Interacting with the external world using comodels (aka runners)

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The plan

- **Computational effects** and **external resources** in PL
- **Runners** – a natural model for **top-level runtime**
- **T-runners** – for also modelling **non-top-level runtimes**
- Turning **T**-runners into a **useful programming construct**
- Some **programming examples**
- Some **implementation details**

Computational effects

and

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Computational effects in PL

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- Using **monads** (as in HASKELL)

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type St a = String → (a,String)

f :: St a → St (a,a)
f c = c   >>=  (\x → c   >>=  (\y → return (x,y)))
```

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- Using **alg. effects** and **handlers** (as in EFF, FRANK, KOKA)

```
effect Get : int
effect Put : int → unit

let g (c:Unit → a!{Get,Put}) =
  with state_handler handle (perform (Put 42); c ())   (* : int → a * int *)
```

Computational effects in PL

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effect Put : int → unit

let g (c:Unit → a!{Get,Put}) =
  with state_handler handle (perform (Put 42); c ())   (* : int → a * int *)
```

- Both are good for **faking comp. effects** in a pure language!

  But what about effects that need access to the **external world**?

External resources in PL

External resources in PL

- Declare a **signature of monads or algebraic effects**, e.g.,

```
(* System.IO *)
type IO a
openFile :: FilePath → IOMode → IO Handle
```

```
(* pervasives.eff *)
effect RandomInt : int → int
effect RandomFloat : float → float
```

- And then **treat them specially** in the compiler, e.g.,

```
(* eff/src/backends/eval.ml *)
let rec top_handle op =
  match op with
  | ...
```

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but there are some issues with that approach . . .

First issue

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- Difficult to cover all possible use cases
  - **external resources hard-coded** into the top-level runtime
  - **non-trivial to change** what's available and how it's implemented

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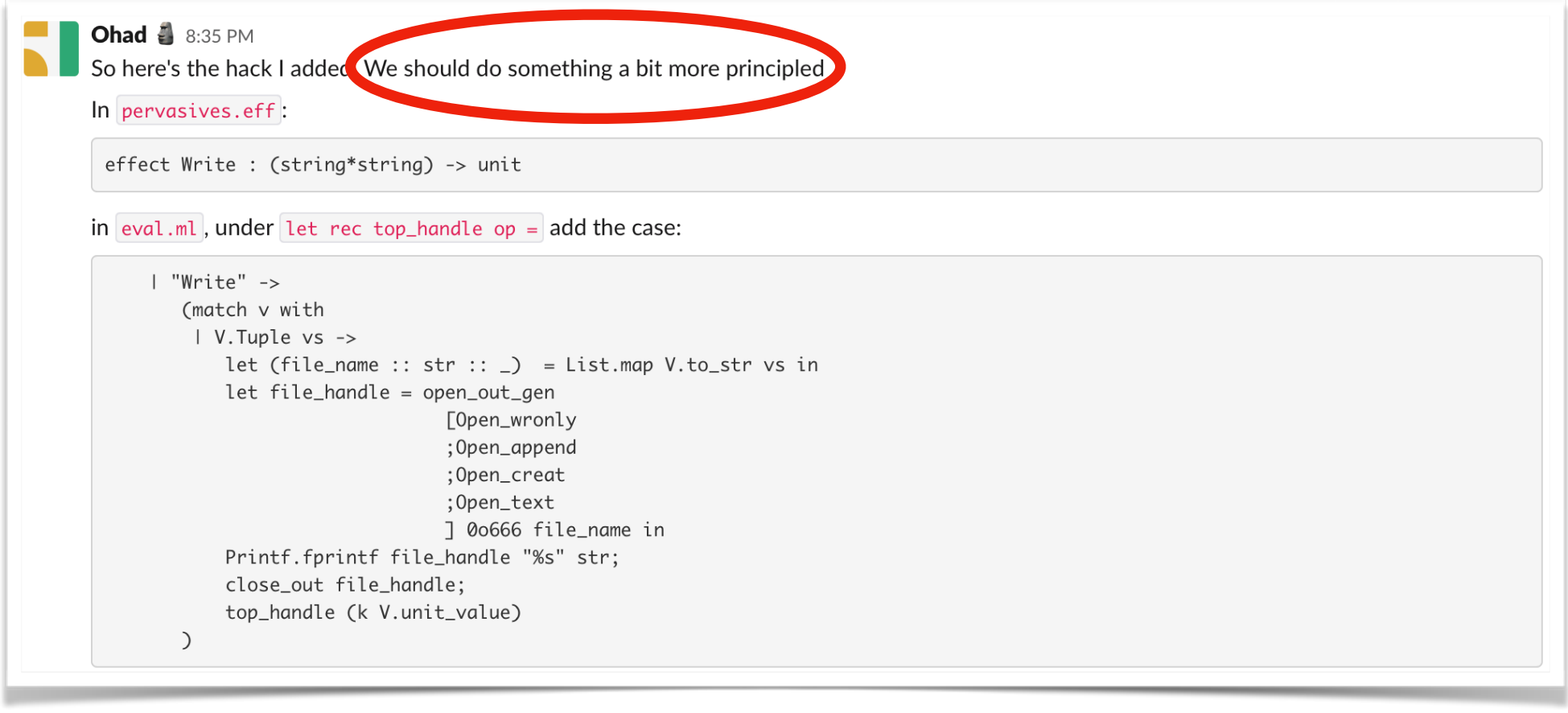

Ohad 8:35 PM

So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
    | "Write" ->
       (match v with
        | V.Tuple vs ->
           let (file_name :: str :: _)  = List.map V.to_str vs in
           let file_handle = open_out_gen
                             [Open_wronly
                             ;Open_append
                             ;Open_creat
                             ;Open_text
                             ] 0o666 file_name in
           Printf.fprintf file_handle "%s" str;
           close_out file_handle;
           top_handle (k V.unit_value)
       )
```

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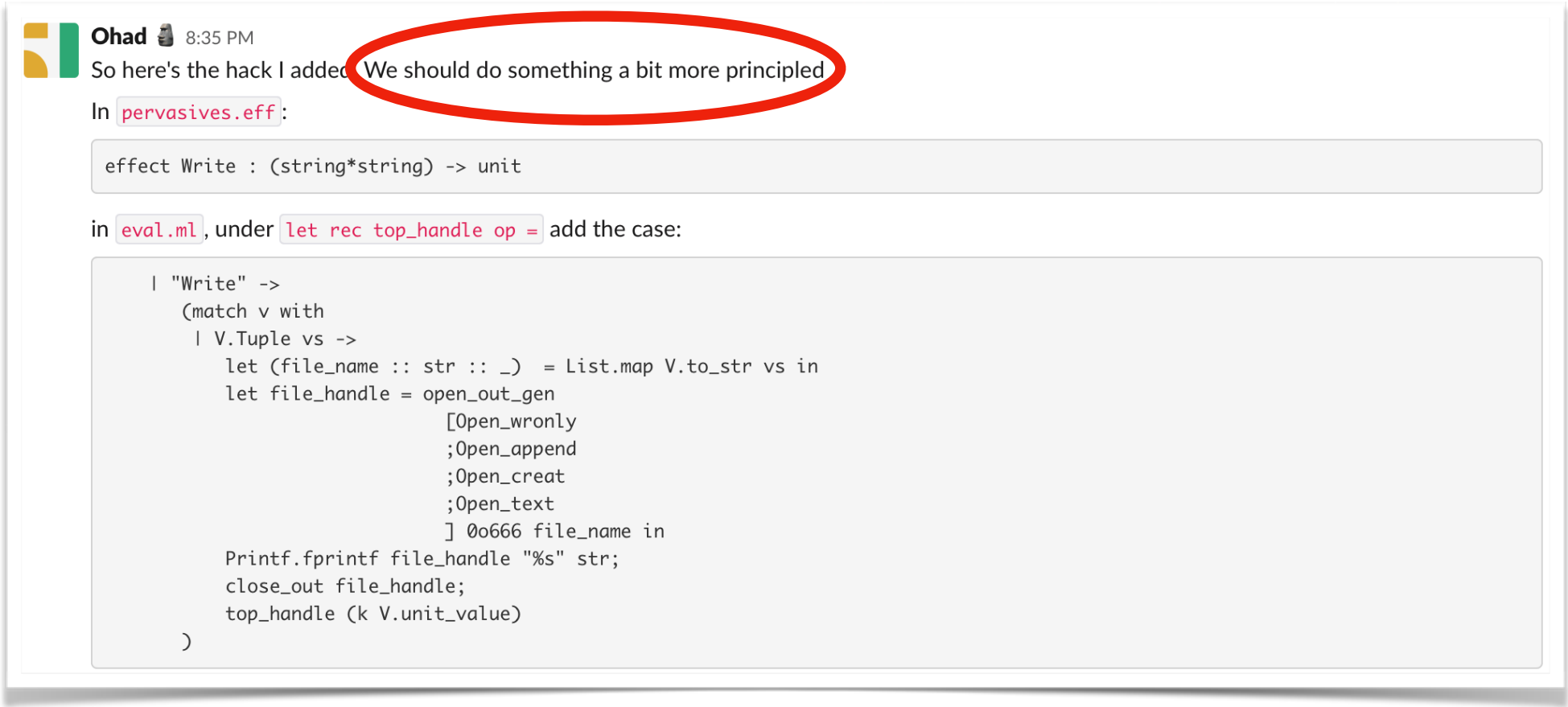

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This talk — a principled modular (co)algebraic approach!

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- **Lack of linearity** for external resources

```
let f (s:string) =
  let fh = fopen "foo.txt" in
  fwrite (fh,s^s);
  fclose fh;
  return fh

let g s =
  let fh = f s in fread fh
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- We shall address these kinds of issues **indirectly**,
  - by **not** introducing a linear typing discipline
  - but instead make it convenient to **hide** external resources

Third issue

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- **Excessive generality** of effect handlers

```
let f (s:string) =
  let fh = fopen "foo.txt" in
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let h = handler { fwrite (fh,s) k → return () }

let f' s = handle (f  "bar") with h
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where misuse of external resources can also be **purely accidental**

```
let g (s:string) =
  let fh = fopen "foo.txt" in
  let b = choose () in
  if b then (fwrite (fh,s)) else (fwrite (fh,s^s));
  fclose fh

let nondet_handler =
  handler { choose () k → return (k true ++ k false) }
```

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- We shall address these kinds of issues **directly**,
  - by proposing a **restricted form** of handlers for resources
  - that support **controlled initialisation** and **finalisation**,
  - and **limit** how general handlers can be used

Runners enter the spotlight

A natural model of top-level runtime

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- Given a **signature**[1] Σ of operation symbols ($A_{\mathsf{op}}, B_{\mathsf{op}}$ countable)

$$\mathsf{op} : A_{\mathsf{op}} \rightsquigarrow B_{\mathsf{op}}$$

a **runner**[2] $\mathcal{R}$ for Σ is given by a carrier $|\mathcal{R}|$ and co-operations

$$\left(\overline{\mathsf{op}}_{\mathcal{R}} : A_{\mathsf{op}} \times |\mathcal{R}| \longrightarrow B_{\mathsf{op}} \times |\mathcal{R}|\right)_{\mathsf{op} \in \Sigma}$$

[1] We consider runners for signatures, but the work generalises to alg. theories.

[2] In the literature also known as **comodels** for Σ (or for an algebraic theory).

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- For example, a natural runner $\mathcal{R}$ for S**-valued state**

$$\mathsf{get} : \mathbb{1} \rightsquigarrow S \qquad \mathsf{set} : S \rightsquigarrow \mathbb{1}$$

  is given by

$$|\mathcal{R}| \stackrel{\text{def}}{=} S \qquad \overline{\mathsf{get}}_{\mathcal{R}}\,(\star, s) \stackrel{\text{def}}{=} (s, s) \qquad \overline{\mathsf{set}}_{\mathcal{R}}\,(s, s) \stackrel{\text{def}}{=} (\star, s)$$

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[2] In the literature also known as **comodels** for Σ (or for an algebraic theory).

A natural model of top-level runtime ctd.

- Runners/comodels have been used for
  - **operational semantics** using tensors of models and comodels [Plotkin and Power '08]

    and
  - **stateful running** of algebraic effects [Uustalu '15]
  - **linear-use state-passing translation** [Møgelberg and Staton '11, '14]

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- The latter explicitly rely on one-to-one correspondence between
  - **runners** $\mathcal{R}$
  - **monad morphisms**[3] $\mathsf{r} : \mathbf{Free}_\Sigma(-) \longrightarrow \mathbf{St}_{|\mathcal{R}|}$

  where

$$\mathbf{St}_C\, X \stackrel{\text{def}}{=} C \Rightarrow X \times C$$

[3] $\mathbf{Free}_\Sigma(X)$ is the free monad ind. defined with leaves $\mathsf{val}\, x$ and nodes $\mathsf{op}(a, \kappa)$.

A natural model of top-level runtime ctd.

- For our purposes, we see runners

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- But what if this runtime is not **the** runtime?
  - hardware vs OS
  - OS vs VMs
  - VMs vs sandboxes

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- Unfortunately, runners, as defined above, are **not readily able to**
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  - **signal failure** caused by unavoidable circumstances

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- But is there a **useful generalisation** that would achieve this?

Effectful runners for modular top-levels

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- Møgelberg and Staton usefully observed that a **runner** $\mathcal{R}$ is equivalently simply a family of **generic effects** for $\mathbf{St}_{|\mathcal{R}|}$, i.e.,

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- The one-to-one correspondence with **monad morphisms**

$$\mathsf{r} : \mathbf{Free}_{\Sigma}(-) \longrightarrow \mathbf{T}$$

now simply amounts to the **univ. property of free models**, e.g.,

$$\mathsf{r}_X\,(\mathsf{val}\,x) = \eta_X\,x \qquad \mathsf{r}_X\,(\mathsf{op}(a,\kappa)) = (\mathsf{r}_X \circ \kappa)^{\dagger}(\overline{\mathsf{op}}_{\mathcal{R}}\,a)$$

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- Observe that κ appears in a **tail call position** on the right!

Effectful runners for modular top-levels ctd.

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- We want a runner to be a bit like a **kernel** of an OS, i.e., to
  - **(i)** provide management of **(internal) resources**
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- **Algebraically** (and pragmatically), this amounts to taking
  - **(i)** getenv : $\mathbb{1} \rightsquigarrow C$, setenv : $C \rightsquigarrow \mathbb{1}$
  - **(ii)** op : $A_{\mathsf{op}} \rightsquigarrow B_{\mathsf{op}}$ ($\mathsf{op} \in \Sigma'$, for some external Σ')
  - **(iii)** kill : $S \rightsquigarrow \mathbb{0}$

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  s.t., **(i)** satisfy state equations; and **(i)** commute with **(ii)** and **(iii)**
- The **induced monad** is then isomorphic to

$$\mathbf{T}\,X \quad \stackrel{\text{def}}{=} \quad C \Rightarrow \mathbf{Free}_{\Sigma'}\big((X \times C) + S\big)$$

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- With this, our **T-runners** $\mathcal{R}$ for Σ are (with "primitive" excs.)

$$\Big(\overline{\mathsf{op}}_{\mathcal{R}} : A_{\mathsf{op}} \longrightarrow \mathbf{K}_C^{\Sigma' ! E_{\mathsf{op}} \lightning S}\, B_{\mathsf{op}}\Big)_{\mathsf{op}\in\Sigma}$$

  where we call $\mathbf{K}_C^{\Sigma ! E \lightning S}$ a **kernel monad**, given by

$$\mathbf{K}_C^{\Sigma ! E \lightning S}\, X \quad \stackrel{\text{def}}{=} \quad C \Rightarrow \mathbf{Free}_{\Sigma}\big(((X + E) \times C) + S\big)$$

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  we can easily accommodate them in a programming language as

  let R = **runner** { $\mathsf{op}_1\ \mathsf{x}_1 \to \mathsf{K}_1$, ... , $\mathsf{op}_n\ \mathsf{x}_n \to \mathsf{K}_n$ } **@** C

  where K_i are **kernel code**, modelled using $\mathbf{K}_C^{\Sigma' ! E_{\mathsf{op}_i} \lightning S}$

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```
let R = runner { op1 x1 → K1 , ... , opn xn → Kn } @ C
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- For instance, we can implement a **write-only file handle** as

```
let RFH = runner {
  write s → if (length s > max)
            then (raise WriteSizeExceeded)
            else  (let fh = getenv () in
                   if (valid fh) then (fwrite (fh,s)) else (kill IOError))
} @ FileHandle
```

  where

$$\left(\mathsf{fwrite} : \mathsf{FileHandle} \times \mathsf{String} \rightsquigarrow 1 + E\right) \in \Sigma'$$

$$\Sigma \stackrel{\text{def}}{=} \{\ \mathsf{write} : \mathsf{String} \rightsquigarrow 1 + E \cup \{\mathsf{WriteSizeExceeded}\}\ \} \qquad \mathsf{IOError} \in S$$

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- Recall that the components r_X of the monad morphism

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  induced by a $\mathbf{T}$-runner $\mathcal{R}$ are all **tail-recursive**

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- We make use of it, to accommodate **running user code**:

```
using R @ M_init
run M
finally {return x @ c → M_ret , ... raise e @ c → M_e ... , ... kill s → M_s ...}
```

  where (a **user monad**)

  - Ms are **user code**, modelled using $\mathbf{U}^{\Sigma!E}\, X \stackrel{\text{def}}{=} \mathbf{Free}_\Sigma(X + E)$

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  - M_{init} produces the **initial kernel state**
  - M is the user code being **run using the runner** R
  - M_{ret}, M_e, M_s **finalise** for return values, exceptions, and signals

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using R **@** $\mathsf{M}_{\mathsf{init}}$
run M
finally {**return** x **@** c $\to$ $\mathsf{M}_{\mathsf{ret}}$, ... **raise** e **@** c $\to$ M_{e} ... , ... **kill** s $\to$ M_{s} ...}

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  - Ms are **user code**, modelled using $\mathbf{U}^{\Sigma!E}\, X \stackrel{\text{def}}{=} \mathbf{Free}_\Sigma(X + E)$
  - $\mathsf{M}_{\mathsf{init}}$ produces the **initial kernel state**
  - M is the user code being **run using the runner** R
  - $\mathsf{M}_{\mathsf{ret}}$, M_{e}, M_{s} **finalise** for return values, exceptions, and signals
  - $\mathsf{M}_{\mathsf{ret}}$ and M_{e} **depend on the final state** c, but M_{s} **does not**

Controlled initialisation and finalisation ctd.

- For instance, we can define a PYTHON-like **with-file construct**

```
with fileName do M
=
using R_FH @ (fopen fileName)
run M
finally {
  return x @ fh → fclose fh; return x ,
  raise  e @ fh → fclose fh; raise e  ,
  kill   s      → return ()  }
```

- Importantly, here
  - the **file handle is hidden** from `M`
  - `M` can only use `write` but not `fopen` and `fclose`
  - `fopen` and `fclose` are limited to **initialisation-finalisation**

A core calculus for programming with runners

Core calculus (syntax)

Core calculus (syntax)

- **Ground types** (types of ops. and kernel state)

$$A, B, C \quad ::= \quad \mathsf{B} \mid 1 \mid 0 \mid A \times B \mid A + B$$

- **Types**

$$\begin{aligned} X, Y \quad ::= \quad & \mathsf{B} \mid 1 \mid 0 \mid X \times Y \mid X + Y \\ \mid \quad & X \xrightarrow{\Sigma} Y \mathbin{!} E \\ \mid \quad & X \xrightarrow{\Sigma} Y \mathbin{!} E \lightning S \mathbin{@} C \\ \mid \quad & \Sigma \Rightarrow \Sigma' \lightning S \mathbin{@} C \end{aligned}$$

- **Values**

$$\Gamma \vdash V : X$$

- **User computations**

$$\Gamma \overset{\Sigma}{\models} M : X \mathbin{!} E$$

- **Kernel computations**

$$\Gamma \overset{\Sigma}{\models} K : X \mathbin{!} E \lightning S \mathbin{@} C$$

$$
\begin{aligned}
M ::= {} & \mathbf{return}\, V \;\mid\; \mathbf{try}\; M \;\mathbf{with}\; \{\; \mathbf{return}\; x \mapsto N_{val}\;,\; \big(\mathbf{raise}\; e \mapsto N_e\big)_{e \in E}\;\} \\
\mid {} & V\, W \;\mid\; \mathbf{match}\; V \;\mathbf{with}\; \{\; \langle x_1, x_2 \rangle \mapsto N\;\} \\
\mid {} & \mathbf{match}\; V \;\mathbf{with}\; \{\}_X \;\mid\; \mathbf{match}\; V \;\mathbf{with}\; \{\; \mathbf{inl}\; x_1 \mapsto N_1\;,\; \mathbf{inr}\; x_2 \mapsto N_2\;\} \\
\mid {} & \mathsf{op}_X\, V\, (x.M)\, (N_e)_{e \in E_{\mathsf{op}}} \;\mid\; \mathbf{raise}_X\, e \\
\mid {} & \mathbf{using}\; V \;@\; W \;\mathbf{run}\; M \;\mathbf{finally}\; \{\; \mathbf{return}\; x \;@\; c \mapsto N_{val}\;, \\
& \qquad\qquad\qquad\qquad \big(\mathbf{raise}\; e \;@\; c \mapsto N_e\big)_{e \in E}\;, \\
& \qquad\qquad\qquad\qquad \big(\mathbf{kill}\; s \mapsto N_s\big)_{s \in S}\;\} \\
\mid {} & \mathbf{exec}\; K \;@\; W \;\mathbf{finally}\; \{\; \mathbf{return}\; x \;@\; c \mapsto N_{val}\;, \\
& \qquad\qquad\qquad \big(\mathbf{raise}\; e \;@\; c \mapsto N_e\big)_{e \in E}\;, \\
& \qquad\qquad\qquad \big(\mathbf{kill}\; s \mapsto N_s\big)_{s \in S}\;\}
\end{aligned}
$$

$$
\begin{aligned}
K ::= {} & \mathbf{return}_C\, V \;\mid\; \mathbf{try}\; K \;\mathbf{with}\; \{\; \mathbf{return}\; x \mapsto L_{val}\;,\; \big(\mathbf{raise}\; e \mapsto L_e\big)_{e \in E}\;\} \\
\mid {} & V\, W \;\mid\; \mathbf{match}\; V \;\mathbf{with}\; \{\; \langle x_1, x_2 \rangle \mapsto L\;\} \\
\mid {} & \mathbf{match}\; V \;\mathbf{with}\; \{\}_{X @ C} \;\mid\; \mathbf{match}\; V \;\mathbf{with}\; \{\; \mathbf{inl}\; x_1 \mapsto L_1\;,\; \mathbf{inr}\; x_2 \mapsto L_2\;\} \\
\mid {} & \mathsf{op}_{X @ C}\, V\, (x.K)\, (L_e)_{e \in E_{\mathsf{op}}} \;\mid\; \mathbf{raise}_{X @ C}\, e \;\mid\; \mathbf{kill}_{X @ C}\, s \\
\mid {} & \mathbf{getenv}_C\, (c.K) \;\mid\; \mathbf{setenv}\, V\, K \\
\mid {} & \mathbf{exec}\; M \;\mathbf{finally}\; \{\; \mathbf{return}\; x \mapsto L_{val}\;,\; \big(\mathbf{raise}\; e \mapsto L_e\big)_{e \in E}\;\}
\end{aligned}
$$

Fig. 1. Syntax of user and kernel computations

Core calculus (type system and eq. theory)

Core calculus (type system and eq. theory)

- For example, the **typing rule for running user comps.** is

$$\frac{\Gamma \vdash V : \Sigma \Rightarrow \Sigma' \not\downarrow S \mathbin{@} C \qquad \Gamma \vdash W : C \qquad \Gamma \overset{\Sigma}{\models} M : X \mathbin{!} E \qquad \Gamma, x : X, c : C \overset{\Sigma'}{\models} N_{ret} : Y \mathbin{!} E' \qquad \left(\Gamma, c : C \overset{\Sigma'}{\models} N_e : Y \mathbin{!} E'\right)_{e \in E} \qquad \left(\Gamma \overset{\Sigma'}{\models} N_s : Y \mathbin{!} E'\right)_{s \in S}}{\Gamma \overset{\Sigma'}{\models} \textbf{using } V \mathbin{@} W \textbf{ run } M \textbf{ finally } \{ \textbf{return } x \mathbin{@} c \mapsto N_{ret} \ , \left(\textbf{raise } e \mathbin{@} c \mapsto N_e\right)_{e \in E} \ , \left(\textbf{kill } s \mapsto N_s\right)_{s \in S} \} : Y \mathbin{!} E'}$$

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- and the **main β-equation for running user comps.** is

$$\begin{array}{l}\Gamma \overset{\Sigma'}{\vDash} \textbf{using } R_C \mathbin{@} W \textbf{ run } (\mathrm{op}_X \, V \, (x.M) \, (M_e)_{e \in E_{\mathrm{op}}}) \textbf{ finally } F \\ \quad \equiv \textbf{exec } R_{op}[V] \mathbin{@} W \textbf{ finally } \{ \\ \qquad \textbf{return } x \mathbin{@} c' \mapsto \textbf{using } R_C \mathbin{@} c' \textbf{ run } M \textbf{ finally } F \ , \\ \qquad \left(\textbf{raise } e \mathbin{@} c' \mapsto \textbf{using } R_C \mathbin{@} c' \textbf{ run } M_e \textbf{ finally } F\right)_{e \in E_{\mathrm{op}}} \ , \\ \qquad \left(\textbf{kill } s \mapsto N_s\right)_{s \in S} \ \} : Y \mathbin{!} E'\end{array}$$

Core calculus (semantics)

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- Monadic semantics, for simplicity in **Set**, using
  - **user monads** $\mathbf{U}^{\Sigma ! E}\, X \stackrel{\text{def}}{=} \mathbf{Free}_{\Sigma}(X + E)$
  - **kernel monads** $\mathbf{K}_{C}^{\Sigma ! E \lightning S}\, X \stackrel{\text{def}}{=} C \Rightarrow \mathbf{Free}_{\Sigma}\big(((X + E) \times C) + S\big)$

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- (At a high level) the **judgements are interpreted** as

$$[\![\Gamma \vdash V : X]\!] : [\![\Gamma]\!] \longrightarrow [\![X]\!]$$

$$[\![\Gamma \stackrel{\Sigma}{\vDash} M : X \mathbin{!} E]\!] : [\![\Gamma]\!] \longrightarrow \mathbf{U}^{\Sigma!E}[\![X]\!]$$

$$[\![\Gamma \stackrel{\Sigma}{\vDash} K : X \mathbin{!} E \mathbin{\lightning} S \mathbin{@} C]\!] : [\![\Gamma]\!] \longrightarrow \mathbf{K}_{[\![C]\!]}^{\Sigma!E\lightning S}[\![X]\!]$$

Core calculus (semantics ctd.)

- However, to prove **coherence** of the semantics (**subtyping!**), we actually give the semantics in the **subset fibration**

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- For instance, **kernel computations** are interpreted as

$$\begin{array}{ccc} [\![\Gamma]\!] & \xrightarrow{[\![\Gamma \overset{\Sigma}{\vdash} K : X \,!\, E \lightning S \,@\, C]\!]} & \mathbf{K}^{\Sigma ! E \lightning S}_{[\![C]\!]}[\![X]\!] \\ \subseteq \downarrow & & \downarrow \subseteq \\ [\![\Gamma^s]\!] & \xrightarrow[[\![\Gamma^s \vdash K : X^s \,@\, C]\!]]{} & \mathbf{K}^{\mathcal{O} ! \mathcal{E} \lightning \mathcal{S}}_{[\![C]\!]}[\![X^s]\!] \end{array}$$

where $\Gamma^s \vdash K : X^s \;@\; C$ is a **skeletal kernel typing judgement**

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  where $\Gamma^s \vdash K : X^s \mathbin{@} C$ is a **skeletal kernel typing judgement**

- No essential obstacles to extending to **Sub**(**Cpo**) and beyond

Core calculus (semantics ctd.)

$$[\![\Gamma \overset{\Sigma'}{\vDash} \textbf{using } V \textbf{ @ } W \textbf{ run } M \textbf{ finally } \{ \textbf{return } x \textbf{ @ } c \mapsto N_{ret} \ , \ \big(\textbf{raise } e \textbf{ @ } c \mapsto N_e\big)_{e \in E} \ , \ \big(\textbf{kill } s \mapsto N_s\big)_{s \in S} \ \} : Y \ ! \ E']\!]_\gamma \overset{\text{def}}{=} \ \ldots$$

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- $[\![V]\!]_\gamma \ = \ \mathcal{R} \ = \ \Big(\overline{\text{op}}_{\mathcal{R}} : [\![A_{\text{op}}]\!] \longrightarrow \mathbf{K}^{\Sigma' ! E_{\text{op}} \not\downarrow S}_{[\![C]\!]} \, [\![B_{\text{op}}]\!]\Big)_{\text{op} \in \Sigma}$
- $[\![W]\!]_\gamma \ \in \ [\![C]\!]$
- $[\![M]\!]_\gamma \ \in \ \mathbf{U}^{\Sigma ! E} \, [\![A]\!]$
- $[\![\textbf{return } \mathrm{x} \textbf{ @ } \mathrm{c} \to N_{ret}]\!]_\gamma \ \in \ [\![A]\!] \times [\![C]\!] \Rightarrow \mathbf{U}^{\Sigma' ! E'} \, [\![B]\!]$
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- allowing us to use the **free model property** to get

$$\mathbf{U}^{\Sigma ! E} [\![A]\!] \xrightarrow{\mathsf{r}_{[\![A]\!] + E}} \mathbf{K}^{\Sigma' ! E \not\downarrow S}_{[\![C]\!]} [\![A]\!] \xrightarrow{(\lambda [\![M_3]\!]_\gamma)^\ddagger} [\![C]\!] \Rightarrow \mathbf{U}^{\Sigma' ! E'} [\![B]\!]$$

and then apply the resulting composite to $[\![M]\!]_\gamma$ and $[\![W]\!]_\gamma$

Runners in action

Runners can be vertically nested

Runners can be vertically nested

- ```
  using R_FH @ (fopen fileName)
  run (
    using R_FC @ (return "")
    run M
    finally {
      return x @ str → write str; return x ,
      raise   e @ str → write str; raise e   }
  )
  finally {
    return x @ fh → fclose fh; return x ,
    raise   e @ fh → fclose fh; raise e   , kill s → return ()}
  ```

where the **file contents runner** (with $\Sigma' = \mathbb{0}$) is defined as

```
let R_FC = runner {
  write str → let str' = getenv () in
              if (length (str^str') > max) then (raise WriteSizeExceeded)
                                           else  (setenv (str^str'))
} @ String
```
```

Runners can be horizontally paired

Runners can be horizontally paired

- Given a runner for Σ

```
let R1 = runner { ... , op1i x → k1i , ... } @ C1
```

and a runner for Σ'

```
let R2 = runner { ... , op2j x → k2j , ... } @ C2
```

we can **pair them** to get a runner for $\Sigma \uplus \Sigma'$

```
let R⊗ = runner {
  ... ,
  op1i x → let (c,c') = getenv () in
           let (x,c'') = k1i x in
           setenv (c'',c');
           return x ,
  ... ,
  op2j x → ... (* analogously to above *) ,
  ...
} @ C1 * C2
```

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} @ C1 * C2
```

- For instance, this way we can build a **runner for multiple files**

Vertical nesting for instrumentation

Vertical nesting for instrumentation

- ```
  using R_Sniffer @ (return 0)
  run M
  finally {
    return x @ c →
          let fh = fopen "nsa.txt" in fwrite (fh,nat_to_str c); fclose fh }
  ```

  where the **instrumenting runner** is defined as

  ```
  let R_Sniffer = runner {
    ... ,
    op a → let c = getenv () in
              setenv (c + 1);
              op a ,                          (* forwards op outwards *)
    ...
  } @ Nat
  ```

- The runner $R_{Sniffer}$ implements the same sig. Σ that `M` is using
- As a result, the runner $R_{Sniffer}$ is **invisible** from `M`'s viewpoint
  ```

Integer state with active monitoring

Integer state with active monitoring

- `type IntHeap = { memory : Nat → Option Int ; next : Nat }`

```
let R_IntState = runner {
  alloc x → ... ,

  deref r → let h = getenv () in
            match (heapSel h r) with
            | Some x → return x
            | None → kill ReferenceDoesNotExistSignal ,

  assign r y → let h = getenv () in
               match (heapUpd h r y) with
               | Some h' → if (rel x y)
                           then (setenv h')
                           else (raise MonotonicityException)
               | None → kill ReferenceDoesNotExistSignal
} @ IntHeap
```

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- This is **runtime verification** for `rel` **-monotonic integer state**

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```

- This is **runtime verification** for `rel` **-monotonic integer state**
- Also possible with **vertical nesting**: MLState ↔ Monotonicity

Other examples

- More general forms of **(ML-style) state** (for general `Ref A`)
  - if the host language allows it, we use GADTs, etc for safety
  - some examples extract a footprint from a larger memory
- **Combinations** of different effects and runners
  - in particular the combination of IO and state
  - good use case for both vertical and horizontal composition
- KOKA-style **ambient values** and **ambient functions**
  - ambient values are essentially mutable variables/parameters
  - ambient functions are executed in their lexical context
  - a runner for amb. funs. treats fun. application as a co-operation
  - amb. funs. are stored in a context-sensitive heap
  - the appl. co-operation restores the heap to the lexical context

Implementing runners

Experimenting with the theory in practice

Experimenting with the theory in practice

- A **small experimental language** COOP[4]
  - Implements the core calculus with few extras
  - The interpreter is directly based on the denotational semantics
  - Top-level containers for running external (OCaml) code

[4] coop [/ku:p/] – a cage where small animals are kept, especially chickens

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- A HASKELL **library** HASKELL-COOP
  - A shallow-embedding of the core calculus in HASKELL
  - Uses one of the Freer monad implementations underneath
  - Again, the operational aspects implement the denot. semantics
  - Top-level containers for arbitrary HASKELL monads
  - Examples make use of HASKELL's features (GADTs, ...)

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  - Examples make use of HASKELL's features (GADTs, ...)
- Both still need some finishing touches, but will be public soon

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Experimenting with the theory in practice

```
module AmbientsTests where

import Control.Runner
import Control.Runner.Ambients

ambFun :: AmbVal Int -> Int -> AmbEff Int
ambFun x y =
  do x <- getVal x;
     return (x + y)

test1 :: AmbEff Int
test1 =
  withAmbVal
    (4 :: Int)
    (\ x ->
      withAmbFun
        (ambFun x)
        (\ f ->
          do rebindVal x 2;
             applyFun f 1))

test2 = ambTopLevel test1
```

Wrapping up

- **Runners** are a natural model of **top-level runtime**
- We proposed **T-runners** to also model **non-top-level runtimes**
- We turned **T**-runners into a **practical programming construct**, that supports controlled initialisation and finalisation
- We showed some **combinators** and **programming examples**
- Two **implementations** in the works, Coop and Haskell-Coop

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Thank you!