MEK 4430 Homework 2. **Fall 2013**

The one-dimensional two-fluid mass momentum equations for the case of steady state, fully developed flow, incompressible flow are given by

$$A_L \frac{\partial P_{IL}}{\partial x} - \tau_L S_L + \tau_i S_i - \rho_L g \sin\beta = 0 \qquad (0.1)$$

$$A_G \frac{\partial P_{IG}}{\partial x} - \tau_G S_G - \tau_i S_i - \rho_G g \sin\beta = 0 \qquad (0.2)$$

Model the interfacial shear using the following

$$\tau_I = \frac{1}{8}\rho_G f_I \bar{u}^2 \quad [N/m^2] \qquad (0.3)$$

where

$$f_I = \theta f_G \, [-] \qquad (0.4)$$

and θ is a model coefficient.

The friction factors for the gas and liquid can be obtained either from the Moody chart or from a known correlation (see page 26)

Question 1. Using the appropriate closure relations, derive the holdup equation.

Question 2. Determine the liquid holdup using the following geometrical variables, physical properties, pipe dimension, inclination and superficial velocities. Use $\theta = 1$. (hint use the bisection method)

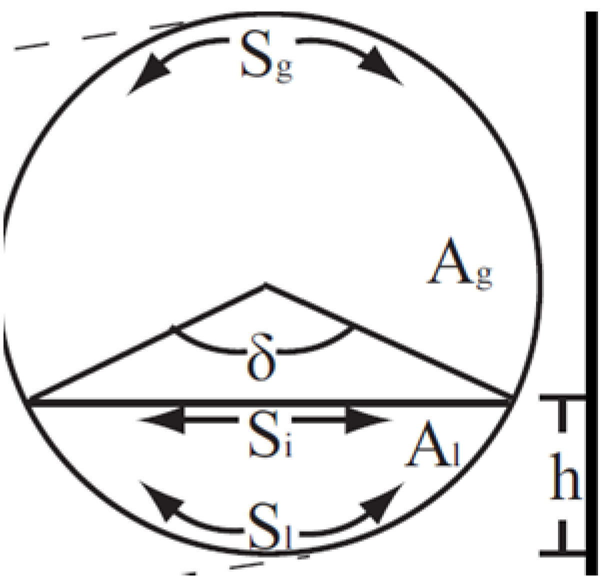

Figure 1. Geometrical variables

Geometrical variables

Angle (see Figure 1.) $[-]$

$$\delta = 2\arccos\left(1 - \frac{2h}{D}\right)$$

Areas $[m^2]$

$$A = \frac{\pi D^2}{4},$$
$$A_l = \frac{A}{2\pi}\left(\delta - \sin\delta\right),$$
$$A_g = A - A_l$$

Perimeters $[m]$

$$S_g = D\left(\pi - \frac{\delta}{2}\right),$$
$$S_l = \frac{D\delta}{2}$$
$$S_i = D\sin\left(\frac{\delta}{2}\right)$$

Hydraulic diameters $[m]$

$$D_l = \frac{4A_l}{S_l},$$
$$D_g = \frac{4A_g}{(S_i + S_g)}$$

Physical Properties

a. Liquid density $\rho_L = 1000\left[\frac{kg}{m^3}\right]$

b. Gas density $\rho_G = 50\left[\frac{kg}{m^3}\right]$

c. Liquid dynamic viscosity $\mu_L = 0.001\left[\frac{kg}{m\cdot s}\right]$

d. Gas dynamic viscosity $\mu_G = 0.00001\left[\frac{kg}{m\cdot s}\right]$

Pipe Dimension

Pipe Diameter $[m]$

$$D = 0.1$$

Pipe inclination $[-]$

$$\beta = 0$$

Superficial velocities $[m/s]$

$$U_{SL} = 0.3$$
$$U_{SG} = 3.0$$

Question 3. What are the in-situ average phase velocities $\overline{u_L}$ and $\overline{u_G}$ $[m/s]$ for the case described in Question 2.

Question 4. Using Eqs. (1.1) and (1.2) determine the pressure gradient using the calculated holdup determined from Question 2.

Question 5. Compare the liquid holdup and average phase velocities from Question 2 with the liquid holdup and average phase velocities when $\theta = 3$.

Question 6. Using $\theta = 1$ and Question 2 once again, what happens to the liquid level when we double the superficial gas velocity (ie. $U_{SG} = 6.0$ $[m/s]$)? Compare the results with Question 2 when we used $U_{SG} = 3.0$ $[m/s]$.

Question 7. Using the original case described in Question 2, how does holdup change when the pipe angle is increased from $\beta = 0$ to $\beta = 10$?