Lagrange Interpolating Polynomial

- polynomial with degree n that passes through a point $(x_0, y_0), (x_1, y_1), \cdots, (x_n, y_n)$
- defined mathematically as

$$f_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

Where:

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

Problem # 1 Determine the value of $f(7)$ for a given set of data below using the alternative cubic spline interpolation method. x should be in increasing number.

x	$f(x)$
1	12
5	-26
8	-14
10	37

$$\begin{aligned} f_3(x) = fx_0(&\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \\ +fx_1(&\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \\ +fx_2(&\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \\ +fx_3(&\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \end{aligned}$$

$$\begin{aligned} f_3(7) = 12(&\frac{(7-5)(7-8)(7-10)}{(1-5)(1-8)(1-10)}) \\ +-26(&\frac{(7-1)(7-8)(7-10)}{(5-1)(5-8)(5-10)} \\ +-14(&\frac{(7-1)(7-5)(7-10)}{(8-1)(8-5)(8-10)} \\ +37(&\frac{(7-1)(7-5)(7-8)}{(10-1)(10-5)(10-8)} \end{aligned}$$

$$
\begin{aligned}
f_3(7) = 12(\frac{(2)(-1)(-3)}{(-4)(-7)(-9)}) \\
+ - 26(\frac{(6)(-1)(-3)}{(4)(-3)(-5)} \\
+ - 14(\frac{(6)(2)(-3)}{(7)(3)(-2)} \\
+37(\frac{(6)(2)(-1)}{(9)(5)(2)}
\end{aligned}
$$

$$
\begin{aligned}
f_3(7) = -0.28571 - 7.8 - 12 - 4.93333 \\
f_3(7) = -25.01904
\end{aligned}
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