Lagrange Interpolation Polynomial

x	$f(x)$
308.6	0.055389
362.6	0.047485
423.3	0.40914
491.4	0.035413

$$\begin{aligned}
f_3(400) = fx_0(&\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}) \\
+fx_1(&\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}) \\
+fx_2(&\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}) \\
+fx_3(&\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)})
\end{aligned}$$

$$\begin{aligned}
f_3(400) = 0.055389(&\frac{(400-362.6)(400-423.3)(400-491.4)}{(308.6-362.6)(308.6-423.3)(308.6-491.4)}) \\
+0.047485(&\frac{(400-308.6)(400-423.3)(400-491.4)}{(362.6-308.6)(362.6-423.3)(362.6-491.4)}) \\
+0.040914(&\frac{(400-308.6)(400-362.6)(400-491.4)}{(423.3-308.6)(423.3-362.6)(423.3-491.4)}) \\
+0.035413(&\frac{(400-308.6)(400-362.6)(400-423.3)}{(491.4-308.6)(491.4-362.6)(491.4-423.3)})
\end{aligned}$$

$$\begin{aligned}
f_3(400) = 0.055389(&\frac{(37.4)(-23.3)(-91.4)}{(-54)(-114.7)(-182.8)}) \\
+0.047485(&\frac{(91.4)(-23.3)(-91.4)}{(54)(-60.7)(-128.8)}) \\
+0.040914(&\frac{(91.4)(37.4)(-91.4)}{(114.7)(60.7)(-68.1)}) \\
+0.035413(&\frac{(91.4)(37.4)(-23.3)}{(182.8)(128.8)(68.1)})
\end{aligned}$$

$$\begin{aligned}
f_3(400) &= -0.00390 + 0.02189 + 0.02696 + (-0.00176) \\
f_3(400) &= 0.04319
\end{aligned}$$

Alternative Cubic Spline Interpolation

x	$f(x)$
1	8
4	12
9	18
10	26

The second derivative at the end points of the curve is equal to 0, therefore:

$f''(x_0) = 0$

$f''(x_1) =?$

$f''(x_2) =?$

$f''(x_3) = 0$

Using the equation above, we can find the value of $f''(x_1)$ *and* $f''(x_2)$*:*

Solving for $f''(x_1)$:

$$(x_i - x_{i-1})f''(x_{i-1}) + 2(x_{i+1} - x_{i-1})f''(x_i) + (x_{i+1} - x_i)f''(x_{i+1})$$
$$= \frac{6}{x_{i+1} - x_i}[f(x_{i+1}) - f(x_i)] + \frac{6}{x_i - x_{i-1}}[f(x_{i-1}) - f(x_i)]$$

$$(x_1 - x_0)f''(x_0) + 2(x_2 - x_0)f''(x_1) + (x_2 - x_1)f''(x_2)$$
$$= \frac{6}{x_2 - x_1}[f(x_2) - f(x_1)] + \frac{6}{x_1 - x_0}[f(x_0) - f(x_1)]$$

$$(4 - 1)(0) + 2(9 - 1)f''(x_1) + (9 - 4)f''(x_2)$$
$$= \frac{6}{9 - 4}[18 - 12] + \frac{6}{4 - 1}[8 - 12]$$
$$= 2(8)f''(x_1) + 5f''(x_2) = \frac{6}{5}[6] + 2[-4]$$
$$= 16f''x_1 + 5f''x_2 = 7.2 + (-8)$$
$$= 16f''x_1 + 5f''x_2 = -0.8(equation1)$$

Solving for $f''(x_2)$:

$$(x_i - x_{i-1})f''(x_{i-1}) + 2(x_{i+1} - x_{i-1})f''(x_i) + (x_{i+1} - x_i)f''(x_{i+1})$$
$$= \frac{6}{x_{i+1} - x_i}[f(x_{i+1}) - f(x_i)] + \frac{6}{x_i - x_{i-1}}[f(x_{i-1}) - f(x_i)]$$

$$(x_2 - x_1)f''(x_1) + 2(x_3 - x_1)f''(x_2) + (x_3 - x_2)f''(x_3)$$
$$= \frac{6}{x_3 - x_2}[f(x_3) - f(x_2)] + \frac{6}{x_2 - x_1}[f(x_1) - f(x_2)]$$

$$(9-4)f''(x_1) + 2(10-4)f''(x_2) + (10-9)(0)$$
$$= \frac{6}{10-9}[26-18] + \frac{6}{9-4}[12-18]$$
$$= 5f''(x_1) + 2(6)f''(x_2) = 6(8) + \frac{6}{5}(-6)$$
$$= 5f''x_1 + 12f''x_2 = 48 + (-7.2)$$
$$= 5f''x_1 + 12f''x_2 = 40.8(equation2)$$

Finding the value of the second derivatives $f''(x_1)$ and $f''(x_2)$

$$16f''x_1 + 5f''x_2 = -0.8$$
$$16f''x_1 = -0.8 - 5f''x_2$$
$$f''x_1 = \frac{-0.8 - 5f''x_2}{16}$$

$$5f''x_1 + 12f''x_2 = 40.8$$
$$5(\frac{-0.8 - 5f''x_2)}{16}) + 12f''x_2 = 40.8$$
$$\frac{-4 - 25f''x_2}{16} + 12f''x_2 = 40.8$$
$$\frac{-4 - 25f''x_2 + 192f''x_2}{16} = 40.8$$
$$\frac{-4 + 167f''x_2}{16} = 40.8$$
$$-4 + 167f''x_2 = 652.8$$
$$167f''x_2 = 656.2.8 + 4$$
$$167f''x_2 = 656.8$$
$$f''x_2 = \frac{656.8}{167}$$
$$f''x_2 = 3.93293$$

$$f''x_1 = \frac{-0.8 - 5f''x_2}{16} f''x_1 = \frac{-0.8 - 5(3.93293)}{16}$$
$$f''x_1 = \frac{-0.8 - 19.66465}{16}$$
$$f''x_1 = \frac{-20.46465}{16}$$
$$f''x_1 = -1.27904$$

Finding the value of $f(6)$

$$\begin{aligned}
f_6 = &\frac{f''(x_{i-1})}{6(x_i - x_{i-1})}(x_i - x)^3 + \frac{f''(x_i)}{6(x_i - x_{i-1})}(x - x_{i-1})^3 \\
&+ \left[\frac{f(x_{i-1})}{x_i - x_{i-1}} - \frac{f''(x_{i-1})(x_i - x_{i-1})}{6}\right](x_i - x) \\
&+ \left[\frac{f(x_i)}{x_i - x_{i-1}} - \frac{f''(x_i)(x_i - x_{i-1})}{6}\right](x - x_{i-1})
\end{aligned}$$

$$\begin{aligned}
x_i - 1 &= 4 \\
x_i &= 9 \\
x &= 6 \\
fx_i - 1 &= 12 \\
fx_i &= 18 \\
f''x_i - 1 &= -1.27904 \\
f''x_i &= 3.93293
\end{aligned}$$

$$\begin{aligned}
f_6 = &\frac{-1.27904}{6(9-4)}(9-6)^3 + \frac{3.93293}{6(9-4)}(6-4)^3 \\
&+ \left[\frac{12)}{9-4} - \frac{(-1.27904)(9-4}{6}\right](9-6) \\
&+ \left[\frac{18}{9-4} - \frac{(3.93293)(9-4)}{6}\right](6-4)
\end{aligned}$$

$$\begin{aligned}
f_6 = &\frac{-1.27904}{6(5)}(3)^3 + \frac{3.93293}{6(5)}(2)^3 \\
&+ \left[\frac{12}{5} - \frac{(-1.27904)(5)}{6}\right](3) \\
&+ \left[\frac{18}{5} - \frac{(3.93293)(5)}{6}\right](2)
\end{aligned}$$

$$\begin{aligned}
f_6 = &\frac{-1.27904}{30}(27) + \frac{3.93293}{30}(8) \\
&+ \left[\frac{12}{5} - \frac{-6.3952}{6}\right](3) \\
&+ \left[\frac{18}{5} - \frac{19.66465}{6}\right](2)
\end{aligned}$$

$$\begin{aligned}
f_6 &= -0.10235 + 10.3976 + 0.64512 \\
f_6 &= -0.10235 + 10.3976 + 0.64512 \\
f_6 &= 10.94037
\end{aligned}$$