# Seminar class 4

**Exercise 1** (exercise 6.6 in the book)

. Consider the class $\mathcal{H}^d_{mcon}$ of monotone Boolean conjunctions over $\{0,1\}^d$. Monotonicity here means that the conjunctions do not contain negations. As in $\mathcal{H}^d_{con}$, the empty conjunction is interpreted as the all-positive hypothesis. We augment $\mathcal{H}^d_{mcon}$ with the all-negative hypothesis $h^-$. Show that $\text{VCdim}(\mathcal{H}^d_{mcon}) = d$.

**Exercise 2** (exercise 6.9 in the book)

Let $\mathcal{H}$ be the class of signed intervals, that is,
$\mathcal{H} = \{h_{a,b,s} : a \le b, s \in \{-1,1\}\}$ where

$$h_{a,b,s}(x) = \begin{cases} s & \text{if } x \in [a,b] \\ -s & \text{if } x \notin [a,b] \end{cases}$$

Calculate $\text{VCdim}(\mathcal{H})$.

**Exercise 3** (exercise 3.13 in the book "Foundations of Machine Learning", 2nd edition)

3.13 VC-dimension of union of $k$ intervals. What is the VC-dimension of subsets of the real line formed by the union of $k$ intervals?

**Exercise 4** (exercise 3.15 in the book "Foundations of Machine Learning", 2nd edition)

VC-dimension of subsets. What is the VC-dimension of the set of subsets $I_\alpha$ of the real line parameterized by a single parameter $\alpha$: $I_\alpha = [\alpha, \alpha+1] \cup [\alpha+2, +\infty)$?

**Exercise 5** (exercise 8.1 in the book)

Let $\mathcal{H}$ be the class of intervals on the line (formally equivalent to axis aligned rectangles in dimension $n = 1$). Propose an implementation of the $\text{ERM}_{\mathcal{H}}$ learning rule (in the agnostic case) that given a training set of size $m$, runs in time $O(m^2)$.
*Hint*: Use dynamic programming.