

# Counting Problems

1.)

Case 1 UUUUA (3 U)  $\rightarrow {}_4C_2 = \frac{4!}{2!2!} = 6$

Case 2 UUUUA (2 U)  $\rightarrow {}_4C_3 = \frac{4!}{3!1!} = 4$

Case 3 UUUUA (1 U)  $\rightarrow {}_4C_4 = \frac{4!}{4!0!} = 1$

Unique subsets = 11 ways  $\leftarrow \overline{11}$

Case 1 3 Us  $\frac{5P_5}{3!} \times {}_4C_2 = 6 \times \frac{5!}{3!} = 120$

Case 2 2 Us  $\frac{5P_5}{2!} \times {}_4C_3 = \frac{5!}{2!} \times 4 = 240$

Case 3 1 U's  $5P_5 = 5! = 120$

Different Strings =  $120 + 240 + 120 = 480$

2.)  $\binom{13}{2} \binom{4}{2} \binom{4}{2} \binom{44}{1} = 123,552$

3.) Case 1: 1 Song  $\rightarrow \binom{15+6-1}{6-1} = \binom{20}{5} = 15504$   
 $n=15 \quad k=6$

Case 2: no songs  $\rightarrow \binom{16+6-1}{6-1} = \binom{21}{5} = 20349$   
 $n=16 \quad k=6$

$20349 + 15504 = 35,853$

4.) 2 value Bst:  $\frac{4!}{3!2!} = 2$

3 value Bst:  $\frac{6!}{4!3!} = 5$

5 value Bst:  $\frac{10!}{6!5!} = 42$

$\frac{(2n)!}{(n+1)!n!}$   
 $(42)(5)(27) = 420$



5.) Case 1: no nurse on break

(3, 3, 2, 2) (1, 1, 1, 1)

(2, 2, 2, 4) (1, 1, 2, 6)

(3, 3, 3, 1) (1, 1, 3, 5)

(1, 2, 3, 4) (1, 1, 4, 4)

(1, 2, 2, 5)

9 cases

Case 2: 1 nurse on break

(1, 1, 8) (1, 3, 6)

(3, 3, 4) (2, 3, 5)

(1, 2, 7) (1, 4, 5)

(2, 2, 6) (2, 4, 4)

8 cases

17 different combinations