Solutions for Sheet 5

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Pattern Matching and Machine Learning
for Audio Signal Processing

18. Mai 2022

Task 5.1

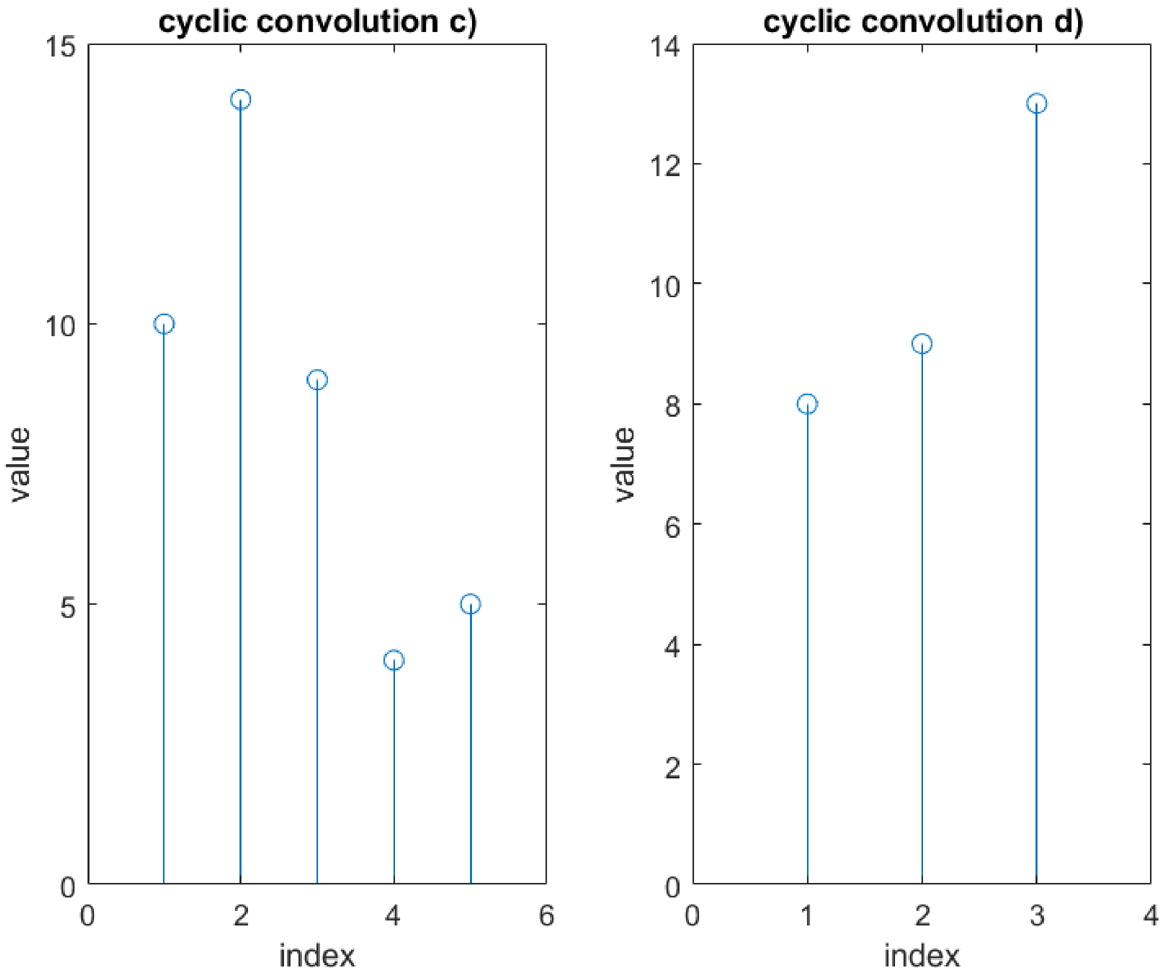
(a) Let $x = [3, 1, 0, 0, 2]$ be a vector. The cyclic autocorrelation of the vector is:

$$\begin{aligned}
ACF[x](0) &= \langle x|T_0(x)\rangle \\
&= \langle [3, 1, 0, 0, 2] \mid [3, 1, 0, 0, 2]\rangle = 13 \\
ACF[x](1) &= \langle x|T_1(x)\rangle \\
&= \langle [3, 1, 0, 0, 2] \mid [2, 3, 1, 0, 0]\rangle = 9 \\
ACF[x](2) &= \langle x|T_2(x)\rangle \\
&= \langle [3, 1, 0, 0, 2] \mid [0, 2, 3, 1, 0]\rangle = 2 \\
ACF[x](3) &= \langle x|T_3(x)\rangle \\
&= \langle [3, 1, 0, 0, 2] \mid [0, 0, 2, 3, 1]\rangle = 2 \\
ACF[x](4) &= \langle x|T_4(x)\rangle \\
&= \langle [3, 1, 0, 0, 2] \mid [1, 0, 0, 2, 3]\rangle = 9
\end{aligned}$$

b) Let $x = [3, 1, 0, 0, 2]$ and $y = [1, 3, 2, 0, 1]$ be two vectors. The cyclic convolution of the vectors is:

$$\begin{aligned}
(x * y)(0) &= \sum_{n=0}^{N-1} x(n) \cdot y(-n \bmod N) \\
&= 3 \cdot y(0) + 1 \cdot y(4) + 0 \cdot y(3) + 0 \cdot y(2) + 2 \cdot y(1) = 3 + 1 + 6 = 10 \\
(x * y)(1) &= \sum_{n=0}^{N-1} x(n) \cdot y(1 - n \bmod N) \\
&= 3 \cdot y(1) + 1 \cdot y(0) + 0 \cdot y(4) + 0 \cdot y(3) + 2 \cdot y(2) = 9 + 1 + 4 = 14 \\
(x * y)(2) &= \sum_{n=0}^{N-1} x(n) \cdot y(2 - n \bmod N) \\
&= 3 \cdot y(2) + 1 \cdot y(1) + 0 \cdot y(0) + 0 \cdot y(4) + 2 \cdot y(3) = 6 + 3 = 9 \\
(x * y)(3) &= \sum_{n=0}^{N-1} x(n) \cdot y(3 - n \bmod N) \\
&= 3 \cdot y(3) + 1 \cdot y(2) + 0 \cdot y(1) + 0 \cdot y(0) + 2 \cdot y(4) = 2 + 2 = 4 \\
(x * y)(4) &= \sum_{n=0}^{N-1} x(n) \cdot y(4 - n \bmod N) \\
&= 3 \cdot y(4) + 1 \cdot y(3) + 0 \cdot y(2) + 0 \cdot y(1) + 2 \cdot y(0) = 3 + 2 = 5
\end{aligned}$$

c) & d)



Task 5.2

The frequency response $H(\omega)$ of

$$h(\omega) = \begin{cases} \frac{1}{3} & \text{if } n = 0, 1, 2 \\ 0 & \text{else.} \end{cases}$$

is given by

$$\begin{aligned} H(\omega) :&= \sum_{k \in \mathbb{Z}} h(k) e^{-2\pi i \omega k} \\ &= h(0) \cdot e^{-2\pi i \omega \cdot 0} + h(1) \cdot e^{-2\pi i \omega \cdot 1} + h(2) \cdot e^{-2\pi i \omega \cdot 2} \\ &= \frac{1}{3}(1 + e^{-2\pi i \omega} + e^{-4\pi i \omega}) \end{aligned}$$

Task 5.3

(a)

Write h as a vector:

$$\begin{pmatrix} h(k_1) \\ h(k_2) \\ \vdots \\ h(k_n) \end{pmatrix}$$

Now write x as a matrix:

$$\begin{pmatrix} x(1-k_1) & x(1-k_2) & \cdots & x(1-k_n) \\ x(2-k_1) & x(2-k_2) & \cdots & x(2-k_n) \\ \vdots & \vdots & \ddots & \vdots \\ x(n-k_1) & x(n-k_2) & \cdots & x(n-k_n) \end{pmatrix}$$

Now we can write $C_h[x]$ as a matrix $\times$ vector product:

$$\begin{aligned} & \begin{pmatrix} x(1-k_1) & x(1-k_2) & \cdots & x(1-k_n) \\ x(2-k_1) & x(2-k_2) & \cdots & x(2-k_n) \\ \vdots & \vdots & \ddots & \vdots \\ x(n-k_1) & x(n-k_2) & \cdots & x(n-k_n) \end{pmatrix} \times \begin{pmatrix} h(k_1) \\ h(k_2) \\ \vdots \\ h(k_n) \end{pmatrix} \\ = & \begin{pmatrix} x(1-k_1) \cdot h(k_1) + x(1-k_2) \cdot h(k_2) + \ldots + x(1-k_n) \cdot h(k_n) \\ x(2-k_1) \cdot h(k_1) + x(2-k_2) \cdot h(k_2) + \ldots + x(2-k_n) \cdot h(k_n) \\ \vdots \\ x(n-k_1) \cdot h(k_1) + x(n-k_2) \cdot h(k_2) + \ldots + x(n-k_n) \cdot h(k_n) \end{pmatrix} \\ = & \begin{pmatrix} (h * x)(1) \\ (h * x)(2) \\ \vdots \\ (h * x)(n) \end{pmatrix} \end{aligned}$$

(b)