

Solutions for Sheet 3

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PATTERN MATCHING AND MACHINE LEARNING FOR AUDIO SIGNAL PROCESSING

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Task 3.1

(a) We have the following formulas for $\cos(z)$ and u_k :

$$\begin{aligned}\cos(z) &= \frac{1}{2}e^{iz} + e^{-iz} \\ u_k &= e^{\frac{2\pi i k n}{N}}\end{aligned}$$

So for f, we get:

$$\begin{aligned}f(t) &= \cos(4\pi t) + 4\cos(20\pi t) + 8\cos(2\pi 20t) = \cos(2\pi 2t) + 4\cos(2\pi 10t) + 8\cos(2\pi 20t) \\ &= \frac{1}{2}(e^{2\pi 2ti} + e^{-2\pi 2ti}) + \frac{4}{2}(e^{2\pi 10ti} + e^{-2\pi 10ti}) + \frac{8}{2}(e^{2\pi 20ti} + e^{-2\pi 20ti}) \\ &= \frac{1}{2}(e^{2\pi 2ti} + e^{-2\pi 2ti}) + 2(e^{2\pi 10ti} + e^{-2\pi 10ti}) + 4(e^{2\pi 20ti} + e^{-2\pi 20ti})\end{aligned}$$

With $t = \frac{k}{N}$ we get:

$$f = \frac{1}{2}(u_2 + u_{N-2}) + 2(u_{10} + u_{N-10}) + 4(u_{20} + u_{N-20})$$

(b) We obtain from f:

$$|\hat{f}(k)| = w_k \cdot \frac{N}{2}$$

with $w_2 = w_{N-2} = 1$, $w_{10} = w_{N-10} = 2$, $w_{20} = w_{N-20} = 4$ and otherwise $w_k = 0$.

c)

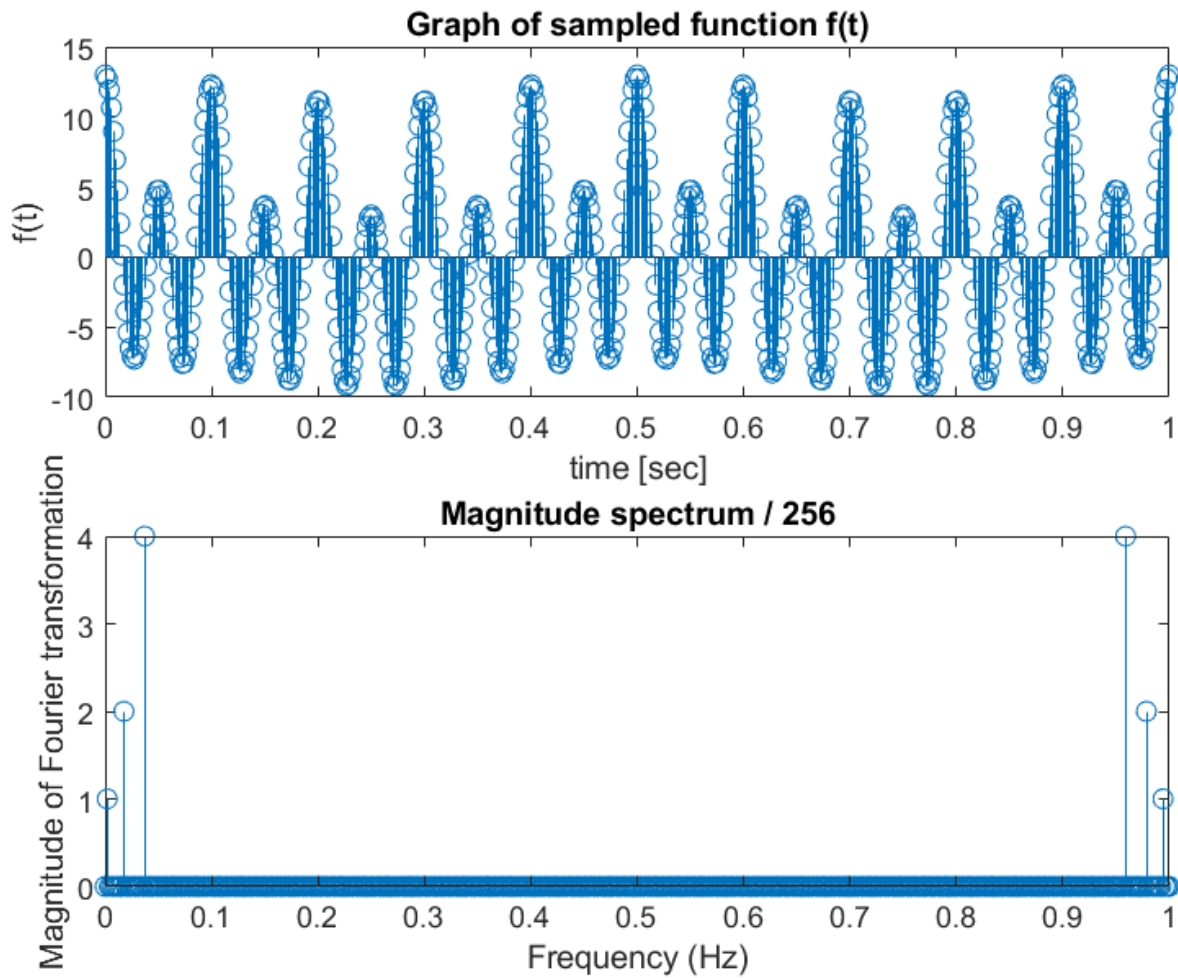


Abbildung 1: $f(t)$ (top) and $|\hat{f}(k)|$ (bottom)

Task 3.2

The spectrum X is defined as $X := (X(0), X(1), \dots, X(N-1))^T \in \mathbb{C}^N$.

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-2\pi i k n / N}$$

The vector $x \in \mathbb{C}^4$ is given as $x := [-2, 1, -1, 3]^T$

Therefore $N = 4$ and

$$x(0) = -2$$

$$x(1) = 1$$

$$x(2) = -1$$

$$x(3) = 3$$

Now we can calculate the spectrum X as follows:

$$X(0) = \sum_{n=0}^3 x(n) \cdot e^{-2\pi i \cdot 0 \cdot n/4} = \sum_{n=0}^3 x(n) = -2 + 1 - 1 + 3 = 1$$

$$\begin{aligned} X(1) &= \sum_{n=0}^3 x(n) \cdot e^{-2\pi i \cdot 1 \cdot n/4} = x(0) \cdot e^{-2\pi i \cdot 0/4} + x(1) \cdot e^{-2\pi i \cdot 1/4} + x(2) \cdot e^{-2\pi i \cdot 2/4} + x(3) \cdot e^{-2\pi i \cdot 3/4} \\ &= 1 \cdot (-2) - i \cdot 1 + 1 + i \cdot 3 = -1 + 2i \end{aligned}$$

$$\begin{aligned} X(2) &= \sum_{n=0}^3 x(n) \cdot e^{-2\pi i \cdot 2 \cdot n/4} \\ &= x(0) \cdot e^{-2\pi i \cdot 2 \cdot 0/4} + x(1) \cdot e^{-2\pi i \cdot 2 \cdot 1/4} + x(2) \cdot e^{-2\pi i \cdot 2 \cdot 2/4} + x(3) \cdot e^{-2\pi i \cdot 2 \cdot 3/4} \\ &= -2 \cdot e^{-2\pi i \cdot 0/4} + 1 \cdot e^{-2\pi i \cdot 2/4} - 1 \cdot e^{-2\pi i \cdot 4/4} + 3 \cdot e^{-2\pi i \cdot 6/4} \\ &= -2 + e^{-2\pi i \cdot 2/4} - e^{-2\pi i \cdot 4/4} + 3 \cdot e^{-2\pi i \cdot 6/4} \\ &= -2 - 1 - 1 + 3 \cdot (-1) \\ &= -7 \end{aligned}$$

$$\begin{aligned} X(3) &= \sum_{n=0}^3 x(n) \cdot e^{-2\pi i \cdot 3 \cdot n/4} \\ &= x(0) \cdot e^{-2\pi i \cdot 3 \cdot 0/4} + x(1) \cdot e^{-2\pi i \cdot 3 \cdot 1/4} + x(2) \cdot e^{-2\pi i \cdot 3 \cdot 2/4} + x(3) \cdot e^{-2\pi i \cdot 3 \cdot 3/4} \\ &= -2 \cdot e^{-2\pi i \cdot 0/4} + 1 \cdot e^{-2\pi i \cdot 3/4} - 1 \cdot e^{-2\pi i \cdot 6/4} + 3 \cdot e^{-2\pi i \cdot 9/4} \\ &= -2 + e^{-2\pi i \cdot 3/4} - e^{-2\pi i \cdot 6/4} + 3 \cdot e^{-2\pi i \cdot 9/4} \\ &= -2 + i + 1 + 3 \cdot (-i) \\ &= -1 - 2i \end{aligned}$$

The spectrum is $X = (1, -1 + 2i, -7, -1 - 2i)^\top$.

Task 3.3