Solutions for Sheet 7

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Pattern Matching and Machine Learning for Audio Signal Processing

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Task 7.1

(a) & b) $C(\mathcal{D}) = \{(1,1),(1,3),(2,4),(3,1),(3,5),(4,2),(5,3),$
$(5,5),(6,1),(6,2),(6,5),(7,4),(8,2),(9,1),(9,4)\}$
$C(\mathcal{Q}) = \{(1,3),(2,1),(2,5),(3,4),(4,2)\}$
For this values we get for the inverted list:

$$L(1) = (1,3,6,9)$$
$$L(2) = (4,6,8)$$
$$L(3) = (1,5)$$
$$L(4) = (2,7,9)$$
$$L(5) = (3,5,6)$$

So the indicator function and the resulting matching functions are:

Query	**L(h) - n**	**indicator functions**								
		-1	0	1	2	3	4	5	6	7
(1,3)	(0,4)	0	1	0	0	0	1	0	0	0
(2,1)	(-1,1,4,7)	1	0	1	0	0	1	0	0	1
(2,5)	(1,3,4)	0	0	1	0	1	1	0	0	0
(3,4)	(-1,4,6)	1	0	0	0	0	1	0	1	0
(4,2)	(0,2,4)	0	1	0	1	0	1	0	0	0
Δ_F		2	2	2	1	1	5	0	1	1

Tabelle 1: indicator function and matching function for $C(\mathcal{Q})$ and $C(\mathcal{D})$

Task 7.2

- $p \in [0,1]$ = probability of a spectral peak to survive in a query audio fragment.
- original document contains in each target zone exactly F spectral peaks

$E_k =$ the anchor point and at least k target points survive

$\Rightarrow P(E_k) = p \cdot P(X \geq k) = p \cdot (1 - P(X < k)) = p \cdot (1 - \sum_{j=0}^{k-1} P(X = j))$

We interpret the peak survival for a frame as a set of Bernoulli experiments. We have F spectral peaks. Therefore we obtain the following binomial distribution:

$$P(X = k) = \binom{F}{k} p^k (1-p)^{F-k}$$

We can simplify the formular for $k = 1$ and $k = 2$.

$k = 1:$

$\Rightarrow P(E_1) = p \cdot (1 - P(X = 0))$

$= p \cdot (1 - \binom{F}{0} p^0 (1-p)^{F-0})$

$= p \cdot (1 - 1 \cdot 1 \cdot (1-p)^F)$

$= p \cdot (1 - (1-p)^F)$

$= p - p \cdot (1-p)^F$

$k = 2$

$\Rightarrow P(E_2) = p \cdot (1 - P(X = 0) - P(X = 1))$

$= p \cdot (1 - \binom{F}{0} p^0 (1-p)^{F-0} - \binom{F}{1} p^1 (1-p)^{F-1})$

$= p \cdot (1 - 1 \cdot 1 \cdot (1-p)^F - F \cdot p(1-p)^{F-1})$

$= p \cdot (1 - (1-p)^F - F \cdot p(1-p)^{F-1})$

$= p - p(1-p)^F - F \cdot p^2 (1-p)^{F-1}$

The probability for $p = 0.5$, $F = 11$ and $k = 2$ is the following:

$P(E_2) = p - p(1-p)^F - F \cdot p^2 (1-p)^{F-1}$

$= 0.5 - 0.5 \cdot 0.5^{11} - 11 \cdot 0.5^2 \cdot 0.5^{10}$

$= 0.5 - 0.5^{12} - 11 \cdot 0.5^{12} \approx 0.497$