0.1 Setting up the problem

Finite element solution of

$$u'' = 1 \quad ; u(0) = u'(1) = 0 \tag{1}$$

and

$$u'' = 1 \quad ; u(0) = u(1) = 0 \tag{2}$$

with

$$V = \left\{ \sin\left(\frac{(i+1)\pi x}{2} \right) \right\}; \quad i = 1, .., N \tag{3}$$

$$V = \{ \sin((i+1)\pi x) \}; \quad i = 1, .., N \tag{4}$$

applied to 1 and 2 respectively. The variational formulation or weak formulation of 1 and 2 read:
find $u \in V$ such that

$$-\int_0^1 u'v' \mathrm{d}x = \int_0^1 v \mathrm{d}x \quad \forall v \in V \tag{5}$$

Using the Galerkin method let $u, v \in V$ be the trial function and the test function given by

$$u = \sum c_j \varphi_j \tag{6}$$

$$v = \varphi_i \tag{7}$$

Inserting the expression of u and v into 5 gives

$$-\left(\int_0^1 \varphi_j' \varphi_i' \mathrm{d}x \right) \sum c_j = \int_0^1 \varphi_j \mathrm{d}x \tag{8}$$

where

$$A_{j,i} = -\int_0^1 \varphi_j' \varphi_i' \mathrm{d}x \tag{9}$$

$$b_i = \int_0^1 \varphi_j \mathrm{d}x \tag{10}$$

and the coefficient c_j are solution of

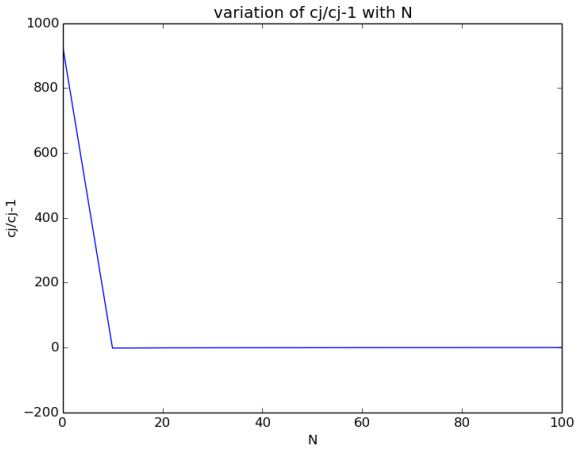
$$Ac = b \tag{11}$$

0.2 Solution of computation result

for $N = 0$ and $x = 1$ the numerical solution of 1 and the analytical solution

$$u(x) = -\frac{1}{2}x + x \tag{12}$$

gives -0.516024550931 and -0.5 respectively. For verification see file $cable - sin.py$. figure 0.2 shows the variation of $\frac{c_j}{c_{j-1}}$ with N. The numerical solution of 1 at $x = 1$ and the numerical solution of 2 at $x = 0.5$ gives the same result -0.516024550931. for verification see file $cable - sin.py$


variation of cj/cj-1 with N
cj/cj-1
N
1000
800
600
400
200
0
−200
0
20
40
60
80
100