Solutions for Sheet 2

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Pattern Matching and Machine Learning for Audio Signal Processing

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Task 2.1

Theorem 1. *Let $1 \neq q \in \mathbb{C}$, then for all positive integers n:* $\sum_{n=0}^{N-1} q^n = \frac{1-q^N}{1-q}$

Proof. Base Case ($N = 1$)

$$\sum_{n=0}^{1-1} q^n = \sum_{n=0}^{0} q^n = q^0 = 1 = \frac{1-q}{1-q} = \frac{1-q^1}{1-q}$$

Induction Hypothesis: Assume that for a given N the following statement holds:

$$\sum_{n=0}^{N-1} q^n = \frac{1-q^N}{1-q}$$

Inductive Step ($N \longrightarrow N+1$)

$$\begin{aligned}
\sum_{n=0}^{N+1-1} q^n = \sum_{n=0}^{N} q^n &= \sum_{n=0}^{N-1} q^n + q^N \\
&\overset{\text{IH}}{=} \frac{1-q^N}{1-q} + q^N \\
&= \frac{1-q^N}{1-q} + \frac{q^N \cdot (1-q)}{1-q} \\
&= \frac{1-q^N}{1-q} + \frac{q^N - q^{N+1}}{1-q} \\
&= \frac{1-q^N+q^N-q^{N+1}}{1-q} \\
&= \frac{1-q^{N+1}}{1-q}
\end{aligned}$$

$\square$