Closed Methods in Finding the root of non-linear system

Bisection

- Determine $x_m = \frac{x_l + x_u}{2}$
- Find $f(x_l), f(x_m), f(x_u)$
- Choose new interval (*find the interval with 2 different signts (+,-)*)

Example Problem

Determine the root of $3x^4 + 7x^3 - 15x^2 + 5x = 17$ between $[0, 2]$. Use Bisection Method for 7 iterations.

Count	x_l	x_m	x_u	$f(x_l)$	$f(x_m)$	$f(x_u)$	*New*
1.	0	1	2	-17	-17	37	[1,2]
2.	1	1.5	2	-17	-4.43750	37	[1.5,2]
3.	1.5	1.75	2	-4.43750	11.46484	37	[1.5,1.75]
4.	1.5	1.625	1.75	-4.43750	2.47144	11.46484	[1.5,1.625]
5.	1.5	1.5625	1.625	-4.43750	-1.22432	2.47144	[1.5625,1.625]
6.	1.5625	1.59375	1.625	-1.22432	0.56087	2.47144	[1.5625,1.59375]
7.	1.5625	1.57813	1.59375	-1.22432	-0.34681	0.56087	[1.57813,1.59375]

First Iteration

$$
\begin{aligned}
x_m &= \frac{x_l + x_u}{2} \\
&= \frac{0+2}{2} \\
x_m &= 1 \\
f(x_l) &= 3(0)^4 + 7(0)^3 - 15(0)^2 + 5(0) - 17 \\
&= -17 \\
f(x_m) &= 3(1)^4 + 7(1)^3 - 15(1)^2 + 5(1) - 17 \\
&= -17 \\
f(x_u) &= 3(2)^4 + 7(2)^3 - 15(2)^2 + 5(2) - 17 \\
&= 37
\end{aligned}
$$

Second Iteration

$$\begin{aligned}
x_m &= \frac{x_l + x_u}{2} \\
&= \frac{1+2}{2} \\
x_m &= 1.5 \\
f(x_l) &= 3(1)^4 + 7(1)^3 - 15(1)^2 + 5(1) - 17 \\
&= -17 \\
f(x_m) &= 3(1.5)^4 + 7(1.5)^3 - 15(1.5)^2 + 5(1.5) - 17 \\
&= -4.43750 \\
f(x_u) &= 3(2)^4 + 7(2)^3 - 15(2)^2 + 5(2) - 17 \\
&= 37
\end{aligned}$$

Third Iteration

$$\begin{aligned}
x_m &= \frac{x_l + x_u}{2} \\
&= \frac{1.5+2}{2} \\
x_m &= 1.75 \\
f(x_l) &= 3(1.5)^4 + 7(1.5)^3 - 15(1.5)^2 + 5(1.5) - 17 \\
&= -4.43750 \\
f(x_m) &= 3(1.75)^4 + 7(1.75)^3 - 15(1.75)^2 + 5(1.75) - 17 \\
&= 11.46484 \\
f(x_u) &= 3(2)^4 + 7(2)^3 - 15(2)^2 + 5(2) - 17 \\
&= 37
\end{aligned}$$

Fourth Iteration

$$\begin{aligned}
x_m &= \frac{x_l + x_u}{2} \\
&= \frac{1.5+1.75}{2} \\
x_m &= 1.625 \\
f(x_l) &= 3(1.5)^4 + 7(1.5)^3 - 15(1.5)^2 + 5(1.5) - 17 \\
&= -4.43750 \\
f(x_m) &= 3(1.625)^4 + 7(1.625)^3 - 15(1.625)^2 + 5(1.625) - 17 \\
&= 2.47144 \\
f(x_u) &= 3(1.75)^4 + 7(1.75)^3 - 15(1.75)^2 + 5(1.75) - 17 \\
&= 11.46484
\end{aligned}$$

Fifth Iteration

$$\begin{aligned}
x_m &= \frac{x_l + x_u}{2} \\
&= \frac{1.5 + 1.625}{2} \\
x_m &= 1.5625 \\
f(x_l) &= 3(1.5)^4 + 7(1.5)^3 - 15(1.5)^2 + 5(1.5) - 17 \\
&= -4.43750 \\
f(x_m) &= 3(1.5625)^4 + 7(1.5625)^3 - 15(1.5625)^2 + 5(1.5625) - 17 \\
&= -1.22432 \\
f(x_u) &= 3(1.625)^4 + 7(1.625)^3 - 15(1.625)^2 + 5(1.625) - 17 \\
&= 2.47144
\end{aligned}$$

Sixth Iteration

$$\begin{aligned}
x_m &= \frac{x_l + x_u}{2} \\
&= \frac{1.5625 + 1.625}{2} \\
x_m &= 1.59375 \\
f(x_l) &= 3(1.5625)^4 + 7(1.5625)^3 - 15(1.5625)^2 + 5(1.5625) - 17 \\
&= -1.22432 \\
f(x_m) &= 3(1.59375)^4 + 7(1.59375)^3 - 15(1.59375)^2 + 5(1.59375) - 17 \\
&= 0.56087 \\
f(x_u) &= 3(1.625)^4 + 7(1.625)^3 - 15(1.625)^2 + 5(1.625) - 17 \\
&= 2.47144
\end{aligned}$$

Seventh Iteration

$$\begin{aligned}
x_m &= \frac{x_l + x_u}{2} \\
&= \frac{1.5625 + 1.59375}{2} \\
x_m &= 1.57813 \\
f(x_l) &= 3(1.5625)^4 + 7(1.5625)^3 - 15(1.5625)^2 + 5(1.5625) - 17 \\
&= -1.22432 \\
f(x_m) &= 3(1.57813)^4 + 7(1.57813)^3 - 15(1.57813)^2 + 5(1.57813) - 17 \\
&= -0.34681 \\
f(x_u) &= 3(1.59375)^4 + 7(1.59375)^3 - 15(1.59375)^2 + 5(1.59375) - 17 \\
&= 0.56087
\end{aligned}$$