VQE Algorithm

Given a hamiltonian $\mathcal{H}$ acting on n-qubits, we wish to find its ground state.

1. Write $\mathcal{H}$ in its Pauli-basis (Pauli group on n-qubits).
2. Start with a random initial vector $|\psi_0\rangle$ created by rotating every qubit in the x and z directions by two random qubit-dependent angles: θ_i and ϕ_i.
3. For $j \in [trials]$, calculate the expectations:

$$E_{\pm} := \langle \psi_j(\pm\varepsilon)|\mathcal{H}|\psi_j(\pm\varepsilon)\rangle$$

   where $\psi_j(\pm\varepsilon)$ is given by setting $\theta_i^j := \theta_i^{j-1} \pm \varepsilon$ (similarly for ϕ_i^j).
   1. We are essentially *rotating each qubit* a little bit at each step, and recomputing its energy.
4. Given the positive expectation and the negative expectation, compute the gradient via finite difference:

$$\nabla \approx \frac{E_+ - E_-}{2\varepsilon}$$

5. Update the angles as:

$$\theta_i^{j+1} := \theta_i^j - \nabla$$

   Similarly for ϕ_i^{j+1}.

6. Repeat and hope you can reach convergence.