

Embracing monotonicity in F^*

Danel Ahman @ INRIA Paris

joint work with

Cătălin Hrițcu and Kenji Maillard @ INRIA Paris

Cédric Fournet, Aseem Rastogi, and Nikhil Swamy @ MSR

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Outline

- F^*
- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F^*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see POPL'18 paper)
- Monadic reification and reflection (if time permits, or see paper)
- Meta-theory and correctness results (see POPL'18 paper)

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- **F*** is
 - a **functional programming language**
 - ML, OCaml, F#, Haskell, ...
 - extracted to OCaml or F#; subset compiled to efficient C code
 - an **interactive proof assistant**
 - Agda, Coq, Lean, Isabelle, ...
 - interactive modes for Emacs and Atom
 - a **semi-automated verifier** of imperative programs
 - Dafny, Why3, FramaC, ...
 - Z3-based SMT automation
- **Application-driven development**
 - Project Everest [project-everest.github.io]
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F* – programming language/proof assistant

```
module Talk
```

```
// Inductive type families
```

```
type vector 'a : nat -> Type =  
  | Nil : vector 'a 0  
  | Cons : #n:nat -> 'a -> vector 'a n -> vector 'a (n + 1)
```

```
// Dependently typed recursive total functions
```

```
val append : #a:Type -> #n:nat -> #m:nat -> vector a n -> vector a m -> Tot (vector a (n + m))  
let rec append #a #n #m xs ys =  
  match xs with  
  | Nil -> ys  
  | Cons #n x xs -> Cons x (append xs ys)
```

```
// Refinement types    x:t{phi x}
```

```
val lkp : #a:Type -> #n:nat -> vector a n -> i:nat{0 < i /\ i <= n} -> a  
let rec lkp #a #n xs i =  
  match xs with  
  | Cons x xs -> if i = n then x else lkp xs i
```

```
// Type-theoretic predicates
```

```
type prefix_of (#a:Type) (#n:nat) (#m:nat{n <= m}) (xs:vector a n) (zs:vector a m) =  
  forall (i:nat) . (0 < i /\ i <= n) ==> lkp xs i == lkp zs i
```

```
// Extrinsic reasoning
```

```
val theorem : #a:Type -> #n:nat -> #m:nat -> xs:vector a n -> ys:vector a m -> Lemma (prefix_of xs (append xs ys))  
let rec theorem #a #n #m xs ys =  
  match xs with  
  | Nil -> ()  
  | Cons x xs -> admit () // need to call an appropriate lemma here
```

```
// Intrinsic reasoning
```

```
val append' : #a:Type -> #n:nat -> #m:nat -> xs:vector a n -> vector a m -> Pure (vector a (n + m)) (requires (True))  
                                                    (ensures (fun zs -> prefix_of xs zs))
```

F* – not just a pure programming language

- Tot, Pure, ... are just some **effects** amongst many
 - Tot t
 - Pure t (requires pre) (ensures post)
 - Lemma (requires pre) (ensures post)
 - Div t (requires pre) (ensures post)
 - Exc t (requires pre_{Exc}) (ensures post_{Exc})
 - ST t (requires pre_{ST}) (ensures post_{ST})
 - ...
- Some connected by **monad morphisms**
- All derived from respective **WP-calculi** (see our POPL'17 paper)

(Global state +) monotonicity is really useful!

Its essence can be captured very neatly!

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Monotonicity in verification

- Consider a program operating on **set-valued state**

```
insert v; complex_procedure(); assert (v ∈ get())
```

- To prove the assertion (say, in a Floyd-Hoare style logic), we could prove that the code maintains a **stateful invariant**

$$\{\lambda s.v \in s\} \text{ complex_procedure() } \{\lambda s.v \in s\}$$

- likely that we have to **carry** $\lambda s.v \in s$ **through** the proof of `c_p`
 - does not guarantee** that $\lambda s.v \in s$ holds at every point in `c_p`
 - sensitive** to proving that `c_p` maintains $\lambda s.w \in s$ for some other `w`
- However, if `c_p` **never removes**, then $\lambda s.v \in s$ is **stable**, and we would like the program logic to give us `v ∈ get()` “for free”

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Monotonicity in programming

- **Programming** also relies on **monotonicity**, even if you don't realise it!
- Consider ML-style typed references $r:\text{ref } a$
 - r is a **proof of existence** of an a -typed value in the heap
- Correctness relies on **monotonicity**!
 - 1) Allocation **stores** an a -typed value in the heap
 - 2) Writes **don't change type** and there is **no deallocation**
 - 3) So, given a ref. r , it is **guaranteed to point** to an a -typed value
- Baked into the memory models of most languages
- We derive them from **global state** + **general monotonicity**

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Monotonicity is really useful!

- In this talk
 - our **motivating example** and **monotonic counters**
 - **typed references** (`ref t`) and **untyped references** (`uref`)
 - more flexibility with **monotonic references** (`mref t rel`)
- See our POPL 2018 paper for more
 - temporarily **violating monotonicity** via snapshots
 - two substantial case studies in F^*
 - a **secure file-transfer** application
 - Ariadne **state continuity** protocol [Strackx, Piessens 2016]
 - pointers to other works in F^* relying on monotonicity for
 - sophisticated **region-based memory models** [fstar-lang.org]
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Key ideas behind our general framework

- We make use of **monotonic programs** and **stable predicates**
 - per verification task, we **choose a preorder rel** on states
 - set inclusion, heap inclusion, increasing counter values, ...

- a stateful program e is **monotonic** (wrt. rel) when

$$\forall s e' s'. (e, s) \rightsquigarrow^* (e', s') \implies \text{rel } s s'$$

- a stateful predicate p is **stable** (wrt. rel) when

$$\forall s s'. p s \wedge \text{rel } s s' \implies p s'$$

- **Our solution:** extend Hoare-style program logics (e.g., F^*) with
 - a means to **witness** the validity of $p s$ in some state s
 - a means for turning a p into a **state-independent proposition**
 - a means to **recall** the validity of $p s'$ in any future state s'
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Recap: Ordinary global state in F*

- F* supports Hoare-style reasoning about state via the **comp. type**

$$\text{ST}_{\text{state}}\ t\ (\text{requires}\ \text{pre})\ (\text{ensures}\ \text{post})$$

where

$$\text{pre} : \text{state} \rightarrow \text{Type} \qquad \text{post} : \text{state} \rightarrow t \rightarrow \text{state} \rightarrow \text{Type}$$

- ST is an abstract pre-postcondition refinement of

$$\text{st}\ t \stackrel{\text{def}}{=} \text{state} \rightarrow t * \text{state}$$

- The global state **actions** have types

$$\text{get} : \text{unit} \rightarrow \text{ST}\ \text{state}\ (\text{requires}\ (\lambda _.\top))\ (\text{ensures}\ (\lambda\ s_0\ s\ s_1.\ s_0 = s = s_1))$$
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- Refs.** and **local state** are defined in F* using **monotonicity**

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New: Monotonic global state in F*

- We capture monotonic state with a new **computational type**

$\text{MST}_{\text{state}, \text{rel}}\ t\ (\text{requires}\ \text{pre})\ (\text{ensures}\ \text{post})$

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- The **get** action is typed as in ST

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- To ensure **monotonicity**, the **put** action gets a precondition

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- So intuitively, MST is an **abstract** pre-postcondition refinement of

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New: Monotonic global state in F*

- We capture monotonic state with a new **computational type**

$$\text{MST}_{\text{state}, \text{rel}} \, t \, (\text{requires} \, \text{pre}) \, (\text{ensures} \, \text{post})$$

where pre and post are typed as in ST

- The **get** action is typed as in ST

$$\begin{aligned} \text{get} : \text{unit} \rightarrow \text{MST} \, \text{state} \, (\text{requires} \, (\lambda _ . \top)) \\ (\text{ensures} \, (\lambda s_0 \, s \, s_1 . s_0 = s = s_1)) \end{aligned}$$

- To ensure **monotonicity**, the **put** action gets a precondition

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New: Recalling a Witness

- We extend F^* with a **logical capability**

$\text{witnessed} : (\text{state} \rightarrow \text{Type}) \rightarrow \text{Type}$

together with a **weakening principle** (functoriality)

$\text{wk} : p, q : (\text{state} \rightarrow \text{Type}) \rightarrow \text{Lemma} (\text{requires } (\forall s. p\ s \implies q\ s))$
 $(\text{ensures } (\text{witnessed } p \implies \text{witnessed } q))$

- Intuitively, think of it as a **necessity modality**

$$\llbracket \text{witnessed } p \rrbracket (s) \stackrel{\text{def}}{=} \forall s'. \text{rel } s\ s' \implies \llbracket p\ s' \rrbracket (s)$$

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Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F^*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see POPL'18 paper)
- Monadic reification and reflection (if time permits, or see paper)
- Meta-theory and correctness results (see POPL'18 paper)

The motivating example revisited

- Recall the program operating on the **set-valued state**

```
insert v; complex_procedure(); assert (v ∈ get())
```

- We pick **set inclusion** \subseteq as our preorder rel on states
- We **prove the assertion** by inserting a witness and recall

```
insert v; witness ( $\lambda s. v \in s$ ); c_p(); recall ( $\lambda s. v \in s$ ); assert (v ∈ get())
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- For any other w, wrapping

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insert w; [ ]; assert (w ∈ get())
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around the program is handled **similarly easily** by

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ML-style typed references (local state)

- First, we define a type of **heaps** as a finite map

```
type heap =
```

```
| H : h : (N → cell) → ctr : N { ∀ n. ctr ≤ n ⇒ h n = Unused } → heap
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where

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type cell =
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```
| Unused : cell
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| Used : a : Type → v : a → cell
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- Next, we define a **preorder** on heaps (**heap inclusion**)

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let heap_inclusion (H h0 _) (H h1 _) = ∀ id. match h0 id, h1 id with
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- As a result, we can define new **local state effect**

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- Next, we define the type of **references** using monotonicity

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- Finally, we define **MLST**'s **actions** using **MST**'s actions

- `let alloc (a:Type) (v:a) : MLST (ref a) ... = ...`
 - get the current heap
 - create a fresh ref., and add it to the heap
 - put the updated heap back
 - witness that the created ref. is in the heap
- `let read (r:ref a) : MLST t ... = ...`
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Adding untyped and monotonic references

- Untyped references (`uref`) with strong updates

- Used heap cells are extended with **tags**

where
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- actions corresponding to urefs have **weaker types** than for refs

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- `mrefs` provide **more flexibility** with ref.-wise monotonicity

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Conclusion

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 - is **useful** for **programming** (refs.) and **verification** (Prj. Everest)
- See our POPL 2018 paper for
 - further **examples** and **case studies**
 - **meta-theory** and **correctness results** for MST
 - based on an instrumented operational semantics
$$(\text{witness } x.\varphi, s, W) \rightsquigarrow (\text{return } (), s, W \cup \{x.\varphi\})$$
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 - first steps towards **monadic reification** for MST
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- In F^* every **abstract ST computation**

$$e : \text{ST } t \text{ (requires pre) (ensures post)}$$

can be **reified** into its **underlying Pure representation**

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and vice versa using **reflection** (see our POPL 2017 paper)

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Appendix: Mon. reification and reflection

- We cannot simply turn an **abstract MST computation**

$$e : \text{MST } t \text{ (requires pre) (ensures post)}$$

into a **state-passing function**

$$s_0 : \text{state} \rightarrow \text{Pure } (t * s_1 : \text{state} \{ \text{rel } s_0 \ s_1 \}) \text{ (req. (pre } s_0))$$
$$\text{(ens. } (\lambda (x, s_1). \text{post } s_0 \ x \ s_1))$$

- For example, consider the **recalling** action

$$\text{recall} : p : (\text{state} \rightarrow \text{Type}) \rightarrow \text{MST unit (requires } (\lambda _. \text{witnessed } p))$$
$$\text{(ensures } (\lambda s_0 \ s_1. s_0 = s_1 \wedge p \ s_1))$$

which we would like to **reduce** as

$$\text{reify (recall } p) \rightsquigarrow \lambda s_0. \text{return } ((), s_0)$$

but we cannot prove $p \ s_0$ from **witnessed** p in the pure logic

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- In our POPL 2018 paper, we support reification and reflection by
 - indexing $\text{MST}_{\text{state}, \text{rel}, \mathbf{b}}$ with a **boolean flag** \mathbf{b} (reifiable?), and
 - **guarding** the pre-postconditions of witness and recall with \mathbf{b}so if $\mathbf{b} = \text{true}$ then witness and recall are **logically no-ops**.
- This **works** but leads to **duplication** of pre- and postconditions!
- Instead, ongoing work is taking (hybrid) **modal logic** seriously

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