Interacting with the external world using comodels (aka runners)

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The plan

- **Computational effects** and **external resources** in PL
- **Runners** – a natural model for **top-level runtime**
- **T-runners** – for also modelling **non-top-level runtimes**
- Turning **T**-runners into a **useful programming construct**
- Some **programming examples**
- Some **implementation details**

Computational effects

and

external resources

Computational effects in PL

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- Using **monads** (as in HASKELL)

```
type St a = String → (a,String)

f :: St a → St (a,a)
f c = c  >>=  (\x → c  >>=  (\y → return (x,y)))
```

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```

- Using **alg. effects** and **handlers** (as in Eff, Frank, Koka)

```
effect Get : int
effect Put : int → unit

let g (c:Unit → a!{Get,Put}) =
  with st_handler handle (perform (Put 42); c ())  (* : int → a * int *)
```

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- Using **alg. effects** and **handlers** (as in EFF, FRANK, KOKA)

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effect Get : int
effect Put : int → unit

let g (c:Unit → a!{Get,Put}) =
  with st_handler handle (perform (Put 42); c ())  (* : int → a * int *)
```

- Both are good for **faking comp. effects** in a pure language!

  But what about effects that need access to the **external world**?

External resources in PL

External resources in PL

- Declare a **signature of monads** **or** **algebraic effects**, e.g.,

```
(* System.IO *)
type IO a
openFile :: FilePath → IOMode → IO Handle
```

```
(* pervasives.eff *)
effect RandomInt : int → int
effect RandomFloat : float → float
```

- And then **treat them specially** in the compiler, e.g.,

```
(* eff/src/backends/eval.ml *)
let rec top_handle op =
  match op with
  | ...
```

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but there are some issues with that approach ...

First issue

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- Difficult to cover all possible use cases
  - **external resources hard-coded** into the top-level runtime
  - **non-trivial to change** what's available and how it's implemented

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Ohad 8:35 PM
So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
    | "Write" ->
      (match v with
       | V.Tuple vs ->
          let (file_name :: str :: _)  = List.map V.to_str vs in
          let file_handle = open_out_gen
                              [Open_wronly
                              ;Open_append
                              ;Open_creat
                              ;Open_text
                              ] 0o666 file_name in
          Printf.fprintf file_handle "%s" str;
          close_out file_handle;
          top_handle (k V.unit_value)
      )
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This talk — a principled modular (co)algebraic approach!

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- **Lack of linearity** for external resources

```
let f (s:string) =
  let fh = fopen "foo.txt" in
  fwrite (fh,s^s);
  fclose fh;
  return fh

let g s =
  let fh = f s in fread fh
```

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```

- We shall address these kinds of issues **indirectly**,
  - by **not** introducing a linear typing discipline
  - but instead make it convenient to **hide external resources** (and address stronger typing disciplines in the future)

Third issue

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- **Excessive generality** of effect handlers

```
let f (s:string) =
  let fh = fopen "foo.txt" in
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let h = handler { fwrite (fh,s) k → return () }

let f' s = handle (f  "bar") with h
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where misuse of external resources can also be **purely accidental**

```
let g (s:string) =
  let fh = fopen "foo.txt" in
  let b = choose () in
  if b then (fwrite (fh,s)) else (fwrite (fh,s^s));
  fclose fh

let nd_handler =
  handler { choose () k → return (k true ++ k false) }
```

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- We shall address these kinds of issues **directly**,
  - by proposing a **restricted form of handlers** for resources
  - that support **controlled initialisation** and **finalisation**,
  - and **limit** how general handlers can be used

Runners enter the spotlight

A natural model of top-level runtime

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- Given a **signature**[1] Σ of operation symbols ($A_{\mathsf{op}}, B_{\mathsf{op}}$ are sets)

  $$\mathsf{op} : A_{\mathsf{op}} \rightsquigarrow B_{\mathsf{op}}$$

  a **runner**[2] $\mathcal{R}$ for Σ is given by a carrier $|\mathcal{R}|$ and co-operations

  $$\Big(\overline{\mathsf{op}}_{\mathcal{R}} : A_{\mathsf{op}} \times |\mathcal{R}| \longrightarrow B_{\mathsf{op}} \times |\mathcal{R}|\Big)_{\mathsf{op} \in \Sigma}$$

[1] We consider runners for signatures, but the work generalises to alg. theories.

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- For example, a natural runner $\mathcal{R}$ for **S-valued state** signature

$$\mathsf{get} : \mathbb{1} \rightsquigarrow S \qquad \mathsf{set} : S \rightsquigarrow \mathbb{1}$$

is given by

$$|\mathcal{R}| \stackrel{\text{def}}{=} S \qquad \overline{\mathsf{get}}_{\mathcal{R}}\,(\star, s) \stackrel{\text{def}}{=} (s, s) \qquad \overline{\mathsf{set}}_{\mathcal{R}}\,(s, s) \stackrel{\text{def}}{=} (\star, s)$$

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A natural model of top-level runtime ctd.

- Runners/comodels have been used for
  - **operational semantics** using tensors of models and comodels
    [Plotkin and Power '08]

    and
  - **stateful running** of algebraic effects [Uustalu '15]
  - **linear-use state-passing translation**
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  - **stateful running** of algebraic effects [Uustalu '15]
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- The latter explicitly rely on one-to-one correspondence between
  - **runners** $\mathcal{R}$
  - **monad morphisms**[3] $\mathsf{r} : \mathbf{Free}_{\Sigma}(-) \longrightarrow \mathbf{St}_{|\mathcal{R}|}$

  where

$$\mathbf{St}_C\, X \stackrel{\text{def}}{=} C \Rightarrow X \times C$$

[3] $\mathbf{Free}_{\Sigma}(X)$ is the free monad ind. defined with leaves $\mathsf{val}\, x$ and nodes $\mathsf{op}(a, \kappa)$.

A natural model of top-level runtime ctd.

- For our purposes, we see runners

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- But what if this runtime is not **the** runtime?
  - hardware vs OS
  - OS vs VMs
  - VMs vs sandboxes

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- Unfortunately, runners, as defined above, are **not readily able to**
  - use **external resources**
  - **signal failure** caused by unavoidable circumstances

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- But is there a **useful generalisation** that would achieve this?

Effectful runners for modular top-levels

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- Møgelberg and Staton usefully observed that a **runner** $\mathcal{R}$ is equivalently simply a family of **generic effects** for $\mathbf{St}_{|\mathcal{R}|}$, i.e.,

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- The one-to-one correspondence with **monad morphisms**

$$\mathsf{r} : \mathbf{Free}_{\Sigma}(-) \longrightarrow \mathbf{T}$$

now simply amounts to the **univ. property of free models**, i.e.,

$$\mathsf{r}_X\,(\mathsf{val}\,x) = \eta_X\,x \qquad\qquad \mathsf{r}_X\,(\mathsf{op}(a,\kappa)) = \underbrace{(\mathsf{r}_X \circ \kappa)^{\dagger}(\overline{\mathsf{op}}_{\mathcal{R}}\ a)}_{\mathsf{op}_{\mathcal{M}}(a,\mathsf{r}_X \circ \kappa)}$$

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- Observe that κ appears in a **tail call position** on the right!

Effectful runners for modular top-levels ctd.

- What would be a **useful class of monads** $\mathbf{T}$ to use?

Effectful runners for modular top-levels ctd.

- What would be a **useful class of monads** $\mathbf{T}$ to use?
- We want a runner to be a bit like a **kernel** of an OS, i.e., to
  - **(i)** provide management of **(internal) resources**
  - **(ii)** use further **external resources**
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- **Algebraically** (and pragmatically), this amounts to taking

  (i) $\mathsf{getenv} : \mathbb{1} \rightsquigarrow C$, $\mathsf{setenv} : C \rightsquigarrow \mathbb{1}$

  (ii) $\mathsf{op} : A_{\mathsf{op}} \rightsquigarrow B_{\mathsf{op}}$ ($\mathsf{op} \in \Sigma'$, for some external Σ')

  (iii) $\mathsf{kill} : S \rightsquigarrow \mathbb{0}$

  s.t., **(i)** satisfy state equations; and **(i)** commute with **(ii)** and **(iii)**

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  - **(iii)** kill : $S \rightsquigarrow \mathbb{0}$

  s.t., **(i)** satisfy state equations; and **(i)** commute with **(ii)** and **(iii)**

- The **induced monad** is then isomorphic to

$$\mathbf{T}\, X \quad \stackrel{\text{def}}{=} \quad C \Rightarrow \mathbf{Free}_{\Sigma'}\big((X \times C) + S\big)$$

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- The corresponding **T-runners** $\mathcal{R}$ for Σ are then of the form

$$\Big(\overline{\mathsf{op}}_{\mathcal{R}} : A_{\mathsf{op}} \longrightarrow C \Rightarrow \mathbf{Free}_{\Sigma'}\big((B_{\mathsf{op}} \times C) + S\big)\Big)_{\mathsf{op} \in \Sigma}$$

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- **Our solution:** consider signatures Σ, Σ' with operation symbols

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- With this, our **T-runners** $\mathcal{R}$ for Σ are (with "primitive" excs.)

$$\Big(\overline{\mathsf{op}}_{\mathcal{R}} : A_{\mathsf{op}} \longrightarrow \mathbf{K}_C^{\Sigma' ! E_{\mathsf{op}} \lightning S}\, B_{\mathsf{op}}\Big)_{\mathsf{op}\in\Sigma}$$

  where we call $\mathbf{K}_C^{\Sigma ! E \lightning S}$ a **kernel monad**, given by

$$\mathbf{K}_C^{\Sigma ! E \lightning S}\, X \quad \stackrel{\text{def}}{=} \quad C \Rightarrow \mathbf{Free}_{\Sigma}\big(((X + E) \times C) + S\big)$$

T-runners as a programming construct

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we accommodate **runners as values** and **co-ops. as kernel code**

```
let R = runner { op₁ x₁ → K₁ , ... , opₙ xₙ → Kₙ } @ C
```

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  we accommodate **runners as values** and **co-ops. as kernel code**

  let R = **runner** { $\mathsf{op}_1\ \mathsf{x}_1 \to \mathsf{K}_1$, ... , $\mathsf{op}_n\ \mathsf{x}_n \to \mathsf{K}_n$ } **@** C

- For instance, we can implement a **write-only file handle** as

```
let R_FH = runner {
  write s → if (length s > max)
             then (raise WriteSizeExceeded)
             else  (let fh = getenv () in
                     if (isValid fh) then (fwrite (fh,s)) else (kill IOError))
} @ FileHandle
```

  where

$$\left(\mathsf{fwrite} : \mathsf{FileHandle} \times \mathsf{String} \rightsquigarrow 1 \ !\ E\right) \in \Sigma'$$

$$\Sigma \stackrel{\text{def}}{=} \{\ \mathsf{write} : \mathsf{String} \rightsquigarrow 1 \ !\ E \cup \{\mathsf{WriteSizeExceeded}\}\ \} \qquad \mathsf{IOError} \in S$$

Controlled initialisation and finalisation

Controlled **initialisation** and **finalisation**

- Recall that the components r_X of the monad morphism

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induced by a $\mathbf{T}$-runner $\mathcal{R}$ are all **tail-recursive**

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- We make use of it to enable one to **run user code**:

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using R @ M_init
run M
finally {return x @ c → M_ret , ... raise e @ c → M_e ... , ... kill s → M_s ...}
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where

(a **user monad**)

- Ms are **user code**, modelled using $\mathbf{U}^{\Sigma!E}\,X \stackrel{\text{def}}{=} \mathbf{Free}_{\Sigma}(X + E)$

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- $\mathsf{M}_{\mathsf{init}}$ produces the **initial kernel state**
- M is the user code being **run using the runner** R
- $\mathsf{M}_{\mathsf{ret}}$, M_{e}, M_{s} **finalise** for return values, exceptions, and signals

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- Ms are **user code**, modelled using $\mathbf{U}^{\Sigma ! E}\, X \stackrel{\text{def}}{=} \mathbf{Free}_{\Sigma}(X + E)$
- $\mathsf{M}_{\mathsf{init}}$ produces the **initial kernel state**
- M is the user code being **run using the runner** R
- $\mathsf{M}_{\mathsf{ret}}$, M_{e}, M_{s} **finalise** for return values, exceptions, and signals
- $\mathsf{M}_{\mathsf{ret}}$ and M_{e} **depend on the final state** c, but M_{s} **does not**

Controlled **initialisation** and **finalisation** ctd.

- For instance, we can define a Python-esque **with construct**

```
with fileName do M
=
using R_FH @ (fopen fileName)
run M
finally {
  return x @ fh → fclose fh; return x ,
  raise  e @ fh → fclose fh; raise e  ,
  kill   s      → return ()  }
```

- Importantly, here
  - the **file handle is hidden** from M
  - M can only use write but not fopen and fclose
    - write : String $\rightsquigarrow$ 1 ! $E \cup \{\text{WriteSizeExceeded}\}$
  - fopen and fclose are limited to **initialisation-finalisation**

A core calculus for programming with runners

Core calculus (syntax)

Core calculus (syntax)

- **Ground types** (types of ops. and kernel state)

$$A, B, C \quad ::= \quad \mathsf{B} \mid 1 \mid 0 \mid A \times B \mid A + B$$

- **Types**

$$\begin{array}{rcl} X, Y & ::= & \mathsf{B} \mid 1 \mid 0 \mid X \times Y \mid X + Y \\ & \mid & X \xrightarrow{\Sigma} Y \mathrel{!} E \\ & \mid & X \xrightarrow{\Sigma} Y \mathrel{!} E \lightning S \mathbin{@} C \\ & \mid & \Sigma \Rightarrow \Sigma' \lightning S \mathbin{@} C \end{array}$$

- **Values**

$$\Gamma \vdash V : X$$

- **User computations**

$$\Gamma \models^{\Sigma} M : X \mathrel{!} E$$

- **Kernel computations**

$$\Gamma \models^{\Sigma} K : X \mathrel{!} E \lightning S \mathbin{@} C$$

Core calculus (user computations)

$$
\begin{aligned}
M, N \;::=\;& \mathsf{return}\ V \\
\mid\;& \mathsf{try}\ M\ \mathsf{with}\ \{\mathsf{return}\ x \mapsto M_{\mathsf{return}}, \\
& \qquad\qquad (\mathsf{raise}\ e \mapsto N_e)_{e \in E}\} \\
\mid\;& V\ W \\
\mid\;& \mathsf{match}\ V\ \mathsf{with}\ \{\langle x, y \rangle \mapsto M\} \\
\mid\;& \mathsf{match}\ V\ \mathsf{with}\ \{\}_X \\
\mid\;& \mathsf{match}\ V\ \mathsf{with}\ \{\mathsf{inl}\ x \mapsto M, \mathsf{inr}\ y \mapsto N\} \\
\mid\;& \mathsf{op}_X(V, (x \,.\, M), (N_e)_{e \in E}) \\
\mid\;& \mathsf{raise}_X\ e \\
\mid\;& \mathsf{using}\ V\ @\ W\ \mathsf{run}\ M\ \mathsf{finally}\ \{\mathsf{return}\ x\ @\ c \mapsto M, \\
& \qquad\qquad\qquad\qquad (\mathsf{raise}\ e\ @\ c \mapsto N_e)_{e \in E} \\
& \qquad\qquad\qquad\qquad (\mathsf{kill}\ s \mapsto N_s)_{s \in S}\} \\
\mid\;& \mathsf{kernel}\ K\ @\ V\ \mathsf{finally}\ \{\mathsf{return}\ x\ @\ c \mapsto M, \\
& \qquad\qquad (\mathsf{raise}\ e\ @\ c \mapsto N_e)_{e \in E} \\
& \qquad\qquad (\mathsf{kill}\ s \mapsto N_s)_{s \in S}\}
\end{aligned}
$$

Core calculus (kernel computations)

$$
\begin{aligned}
K, L \; ::= \;& \mathsf{return}_C\, V \\
\mid\;& \mathsf{try}\ K\ \mathsf{with}\ \{\mathsf{return}\, x \mapsto K_{\mathsf{return}}, \\
& \qquad\qquad\qquad (\mathsf{raise}\, e \mapsto L_e)_{e \in E}\} \\
\mid\;& V\, W \\
\mid\;& \mathsf{match}\ V\ \mathsf{with}\ \{\langle x, y\rangle \mapsto K\} \\
\mid\;& \mathsf{match}\ V\ \mathsf{with}\ \{\}_{X @ C} \\
\mid\;& \mathsf{match}\ V\ \mathsf{with}\ \{\mathsf{inl}\ x \mapsto K, \mathsf{inr}\ y \mapsto L\} \\
\mid\;& \mathsf{op}_{X @ C}(V, (x \,.\, K), (L_e)_{e \in E}) \\
\mid\;& \mathsf{raise}_{X @ C}\, e \\
\mid\;& \mathsf{kill}_{X @ C}\, s \\
\mid\;& \mathsf{getenv}_C(c \,.\, K) \\
\mid\;& \mathsf{setenv}(V, K) \\
\mid\;& \mathsf{user}\ M\ \mathsf{with}\ \{\mathsf{return}\, x \mapsto K_{\mathsf{return}}, \\
& \qquad\qquad\qquad (\mathsf{raise}\, e \mapsto L_e)_{e \in E}\}
\end{aligned}
$$

Core calculus (type system and eq. theory)

Core calculus (type system and eq. theory)

- For example, the **typing rule for running user comps.** is

$$
\frac{
\begin{array}{c}
\Gamma \vdash V : \Sigma \Rightarrow \Sigma' \not\downarrow S \mathbin{@} C \qquad \Gamma \vdash W : C \\
\Gamma \models^{\Sigma} M : X \mathrel{!} E \qquad \Gamma, x\!:\!X, c\!:\!C \models^{\Sigma'} N_{ret} : Y \mathrel{!} E' \\
\left(\Gamma, c\!:\!C \models^{\Sigma'} N_e : Y \mathrel{!} E'\right)_{e \in E} \qquad \left(\Gamma \models^{\Sigma'} N_s : Y \mathrel{!} E'\right)_{s \in S}
\end{array}
}{
\begin{array}{l}
\Gamma \models^{\Sigma'} \mathbf{using}\ V \mathbin{@} W\ \mathbf{run}\ M\ \mathbf{finally}\ \{\ \mathbf{return}\ x \mathbin{@} c \mapsto N_{ret}\ , \\
\qquad\qquad\qquad\qquad\qquad\qquad \left(\mathbf{raise}\ e \mathbin{@} c \mapsto N_e\right)_{e \in E}\ , \\
\qquad\qquad\qquad\qquad\qquad\qquad \left(\mathbf{kill}\ s \mapsto N_s\right)_{s \in S}\ \} : Y \mathrel{!} E'
\end{array}
}
$$

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$$\frac{\Gamma \vdash V : \Sigma \Rightarrow \Sigma' \lightning S \mathbin{@} C \qquad \Gamma \vdash W : C \qquad \Gamma \models^{\Sigma} M : X \mathbin{!} E \qquad \Gamma, x\!:\!X, c\!:\!C \models^{\Sigma'} N_{ret} : Y \mathbin{!} E' \qquad \big(\Gamma, c\!:\!C \models^{\Sigma'} N_e : Y \mathbin{!} E'\big)_{e \in E} \qquad \big(\Gamma \models^{\Sigma'} N_s : Y \mathbin{!} E'\big)_{s \in S}}{\Gamma \models^{\Sigma'} \mathbf{using}\ V \mathbin{@} W\ \mathbf{run}\ M\ \mathbf{finally}\ \{\ \mathbf{return}\ x \mathbin{@} c \mapsto N_{ret}\ ,\ \big(\mathbf{raise}\ e \mathbin{@} c \mapsto N_e\big)_{e \in E}\ ,\ \big(\mathbf{kill}\ s \mapsto N_s\big)_{s \in S}\ \} : Y \mathbin{!} E'}$$

- and the **main β-equation for running user comps.** is

$$\begin{aligned}
&\Gamma \models^{\Sigma'} \mathbf{using}\ R_C \mathbin{@} W\ \mathbf{run}\ (\mathrm{op}_X\,(V, (x.M), (M_e)_{e \in E_{\mathrm{op}}}))\ \mathbf{finally}\ F \\
&\quad \equiv \mathbf{kernel}\ R_{op}[V] \mathbin{@} W\ \mathbf{finally}\ \{ \\
&\qquad\qquad \mathbf{return}\ x \mathbin{@} c' \mapsto \mathbf{using}\ R_C \mathbin{@} c'\ \mathbf{run}\ M\ \mathbf{finally}\ F\ , \\
&\qquad\qquad \big(\mathbf{raise}\ e \mathbin{@} c' \mapsto \mathbf{using}\ R_C \mathbin{@} c'\ \mathbf{run}\ M_e\ \mathbf{finally}\ F\big)_{e \in E_{\mathrm{op}}}\ , \\
&\qquad\qquad \big(\mathbf{kill}\ s \mapsto N_s\big)_{s \in S}\ \} : Y \mathbin{!} E'
\end{aligned}$$

Core calculus (type system and eq. theory)

- The calculus also includes **subtyping**, and **subsumption rules**

$$\frac{\Gamma \vdash V : A \qquad A <: B}{\Gamma \vdash V : B}$$

$$\frac{\Gamma \models^{\Sigma} M : A \,!\, E \qquad \Sigma \subseteq \Sigma' \qquad A <: B \qquad E \subseteq E'}{\Gamma \models^{\Sigma'} M : B \,!\, E'}$$

$$\frac{\begin{array}{c}\Gamma \models^{\Sigma} K : A \,!\, E \lightning S \,\texttt{@}\, C \qquad \Sigma \subseteq \Sigma' \\ A <: B \qquad E \subseteq E' \qquad S \subseteq S' \qquad C = C'\end{array}}{\Gamma \models^{\Sigma'} K : B \,!\, E' \lightning S' \,\texttt{@}\, C'}$$

Core calculus (type system and eq. theory)

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- We use $C = C'$ to have (standard) **proof-irrelevant subtyping**
- Otherwise, instead of just $C <: C'$, we would need a **lens** $C' \leftrightarrow C$

Core calculus (semantics)

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- **Monadic semantics**, for simplicity in **Set**, using
  - **user monads** $\mathbf{U}^{\Sigma ! E}\, X \stackrel{\text{def}}{=} \mathbf{Free}_{\Sigma}(X + E)$
  - **kernel monads** $\mathbf{K}_{C}^{\Sigma ! E \lightning S}\, X \stackrel{\text{def}}{=} C \Rightarrow \mathbf{Free}_{\Sigma}\big(((X + E) \times C) + S\big)$

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- (At a high level) the **judgements are interpreted** as

$$[\![\Gamma \vdash V : X]\!] : [\![\Gamma]\!] \longrightarrow [\![X]\!]$$

$$[\![\Gamma \mathrel{\overset{\Sigma}{\models}} M : X \,!\, E]\!] : [\![\Gamma]\!] \longrightarrow \mathbf{U}^{\Sigma!E}[\![X]\!]$$

$$[\![\Gamma \mathrel{\overset{\Sigma}{\models}} K : X \,!\, E \lightning S \,@\, C]\!] : [\![\Gamma]\!] \longrightarrow \mathbf{K}_{[\![C]\!]}^{\Sigma!E\lightning S}[\![X]\!]$$

Core calculus (semantics ctd.)

- However, to prove **coherence** of the semantics (**subtyping!**), we actually give the semantics in the **subset fibration**

Core calculus (semantics ctd.)

- However, to prove **coherence** of the semantics (**subtyping!**), we actually give the semantics in the **subset fibration**

- For instance, **kernel computations** are interpreted as

$$\begin{tikzcd}
\llbracket \Gamma \rrbracket \arrow[rr, "\llbracket \Gamma \vDash^{\Sigma} K \,:\, X \,!\, E \lightning S \,@\, C \rrbracket"] \arrow[d, "\subseteq"'] & & \mathbf{K}^{\Sigma ! E \lightning S}_{\llbracket C \rrbracket} \llbracket X \rrbracket \arrow[d, "\subseteq"] \\
\llbracket\llbracket \Gamma^s \rrbracket\rrbracket \arrow[rr, "\llbracket\llbracket \Gamma^s \vdash K \,:\, X^s \,@\, C \rrbracket\rrbracket"'] & & \mathbf{K}^{\mathcal{O} ! \mathcal{E} \lightning \mathcal{S} + \{\lightning\}}_{\llbracket\llbracket C \rrbracket\rrbracket} \llbracket\llbracket X^s \rrbracket\rrbracket
\end{tikzcd}$$

where $\Gamma^s \vdash K : X^s \;@\; C$ is a **skeletal kernel typing judgement**

Core calculus (semantics ctd.)

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$$
\begin{tikzcd}
[\![\Gamma]\!] \arrow[rr, "{[\![\Gamma \models^{\Sigma} K : X \,!\, E \lightning S \,@\, C]\!]}"] \arrow[d, "\subseteq"'] & & \mathbf{K}^{\Sigma ! E \lightning S}_{[\![C]\!]} [\![X]\!] \arrow[d, "\subseteq"] \\
[\![\![\Gamma^s]\!]\!] \arrow[rr, "{[\![\![\Gamma^s \vdash K : X^s \,@\, C]\!]\!]}"'] & & \mathbf{K}^{\mathcal{O} ! \mathcal{E} \lightning \mathcal{S} + \{\dagger\}}_{[\![\![C]\!]\!]} [\![\![X^s]\!]\!]
\end{tikzcd}
$$

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- No essential obstacles to extending to $\mathbf{Sub}(\mathbf{Cpo})$ and beyond

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\end{tikzcd}
$$

  where $\Gamma^s \vdash K : X^s \,@\, C$ is a **skeletal kernel typing judgement**

- No essential obstacles to extending to **Sub**(**Cpo**) and beyond

- **Ground type restriction** on C needed to stay within **Sub**$(\ldots)$
  - Otherwise, analogously to subtyping, we'd need **lenses** instead

Runners in action

Runners can be vertically nested

Runners can be **vertically nested**

- ```
  using R_FH @ (fopen fileName)
  run (
    using R_FC @ (return "")
    run M
    finally {
      return x @ str → write str; return x ,
      raise  e @ str → raise e  }
  )
  finally {
    return x @ fh → fclose fh; return x ,
    raise  e @ fh → fclose fh; raise e  , kill IOError → return ()}
  ```

  where the **file contents runner** (with $\Sigma' = \mathbb{0}$) is defined as

  ```
  let R_FC = runner {
    write str → let str' = getenv () in
                if (length (str^str') > max) then (raise WriteSizeExceeded)
                                                else  (setenv (str^str'))
  } @ String
  ```
  ```

Vertical nesting for instrumentation

Vertical nesting for **instrumentation**

- ```
  using R_Sniffer @ (return 0)
  run M
  finally {
    return x @ c →
         let fh = fopen "nsa.txt" in fwrite (fh,toStr c); fclose fh }
  ```

  where the **instrumenting runner** is defined as

  ```
  let R_Sniffer = runner {
    ... ,
    op a → let c = getenv () in
              setenv (c + 1);
              op a ,                        (* forwards op outwards *)
    ...
  } @ Nat
  ```

- The runner R_{Sniffer} implements the same sig. Σ that `M` is using
- As a result, the runner R_{Sniffer} is **invisible** from `M`'s viewpoint
  ```

Vertical nesting for active monitoring

Vertical nesting for **active monitoring**

- First, we define a runner for **integer-valued ML-style state** as

```
type IntHeap = (Nat → (Int + 1)) × Nat                    type Ref = Nat
```

```
let R_IntState = runner {
  alloc x → let h = getenv () in                  (* alloc : Int ⇝ Ref ! 𝟘 *)
            let (r,h') = heapAlloc h x in
            setenv h';
            return r ,

  deref r → let h = getenv () in                  (* deref : Ref ⇝ Int ! 𝟘 *)
            match (heapSel h r) with
            | inl x → return x
            | inr () → kill ReferenceDoesNotExist ,

  assign r y → let h = getenv () in           (* assign : Ref × Int ⇝ 1 ! 𝟘 *)
               match (heapUpd h r y) with
               | inl h' → setenv h'
               | inr () → kill ReferenceDoesNotExist
} @ IntHeap
```

Vertical nesting for **active monitoring** ctd.

- Next we define a runner for **monotonicity layer** on top of $\mathsf{R}_{\mathsf{IntState}}$

Vertical nesting for **active monitoring** ctd.

- Next we define a runner for **monotonicity layer** on top of $\mathsf{R}_{\mathsf{IntState}}$

```
type MonMemory = Ref → ((Int → Int → Bool) + 1)
```

```
let R_MonState = runner {
  monAlloc x rel → let r = alloc x in                    (* : Int × Ord ⇝ Ref ! 𝟘 *)
                   let m = getenv () in
                   setenv (memAdd m r rel);
                   return r,

  monDeref r → deref r ,                             (* monDeref : Ref ⇝ Int ! 𝟘 *)

  monAssign r y → let x = deref r in                  (* : Ref × Int ⇝ 1 ! {MV} *)
                  let m = getenv () in
                  match (memSel m r) with
                  | inl rel → if (rel x y)
                              then (assign r y)
                              else (raise MonotonicityViolation)
                  | inr → kill PreorderDoesNotExist
} @ IntHeap
```

Vertical nesting for **active monitoring** ctd.

- We can then perform **runtime monotonicity verification** as

Vertical nesting for active monitoring ctd.

- We can then perform **runtime monotonicity verification** as

```
using R_IntState @ ((fun _ → inr ()) , 0)          (* init empty ML−style heap *)
run (
  using R_MonState @ (fun _ → inr ())         (* init empty preorders memory *)
  run (
    let r = monAlloc 0 (⩽) in
    monAssign r 1;
    try (monAssign r 0) with {                      (* R_MonState raises exception *)
      return _ → monAssign r 3 ,
      raise MonotonicityViolation → return ()};
    monAssign r 2
  )
  finally {return x @ m → return x ,
           raise MonotonicityViolation @ m → ... ,
           kill PreorderDoesNotExist → ... }
)
finally {return x @ h → print h; return x ,
         kill ReferenceDoesNotExist → ... }
```

Runners can also be horizontally paired

Runners can also be **horizontally paired**

- Given a runner for Σ

  ```
  let R₁ = runner { ... , op₁ᵢ x → k₁ᵢ , ... } @ C₁
  ```

  and a runner for Σ'

  ```
  let R₂ = runner { ... , op₂ⱼ x → k₂ⱼ , ... } @ C₂
  ```

  we can **pair them** to get a runner for $\Sigma \uplus \Sigma'$

  ```
  let R = runner {
    ... ,
    op₁ᵢ x → let (c,c') = getenv () in
             let (x,c'') = k₁ᵢ x in
             setenv (c'',c');
             return x ,
    ... ,
    op₂ⱼ x → ... (* analogously to above *) ,
    ...
  } @ C₁ × C₂
  ```

Runners can also be **horizontally paired**

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```
let R₁ = runner { ... , op₁ᵢ x → k₁ᵢ , ... } @ C₁
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  and a runner for Σ'

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```
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           let (x,c'') = k₁ᵢ x in
           setenv (c'',c');
           return x ,
  ... ,
  op₂ⱼ x → ... (* analogously to above *) ,
  ...
} @ C₁ × C₂
```

- For instance, this way we can build a **runner for IO and state**

Other examples

Other examples

- More general forms of **(ML-style) state** (for general `Ref A`)
  - if the host language allows it, we use GADTs, etc for safety
  - some examples extract a footprint from a larger memory

- **Combinations** of different effects and runners
  - in particular the combination of IO and state
  - good use case for both vertical and horizontal composition

- KOKA-style **ambient values** and **ambient functions**
  - **ambient values** are essentially **mutable variables/parameters**
  - **ambient functions** are **applied in their lexical context**
  - a runner that treats **amb. fun. application as a co-operation**
  - amb. funs. are stored in a context-depth-sensitive heap
  - the appl. co-operation restores the heap to the lexical context

Implementing runners

Experimenting with the theory in practice

Experimenting with the theory in practice

- A **small experimental language** COOP[4]
  - Implements the core calculus with few extras
  - The interpreter is directly based on the denotational semantics
  - Top-level containers for running external (OCaml) code

[4] coop [/ku:p/] – a cage where small animals are kept, especially chickens

Experimenting with the **theory in practice**

- A **small experimental language** COOP[4]
    - Implements the core calculus with few extras
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    - Top-level containers for running external (OCaml) code
- A HASKELL **library** HASKELL-COOP
    - A shallow-embedding of the core calculus in HASKELL
    - Uses one of the Freer monad implementations underneath
    - Again, the operational aspects implement the denot. semantics
    - Top-level containers for arbitrary HASKELL monads
    - Examples make use of HASKELL's features (GADTs, ...)

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  - Examples make use of HASKELL's features (GADTs, ...)
- Both still need some finishing touches, but will be public soon

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Experimenting with the theory in practice

```haskell
module AmbientsTests where

import Control.Runner
import Control.Runner.Ambients

ambFun :: AmbVal Int -> Int -> AmbEff Int
ambFun x y =
  do x <- getVal x;
     return (x + y)

test1 :: AmbEff Int
test1 =
  withAmbVal
    (4 :: Int)
    (\ x ->
      withAmbFun
        (ambFun x)
        (\ f ->
          do rebindVal x 2;
             applyFun f 1))

test2 = ambTopLevel test1
```

Wrapping up

- **Runners** are a natural model of **top-level runtime**
- We propose **T-runners** to also model **non-top-level runtimes**
- We have turned **T**-runners into a **(practical ?) programming construct**, that supports controlled initialisation and finalisation
- I showed you some **combinators** and **programming examples**
- Two **implementations** in the works, Coop & Haskell-Coop
- **Future:** lenses in subtyping and semantics, category of runners, handlers, larger case studies, refinement typing, compilation, ...

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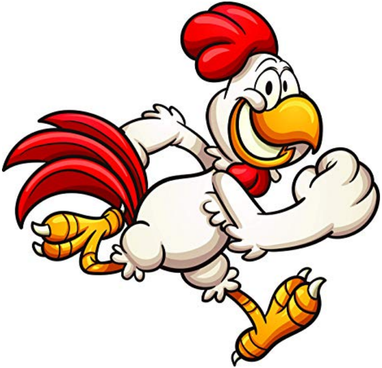
Thank you!



Core calculus (semantics ctd.)

$$
\begin{aligned}
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& \left(\textbf{raise } e \mathbin{@} c \mapsto N_e\right)_{e \in E} , \\
& \left(\textbf{kill } s \mapsto N_s\right)_{s \in S} \} : Y \;!\; E']\!]_\gamma \stackrel{\text{def}}{=} \ \ldots
\end{aligned}
$$

- $[\![V]\!]_\gamma = \mathcal{R} = \left(\overline{\mathsf{op}}_{\mathcal{R}} : [\![A_{\mathsf{op}}]\!] \longrightarrow \mathbf{K}_{[\![C]\!]}^{\Sigma'!E_{\mathsf{op}} \lightning S} [\![B_{\mathsf{op}}]\!]\right)_{\mathsf{op} \in \Sigma}$
- $[\![W]\!]_\gamma \in [\![C]\!]$
- $[\![M]\!]_\gamma \in \mathbf{U}^{\Sigma!E} [\![A]\!]$
- $[\![\textbf{return } x \mathbin{@} c \to N_{ret}]\!]_\gamma \in [\![A]\!] \times [\![C]\!] \longrightarrow \mathbf{U}^{\Sigma'!E'} [\![B]\!]$
- $[\![(\textbf{raise } e \mathbin{@} c \to N_e)_{e \in E}]\!]_\gamma \in E \times [\![C]\!] \longrightarrow \mathbf{U}^{\Sigma'!E'} [\![B]\!]$
- $[\![(\textbf{kill } s \to N_s)_{s \in S}]\!]_\gamma \in S \longrightarrow \mathbf{U}^{\Sigma'!E'} [\![B]\!]$
- allowing us to use the **free model property** to get

$$
\mathbf{U}^{\Sigma!E}[\![A]\!] \xrightarrow{\mathsf{r}_{[\![A]\!]+E}} \mathbf{K}_{[\![C]\!]}^{\Sigma'!E \lightning S}[\![A]\!] \xrightarrow{(\lambda [\![N_{ret}]\!]_\gamma)^{\ddagger}} [\![C]\!] \Rightarrow \mathbf{U}^{\Sigma'!E'}[\![B]\!]
$$

and then apply the resulting composite to $[\![M]\!]_\gamma$ and $[\![W]\!]_\gamma$