MEK4430 Homework 2

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0.1 Derivation of the holdup equation

Consider the one-dimension two fluid mass momentum equation for the case of steady state, fully developed incompressible flow:

$$A_L \frac{\partial P_{IL}}{\partial x} - \tau_L S_L + \tau_I S_I - \rho_L g \sin(\beta) = 0 \tag{1}$$

$$A_G \frac{\partial P_{IG}}{\partial x} - \tau_G S_G - \tau_I S_I - \rho_G g \sin(\beta) = 0 \tag{2}$$

The interfacial shear is given by

$$\tau_I = \frac{1}{8} \rho_G f_G \bar{u}^2 \tag{3}$$

where the interfacial friction factor f is given by

$$f_I = \theta f_G \tag{4}$$

The interfacial boundary condition for pressure is given by

$$P_{LI} = P_{GI} = P_I \tag{5}$$

Inserting 5 in to 1 and 2 we get

$$A_L \frac{\partial P_I}{\partial x} - \tau_L S_L + \tau_I S_I - \rho_L g \sin(\beta) = 0 \tag{6}$$

$$A_G \frac{\partial P_I}{\partial x} - \tau_G S_G - \tau_I S_I - \rho_G g \sin(\beta) = 0 \tag{7}$$

Dividing 6 and 7 by A_L and A_G respectively we get

$$\frac{\partial P_I}{\partial x} - \frac{\tau_L S_L}{A_L} + \frac{\tau_I S_I}{A_L} - \frac{\rho_L g \sin(\beta)}{A_L} = 0 \tag{8}$$

$$\frac{\partial P_I}{\partial x} - \frac{\tau_G S_G}{A_G} - \frac{\tau_I S_I}{A_G} - \frac{\rho_G g \sin(\beta)}{A_G} = 0 \tag{9}$$

Multiplying equation 9 by -1 and adding 8 and 9 makes the interfacial pressure gradient fall out and we get the holdup equation:

$$\left(\frac{\tau_G S_G}{A_G} - \frac{\tau_L S_L}{A_L} \right) + \tau_I S_I \left(\frac{1}{A_G} + \frac{1}{A_L} \right) + g \sin(\beta) \left(\frac{\rho_G}{A_G} - \frac{\rho_L}{A_L} \right) = 0 \tag{10}$$

Using the hydraulic approximation the liquid, gas and interfacial shear stress τ_L, τ_G, τ_I in equation 10 are given by

$$\tau_L = \frac{1}{8} \rho_L f_L \bar{U_L}^2$$

$$\tau_G = \frac{1}{8} \rho_G f_G \bar{U_G}^2$$

$$\tau_I = \frac{1}{8} \rho_G \theta f_G (\bar{u_G} - \bar{u_L})^2$$

where the in-situ phase velocity are related to the superficial phase velocity as

$$\bar{U_L} = \frac{U_{SL}}{\alpha_L} \tag{11}$$

$$\bar{U_G} = \frac{U_{SG}}{(1 - \alpha_L)} \tag{12}$$

where α_L is the liquid holdup given by

$$\alpha_L = \frac{A_L}{A} \tag{13}$$

0.2 Computation of the liquid holdup

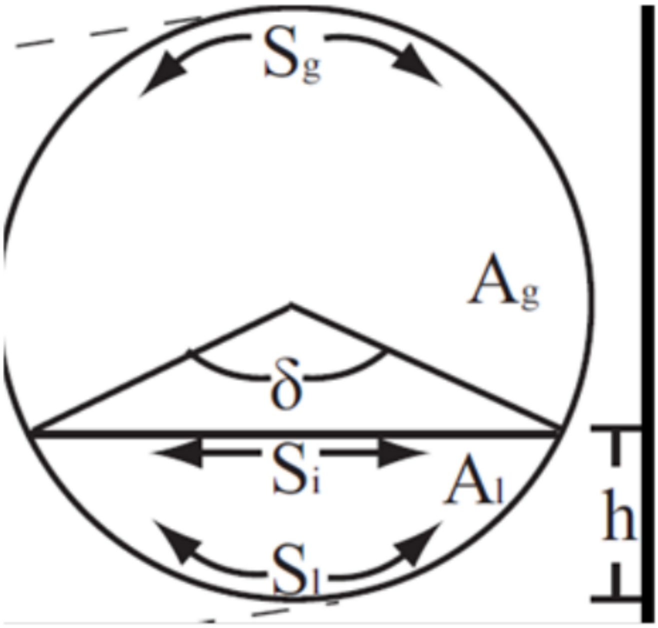

Figure 1: geometrycal variables

Using $\theta = 1$ we derive the liquid holdup in this section. Given the following parameters:

$$\delta = 2\arccos\left(1 - \frac{2h}{D}\right)$$

$$A = \frac{\pi D^2}{4}$$

$$A_L = \frac{A}{2\pi}(\delta - \sin(\delta)) \tag{14}$$
$$A_G = A - A_L$$

$$S_G = D\left(\pi - \frac{\delta}{2}\right)$$

$$S_L = \frac{D\delta}{2}$$

$$S_I = D\sin\left(\frac{\delta}{2}\right)$$

$$D_L = \frac{4A_L}{S_L}$$

$$D_L = \frac{4A_G}{(S_I + S_G)}$$

$\rho_L = 1000$ si, $\rho_G = 50$ si, $\mu_L = 0.001$ si, $\mu_G = 0.00001$ si are the physical properties of the phases. The liquid holdup is by definition given by

$$\alpha_L = \frac{A_L}{A} \tag{15}$$

and given the definition of A_L from (14) we get

$$\alpha_L = \frac{\delta - \sin(\delta)}{2\pi} \tag{16}$$

All the parameters in equation (10) depend on δ so we can solve equation (10) for δ and substitute it value in (16) to find the liquid holdup. To this end we implement a function in python called liquidHoldup() which solve equation (10) for δ by using the bisection method for $0 \leq \delta \leq \pi/2$. The liquid holdup is about

$$\alpha_L = 0.84$$

the program out put:
parameters entered:
superficial liquid velocity $Usl = 0.3$
superficial gas velocity $Usg = 3$
angle $\beta = 0$
$\theta = 1$

computation results:
liquid holdup = 0.837510724347
gas holdup = 0.162489275653

in-situ liquid velocity = 0.358204368349 m/s
in-situ gas velocity = 18.4627569293 m/s

0.3 In-situ average phase velocities of gas and liquid

the in-situ average phase velocity are given by

$$\bar{U}_L = \frac{U_{sl}}{\alpha_L} \tag{17}$$

$$\bar{U}_G = \frac{U_{sg}}{(1-\alpha_L)} \tag{18}$$

From the preview computation, $\alpha_L = 0.84$. with $U_{sl} = 0.3\ m/s$ and $U_{sg} = 3\ m/s$. we get

$$U_L = 0.36 \quad m/s$$

$$U_G = 18.5 \quad m/s$$

0.4 Pressure gradient

from equation (8) and (9) the pressure gradient is given by

$$\frac{\partial P_I}{\partial x} = \frac{\tau_L S_L}{A_L} - \frac{\tau_I S_I}{A_L} + \frac{\rho_L g \sin(\beta)}{A_L} \tag{19}$$

$$\frac{\partial P_I}{\partial x} = \frac{\tau_G S_G}{A_G} + \frac{\tau_I S_I}{A_G} + \frac{\rho_G g \sin(\beta)}{A_G} \tag{20}$$

In both case the pressure gradient is about

$$\frac{\partial P_I}{\partial x} = 173.2 \tag{21}$$

0.5 Comparison of liquid hold up and in-situ velocities

when θ is increased from 1 to 3 we have a very small change in liquid holdup and superficial velocities. This increase has little effect on the liquid holdup and the superficial velocities:
The out put of the program gives:

parameters entered:
superficial liquid velocity $Usl = 0.3$
superficial gas velocity $Usg = 3$
angle $\beta = 0$

$\theta = 1$

computation results:
liquid holdup = 0.837510724346
gas holdup = 0.162489275654

in-situ liquid velocity = 0.358204368349 m/s
in-situ gas velocity = 18.4627569292 m/s

parameters entered:
superficial liquid velocity $Usl = 0.3$
superficial gas velocity $Usg = 3$
angle $\beta = 0$
$\theta = 3$

computation results:
liquid holdup = 0.837105384959
gas holdup = 0.162894615041

in-situ liquid velocity = 0.35837781645 m/s
in-situ gas velocity = 18.4168150632 m/s

The liquid hold up decrease by about $4 * 10^{-4}$ while the in-situ phase velocity of liquid increase by about $2 * 10^{-4}$ and the in-situ phase velocity of gas decrease by about $4.5 * 10^{-2}$. We can say that there is a very small change in the liquid holdup and the in-situ phase velocity when θ changes from 1 to 3

0.6 Increasing the gas superficial phase velocity

By doubling the gas superficial velocity we observe a decrease in the liquid holdup. Since the gas is moving faster in the pipe, to maintain the mass flow rate constant, the liquid level must decrease.

program output:
parameters entered:
superficial liquid velocity $Usl = 0.3$
superficial gas velocity $Usg = 6$
angle $\beta = 0$
$\theta = 1$

computation results:
liquid holdup = 0.696680891221
gas holdup = 0.303319108779

in-situ liquid velocity = 0.4306132173 m/s
in-situ gas velocity = 19.7811474 m/s

the liquid holdup decrease by 0.14

0.7 Change in liquid holdup by increasing the angle

for $\theta = 10$ the liquid holdup is very small : 0.004. The gas occupied all the pipe according to the simulation result. This correspond to annular flow regime where the gas occupied most of the pipe. Bellow is the computation result:

parameters entered:
superficial liquid velocity $Usl = 0.3$
superficial gas velocity $Usg = 3$
angle $\beta = 10$
$\theta = 1$

computation results:
liquid holdup = 0.00371854672427
gas holdup = 0.996281453276

in-situ liquid velocity = 80.6766788869 m/s
in-situ gas velocity = 3.01119727777 m/s

The gas superficial velocity is equal to the gas in-situ velocity. This is because the gas occupied all the pipe.

Furthermore according $\alpha_G \approx 1$

$$U_G = \frac{U_{sg}}{\alpha_G} = U_{sg}$$

The liquid in-situ velocity is higher than it superficial velocity. Recall that the in-situ velocity of liquid is

$$U_L = \frac{Q_L}{A_L} = \frac{U_{sl}}{\alpha_L}$$

Since the liquid holdup decrease, the cross sectional areal of liquid decrease as well. To maintain a constant mass flow Q_L, the in-situ velocity must increase. Similarly since the holdup decrease, to maintain a constant superficial velocity U_{sl}, the in-situ velocity U_L must increase

0.8 python program

In the program, the python function bisect() is used for root finding:

```
"""
solve the holdup equation for sigma

"""
import numpy as np
import pylab as pl
from math import pi, sin, cos
from scipy.optimize import fsolve, bisect

def liquidHoldup(theta,Usg,Usl,beta,string=None):
    """
    this function takes as input the parameters theta
    the superficial velocity of liquid and gas: Usl, Usg
    and the string "print".

    usage:
    to print results, enter the string "print":

    thata = 1; Usg = 3; Usl = 0.3; beta = 0
    liquidHoldup(thata,Usg,Usl,beta,"print")

    output:
    ################################################
    parameters entered:
    superficial liquid velocity Usl = 0.3
    superficial gas velocity Usg =  3
    angle beta                   =  0
    #################################################
    computation results:
    liquid holdup           = 0.837510724346
    gas holdup              = 0.162489275654

    in-situ liquid velocity = 0.358204368349 m/s
    in-situ gas velocity    = 18.4627569292 m/s
    ################################################

    to return the value of sigma to be used later in a computation:
    do not specify the string print:

    usage:
    sigma = liquidHoldup(thata,Usg,Usl,beta)

    """

    def holdupEquation(sigma):
        """
        This function return the holdup equation as a function of
        sigma. The holdup equation is used as an input
        by the fsolve() python function to return sigma.
```

```
    the python function fsolve() solves nonlinear equation.
    """
    #physical parameter values
    rhoL, rhoG = 1000, 50    # density

    D          = 0.1             # pipe diameter
    muL, muG  = 0.001, 0.00001 # dynamic viscosity

    # in-situ velocity
    alphaL = (sigma-sin(sigma))/(2*pi)
    UL = Usl/(alphaL)
    UG = Usg/(1-alphaL)

    # physical boundary of the pipe
    A  = (pi*D**2)/4                 # cross sectional area of pip
    AL = (A/2*pi)*(sigma-sin(sigma)) # cross sectional area of liquid
    AG = A-AL                        # cross sectional area of gas

    SL = (D*sigma)/2
    SG = D*(pi-0.5*sigma)
    SI = D*sin(0.5*sigma)

    DL = (4*AL)/SL
    DG = (4*AG)/(SI+SG)

    #shear stress
    ReG = (rhoG*UG*DG)/muG # gas Reynold number
    ReL = (rhoL*UL*DL)/muL # liquid Reynold number

    #blasius friction factor
    fG = 0.316/(ReG**0.25)        # gas friction factor
    fL = 0.316/(ReL**0.25)        # liquid friction factor

    #shear stress
    tauG = (1./8)*rhoG*fG*UG**2      # gas shear sress
    tauL = (1./8)*rhoL*fL*UL**2      # liquid shear stress
    tauI = (1./8)*rhoG*theta*fG*(UL-UG)**2 # interfacial shear stress
    g = 9.81 # gavitational constant

    #holdup equation as a function of sigma
    equation = ( ((tauG*SG)/AG)-((tauL*SL)/AL) ) + tauI*SI*((1./AG)+(1./AL)
        )+g*sin(beta)*((rhoG/AG)-(rhoL/AL) )
    return equation


sig = bisect(holdupEquation,0.1,pi/2.)
#sig = fsolve(holdupEquation,0.8)

#compute liquid holdup
alphaL = (sig-sin(sig))/2.*pi

#compute in-situ velocities
UL = Usl/alphaL
UG = Usg/(1-alphaL)

#print result if string print is an iput to function liquidHolup()
if string !=None:
    print"##################################################"
    print"parameters entered:"
    print"superficial liquid velocity Usl = ", Usl
    print"superficial gas velocity Usg = ", Usg
    print"angle beta                    = ", beta
    print"theta                         = ", theta
    print"###################################################"
    print"computation results:"
    print"liquid holdup          =", alphaL
```

```
        print"gas holdup              =", 1-alphaL
        print""
        print"in-situ liquid velocity =",UL, "m/s"
        print"in-situ gas velocity   =",UG, "m/s"
        print"################################################"
        print"h", (0.1/2)*(1-cos(sig/2))



    return sig


def pressureGradient():
    """
    given the sigma compute the pressure gradient
    """
    theta, Usg, Usl, beta = 1,3,0.3,0
    sigma = liquidHoldup(theta,Usg,Usl,beta)
    rhoL, rhoG = 1000, 50    # density

    D         = 0.1           # pipe diameter
    muL, muG  = 0.001, 0.00001 # dynamic viscosity

    # in-situ velocity
    alphaL = (sigma-sin(sigma))/(2*pi)
    UL = Usl/(alphaL)
    UG = Usg/(1-alphaL)

    # physical boundary of the pipe
    A  = (pi*D**2)/4                # cross sectional area of pip
    AL = (A/2*pi)*(sigma-sin(sigma)) # cross sectional area of liquid
    AG = A-AL                       # cross sectional area of gas

    SL = (D*sigma)/2
    SG = D*(pi-0.5*sigma)
    SI = D*sin(0.5*sigma)

    DL = (4*AL)/SL
    DG = (4*AG)/(SI+SG)

    #shear stress
    ReG = rhoG*UG*DG/muG # gas Reynold number
    ReL = rhoL*UL*DL/muL # liquid Reynold number
    print ReG,ReL

    fG = 0.316/(ReG**0.25)        # gas friction factor
    fL = 0.316/(ReL**0.25)        # liquid friction factor

    tauG = (1./8)*rhoG*fG*UG**2     # gas shear sress
    tauL = (1./8)*rhoL*fL*UL**2     # liquid shear stress
    tauI = (1./8)*rhoG*theta*fG*(UL-UG)**2 # interfacial shear stress
    g = 9.81 # gavitational constant

    PLI = (1./AL)*(tauL*SL-tauI*SI) +(rhoL*g*sin(beta))/(AL)

    PGI = (1./AG)*(tauG*SG+tauI*SI) +(rhoG*g*sin(beta))/(AG)

    print"parameters used for computation:"
    print"theta = ", theta
    print"Usl  = ", Usl
    print"Usg  = ", Usg
    print"beta = ", beta
    print"####################################################"
    print"pressure gradient for liquid dPLi/dx", PLI
    print"pressure gradient for Gas dPGi/dx ", PGI
    print"#####################################################"
```

```
    print"DL",DL
    print"DG", DG



#pressureGradient()
theta, Usg,Usl,beta = 1,3,0.3,10
liquidHoldup(theta,Usg,Usl,beta,"print")
```