# Solutions for Sheet 6

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## Task 6.1

The time-shift operator $T^k$, $k \in \mathbb{Z}$ is defined as $T^k[x](n) = x^k(n) := x(n-k)$.

A system $S$ is time invariant $\Leftrightarrow \forall k \in \mathbb{Z}, \forall x \in \ell^p(\mathbb{Z}) : S[x^k] = S[x]^k$.

### (a)

The upsampling operator is defined as $(\uparrow M)[x](n) = \begin{cases} x(n/M), & \text{if } M \text{ divides } n, \\ 0, & \text{otherwise.} \end{cases}$

$\Rightarrow (\uparrow M)[x^k](n) = \begin{cases} x^k(n/M), & \text{if } M \text{ divides } n, \\ 0, & \text{otherwise.} \end{cases}$

$= \begin{cases} x(n/M-k), & \text{if } M \text{ divides } n, \\ 0, & \text{otherwise.} \end{cases}$

$\neq (\uparrow M)[x]^k(n) = (\uparrow M)[x](n-k) = \begin{cases} x((n-k)/M), & \text{if } M \text{ divides } (n-k), \\ 0, & \text{otherwise.} \end{cases}$

$\Rightarrow$ The upsampling operator is not time invariant.

## (b)

The frequency-shift operator is defined as $E_w[x](n) := e^{-2\pi iwn}x(n)$.

$\Rightarrow E_w[x^k](n) = e^{-2\pi iwn}x^k(n) = e^{-2\pi iwn}x(n-k) \neq e^{-2\pi iw(n-k)}x(n-k) = E_w[x](n-k) = E_w[x]^k(n)$

$\Rightarrow$ The frequency-shift operator is not time invariant.