

Solutions for Sheet 1

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PATTERN MATCHING AND MACHINE LEARNING FOR AUDIO SIGNAL PROCESSING

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Task 1.1

$$(a) \quad z = \left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^3 = \left(\frac{\sqrt{2}}{2}\right)^3 + 3\left(\frac{\sqrt{2}}{2}\right)^2\left(i\frac{\sqrt{2}}{2}\right) + 3\left(\frac{\sqrt{2}}{2}\right)\left(i\frac{\sqrt{2}}{2}\right)^2 + \left(i\frac{\sqrt{2}}{2}\right)^3 = \frac{1}{2}(-1 + i)\sqrt{2} = \frac{1}{\sqrt{2}}(-1 + i) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

$$|z| = \sqrt{\left(-\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

$$\cos\varphi = \frac{\operatorname{Re}[z]}{|z|} = \frac{-\frac{1}{\sqrt{2}}}{1} = -\frac{1}{\sqrt{2}}$$

$$\varphi = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}$$

$$\Rightarrow z = |z| \cdot e^{i\varphi} = e^{\frac{3\pi i}{4}}$$

(b)

$$(c) \quad \operatorname{Re}[5e^{\frac{\pi}{4}i}] + \sqrt{2}e^{\pi i} =$$

(d)

Task 1.2

If we look at both spectrograms, we see that there are no differences between the two waves. This is due to the fact that a cosine wave is only a phase-shifted sine wave.