

# Closed Methods in Finding the root of non-linear system

## Bisection Method

1. Determine  $x_m = \frac{x_l + x_u}{2}$
2. Find  $f(x_l), f(x_m), f(x_u)$
3. Choose new interval (*find the interval with 2 different signs (+,-)*)

### Problem 1

Determine the root of  $3x^4 + 7x^3 - 15x^2 + 5x = 17$  between  $[0, 2]$ . Use Bisection Method for 7 iterations. The equation becomes  $f(x) = 3x^4 + 7x^3 - 15x^2 + 5x - 17$ . *17 is transposed so it became -17*

Count	$x_l$	$x_m$	$x_u$	$f(x_l)$	$f(x_m)$	$f(x_u)$	New
1.	0	1	2	-17	-17	37	[1,2]
2.	1	1.5	2	-17	-4.43750	37	[1.5,2]
3.	1.5	1.75	2	-4.43750	11.46484	37	[1.5,1.75]
4.	1.5	1.625	1.75	-4.43750	2.47144	11.46484	[1.5,1.625]
5.	1.5	1.5625	1.625	-4.43750	-1.22432	2.47144	[1.5625,1.625]
6.	1.5625	1.59375	1.625	-1.22432	0.56087	2.47144	[1.5625,1.59375]
7.	1.5625	1.57813	1.59375	-1.22432	-0.34681	0.56087	[1.57813,1.59375]

To get the root after the 7th iteration, use the formula of getting  $x_m$ . So,  
 $x_m = \frac{1.57813 + 1.59375}{2} = 1.58594$

### First Iteration

$$x_m = \frac{x_l + x_u}{2} = \frac{0+2}{2} = 1$$

$$f(x_l) = 3(0)^4 + 7(0)^3 - 15(0)^2 + 5(0) - 17 = -17$$

$$f(x_m) = 3(1)^4 + 7(1)^3 - 15(1)^2 + 5(1) - 17 = -17$$

$$f(x_u) = 3(2)^4 + 7(2)^3 - 15(2)^2 + 5(2) - 17 = 37$$

### Second Iteration

$$x_m = \frac{x_l + x_u}{2} = \frac{1+2}{2} = 1.5$$

$$f(x_l) = 3(1)^4 + 7(1)^3 - 15(1)^2 + 5(1) - 17 = -17$$

$$f(x_m) = 3(1.5)^4 + 7(1.5)^3 - 15(1.5)^2 + 5(1.5) - 17 = -4.43750$$

$$f(x_u) = 3(2)^4 + 7(2)^3 - 15(2)^2 + 5(2) - 17 = 37$$

### Third Iteration

$$x_m = \frac{x_l + x_u}{2} = \frac{1.5+2}{2} = 1.75$$

$$f(x_l) = 3(1.5)^4 + 7(1.5)^3 - 15(1.5)^2 + 5(1.5) - 17 = -4.43750$$

$$f(x_m) = 3(1.75)^4 + 7(1.75)^3 - 15(1.75)^2 + 5(1.75) - 17 = 11.46484$$

$$f(x_u) = 3(2)^4 + 7(2)^3 - 15(2)^2 + 5(2) - 17 = 37$$

#### Fourth Iteration

$$x_m = \frac{x_l + x_u}{2} = \frac{1.5 + 1.75}{2} = 1.625$$

$$f(x_l) = 3(1.5)^4 + 7(1.5)^3 - 15(1.5)^2 + 5(1.5) - 17 = -4.43750$$

$$f(x_m) = 3(1.625)^4 + 7(1.625)^3 - 15(1.625)^2 + 5(1.625) - 17 = 2.47144$$

$$f(x_u) = 3(1.75)^4 + 7(1.75)^3 - 15(1.75)^2 + 5(1.75) - 17 = 11.46484$$

#### Fifth Iteration

$$x_m = \frac{x_l + x_u}{2} = \frac{1.5 + 1.625}{2} = 1.5625$$

$$f(x_l) = 3(1.5)^4 + 7(1.5)^3 - 15(1.5)^2 + 5(1.5) - 17 = -4.43750$$

$$f(x_m) = 3(1.5625)^4 + 7(1.5625)^3 - 15(1.5625)^2 + 5(1.5625) - 17 = -1.22432$$

$$f(x_u) = 3(1.625)^4 + 7(1.625)^3 - 15(1.625)^2 + 5(1.625) - 17 = 2.47144$$

#### Sixth Iteration

$$x_m = \frac{x_l + x_u}{2} = \frac{1.5625 + 1.625}{2} = 1.59375$$

$$f(x_l) = 3(1.5625)^4 + 7(1.5625)^3 - 15(1.5625)^2 + 5(1.5625) - 17 = -1.22432$$

$$f(x_m) = 3(1.59375)^4 + 7(1.59375)^3 - 15(1.59375)^2 + 5(1.59375) - 17 = 0.56087$$

$$f(x_u) = 3(1.625)^4 + 7(1.625)^3 - 15(1.625)^2 + 5(1.625) - 17 = 2.47144$$

#### Seventh Iteration

$$x_m = \frac{x_l + x_u}{2} = \frac{1.5625 + 1.59375}{2} = 1.57813$$

$$f(x_l) = 3(1.5625)^4 + 7(1.5625)^3 - 15(1.5625)^2 + 5(1.5625) - 17 = -1.22432$$

$$f(x_m) = 3(1.57813)^4 + 7(1.57813)^3 - 15(1.57813)^2 + 5(1.57813) - 17 = -0.34681$$

$$f(x_u) = 3(1.59375)^4 + 7(1.59375)^3 - 15(1.59375)^2 + 5(1.59375) - 17 = 0.56087$$

#### Problem 2

Determine the root of  $f(x) = x^3 - 2x - 5$  between  $[2, 3]$ . Use Bisection for 5 iterations.

Count	$x_l$	$x_m$	$x_u$	$f(x_l)$	$f(x_m)$	$f(x_u)$	New
1.	2	2.5	3	-1	5.625	16	[2, 2.5]
2.	2	2.25	2.5	-1	6.89063	5.625	[2, 2.25]
3.	2	2.125	2.25	-1	0.34570	6.89063	[2, 2.125]
4.	2	2.0625	2.125	-1	8.96289	0.34570	[2, 2.0625]
5.	2	2.03125	2.0625	-1	-0.68161	8.96289	[2.03125, 2.0625]

To get the root after the 5th iteration, use the formula of getting  $x_m$ . So,  
 $x_m = \frac{2.03125 + 2.0625}{2} = 2.046875$

**First Iteration**

$$x_m = \frac{x_l + x_u}{2} = \frac{2+3}{2} = 1$$

$$f(x_l) = 2^3 - 2(2) - 5 = -1$$

$$f(x_m) = 2.5^3 - 2(2.5) - 5 = 5.625$$

$$f(x_u) = 3^3 - 2(3) - 5 = 16$$

**Second Iteration**

$$x_m = \frac{x_l + x_u}{2} = \frac{2+2.5}{2} = 2.25$$

$$f(x_l) = 2^3 - 2(2) - 5 = -1$$

$$f(x_m) = 2.25^3 - 2(2.25) - 5 = 6.89063$$

$$f(x_u) = 2.5^3 - 2(2.5) - 5 = 5.625$$

**Third Iteration**

$$x_m = \frac{x_l + x_u}{2} = \frac{2+2.25}{2} = 2.125$$

$$f(x_l) = 2^3 - 2(2) - 5 = -1$$

$$f(x_m) = 2.125^3 - 2(2.125) - 5 = 0.34570$$

$$f(x_u) = 2.25^3 - 2(2.25) - 5 = 6.89063$$

**Fourth Iteration**

$$x_m = \frac{x_l + x_u}{2} = \frac{2+2.125}{2} = 2.0625$$

$$f(x_l) = 2^3 - 2(2) - 5 = -1$$

$$f(x_m) = 2.0625^3 - 2(2.0625) - 5 = 8.96289$$

$$f(x_u) = 2.125^3 - 2(2.125) - 5 = 0.34570$$

**Fifth Iteration**

$$x_m = \frac{x_l + x_u}{2} = \frac{2+2.0625}{2} = 2.03125$$

$$f(x_l) = 2^3 - 2(2) - 5 = -1$$

$$f(x_m) = 2.03125^3 - 2(2.03125) - 5 = -0.68161$$

$$f(x_u) = 2.0625^3 - 2(2.0625) - 5 = 8.96289$$

**False Position Method**

1. Determine  $f(x_u)$  and  $f(x_l)$
2. Determine  $x_m$
3. Determine  $f(x_m)$
4. Choose new interval

The formula for finding  $x_m$  is:  $x_m = \frac{x_l(f(x_u)) - x_u(f(x_l))}{f(x_u) - f(x_l)}$

### Problem 1

Determine the root of  $3x^4 + 7x^3 - 15x^2 + 5x = 17$  between  $[0, 2]$ . Use False Position Method for 7 iterations. The equation becomes  $f(x) = 3x^4 + 7x^3 - 15x^2 + 5x - 17$ . *17 is transposed so it became -17*

Count	$x_l$	$x_m$	$x_u$	$f(x_l)$	$f(x_m)$	$f(x_u)$	New
1.	0	0.62963	2	-17	-17.57963	37	[0.62963, 2]
2.	0.62963	1.07101	2	-17.57963	-16.30402		

### First Iteration

$$f(x_l) = 3(0)^4 + 7(0)^3 - 15(0)^2 + 5(0) - 17 = -17$$

$$f(x_u) = 3(2)^4 + 7(2)^3 - 15(2)^2 + 5(2) - 17 = 37$$

$$x_m = \frac{x_l(f(x_u)) - x_u(f(x_l))}{f(x_u) - f(x_l)} = \frac{0(37) - 2(-17)}{37 - (-17)} = 0.62963$$

$$f(x_m) = 3(0.62963)^4 + 7(0.62963)^3 - 15(0.62963)^2 + 5(0.62963) - 17 = -17.57963$$

### Second Iteration

$$f(x_l) = 3(0.62963)^4 + 7(0.62963)^3 - 15(0.62963)^2 + 5(0.62963) - 17 = -17.57963$$

$$f(x_u) = 3(2)^4 + 7(2)^3 - 15(2)^2 + 5(2) - 17 = 37$$

$$x_m = \frac{x_l(f(x_u)) - x_u(f(x_l))}{f(x_u) - f(x_l)} = \frac{0.62963(37) - 2(-17.57963)}{37 - (-17.57963)} = 1.07101$$

$$f(x_m) = 3(1.07101)^4 + 7(1.07101)^3 - 15(1.07101)^2 + 5(1.07101) - 17 = -16.30402$$