0.1 Exercise 16

$$\frac{\partial \mid\mid \nabla u \mid\mid^q}{\partial c_j} = \int_\Omega \left(\frac{\partial \mid \nabla u \mid^q}{\partial \mid \nabla u \mid} \right) \left(\frac{\partial \mid \nabla u \mid}{\partial \nabla u} \right) \left(\frac{\partial \nabla u}{\partial c_j} \right) \tag{1}$$

with

$$\frac{\partial \mid \nabla u \mid^q}{\partial \mid \nabla u \mid} = q \mid \nabla u \mid^{q-1} \tag{2}$$

$$\frac{\partial \mid \nabla u \mid}{\partial \nabla u} = \frac{\nabla u}{\mid \nabla u \mid} \tag{3}$$

$$\frac{\partial \nabla u}{\partial c_j} = \nabla \phi_j \tag{4}$$

we get

$$\frac{\partial \mid\mid \nabla u \mid\mid^q}{\partial c_j} = \int_\Omega \left(q \mid \nabla u \mid^{q-1} \right) \left(\frac{\nabla u}{\mid \nabla u \mid} \right) (\nabla \phi_j) \tag{5}$$

$$= \int_\Omega \left(q \mid \nabla u \mid^{q-2} \right) \nabla u \cdot \nabla \phi_j \tag{6}$$

$$= \nabla u \cdot \nabla \phi_j \int_\Omega \left(q \mid \nabla u \mid^{q-2} \right) \tag{7}$$

$$\tag{8}$$

since

$$\nabla u \cdot \nabla \phi_j = \sum_j c_j \mid\mid \nabla \phi_j \mid\mid^2 \tag{9}$$

so that

$$\frac{\partial \mid \nabla u \mid^q}{\partial \mid \nabla u \mid} = q \mid\mid \nabla u \mid\mid^{q-2} \nabla u \cdot \nabla \phi_j \tag{10}$$