Department of Computer Science IV, University of Bonn
apl. Prof. Dr. Frank Kurth
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Pattern Matching and Machine Learning for Audio Signal Processing
Exercise sheet 2

To be uploaded in eCampus till: 23-04-2022 22:00 (strict deadline)

Exercise 2.1 [4 points]

Let $1 \neq q \in \mathbb{C}$. Prove that for all positive integers N

$$\sum_{n=0}^{N-1} q^n = \frac{1-q^N}{1-q}.$$

Exercise 2.2 [$9+3=12$ points]

For a fixed positive integer N, we consider two particular bases in the space $\mathbb{C}^N$ of length N signals:

- the ON-basis $\boldsymbol{e}_0, \ldots, \boldsymbol{e}_{N-1}$ of unit vectors (representing an extremely local view) and
- the unnormalized Fourier basis $\boldsymbol{u}_0, \ldots, \boldsymbol{u}_{N-1}$ (representing an extremely global view).

Let $\rho := \exp(2\pi i/N)$ and $\omega := \exp(-2\pi i/N)$ denote two primitive Nth roots of unity. Recall that $\boldsymbol{u}_k(n) := \exp(2\pi ikn/N) = \rho^{kn}$, for $k, n \in [0 : N-1]$. Furthermore, the $N \times N$ DFT matrix DFT_N is defined by $\mathrm{DFT}_N(k,n) := \omega^{kn}$, for $k, n \in [0 : N-1]$.

(a) Prove that for $k \in [0 : N-1]$ the following holds:

(i) $\overline{\boldsymbol{u}_k} = \boldsymbol{u}_{N-k}$.

(ii) $\mathrm{DFT}_N \boldsymbol{e}_k = \boldsymbol{u}_{N-k} = \overline{\boldsymbol{u}_k}$.

(iii) $\mathrm{DFT}_N \boldsymbol{u}_k = N \cdot \boldsymbol{e}_k$.

(b) Calculate explicitly DFT_3 explaining all the steps you use to obtain the result.

Exercise 2.3 [5 points]

Let $\boldsymbol{b}_0, \ldots, \boldsymbol{b}_{N-1}$ denote an ON-basis of $\mathbb{C}^N$. Prove that every $\boldsymbol{x} \in \mathbb{C}^N$ has the following **Fourier representation** (with respect to this ON-basis):

$$\boldsymbol{x} = \sum_{k=0}^{N-1} \langle \boldsymbol{x} | \boldsymbol{b}_k \rangle \boldsymbol{b}_k.$$

$(\langle \boldsymbol{x} | \boldsymbol{b}_0 \rangle, \ldots, \langle \boldsymbol{x} | \boldsymbol{b}_{N-1} \rangle)^\top$ is the vector of **Fourier coefficients** (w.r.t. this ON-basis).