Comodels as a gateway for interacting with the external world

Danel Ahman

(joint work with Andrej Bauer)

Ljubljana, 21 March 2019

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What is a comodel?

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- A **signature** Σ is a set of operation symbols

$$\mathsf{op} : A \rightsquigarrow B \qquad\qquad (\text{in univ. alg., } \mathsf{op} : 1 \rightsquigarrow n)$$

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- Intutively, comodels describe a notion of **state/world**, e.g.,
  - **Operational semantics** as a tensor of a model and comodel (Plotkin & Power, Abou-Saleh & Pattinson)
  - **Stateful runners** of effectful programs (Uustalu)
  - Default **top-level behaviour** of alg. effects (Bauer & Pretnar)

Computational effects in FP

Computational effects in FP

- Using **monads** (e.g., as in HASKELL)

```
type St a = String → (a,String)

f :: St a → St (a,a)
f c = c >>= (\x → c >>= (\y → return (x,y)))
```

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type St a = String → (a,String)

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```

- Using **algebraic effects** and **handlers** (e.g., as in EFF)

```
effect Get : int
effect Put : int → unit
                                    (*: int → a*int!{} *)
let g (c:unit → a!{Get,Put}) =
  with st_h handle (perform (Put 42); c ())
```

Computational effects in FP

- Using **monads** (e.g., as in Haskell)

```haskell
type St a = String → (a,String)

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- Using **algebraic effects** and **handlers** (e.g., as in Eff)

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effect Put : int → unit
                                          (*: int → a*int!{} *)
let g (c:unit → a!{Get,Put}) =
  with st_h handle (perform (Put 42); c ())
```

- Works well for effects that can be **represented** as pure data!

  But what about effects that need access to the **external world**?

External world in FP

- Declare a **signature** of monads/effects

```
type IO a

openFile  :: FilePath → IOMode → IO Handle
hGetLine  :: Handle → IO String
hClose    :: Handle → IO ()
```

External world in FP

- Declare a **signature** of monads/effects

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type IO a

openFile  :: FilePath → IOMode → IO Handle
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```
effect Read  : string
effect Raise : string → empty

effect RandomInt   : int → int
effect RandomFloat : float → float
```

External world in FP

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type IO a

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```

```
effect Read  : string
effect Raise : string → empty

effect RandomInt   : int → int
effect RandomFloat : float → float
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- And then treat it **specially** in the compiler, e.g.,

```
let rec top_handle op =
  match op with
  | ...
```

External world in FP

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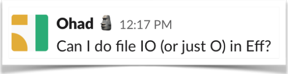

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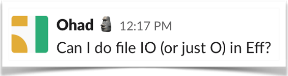

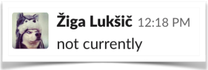

External world in FP

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Can I do file IO (or just O) in Eff?

Žiga Lukšič 12:18 PM
not currently

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So here's the hack I added. We should do something a bit more principled.
In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
     | "Write" ->
       (match v with
        | V.Tuple vs ->
          let (file_name :: str :: _)  = List.map V.to_str vs in
          let file_handle = open_out_gen
                              [Open_wronly
                              ;Open_append
                              ;Open_creat
                              ;Open_text
                              ] 0o666 file_name in
          Printf.fprintf file_handle "%s" str;
          close_out file_handle;
          top_handle (k V.unit_value)
       )
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This talk — a principled (co)algebraic approach!

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- ```
  let f (s:string) =
    let fh = fopen "foo.txt" in
    fwrite fh (s^s);
    fclose fh;
    return fh

  let g s =
    let fh = f s in fread fh
  ```
  ```

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- We could resolve this by typing file handles **linearly**
  - But we want other values (e.g., strings) to be used **non-linearly**!
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- But what if we wrap `f` in a **handler**?

  ```
  let h = handler
          | effect (FWrite fh s k) → return s

  let g s = with h handle f s
  ```
  ```

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  ```
  let h = handler
          | effect (FWrite fh s k) → return s

  let g s = with h handle f s  (* dangling fh ! *)
  ```
  ```

So, how could we solve these issues?

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- Using **existing programming mechanisms**, e.g.,
  - Modules and abstraction

```
module System.IO where

type IO a

hClose :: Handle → IO ()
```

  - Linear (and non-linear) types

```
linear type fhandle

effect FClose : (linear fhandle) → unit

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  - Linear (and non-linear) types

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- **Problem:** They don't really explain the **essence of the solution**

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- Let's look at HASKELL's **IO monad** again
- A common explanation is to think of functions

$$\mathsf{a} \to \mathsf{IO}\ \mathsf{b}$$

as

$$\mathsf{a} \to (\mathsf{RealWorld} \to (\mathsf{b}, \mathsf{RealWorld}))$$

which is the same as

$$(\mathsf{a}, \mathsf{RealWorld}) \to (\mathsf{b}, \mathsf{RealWorld})$$

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  which is the same as

$$(\mathsf{a}, \mathsf{RealWorld}) \rightarrow (\mathsf{b}, \mathsf{RealWorld})$$

- With the `System.IO` **module abstraction** ensuring that
  - We can't get our hands on RealWorld — it's an idea of the world
  - The **real world** is affected linearly
  - We don't ask more from the **real world** than it can provide

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which is the same as

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But wait a minute, RealWorld looks a lot like a **comodel**!

$$\mathsf{hGetLine} : (\mathsf{Handle}, \mathsf{RealWorld}) \to (\mathsf{String}, \mathsf{RealWorld})$$

$$\mathsf{hClose} \quad : (\mathsf{Handle}, \mathsf{RealWorld}) \to ((), \mathsf{RealWorld})$$

So, **IO** is more about in which **external world** our program is in!

Comodels as a gateway to the external world

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- ```
  let f (s:string) =
    using IO run
      let fh = fopen "foo.txt" in
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Comodels as a gateway to the external world

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      fwrite fh (s^s)                  (* @ IO : unit *)
    ending_with (fclose fh)
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- **Solution:** Modular treatment of **external worlds**

Comodels as a gateway to the external world

- Examples of **modularity** we might want from comodels

$$\begin{tikzcd} & \mathsf{IO} \arrow[dl, <->] \arrow[d, <->] \arrow[dr, <->] & \\ \mathsf{Fh} \arrow[d, <->] & \mathsf{IO} + \mathsf{CallStatistics} & \ldots \\ \mathsf{Str} & & \end{tikzcd}$$

Comodels as a gateway to the external world

- Examples of **modularity** we might want from comodels

$$
\begin{tikzcd}
& \mathsf{IO} \arrow[dl, leftrightarrow] \arrow[d, leftrightarrow] \arrow[dr, leftrightarrow] & \\
\mathsf{Fh} \arrow[d, leftrightarrow] & \mathsf{IO} + \mathsf{CallStatistics} & \dots \\
\mathsf{Str} & &
\end{tikzcd}
$$

- Fh — "world which consists of exactly one `fh`"
- IO $\longrightarrow$ Fh — "call `fopen` with `foo.txt`, store returned `fh`"
- Fh $\longrightarrow$ IO — "call `fclose` with stored `fh`"

- **Observation:** IO $\longleftrightarrow$ Fh and other $\longleftrightarrow$ look a lot like **lenses**

Comodels as a gateway to the external world

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- Our **general framework** on the file operations example

```
let f (s:string) =                 (* @ IO : unit *)
  using Fh
  starting_with (fopen_of_io "foo.txt")
  run
    fwrite_of_fh (s^s)             (* @ Fh : unit *)
  ending_with (fun _ fh → fclose_of_io fh)
```

Comodels as a gateway to the external world

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```

where

```
Fh =
  ⟨W = fhandle,
   co_fread ((),fh) = ...,
   co_fwrite (s,fh) = fwrite_of_io s fh;
                      return ((),fh)⟩

 (* co_fread : (unit * W) → (string * W) @ IO *)
(* co_fwrite : (string * W) → (unit * W) @ IO *)
```

Comodels as a gateway to the external world

Comodels as a gateway to the external world

- The **modularity aspect** of our general framework

```
let f (s:string) =                    (* @ IO : unit *)
  using Fh
  starting_with (fopen_of_io "foo.txt")
  run

    using Str
    starting_with (fread_of_fh ())
    run
      fwrite_of_str (s^s)     (* @ Str : unit *)
    ending_with (fun _ s → fwrite_of_fh s)

  ending_with (fun _ fh → fclose_of_io fh)
```

where

```
Str = ⟨W = string, ... ⟩
```

Comodels as a gateway to the external world

Comodels as a gateway to the external world

- Comodels can also **extend** the (intermediate) external world(s)

```
let f (s:string) =                     (* @ IO : unit *)
  using Stats
  starting_with (fopen_of_io "foo.txt")
  run
    fwrite_of_stats (s^s)  (* @ Stats : unit *)
  ending_with
    (fun _ (fh,c) →
      let fh' = fopen_of_io "stats.txt" in
      fwrite_of_io fh' c;
      fclose_of_io fh'; fclose_of_io fh)
```

where

```
Stats = ⟨W = (fhandle * nat), ... ⟩
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Comodels as a gateway to the external world

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where

```
Stats = ⟨W = (fhandle * nat), ... ⟩
```

- We can also track of **nondet./prob. choice results**, and alike
- Could we also use comodels for **dynamic (NI) monitoring**?

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- The two central **typing rules** are (U is the "universe", aka IO)

$$\dfrac{\Gamma \vdash \mathsf{D}\ \text{comodel}\ \texttt{@}\ \vec{\mathsf{C}} \qquad \mathsf{D} \neq \mathsf{U} \\ \Gamma \vdash c_s\ \texttt{@}\ \vec{\mathsf{C}} : \mathsf{D}.W \qquad \Gamma \vdash c\ \texttt{@}\ \vec{\mathsf{C}}, \mathsf{D} : A \qquad \Gamma, x\!:\!A, w\!:\!\mathsf{D}.W \vdash c_e\ \texttt{@}\ \vec{\mathsf{C}} : A}{\begin{array}{l} \Gamma \vdash \textbf{using}\ \mathsf{D} \\ \qquad \textbf{starting\_with}\ c_s \\ \qquad \textbf{run}\ c \\ \qquad \textbf{ending\_with}\ (x.w.c_e) \quad \texttt{@}\ \vec{\mathsf{C}} : A \end{array}}$$

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and

$$\frac{\Gamma \vdash \mathsf{D}\ \text{comodel} \mathbin{@} \vec{\mathsf{C}} \qquad \mathsf{op} : A \leadsto B \in \mathsf{D}.\Sigma \qquad \Gamma \vdash v : A}{\Gamma \vdash \mathsf{op}\ v \mathbin{@} \vec{\mathsf{C}}, \mathsf{D} : B}$$

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- Regarding the **denotational semantics**, the idea is to interpret

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- Regarding **operational semantics**, the idea is to consider confs.

  $$(\; \overrightarrow{(\mathsf{C}, w)} \;,\; c \;)$$

  either in a **big-** or **small-step** style

  - where $\overrightarrow{(\mathsf{C}, w)}$ is a stack of worlds

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}{
\begin{array}{c}
(\ (\overrightarrow{(\mathsf{C}, w_0)}, (\mathsf{C}', w_0'))\ ,\ \textbf{using}\ \mathsf{D}\ \textbf{s\_w}\ c_s\ \textbf{run}\ c\ \textbf{e\_w}\ (x.w.c_e)\ ) \\[0.5em]
\Downarrow \\[0.5em]
(\ (\overrightarrow{(\mathsf{C}, w_3)}, (\mathsf{C}', w_3'))\ ,\ \textbf{return}\ v'\ )
\end{array}
}
$$

- The interpretation of **operations** uses the **co-operations** of Cs, naturally traversing the stack of (intermediate) external worlds

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ending_with (fun x w → c_e)
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  - How do **multi-handlers** fit in the picture? Tensors of some sort?

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- `System.IO`, KOKA's **`initially`** & **`finally`**, PYTHON's **`with`**, ...

Ongoing and future work

- Work out all the **formal details** of what I have shown you today
- **Algebraic effects** and **(multi-)handlers**
- More **examples** and **use cases** (Matija, the Eff Architecture?)
- Clarify the connection with **(effectful) lenses**
- **Combinatorics** of comodels and their lens-like relationships