A linear system is a set of n equations that can solve n number of unknowns. This can be written as:

$A_{11}X_1 + A_{12}X_2 + ... + A_{1n}X_n = B_1$

$A_{21}X_1 + A_{22}X_2 + ... + A_{2n}X_n = B_2$

$A_{n1}X_1 + A_{n2}X_2 + ... + A_{nn}X_n = B_n$

The set $x = X_1X_2, ..., X_n$ is considered the solution of the linear system if they are **correct in all equations.**

Methods of Solution

Direct Method

Gauss - Elimination

- Consists of 2 phases:
    1. **Elimination**: Series or row operations are performed in the coefficient matrix until it becomes an **upper triangular matrix**.

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ 0 & x_5 & x_6 \\ 0 & 0 & x_9 \end{bmatrix}$$

    2. **Back-substitution**: Solving each element of the solution will start at the last row and move up to the first row.

Problem #1 Determine the solution of the linear system shown below using Gauss Elimination. $$ 2X_1 - 4X_2 + X_3 = -11

X_1 + 3X_2 + 2X_3 -4

3X_1 + 5X_2 + 2X_3 = -2 $$

Elimination Phase:

$$\left|\begin{matrix} 2 & -4 & 1 \\ 1 & 3 & 2 \\ 3 & 5 & 2 \end{matrix}\right| \left|\begin{matrix} -11 \\ -4 \\ -2 \end{matrix}\right| \quad \begin{matrix} R_2 \to R_2 - \left(\frac{1}{2}\right) R_1 \\ R_3 \to R_3 - \left(\frac{3}{2}\right) R_1 \end{matrix} \quad \left|\begin{matrix} 2 & -4 & 1 \\ 0 & 5 & \frac{3}{2} \\ 0 & 11 & \frac{1}{2} \end{matrix}\right| \left|\begin{matrix} -11 \\ \frac{3}{2} \\ \frac{29}{2} \end{matrix}\right|$$

*The goal here is to create an **upper triangular matrix**, where all elements below the main diagonal are zeros. To achieve this, we need to perform row operations on R_2 and R_3 in order to eliminate the first element of both rows (below the diagonal). The number 2 is derived from the 1st element of R_0.*