Comodels as a gateway for interacting with the external world

Danel Ahman

(joint work with Andrej Bauer)

Ljubljana, 21 March 2019

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What is a comodel?

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- A **signature** Σ is a set of operation symbols

$$\mathsf{op} : A \rightsquigarrow B \qquad \text{(in univ. alg., } |A|\text{-many op} : |B|\text{)}$$

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  $$\mathcal{W} = \langle\ W : \mathsf{Set}\ ,\ \{\overline{\mathsf{op}}_{\mathcal{W}} : A \times W \longrightarrow B \times W\}_{\mathsf{op} \in \Sigma}\ \rangle$$

- Intutively, comodels describe a notion of **state/world**, e.g.,
  - **Operational semantics** using a tensor of a model and a comodel (Plotkin & Power, Abou-Saleh & Pattinson)
  - **Stateful runners** of effectful programs (Uustalu)
  - Default **top-level behaviour** of alg. effects (Bauer & Pretnar)

Computational effects in FP

Computational effects in FP

- Using **monads** (e.g., as in HASKELL)

```
type St a = String → (a, String)

f :: St a → St (a,a)
f c = c >>= (\x → c >>= (\y → return (x,y)))
```

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- Using **algebraic effects** and **handlers** (e.g., as in EFF)

```
effect Get : int
effect Put : int → unit
                                (*: int → a*int!{} *)
let g (c:unit → a!{Get, Put}) =
  with st_h handle (perform (Put 42); c ())
```

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```

- Works well for effects that can be **represented** as pure data!

  But what about effects that need access to the **external world**?

External world in FP

- Declare a **signature** of monads/effects

```
type IO a

openFile  :: FilePath → IOMode → IO Handle
hGetLine  :: Handle → IO String
hClose    :: Handle → IO ()
```

External world in FP

- Declare a **signature** of monads/effects

```
type IO a

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```

```
effect Read   : string
effect Raise  : string → empty

effect RandomInt    : int → int
effect RandomFloat  : float → float
```

External world in FP

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```
effect Read   : string
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effect RandomInt    : int → int
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```

- And then treat it **specially** in the compiler, e.g.,

```
let rec top_handle op =
  match op with
  | ...
```

External world in FP

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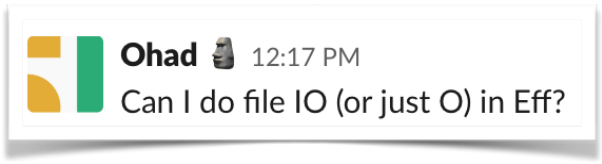
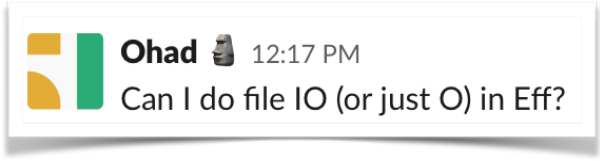
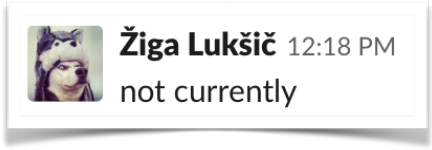


External world in FP





External world in FP

Ohad 12:17 PM
Can I do file IO (or just O) in Eff?

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not currently

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So here's the hack I added. We should do something a bit more principled :)

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
  | "Write" ->
     (match v with
      | V.Tuple vs ->
         let (file_name :: str :: _)  = List.map V.to_str vs in
         let file_handle = open_out_gen
                           [Open_wronly
                           ;Open_append
                           ;Open_creat
                           ;Open_text
                           ] 0o666 file_name in
         Printf.fprintf file_handle "%s" str;
         close_out file_handle;
         top_handle (k V.unit_value)
     )
```

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This talk — a principled (co)algebraic approach!

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- ```
  let f (s:string) =
    let fh = fopen "foo.txt" in
    fwrite fh (s^s);
    fclose fh;
    return fh

  let g s =
    let fh = f s in fread fh
  ```
  ```

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- We could resolve this by typing file handles **linearly**
  - But we want other values (e.g., strings) to be used **non-linearly**!
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- But what if we wrap `f` in a **handler**?

```
let h = handler
        | effect (FWrite fh s k) → return s

let g s = with h handle f s
```
```

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- But what if we wrap `f` in a **handler**?

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let h = handler
        | effect (FWrite fh s k) → return s

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```
```

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- Using existing programming mechanisms, e.g.,
  - **Modules** and **abstraction**, e.g., `System.IO`

```
type IO a

hClose :: Handle → IO ()
```

  - **Linear** (and **non-linear**) **types** and **effects**

```
linear type fhandle

effect FClose : (linear fhandle) → unit

linear effect FClose : fhandle → unit
```

  - Handlers with **finally** **clauses**

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```
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effect FClose : (linear fhandle) → unit

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  - Handlers with **finally** **clauses**

- **Problem:** They don't really explain the **essence of the problem**

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- A common explanation is to think of functions

$$\mathsf{a} \to \mathsf{IO\ b}$$

as

$$\mathsf{a} \to (\mathsf{RealWorld} \to (\mathsf{b}, \mathsf{RealWorld}))$$

which is the same as

$$(\mathsf{a}, \mathsf{RealWorld}) \to (\mathsf{b}, \mathsf{RealWorld})$$

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$$(a, RealWorld) \to (b, RealWorld)$$

- With the `System.IO` **module abstraction** ensuring that
  - We can't get our hands on RealWorld — it's an idea of the world
  - The **real world** is affected linearly
  - We don't ask more from the **real world** than it can provide

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- A common explanation is to think of functions

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But wait a minute, RealWorld looks a lot like a **comodel**!

$$hGetLine : (Handle, RealWorld) \to (String, RealWorld)$$

$$hClose\ \ : (Handle, RealWorld) \to ((), RealWorld)$$

So, **IO** is more about in which **external world** our program is in!

Comodels as a gateway to the external world

Comodels as a gateway to the external world

- ```
  let f (s:string) =
    using IO run
      let fh = fopen "foo.txt" in
      fwrite fh (s^s);
      fclose fh
  ```
  ```

Comodels as a gateway to the external world

- ```
  let f (s:string) =
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- ```
  let f (s:string) =
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    ending_with (fclose fh)
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  ```
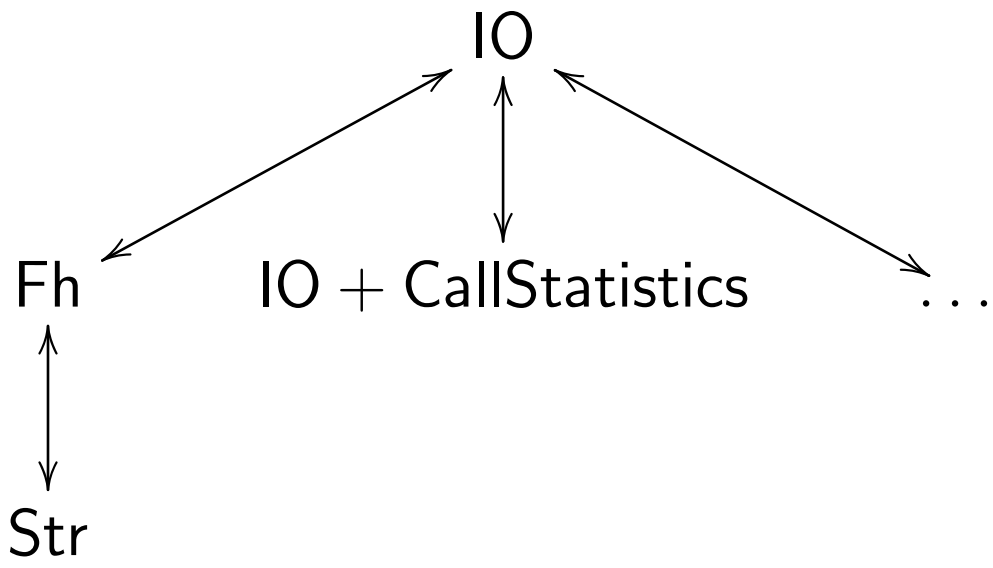
- Better, but **have to explicitly open** and **thread through** `fh`
- **Solution:** Modular treatment of **external worlds**

Comodels as a gateway to the external world

- Examples of **modularity** we might want from comodels



Comodels as a gateway to the external world

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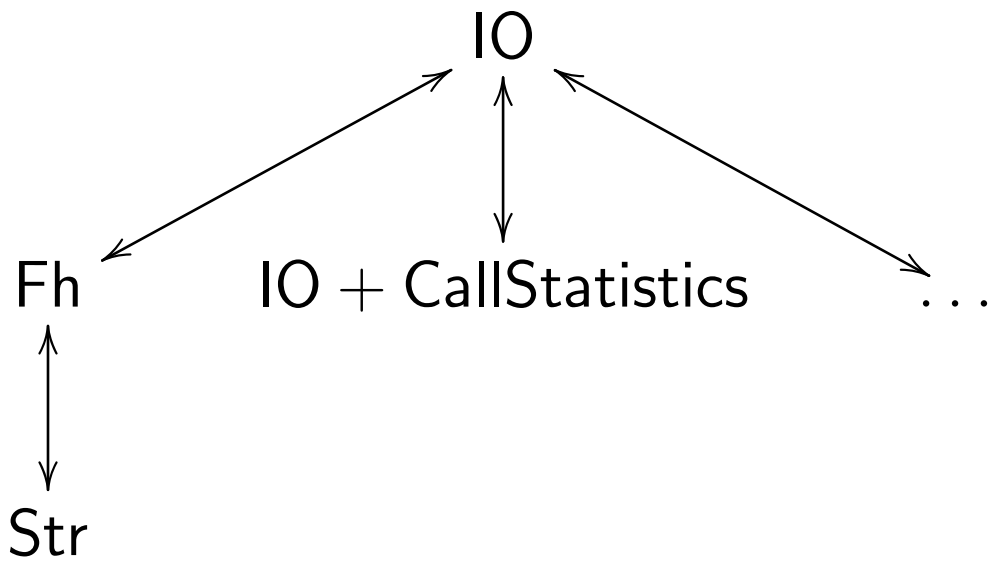

- Fh — "world which consists of exactly one `fh`"
- IO $\longrightarrow$ Fh — "call `fopen` with `foo.txt`, store returned `fh`"
- Fh $\longrightarrow$ IO — "call `fclose` with stored `fh`"

- **Observation:** IO $\longleftrightarrow$ Fh and other $\longleftrightarrow$ look a lot like **lenses**

Comodels as a gateway to the external world

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- Our **general framework** on the file operations example

```
let f (s:string) =                    (* @ IO : unit *)
  using Fh
  starting_with (fopen_of_io "foo.txt")
  run
    fwrite_of_fh (s^s)                (* @ Fh : unit *)
  ending_with (fun _ fh → fclose_of_io fh)
```

Comodels as a gateway to the external world

- Our **general framework** on the file operations example

```
let f (s:string) =                          (* @ IO : unit *)
  using Fh
  starting_with (fopen_of_io "foo.txt")
  run
    fwrite_of_fh (s^s)                      (* @ Fh : unit *)
  ending_with (fun _ fh → fclose_of_io fh)
```

where

```
Fh =
  ⟨W = fhandle,
   co_fread ((),fh) = ...,
   co_fwrite (s,fh) = fwrite_of_io s fh;
                      return ((),fh)⟩

(* co_fread : (unit * W) → (string * W) @ IO *)
(* co_fwrite : (string * W) → (unit * W) @ IO *)
```

Comodels as a gateway to the external world

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- The **modularity aspect** of our general framework

```
let f (s:string) =                          (* @ IO : unit *)
  using Fh
  starting_with (fopen_of_io "foo.txt")
  run

    using Str
    starting_with (fread_of_fh ())
    run
      fwrite_of_str (s^s)      (* @ Str : unit *)
    ending_with (fun _ s → fwrite_of_fh s)

  ending_with (fun _ fh → fclose_of_io fh)
```

where

```
Str = ⟨W = string , ... ⟩
```

Comodels as a gateway to the external world

Comodels as a gateway to the external world

- Comodels can also **extend** the (intermediate) external world(s)

```
let f (s:string) =                          (* @ IO : unit *)
  using Stats
  starting_with (fopen_of_io "foo.txt")
  run
    fwrite_of_stats (s^s)  (* @ Stats : unit *)
  ending_with
    (fun _ (fh,c) →
      let fh' = fopen_of_io "stats.txt" in
      fwrite_of_io fh' c;
      fclose_of_io fh'; fclose_of_io fh)
```

where

```
Stats = ⟨W = (fhandle * nat), ... ⟩
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```

where

```
Stats = ⟨W = (fhandle * nat), ... ⟩
```

- We can also track of **nondet./prob. choice results**, and alike
- Could we also use comodels for **dynamic (NI) monitoring**?

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- The two central **typing rules** are (U is the "universe", aka IO)

$$\frac{\Gamma \vdash \mathsf{D} \text{ comodel } @ \ \vec{C} \qquad \mathsf{D} \neq \mathsf{U} \qquad \Gamma \vdash c_s \ @ \ \vec{C} : \mathsf{D}.W \qquad \Gamma \vdash c \ @ \ \vec{C}, \mathsf{D} : A \qquad \Gamma, x : A, w : \mathsf{D}.W \vdash c_e \ @ \ \vec{C} : A}{\begin{array}{l}\Gamma \vdash \textbf{using } \mathsf{D} \\ \quad \textbf{starting\_with } c_s \\ \quad \textbf{run } c \\ \quad \textbf{ending\_with } (x.w.c_e) \qquad @ \ \vec{C} : A\end{array}}$$

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and

$$\frac{\Gamma \vdash \mathsf{D}\ \text{comodel} \mathbin{@} \vec{C} \qquad \mathsf{op} : A \rightsquigarrow B \in \mathsf{D}.\Sigma \qquad \Gamma \vdash v : A}{\Gamma \vdash \mathsf{op}\ v \mathbin{@} \vec{C}, \mathsf{D} : B}$$

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- Regarding the **denotational semantics**, the idea is to interpret

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$$[\![\Gamma \vdash c \mathbin{@} \vec{C} : A]\!] \;:\; [\![\vec{C}]\!] \longrightarrow [\![A]\!] \times [\![\vec{C}]\!]$$

which in its essence is very similar to Møgelberg and Staton's comodels-based **linear state-passing transformation**

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- Regarding **operational semantics**, the idea is to consider confs.

$$\left(\overrightarrow{(C, w)} \;,\; c \right)$$

either in a **big-** or **small-step** style

  - where $\overrightarrow{(C, w)}$ is a stack of worlds

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$$\left(\left(\overrightarrow{(\mathsf{C}, w_1)}\right), (\mathsf{C}', w_1'), (\mathsf{D}, w_0'') \right) \ , \ c \ \Big) \Downarrow \left(\left(\overrightarrow{(\mathsf{C}, w_2)}\right), (\mathsf{C}', w_2'), (\mathsf{D}, w_1'') \right) \ , \ \textbf{return } v \ \Big)$$

$$\left(\left(\overrightarrow{(\mathsf{C}, w_2)}\right), (\mathsf{C}', w_2') \right) \ , \ c_e[v/x, w_1''/w] \ \Big) \Downarrow \left(\left(\overrightarrow{(\mathsf{C}, w_3)}\right), (\mathsf{C}', w_3') \right) \ , \ \textbf{return } v' \ \Big)$$

$$\left(\left(\overrightarrow{(\mathsf{C}, w_0)}\right), (\mathsf{C}', w_0') \right) \ , \ \textbf{using } \mathsf{D} \ \textbf{s\_w } c_s \ \textbf{run } c \ \textbf{e\_w } (x.w.c_e) \ \Big)$$

$$\Downarrow$$

$$\left(\left(\overrightarrow{(\mathsf{C}, w_3)}\right), (\mathsf{C}', w_3') \right) \ , \ \textbf{return } v' \ \Big)$$

So what's happening more formally?

- For example, consider the **big-step evaluation** of **using** D ...

$$(\overrightarrow{((\mathsf{C}, w_0), (\mathsf{C}', w_0'))} , c_s) \Downarrow (\overrightarrow{((\mathsf{C}, w_1), (\mathsf{C}', w_1'))} , \textbf{return } w_0'')$$

$$(\overrightarrow{((\mathsf{C}, w_1), (\mathsf{C}', w_1'), (\mathsf{D}, w_0''))} , c) \Downarrow (\overrightarrow{((\mathsf{C}, w_2), (\mathsf{C}', w_2'), (\mathsf{D}, w_1''))} , \textbf{return } v)$$

$$(\overrightarrow{((\mathsf{C}, w_2), (\mathsf{C}', w_2'))} , c_e[v/x, w_1''/w]) \Downarrow (\overrightarrow{((\mathsf{C}, w_3), (\mathsf{C}', w_3'))} , \textbf{return } v')$$

$$(\overrightarrow{((\mathsf{C}, w_0), (\mathsf{C}', w_0'))} , \textbf{using } \mathsf{D} \textbf{ s\_w } c_s \textbf{ run } c \textbf{ e\_w } (x.w.c_e))$$

$$\Downarrow$$

$$(\overrightarrow{((\mathsf{C}, w_3), (\mathsf{C}', w_3'))} , \textbf{return } v')$$

- The interpretation of **operations** uses the **co-operations** of Cs, naturally traversing the stack of (intermediate) external worlds

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starting_with c_s
run c
ending_with (fun x w → c_e)
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  - One can use **algebraic operations** (in the sense of Eff) in `c`, but they must not be allowed to escape `run` (for linearity)

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- How do **multi-handlers** fit in here? Tensor products of some sort?

Conclusions

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Ongoing and future work

- Work out all the **formal details** of what I have shown you today
- **Algebraic effects** and **(multi-)handlers**
- More **examples** and **use cases** (Matija, the Eff Architecture?)
- Clarify the connection with **(effectful) lenses**
- **Combinatorics** of comodels and their lens-like relationships