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BSCS3

Midterm Activity1 - Polynomial Interpolation

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Lagrange Interpolation for (f(7))

$$\begin{aligned}
x_0 &= 1, & f(x_0) &= 12 \\
x_1 &= 5, & f(x_1) &= -26 \\
x_2 &= 8, & f(x_2) &= -14 \\
x_3 &= 10, & f(x_3) &= 37
\end{aligned}$$

Lagrange interpolation formula:

$$f(x) = \sum_{i=0}^{3} f(x_i) \cdot L_i(x)$$

Where:

$$L_i(x) = \prod_{j=0, j \neq i} \frac{x - x_j}{x_i - x_j}$$

Calculate $L_i(7)$

$L_0(7)$**:**

$$L_0(7) = \frac{(7-5)(7-8)(7-10)}{(1-5)(1-8)(1-10)} = \frac{(2)(-1)(-3)}{(-4)(-7)(-9)} = \frac{6}{252} = \frac{1}{42}$$

$L_1(7)$**:**

$$L_1(7) = \frac{(7-1)(7-8)(7-10)}{(5-1)(5-8)(5-10)} = \frac{(6)(-1)(-3)}{(4)(-3)(-5)} = \frac{18}{60} = \frac{3}{10}$$

$L_2(7)$**:**

$$L_2(7) = \frac{(7-1)(7-5)(7-10)}{(8-1)(8-5)(8-10)} = \frac{(6)(2)(-3)}{(7)(3)(-2)} = \frac{-36}{-42} = \frac{6}{7}$$

$L_3(7)$**:**

$$L_3(7) = \frac{(7-1)(7-5)(7-8)}{(10-1)(10-5)(10-8)} = \frac{(6)(2)(-1)}{(9)(5)(2)} = \frac{-12}{90} = -\frac{2}{15}$$

Calculate $f(7)$

The interpolation formula is:

$$f(7) = f(x_0) \cdot L_0(7) + f(x_1) \cdot L_1(7) + f(x_2) \cdot L_2(7) + f(x_3) \cdot L_3(7)$$

Substitute the values:

$$f(7) = 12 \cdot \frac{1}{42} + (-26) \cdot \frac{3}{10} + (-14) \cdot \frac{6}{7} + 37 \cdot \left(-\frac{2}{15}\right)$$

Simplify

Each term evaluates as follows:

$$12 \cdot \frac{1}{42} = \frac{12}{42} = \frac{2}{7} \approx 0.28571$$

$$-26 \cdot \frac{3}{10} = -\frac{78}{10} = -7.8$$

$$-14 \cdot \frac{6}{7} = -12$$

$$37 \cdot \left(-\frac{2}{15}\right) = -\frac{74}{15} \approx -4.93333$$

Which adds to:

$$f(7) = 0.28571 - 7.8 - 12 - 4.93333$$

$$f(7) \approx -24.44762$$