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BSCS3

Midterm Activity 1

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## Gauss Elimination

$$a + b + c + d = 12$$

$$125a + 25b + 5c + d = -26$$

$$512a + 64b + 8c + d = -14$$

$$1000a + 100b + 10c + d = 37$$

### Elimination Phase

$$\left[ \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 12 \\ 125 & 25 & 5 & 1 & -26 \\ 512 & 64 & 8 & 1 & -14 \\ 1000 & 100 & 10 & 1 & 37 \end{array} \right] \begin{array}{l} R_2 \rightarrow R_2 - 125R_1 \\ R_3 \rightarrow R_3 - 512R_1 \\ R_4 \rightarrow R_4 - 1000R_1 \end{array}$$

R2 Operations:

$$R_2[1] = 125 - 125(1) = 0$$

$$R_2[2] = 25 - 125(1) = -100$$

$$R_2[3] = 5 - 125(1) = -120$$

$$R_2[4] = 1 - 125(1) = -124$$

$$R_2[5] = -26 - 125(12) = -1526$$

R3 Operations:

$$R_3[1] = 512 - 512(1) = 0$$

$$R_3[2] = 64 - 512(1) = -448$$

$$R_3[3] = 8 - 512(1) = -504$$

$$R_3[4] = 1 - 512(1) = -511$$

$$R_3[5] = -14 - 512(12) = -6158$$

R4 Operations:

$$R_4[1] = 1000 - 1000(1) = 0$$

$$R_4[2] = 100 - 1000(1) = -900$$

$$R_4[3] = 10 - 1000(1) = -990$$

$$R_4[4] = 1 - 1000(1) = -999$$

$$R_4[5] = 37 - 1000(12) = -11963$$

**New Matrix**

$$\left[ \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 12 \\ 0 & -100 & -120 & -124 & -1526 \\ 0 & -448 & -504 & -511 & -6158 \\ 0 & -900 & -990 & -999 & -11963 \end{array} \right] R_2 \rightarrow \frac{R_2}{-100}$$

R2 Operations:

$$R_2[1] = \frac{0}{-100} = 0$$

$$R_2[2] = \frac{-100}{-100} = 1$$

$$R_2[3] = \frac{-120}{-100} = 1.2$$

$$R_2[4] = \frac{-124}{-100} = 1.24$$

$$R_2[5] = \frac{-1526}{-100} = 15.26$$

**New Matrix**

$$\left[ \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 12 \\ 0 & 1 & 1.2 & 1.24 & 15.26 \\ 0 & -448 & -504 & -511 & -6158 \\ 0 & -900 & -990 & -999 & -11963 \end{array} \right] \begin{array}{l} R_3 \rightarrow R_3 - (-448)R_2 \rightarrow R_3 + 448R_2 \\ R_4 \rightarrow R_4 - (-900)R_2 \rightarrow R_4 + 900R_2 \end{array}$$

Row 3 Operations:

$$R_3[1] = 0 + 448(0) = 0$$

$$R_3[2] = -448 + 448(1) = 0$$

$$R_3[3] = -504 + 448(1.2) = 33.6$$

$$R_3[4] = -511 + 448(1.24) = 44.52$$

$$R_3[5] = -6158 + 448(15.26) = -678.48$$

Row 4 Operations:

$$R_4[1] = 0 + 900(0) = 0$$

$$R_4[2] = -900 + 900(1) = 0$$

$$R_4[3] = -990 + 900(1.2) = 90$$

$$R_4[4] = -999 + 900(1.24) = 117$$

$$R_4[5] = -11963 + 900(15.26) = 1771$$

**New Matrix**

$$\left[ \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 12 \\ 0 & 1 & 1.2 & 1.24 & 15.26 \\ 0 & 0 & 33.6 & 44.52 & 678.48 \\ 0 & 0 & 90 & 117 & 1771 \end{array} \right] R_3 \rightarrow \frac{R_3}{33.6}$$

Row 3 Operations:

$$R_3[1] = \frac{0}{33.6} = 0$$

$$R_3[2] = \frac{0}{33.6} = 0$$

$$R_3[3] = \frac{33.6}{33.6} = 1$$

$$R_3[4] = \frac{44.52}{33.6} = 1.325$$

$$R_3[5] = \frac{678.48}{33.6} = 20.19286$$

**New Matrix**

$$\left[ \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 12 \\ 0 & 1 & 1.2 & 1.24 & 15.26 \\ 0 & 0 & 1 & 1.325 & 20.19286 \\ 0 & 0 & 90 & 117 & 1771 \end{array} \right] R_4 \rightarrow R_4 - 90R_3$$

Row 4 Operations:

$$R_4[1] = 0 - 90(0) = 0$$

$$R_4[2] = 0 - 90(0) = 0$$

$$R_4[3] = 90 - 90(1) = 0$$

$$R_4[4] = 117 - 90(1.325) = -2.25$$

$$R_4[5] = 1771 - 90(20.19286) = -46.3574$$

**New Matrix**

$$\left[ \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 12 \\ 0 & 1 & 1.2 & 1.24 & 15.26 \\ 0 & 0 & 1 & 1.325 & 20.19286 \\ 0 & 0 & 0 & -2.25 & -46.3574 \end{array} \right]$$

## Back Substitution

Solving for  $d$ :

$$-2.25d = -46.3574$$

$$\frac{-2.25d}{-2.25} = \frac{-46.3574}{-2.25}$$

$$d = 20.60329$$

Solving for  $c$ :

$$c + 1.325d = 20.19286$$

$$c + 1.325(20.60329) = 20.19286$$

$$c + 27.29936 = 20.19286$$

$$c = 20.19286 - 27.29936$$

$$c = -7.1065$$

Solving for  $b$ :

$$b + 1.2c + 1.24d = 15.26$$

$$b + 1.2(-7.1065) + 1.24(20.60329) = 15.26$$

$$b - 8.5278 + 25.54808 = 15.26$$

$$b + 17.02028 = 15.26$$

$$b = 15.26 - 17.02028$$

$$b = -1.76028$$

Solving for  $a$ :

$$a + b + c + d = 12$$

$$a - 1.76028 - 7.1065 + 20.60329 = 12$$

$$a + 11.73651 = 12$$

$$a = 12 - 11.73651$$

$$a = 0.26349$$

## Final Answer

$$a = 0.26349$$

$$b = -1.76028$$

$$c = -7.1065$$

$$d = 20.60329$$