ACTL 2102 ASSIGNMENT T3 2024

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Executive Summary

This report assesses the current Bonus Malus System (BMS) implemented by Fantastic Insurance (FI) whilst also assessing two other potential BMS which may be implemented in place of the existing system.

To assess each BMS option, the probability transition matrix for each scheme had to be determined. Assuming that the rate of claim by a policy holder is Poisson Distributed, we can calculate the transition matrices for each scheme. To assess the current scheme I simulated the following year in accordance with the transition matrices 1500 times to generate an estimation for the coming years distribution of policy holders in each BMS level, the total premiums collected and the expected profit for FI in 2025. My simulations led me to estimate a **total premium of $4,534,428.74** and **expected profit of $1,234,418**. I had also found that the existing BMS had a very low variance with respect to the mean, indicating that the use of this scheme is low risk, with a negligible probability of recording a loss in 2025.

Then, to develop a clearer picture on the effectiveness of each scheme, we tested the Loimaranta efficiency which measures the effectiveness of a scheme to apply premiums by risk parameter. A higher Loimaranta efficiency tells us that the scheme is efficient at allocating premium levels by risk rate. My findings demonstrated that the Complex scheme applied to young policy holders is the most efficiency, followed by the existing and then the simplified. However, at the current rate parameters for our policy holders, the efficiency is marginally different. Further, I was able to estimate the **long run annual profit** for each scheme to be **$611,987.20 for Option 0**, **$1,078,844.00 for Option 1** and **$361,686.70 for Option 2.**

Thus, my final recommendation is for FI to **maintain their current BMS** as it effectively manages the balance between efficiently allocating premiums by risk and generating profit.

Introduction

The purpose of this report is to analyse FI's current BMS and analyse two other BMS options currently being considered by FI. FI currently provides Car Insurance to 10,000 policy holders, 5,000 of which are between the ages of 20 and 25, the other 5,000 between the ages of 50 and 55. FI are currently modelling that a policy holder making claim follows a Poisson distribution with rate parameter $\lambda_{old} = 0.12$ and $\lambda_{young} = 0.18$. Currently, all three BMS are modelled in a way that no claims for a year will result in a level increase, one claim in a year results in a level decrease, and two or more claims results in a two level decrease. Further, the base premium for the group of old policy holders is $450 compared to $500 for the young group. The discounts for each scheme are as follows:

Level	-2	-1	0	1	2
Discount (%)	-30	-15	0	10	20

Table 1. Existing Scheme

Level	-1	0	1
Discount (%)	-15	0	10

Table 2. Simplified Scheme for all Policy Holders

Level	-3	-2	-1	0	1	2	3
Discount (%)	-45	-30	-15	0	10	20	30

Table 3. Complex Scheme for Young Policy Holders

This report aims to assess the feasibility of the three schemes to find the most effective scheme for FI going forward.

1. **Task 1**

1.1. **Probability Transition Matrices for Each Option.**

The following are the probability transition matrices for each of the BMS schemes and each group of policy holders young and old. These matrices were calculated assuming that claims are modelled by a Poisson distribution with $\lambda_{old} = 0.12$ and $\lambda_{young} = 0.18$.

<u>Existing Scheme</u>:

$$\mathbb{P}_{old} = \begin{pmatrix} 0.1131 & 0.8869 & 0 & 0 & 0 \\ 0.1131 & 0 & 0.8869 & 0 & 0 \\ 0.0066 & 0.1064 & 0 & 0.887 & 0 \\ 0 & 0.0066 & 0.1064 & 0 & 0.887 \\ 0 & 0 & 0.0066 & 0.1064 & 0.887 \end{pmatrix},$$

and

$$\mathbb{P}_{young} = \begin{pmatrix} 0.1647 & 0.8353 & 0 & 0 & 0 \\ 0.1647 & 0 & 0.8353 & 0 & 0 \\ 0.0144 & 0.1503 & 0 & 0.8353 & 0 \\ 0 & 0.0144 & 0.1503 & 0 & 0.8353 \\ 0 & 0 & 0.0144 & 0.1503 & 0.83531 \end{pmatrix}.$$

<u>Scheme Option 1</u>:

$$\mathbb{P}_{old} = \begin{pmatrix} 0.1131 & 0.8869 & 0 \\ 0.1131 & 0 & 0.8869 \\ 0.0066 & 0.1064 & 0.8869 \end{pmatrix},$$

and

$$\mathbb{P}_{young} = \begin{pmatrix} 0.1647 & 0.8353 & 0 \\ 0.1647 & 0 & 0.8353 \\ 0.0144 & 0.1503 & 0.8353 \end{pmatrix}.$$

<u>Scheme Option 2</u>:

$$\mathbb{P}_{old} = \begin{pmatrix} 0.1131 & 0.8869 & 0 & 0 & 0 \\ 0.1131 & 0 & 0.8869 & 0 & 0 \\ 0.0066 & 0.1064 & 0 & 0.887 & 0 \\ 0 & 0.0066 & 0.1064 & 0 & 0.887 \\ 0 & 0 & 0.0066 & 0.1064 & 0.887 \end{pmatrix},$$

and

$$\mathbb{P}_{young} = \begin{pmatrix} 0.1647 & 0.8353 & 0 & 0 & 0 & 0 & 0 \\ 0.1647 & 0 & 0.8353 & 0 & 0 & 0 & 0 \\ 0.0144 & 0.1503 & 0 & 0.8353 & 0 & 0 & 0 \\ 0 & 0.0144 & 0.1503 & 0 & 0.8353 & 0 & 0 \\ 0 & 0 & 0.0144 & 0.1503 & 0 & 0.83531 & 0 \\ 0 & 0 & 0 & 0.0144 & 0.1503 & 0 & 0.8353 \\ 0 & 0 & 0 & 0 & 0.0144 & 0.1503 & 0.8353 \end{pmatrix}.$$

1.2. **One Year Forecast of Existing Scheme.**

After simulating the existing scheme 1500 times, here are the average number of policy holders in each NDC level.

-2	-1	0	1	2
557.69	1771.71	2177.97	2104.48	3388.15

Here are some summary statistics for the distribution of the total premium under the existing scheme:

Mean	SD	Skewness	Kurtosis	Min	Max
\$4,534,428.74	\$3,893.678	-0.05155	3.2549	\$4,520,770.00	\$4,547,212.50

As we can see from the summary statistics, the distribution of the total premium is essentially symmetrical, and possesses a kurtosis of slightly over 3; thus, we can assume that the distribution roughly follows a normal distribution. We can also note that the relative size of the SD to the Mean is very low, demonstrating a tightly clustered distribution which implies relatively low risk in the existing scheme.
By considering the cost associated with a claim to be \$2,200 per claim, I have calculated the **expected profit** for FI for 2025 to be approximately **\$1,234,418.**

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2. **Task 2**

2.1. **Loimaranta Efficiency of Each Scheme.**

I have calculated the Loimaranta efficiency for all three BMS schemes. The Loimaranta efficiency represents the fractional change in the mean premium paid divided by the fractional change in the rate parameter. This measure will help us understand for what rate parameter is a given scheme most efficient at allocating an accurate premium. Here is a graphical representation of the Loimaranta efficiency vs Rate Parameter for the three schemes:

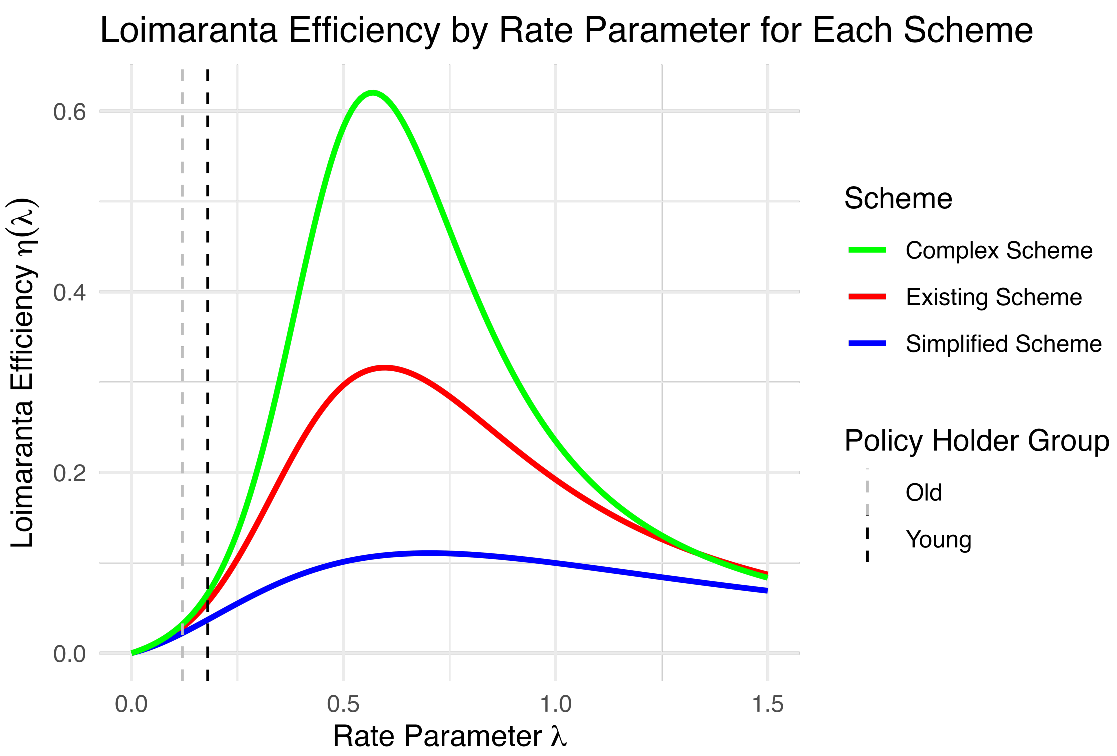

FIGURE 1.

By inspecting Figure 1, we can see that the complex scheme is the most efficient at allocating premium to risk, followed by the existing scheme and then the simplified scheme. However, we can note that the maximum efficiency for all three schemes occurs when the Rate Parameter is in the range of 0.55 to 0.6. We must note that the rate parameter for our two sets of policy holders are 0.18 for youth and 0.12 for the older group, as indicated by the vertical dashing lines in Figure 1. When inspecting the efficiency, there is minimal difference in the efficiency of each scheme for older policy holders, and a marginal difference in efficiency for the young policy holders.

2.2. **Long Run Annual Profit of Each Scheme.**

Further, for the three options we can calculate the long run expected profit for each of the 3 possible options by considering the long run annual revenue to be the long run proportion in each level multiplied by the premiums at their respective discounts, and the long run annual cost to be $2200 multiplied by the expected number of claims per year.

For **Option 0**, where the existing scheme is used for both young and old policy holders, we can compute the long run annual profit to be **$611,987.20**. This profit is made up from a **$94,541** profit from young policy holders and **$517,446** profit from the old group of policy holders.

For **Option 1**, where the simplified scheme is used for both young and old policy holders, we can compute the long run annual profit to be **$1,078,844.00**. This profit is made up from a **$337,320** profit from young policy holders and **$741,523** profit from the old group of policy holders.

For **Option 2**, where the existing scheme is used for the old policy holders and the complex scheme used for the young policy holders, we can compute the long run annual profit to be **$361,686.70**. This profit is made up from a **-$155,759.3** deficit from young policy holders and **$517,446** profit from the old group of policy holders.

2.3. **System Recommendation.**
Looking through each of the options, Option 0 is the best BMS available as it balances Loimaranta efficiency and profit well compared to the other options. Option 1 is very effective at generating profits, however, its Loimaranta efficiency is extremely low in comparison to options 0 and 2. Although profit may be the driving goal, it may deter customers if it is made evident that the premiums do not effectively reflect that policy holders risk, this may cause a growing sentiment against FI which may attract less future customers and even cause existing policy holders to change insurance companies. Finally, Option 2 is extremely effcicient at allocating premiums with relation to a policy holders risk, however, it is noted that within Option 2, the complex scheme is only applied to young policy holders. With the current model for the complex scheme, it currently records a loss for the group of young policy holders, and therefore should most definitely be eliminated from consideration for the replacement BMS.
Thus, the final recommendation for FI is to **maintain their existing BMS**. If they are interested in increasing their schemes efficiency similar to the complex scheme it would have to impose harsher penalties for the lower levels of the scheme and be less generous with the discounts given at higher levels. Another way to improve their BMS system would be to hold larger amounts of older policy holders as they are much more profitable than the younger group for all scheme options.

2.4. **Assumptions.**
One of the key assumptions that was made in this report was that the claim rate is independent of the number of claims previously made. This would make the calculations more complex but would generate a much more accurate representation of risk within each policy holder. Perhaps the policy holder groups should be subdivided further than just age, taking into account driving experience or each policy holders previous driving record. This would allow FI to generate a more accurate model allowing the company to make more informed business decisions in the future leading to increased shareholder value.

It was also assumed that the risk profile of the policy holders that FI would hold would remain constant when computing long run estimations. In reality, their policy holder base would be changing year to year. A more dynamic rate parameter for each policy holder group could combat this issue allowing for more nuanced scheme which can account for more variation.

Conclusion
Thus, my final recommendation is for FI to **maintain their current BMS** as it effectively manages the balance between efficiently allocating premiums by risk and generating profit.

References

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