2pointTrailingMovingAverage	793.036740	NaN	NaN
4pointTrailingMovingAverage	1155.559612	NaN	NaN
6pointTrailingMovingAverage	1302.932089	NaN	NaN
9pointTrailingMovingAverage	1388.801927	NaN	NaN
Alpha=0.0496,SimpleExponentialSmoothing	1343.723719	NaN	NaN
Alpha=0.0702,SES	1358.940488	NaN	NaN
Alpha=0.66499,Beta=0.0001:DES	4879.866573	NaN	NaN
Alpha=0.1112,Beta=0.01235,Gamma=0.460717:TES	387.165102	NaN	NaN

NaN MAPE	NaN Test RMSE	407.410296 R MS E	Alpha=0.111,Beta=0.04931,Gamma=0.36244:TES
43.893227	1329.406019	NaN	ARIMA(2,1,2)
22.966430	572.019362	NaN	SARIMA(0,1,2)(2,0,2,6)
19.659414	522.981539	NaN	SARIMA(1,1,2)(1,0,2,12)
41.714289	1325.920594	NaN	ARIMA(3,1,2)
35.675759	911.014908	NaN	SARIMA(3,1,2)(1,0,1,6)
23.827279	581.269173	NaN	SARIMA(3,1,2)(1,0,1,12)

Least RMSE is for Alpha=0.1112, Beta=0.01235, Gamma=0.460717:TES which is 387.165102.

So, we design the full modle to evaluate RMSE along with other SARIMA models.

```
In [190]:
# Initializing the Double Exponential Smoothing Model
model TES = ExponentialSmoothing(train, trend='additive', seasonal='additive', initializatio
n method='estimated')
# Fitting the model
model TES = model TES.fit()
print('')
print('==Holt Winters model Exponential Smoothing Estimated Parameters ==')
print('')
print(model TES.params)
# Forecasting using this model for the duration of the test set
TES predict = model TES.forecast(len(df))
TES predict
## Plotting the Training data, Test data and the forecasted values
print('TES RMSE:', mean squared error(df.values, TES predict.values, squared=False))
==Holt Winters model Exponential Smoothing Estimated Parameters ==
{'smoothing level': 0.11127217746964548, 'smoothing trend': 0.012360783178126748, 'smooth
ing_seasonal': 0.46071776538894016, 'damping_trend': nan, 'initial level': 2356.578364436
942, 'initial trend': -0.018474239476475418, 'initial seasons': array([-636.23349217, -72
2.98352055, -398.6435547 , -473.43067518,
       -808.42508553, -815.35030575, -384.23066985,
                                                      72.99524975,
       -237.44267238, 272.32595851, 1541.37820976, 2590.07753817]), 'use boxcox': False,
'lamda': None, 'remove bias': False}
TES RMSE: 505.90734910682886
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa model.py:
524: ValueWarning: No frequency information was provided, so inferred frequency M will be
 warnings.warn('No frequency information was'
```

The next better model is SARIMA (1,1,2) (1,0,2,12) with RMSE 522.981539. So, we built model for whole data using SARIMA (1,1,2) (1,0,2,12)

```
In [191]:
```

used.

warnings.warn('No frequency information was'
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py:
524: ValueWarning: No frequency information was provided, so inferred frequency M will be used.

warnings.warn('No frequency information was'

SARIMAX Results

Dep. Variable: Sparkling 1 No. Observations: 80 SARIMAX(1, 1, 2) \times (1, 0, 2, 12) Model: Log Likelihood -1124.5 60 Sun, 23 May 2021 2263.1 Date: AIC 20 Time: 23:06:09 BIC 2284. 288 Sample: 01-31-1980 HOIC 2271.7 19 - 12-31-1994

Covariance Type:

opg

=======	coef	std err	======= Z	========= P> z	[0.025	0.975]
ar.I ₁ 1	 -0.6678	0.250	 -2.671	0.008	 -1.158	-0.178
ma.L1	-0.1759	0.227	-0.775	0.438	-0.621	0.269
ma.L2	-0.7427	0.199	-3.732	0.000	-1.133	-0.353
ar.S.L12	1.0152	0.013	80.000	0.000	0.990	1.040
ma.S.L12 ma.S.L24	-0.5700 -0.0695	0.080	-7.159 -0.804	0.000 0.421	-0.726 -0.239	-0.414 0.100
sigma2	1.493e+05	1.58e+04	9.460	0.000	1.18e+05	1.8e+05
======= Ljung-Box	(L1) (Q):		0.00	Jarque-Bera	======== (JB):	23.8
Prob(Q):			0.99	Prob(JB):		0.0
	lasticity (H)	:	1.26	Skew:		0.5
Prob(H) (t	wo-sided):		0.41	Kurtosis:		4.6

Warnings:

[1] Covariance matrix calculated using the outer product of gradients (complex-step).

Note: If we have a scenario where our training data was stationary but our full data was not stationary. We can still use the same parameters as our training data but with adding an appropriate level of differencing or transformation which is needed for the data to be stationary.

Evaluate the model on the whole data and predict 12 months into the future (till the end of next year).

```
In [192]:
```

```
predicted_manual_SARIMA_full_data = results_full_data_model.get_forecast(steps=12)
```

In [193]:

```
pred_full_manual_SARIMA_date = predicted_manual_SARIMA_full_data.summary_frame(alpha=0.05)
pred_full_manual_SARIMA_date.head()
```

Out[193]:

Sparkling	mean	mean_se	mean_ci_lower	mean_ci_upper
1995-01-31	1328.183351	386.410321	570.833038	2085.533663
1995-02-28	1766.092479	391.101507	999.547611	2532.637346
1995-03-31	1756.431439	391.202001	989.689607	2523.173270
1995-04-30	1771.496613	392.983257	1001.263583	2541.729644

Sparkling 1625.496905 393.036776 mean_ci_lower mean_ci_upper 2395.834831

In [194]:

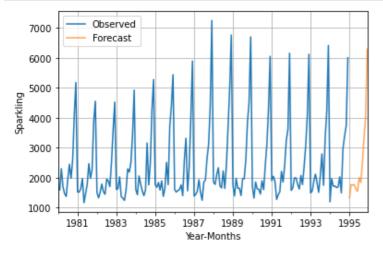
```
rmse = mean_squared_error(df['Sparkling'], results_full_data_model.fittedvalues, squared=F
alse)
print('RMSE of the Full Model', rmse)
```

RMSE of the Full Model 546.3764684499913

In [195]:

```
axis = df['Sparkling'].plot(label='Observed')
pred_full_manual_SARIMA_date['mean'].plot(ax=axis, label='Forecast', alpha=0.7)

axis.set_xlabel('Year-Months')
axis.set_ylabel('Sparkling')
plt.legend(loc='best')
plt.grid();
```



In [196]:

results_table_s

Out[196]:

	RMSE	Test RMSE	MAPE
RegressionOnTime	1378.580256	NaN	NaN
NaiveModel	3780.226721	NaN	NaN
SimpleAverageModel	1329.929685	NaN	NaN
2pointTrailingMovingAverage	793.036740	NaN	NaN
4pointTrailingMovingAverage	1155.559612	NaN	NaN
6pointTrailingMovingAverage	1302.932089	NaN	NaN
9pointTrailingMovingAverage	1388.801927	NaN	NaN
Alpha=0.0496,SimpleExponentialSmoothing	1343.723719	NaN	NaN
Alpha=0.0702,SES	1358.940488	NaN	NaN
Alpha=0.66499,Beta=0.0001:DES	4879.866573	NaN	NaN
Alpha=0.1112,Beta=0.01235,Gamma=0.460717:TES	387.165102	NaN	NaN
Alpha=0.111,Beta=0.04931,Gamma=0.36244:TES	407.410296	NaN	NaN
ARIMA(2,1,2)	NaN	1329.406019	43.893227
SARIMA(0,1,2)(2,0,2,6)	NaN	572.019362	22.966430
SARIMA(1,1,2)(1,0,2,12)	NaN	522.981539	19.659414
ARIMA(3,1,2)	NaN	1325.920594	41.714289
SARIMA(3.1.2)(1.0.1.6)	NaN	911.014908	35.675759

RMSE Test RMSE MAPE
SARIMA(3,1,2)(1,0,1,12) NaN 581.269173 23.827279

Also we observe for other SARIMA models with nearly similar RMSE (less than 600) for thane whole data and we plot the forecast for them

```
In [197]:
```

C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py: 524: ValueWarning: No frequency information was provided, so inferred frequency M will be used.

warnings.warn('No frequency information was'

C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py: 524: ValueWarning: No frequency information was provided, so inferred frequency M will be used.

warnings.warn('No frequency information was'

SARIMAX Results

==========	-======================================		=======================================
Dep. Variable:	Sparkling	No. Observations:	18
Model:	SARIMAX(0, 1, 2) \times (2, 0, 2, 6)	Log Likelihood	-1209.57
Date:	Sun, 23 May 2021	AIC	2433.14
Time:	23:06:10	BIC	2454.84
4 Sample:	01-31-1980	HQIC	2441.95
4	- 12-31-1994		

Covariance Type:

opg

========			=======		========	=======	
	coef	std err	Z	P> z	[0.025	0.975]	
ma.L1	-0.9551	0.109	-8.726	0.000	-1.170	-0.741	
ma.L2	-0.1178	0.090	-1.313	0.189	-0.294	0.058	
ar.S.L6	0.0057	0.019	0.306	0.760	-0.031	0.042	
ar.S.L12	1.0155	0.012	82.035	0.000	0.991	1.040	
ma.S.L6	-0.0484	0.090	-0.536	0.592	-0.226	0.129	
ma.S.L12	-0.6297	0.071	-8.832	0.000	-0.769	-0.490	
sigma2	1.265e+05	1.41e+04	8.982	0.000	9.89e+04	1.54e+05	
======= Ljung-Box	(L1) (Q):		0.00	Jarque-Bera	========= (JB):	48.	.70
Prob(Q):			0.95	Prob(JB):		0.	.00
Heteroskedasticity (H):		1.32	Skew:		0.	. 61	
Prob(H) (t	wo-sided):		0.30	Kurtosis:		5.	.38
========							

Warnings:

[1] Covariance matrix calculated using the outer product of gradients (complex-step).

Note: If we have a scenario where our training data was stationary but our full data was not stationary. We can still use the same parameters as our training data but with adding an appropriate level of differencing or transformation which is needed for the data to be stationary.

Evaluate the model on the whole data and predict 12 months into the future (till the end of next year).

predicted_manual_SARIMA_full_data = results_full_data_model.get_forecast(steps=12)

In [199]:

```
pred_full_manual_SARIMA_date = predicted_manual_SARIMA_full_data.summary_frame(alpha=0.05)
pred_full_manual_SARIMA_date.head()
```

Out[199]:

Sparkling	mean	mean_se	mean_ci_lower	mean_ci_upper
1995-01-31	1346.741629	379.032410	603.851757	2089.631500
1995-02-28	1698.024252	384.609920	944.202661	2451.845843
1995-03-31	1756.114644	385.483163	1000.581527	2511.647761
1995-04-30	1770.004992	386.354317	1012.764445	2527.245539
1995-05-31	1691.209039	387.223522	932.264883	2450.153196

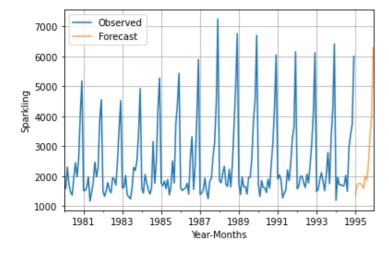
In [200]:

```
rmse = mean_squared_error(df['Sparkling'], results_full_data_model.fittedvalues, squared=F
alse)
print('RMSE of the Full Model', rmse)
```

RMSE of the Full Model 541.0028199891531

In [201]:

```
axis = df['Sparkling'].plot(label='Observed')
pred_full_manual_SARIMA_date['mean'].plot(ax=axis, label='Forecast', alpha=0.7)
axis.set_xlabel('Year-Months')
axis.set_ylabel('Sparkling')
plt.legend(loc='best')
plt.grid();
```



In []:

RMSE values of SARIMA models for the entire dataset considering least RMSE in the dataset are summarised as:

TES RMSE: 505.90734910682886 SARIMA(1,1,2) (1,0,2,12) - RMSE = 546.367 SARIMA(0,1,2) (2,0,2,6) - RMSE = 541.0028 SARIMA(3,1,2) (1,0,1,12) - RMSE = 541.97

For the test data, we have RMSE as

RMSE values computed for ARIMA models for test data are: SARIMA(1,1,2) (1,0,2,12) - RMSE = 522.981

SAMINIA(U, 1,2) (2,U,2,0) - MINISE = 3/2.U 19 SAMINIA(3, 1,2) (1,U, 1, 12) - MINISE = 381.20

The best model based on RMSE is

SARIMA(1,1,2)(1,0,2,12)

as it has least RMSE among the ARIMA/SARIMA models designed for test data.

The best model overall considering test and whole dataset is

Alpha=0.1112,Beta=0.01235,Gamma=0.460717:TES

For the whole dataset, we can observe that

SARIMA(0,1,2) (2,0,2,6)

has the least RMSE among SARIMA in the entire dataset.

Based on MAPE values, the best ARIMA model is ARIMA(3,1,2) with MAPE 41.714 and the best SARIMA model based on MAPE is SARIMA(1,1,2)(1,0,2,12).

Hence, it is concluded that best models are SARIMA(1,1,2)(1,0,2,12) and Alpha=0.1112,Beta=0.01235,Gamma=0.460717:TES based on the observations from the table.

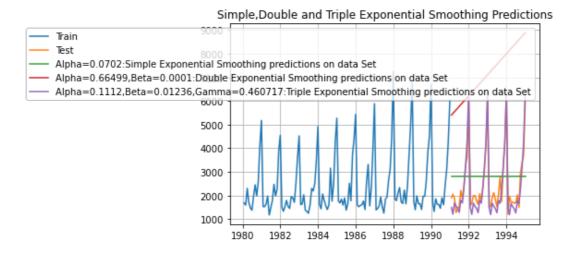
The plots of output curves for Alpha=0.1112,Beta=0.01235,Gamma=0.460717:TES and SARIMA(1,1,2)(1,0,1,12) are plotted.

```
In [202]:
```

```
### Holt-Winters - ETS(A, A, A) - Holt Winter's linear method with additive errors
# Initializing the Double Exponential Smoothing Model
model TES = ExponentialSmoothing(train, trend='additive', seasonal='additive', initializatio
n method='estimated')
# Fitting the model
model TES = model TES.fit()
print('')
print('==Holt Winters model Exponential Smoothing Estimated Parameters ==')
print('')
print(model TES.params)
# Forecasting using this model for the duration of the test set
TES predict = model TES.forecast(len(test))
TES predict
## Plotting the Training data, Test data and the forecasted values
plt.plot(train, label='Train')
plt.plot(test, label='Test')
plt.plot(SES predict, label='Alpha=0.0702:Simple Exponential Smoothing predictions on dat
a Set')
plt.plot(DES predict, label='Alpha=0.66499, Beta=0.0001: Double Exponential Smoothing predi
ctions on data Set')
plt.plot(TES predict, label='Alpha=0.1112, Beta=0.01236, Gamma=0.460717: Triple Exponential
Smoothing predictions on data Set')
plt.legend(loc='best')
plt.grid()
plt.title('Simple, Double and Triple Exponential Smoothing Predictions');
print('TES RMSE:', mean squared error(test.values, TES predict.values, squared=False))
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa model.py:
521. ValuaWarning. No fraguency information was provided so inferred fraguency M will be
```

```
used.
warnings.warn('No frequency information was provided, so interfed frequency m will be used.
```

```
==Holt Winters model Exponential Smoothing Estimated Parameters ==
```



warnings.warn('No frequency information was'

In [203]:

```
full data model = sm.tsa.statespace.SARIMAX(df['Sparkling'],
                                order=(1,1,2),
                                seasonal order=(1, 0, 2, 12),
                                enforce stationarity=False,
                                enforce invertibility=False)
results full data model = full data model.fit(maxiter=1000)
print(results full data model.summary())
###### Note: If we have a scenario where our training data was stationary but our full da
ta was not stationary. We can still use the same parameters as our training data but with
adding an appropriate level of differencing or transformation which is needed for the dat
a to be stationary.
#### Evaluate the model on the whole data and predict 12 months into the future (till the
end of next year).
predicted manual SARIMA full data = results full data model.get forecast(steps=12)
pred full manual SARIMA date = predicted manual SARIMA full data.summary frame(alpha=0.05
pred full manual SARIMA date.head()
rmse = mean squared error(df['Sparkling'], results full data model.fittedvalues, squared=F
alse)
print('RMSE of the Full Model', rmse)
axis = df['Sparkling'].plot(label='Observed')
pred full manual SARIMA date['mean'].plot(ax=axis, label='Forecast', alpha=0.7)
axis.set xlabel('Year-Months')
axis.set ylabel('Sparkling')
plt.legend(loc='best')
plt.grid();
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa_model.py:
524: ValueWarning: No frequency information was provided, so inferred frequency M will be
```

C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa model.py:

524: ValueWarning: No frequency information was provided, so inferred frequency M will be used

warnings.warn('No frequency information was'

SARIMAX Results

============			===============
=			
Dep. Variable:	Sparkling	No. Observations:	1
80			
Model:	SARIMAX(1, 1, 2) \times (1, 0, 2, 12)	Log Likelihood	-1124.5
60			
Date:	Sun, 23 May 2021	AIC	2263.1
20			
Time:	23:06:13	BIC	2284.
288			
Sample:	01-31-1980	HQIC	2271.7
19			

- 12-31-1994

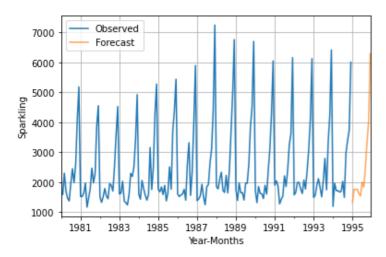
Covariance Type:

opg

	coef	std err	z	P> z	[0.025	0.975]
ar.L1	-0.6678	0.250	-2.671	0.008	-1.158	-0.178
ma.L1	-0.1759	0.227	-0.775	0.438	-0.621	0.269
ma.L2	-0.7427	0.199	-3.732	0.000	-1.133	-0.353
ar.S.L12	1.0152	0.013	80.000	0.000	0.990	1.040
ma.S.L12	-0.5700	0.080	-7.159	0.000	-0.726	-0.414
ma.S.L24	-0.0695	0.086	-0.804	0.421	-0.239	0.100
sigma2	1.493e+05	1.58e+04	9.460	0.000	1.18e+05	1.8e+05
======= Ljung-Box	(L1) (Q):	=======	0.00	Jarque-Bera	(JB):	======================================
Prob(Q):			0.99	Prob(JB):		0.00
Heteroskedasticity (H):			1.26	Skew:		0.52
Prob(H) (t	wo-sided):		0.41	Kurtosis:		4.64
Prob(H) (U	.wo-sided): =========		U.41 ========	Kurtosis:		4 ==========

Warnings:

[1] Covariance matrix calculated using the outer product of gradients (complex-step). RMSE of the Full Model 546.3764684499913



Q10. Comment on the model thus built and report your findings and suggest the measures that the company should be taking for future sales.

MODELS BUILT: SARIMA(1,1,2)(1,0,2,12) and Alpha=0.1112,Beta=0.01235,Gamma=0.460717:TES

In [204]:

```
# Initializing the Double Exponential Smoothing Model
model TES = ExponentialSmoothing(train, trend='additive', seasonal='additive', initializatio
n method='estimated')
# Fitting the model
model TES = model TES.fit()
print('')
print('==Holt Winters model Exponential Smoothing Estimated Parameters ==')
print('')
print(model TES.params)
# Forecasting using this model for the duration of the test set
TES_predict = model_TES.forecast(len(df))
TES predict
## Plotting the Training data, Test data and the forecasted values
print('TES RMSE:', mean squared error(df.values, TES predict.values, squared=False))
==Holt Winters model Exponential Smoothing Estimated Parameters ==
{'smoothing level': 0.11127217746964548, 'smoothing trend': 0.012360783178126748, 'smooth
ing seasonal': 0.46071776538894016, 'damping trend': nan, 'initial level': 2356.578364436
942, 'initial trend': -0.018474239476475418, 'initial seasons': array([-636.23349217, -72
2.98352055, -398.6435547 , -473.43067518,
       -808.42508553, -815.35030575, -384.23066985, 72.99524975,
       -237.44267238, 272.32595851, 1541.37820976, 2590.07753817]), 'use boxcox': False,
'lamda': None, 'remove bias': False}
TES RMSE: 505.90734910682886
C:\Users\DANDE SWAPNA SREE\anaconda3\lib\site-packages\statsmodels\tsa\base\tsa model.py:
524: ValueWarning: No frequency information was provided, so inferred frequency M will be
 warnings.warn('No frequency information was'
In [205]:
import statsmodels.api as sm
manual SARIMA 12 = sm.tsa.statespace.SARIMAX(train['Sparkling'].values,
                                order=(1, 1, 2),
                                seasonal order=(1, 0, 2, 12),
                                enforce stationarity=False,
                                enforce invertibility=False)
results manual SARIMA 12 = manual SARIMA 12.fit(maxiter=1000)
print(results manual SARIMA 12.summary())
```

```
results manual SARIMA 12.plot diagnostics()
plt.show()
```

SARIMAX Results

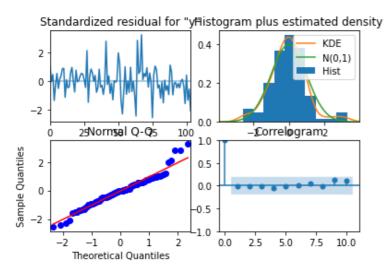
Dep. Variable: No. Observations: 1 32 SARIMAX(1, 1, 2)x(1, 0, 2, 12)-770.7 Model: Log Likelihood 92 Sun, 23 May 2021 Date: AIC 1555.5 84 Time: 23:06:15 BIC 1574. 095 Sample: 0 HOIC 1563. 083 - 132 Covariance Type: opg

coef std err z P>|z| [0.025 0.975] ar. $\overline{1}$ 1 -0.6283 0.255 -2.464 0.014 -1.128 -0.128

```
0.225
                            -0.463
                                     0.644
                                                        0.337
ma.L1
          -0.1040
                                              -0.545
ma.L2
          -0.7277
                    0.154
                            -4.736
                                     0.000
                                              -1.029
                                                       -0.427
                            72.834
                                              1.016
ar.S.L12
          1.0439
                    0.014
                                     0.000
                                                        1.072
ma.S.L12
          -0.5550
                    0.098
                            -5.663
                                     0.000
                                              -0.747
                                                       -0.363
ma.S.L24
                    0.120
                            -1.133
                                     0.257
                                              -0.370
                                                        0.099
          -0.1354
         1.506e+05
                                     0.000
sigma2
                  2.03e+04
                             7.401
                                            1.11e+05
                                                      1.9e + 05
______
Ljung-Box (L1) (Q):
                             0.04
                                  Jarque-Bera (JB):
                                                            11.72
Prob(Q):
                             0.84
                                                            0.00
                                  Prob(JB):
                             1.47
Heteroskedasticity (H):
                                  Skew:
                                                            0.36
Prob(H) (two-sided):
                             0.26
                                  Kurtosis:
                                                            4.48
______
```

Warnings:

[1] Covariance matrix calculated using the outer product of gradients (complex-step).



In [206]:

SARIMAX Results

```
Dep. Variable:
                                                       No. Observations:
                                                                                               1
32
                    SARIMAX (1, 1, 2) \times (1, 0, 2, 12)
                                                        Log Likelihood
                                                                                          -770.7
Model:
92
Date:
                                    Sun, 23 May 2021
                                                                                          1555.5
84
                                             23:06:16
                                                                                          1574.
Time:
                                                        BIC
095
                                                    0
                                                         HQIC
                                                                                          1563.
Sample:
083
                                                - 132
```

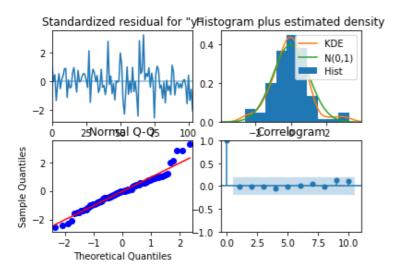
Covariance Type: opg

	coef	std err	Z	P> z	[0.025	0.975]
ar.L1	-0.6283 -0.1040	0.255 0.225	-2.464 -0.463	0.014 0.644	-1.128 -0.545	-0.128 0.337
Ŧ ^	A 1011	O 1 F 4	4 700	0 000	1 000	0 407

ma.LZ ar.S.L12 ma.S.L12 ma.S.L24 sigma2	-0.7277 1.0439 -0.5550 -0.1354 1.506e+05	0.154 0.014 0.098 0.120 2.03e+04	-4./36 72.834 -5.663 -1.133 7.401	0.000 0.000 0.000 0.257 0.000	1.016 -0.747 -0.370 1.11e+05	-0.42/ 1.072 -0.363 0.099 1.9e+05
	(L1) (Q): dasticity (H):	:	0.04 0.84 1.47 0.26	Jarque-Bera Prob(JB): Skew: Kurtosis:	(JB):	11.72 0.00 0.36 4.48

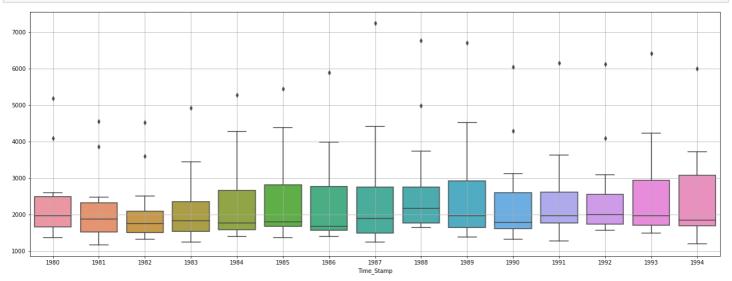
Warnings:

[1] Covariance matrix calculated using the outer product of gradients (complex-step).



In [207]:

```
## YEARLY BOXPLOT
_, ax = plt.subplots(figsize=(22,8))
sns.boxplot(x = df.index.year,y = df.values[:,0],ax=ax)
plt.grid();
```



In [208]:

```
### MONTHLY BOXPLOT

_, ax = plt.subplots(figsize=(22,8))
sns.boxplot(x = df.index.month_name(),y = df.values[:,0],ax=ax)
plt.grid();
```

