

Solutions for Sheet 3

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PATTERN MATCHING AND MACHINE LEARNING FOR AUDIO SIGNAL PROCESSING

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Task 3.1

(a) We have the following formulas for $\cos(z)$ and u_k :

$$\cos(z) = \frac{1}{2}e^{iz} + e^{-iz}$$
$$u_k = e^{\frac{2\pi i k n}{N}}$$

So for f, we get:

$$\begin{aligned} f(t) &= \cos(4\pi t) + 4\cos(20\pi t) + 8\cos(2\pi 20t) = \cos(2\pi 2t) + 4\cos(2\pi 10t) + 8\cos(2\pi 20t) \\ &= \frac{1}{2} \left(e^{2\pi 2ti} + e^{-2\pi 2ti} \right) + \frac{4}{2} \left(e^{2\pi 10ti} + e^{-2\pi 10ti} \right) + \frac{8}{2} \left(e^{2\pi 20ti} + e^{-2\pi 20ti} \right) \\ &= \frac{1}{2} \left(e^{2\pi 2ti} + e^{-2\pi 2ti} \right) + 2 \left(e^{2\pi 10ti} + e^{-2\pi 10ti} \right) + 4 \left(e^{2\pi 20ti} + e^{-2\pi 20ti} \right) \end{aligned}$$

With $t = \frac{k}{N}$ we get:

$$f = \frac{1}{2} (u_2 + u_{N-2}) + 2 (u_{10} + u_{N-10}) + 4 (u_{20} + u_{N-20})$$

(b) We obtain from f:

$$|\hat{f}(k)| = w_k \cdot \frac{N}{2}$$

with $w_2 = w_{N-2} = 1$, $w_{10} = w_{N-10} = 2$, $w_{20} = w_{N-20} = 4$ and otherwise $w_k = 0$.

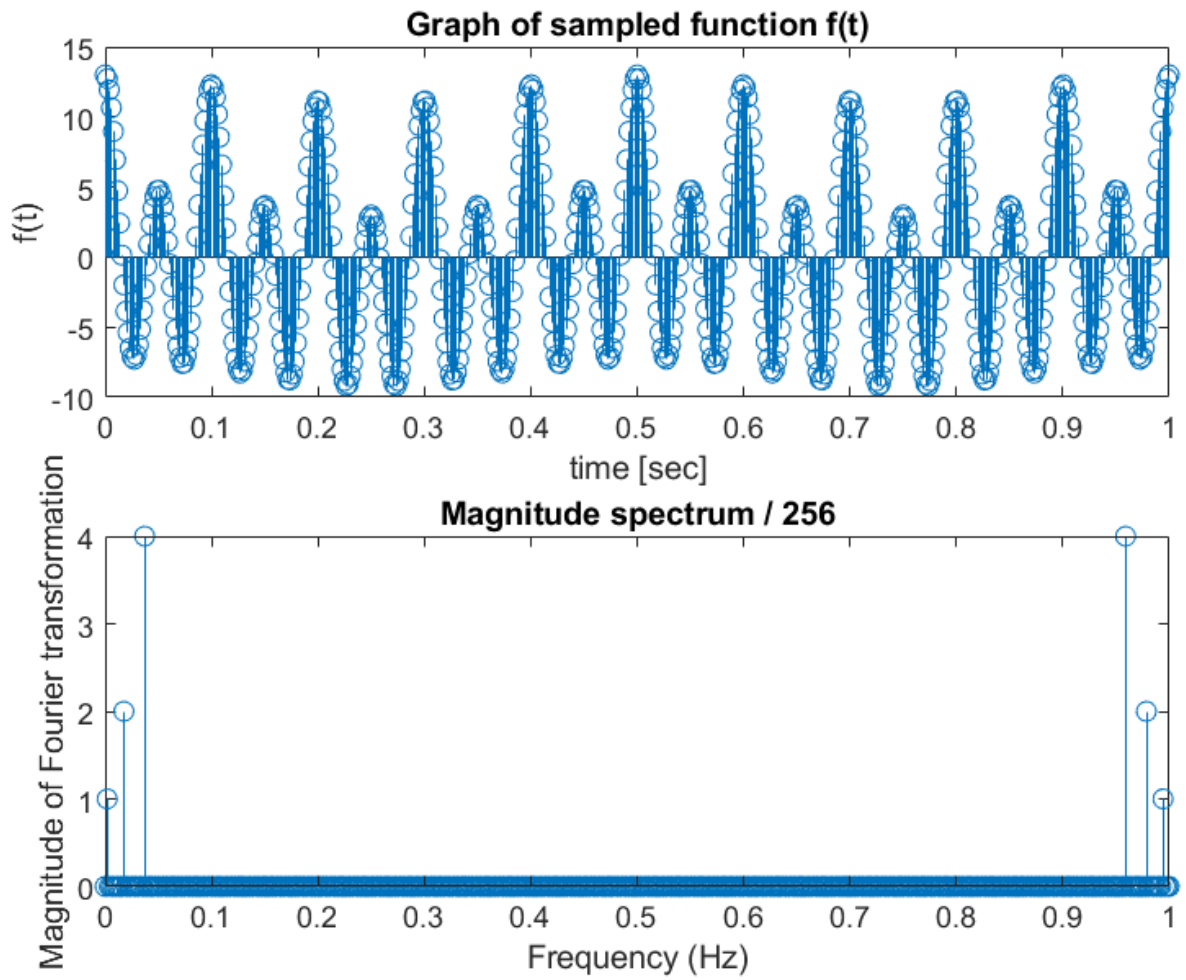


Abbildung 1: $f(t)$ (top) and $|\hat{f}(k)|$ (bottom)

c)