Solutions for Sheet 1

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Pattern Matching and Machine Learning for Audio Signal Processing

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Task 1.1

(a) $z = (\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2})^3 = (\frac{\sqrt{2}}{2})^3 + 3(\frac{\sqrt{2}}{2})^2(i\frac{\sqrt{2}}{2}) + 3(\frac{\sqrt{2}}{2})(i\frac{\sqrt{2}}{2})^2 + (i\frac{\sqrt{2}}{2})^3 = \frac{1}{2}(-1+i)\sqrt{2} = \frac{1}{\sqrt{2}}(-1+i) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

$|z| = \sqrt{(-\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2} = 1$

$cos\varphi = \frac{Re[z]}{|z|} = \frac{-\frac{1}{\sqrt{2}}}{1} = -\frac{1}{\sqrt{2}}$

$\varphi = cos^{-1}(-\frac{1}{\sqrt{2}}) = \frac{3\pi}{4}$

$\Rightarrow z = |z| \cdot e^{i\varphi} = e^{\frac{3\pi i}{4}}$

(b)

(c) $Re[5e^{\frac{\pi}{4}i}] + \sqrt{2e^{\pi i}} =$

(d) Using Euler's Formula, prove the following statement:
$sin(z)^2 + cos(z)^2 = 1, z \in \mathbb{C}$
Proof:
Euler's formula is as follows: $e^{iz} = cos(z) + i \cdot sin(z), z \in \mathbb{C}$
For the opposite angle $-z$, we have $e^{-iz} = cos(z) - i \cdot sin(z)$
With these two, we can derive the following for $i \cdot sin(z), cos(z)$:
$i \cdot sin(z) = e^{iz} - cos(z) = cos(z) - e^{-iz}$
$cos(z) = e^{iz} - i \cdot sin(z) = e^{-iz} + i \cdot sin(z)$
And by plugging in, we can write $sin(z), cos(z)$ only in terms of e^{iz}, e^{-iz}:
$cos(z) = e^{iz} - i \cdot sin(z) = e^{iz} - cos(z) + e^{-iz} = \frac{e^{iz}+e^{-iz}}{2}$
$sin(z) = \frac{e^{iz}-cos(z)}{i} = \frac{e^{iz}-e^{-iz}-i \cdot sin(z)}{i} = \frac{e^{iz}-e^{-iz}}{2i}$
All that's left is to square and plug in once more:
$sin(z)^2 = \frac{1}{-4}((e^{iz})^2 - 2e^{iz}e^{-iz} + (e^{-iz})^2)$

$cos(z)^2 = \frac{1}{4}((e^{iz})^2 + 2e^{iz}e^{-iz} + (e^{-iz})^2)$
$sin(z)^2 + cos(z)^2 = \frac{1}{4}4e^{iz}e^{-z} = 1$

Task 1.2

If we look at both spectrograms, we see that there are no differences between the two waves. This is due to the fact that a cosine wave is only a phase-shifted sine wave. Therefore, both time-frequency representations are the same.