Recalling a Witness

Foundations and Applications of Monotonic State

Danel Ahman @ INRIA Paris

Cătălin Hrițcu and Kenji Maillard @ INRIA Paris

Cédric Fournet, Aseem Rastogi, and Nikhil Swamy @ MSR

POPL 2018

January 12, 2018

Monotonicity is really useful!

Its essence can be captured very neatly!

Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

Monotonicity in program verification

- Consider a program operating on **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert (v} \in \texttt{get())}$$

- To prove the assertion (say, in a Floyd-Hoare style logic), we could prove that the code maintains a **stateful invariant**

$$\{\lambda\,\mathtt{s}\,.\,\mathtt{v} \in \mathtt{s}\}\ \texttt{complex\_procedure()}\ \{\lambda\,\mathtt{s}\,.\,\mathtt{v} \in \mathtt{s}\}$$

  - likely that we have to **carry** $\lambda\,\mathtt{s}\,.\,\mathtt{v} \in \mathtt{s}$ **through** the proof of c_p
    - **does not guarantee** that $\lambda\,\mathtt{s}\,.\,\mathtt{v} \in \mathtt{s}$ holds at every point in c_p
    - **sensitive** to proving that c_p maintains $\lambda\,\mathtt{s}\,.\,\mathtt{w} \in \mathtt{s}$ for some other $\mathtt{w}$

- However, if c_p **never removes**, then $\lambda\,\mathtt{s}\,.\,\mathtt{v} \in \mathtt{s}$ is **stable**, and we would like the program logic to give us $\mathtt{v} \in \mathtt{get()}$ "**for free**"

Monotonicity in program verification

- Consider a program operating on **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert } (\mathtt{v} \in \mathtt{get}())$$

- To prove the assertion (say, in a Floyd-Hoare style logic), we could prove that the code maintains a **stateful invariant**

$$\{\lambda\, \mathtt{s} \,.\, \mathtt{v} \in \mathtt{s}\}\ \texttt{complex\_procedure()}\ \{\lambda\, \mathtt{s} \,.\, \mathtt{v} \in \mathtt{s}\}$$

  - likely that we have to **carry** $\lambda\, \mathtt{s} \,.\, \mathtt{v} \in \mathtt{s}$ **through** the proof of c_p
    - **does not guarantee** that $\lambda\, \mathtt{s} \,.\, \mathtt{v} \in \mathtt{s}$ holds at every point in c_p
    - **sensitive** to proving that c_p maintains $\lambda\, \mathtt{s} \,.\, \mathtt{w} \in \mathtt{s}$ for some other $\mathtt{w}$

- However, if c_p **never removes**, then $\lambda\, \mathtt{s} \,.\, \mathtt{v} \in \mathtt{s}$ is **stable**, and we would like the program logic to give us $\mathtt{v} \in \mathtt{get}()$ "**for free**"

Monotonicity in program verification

- Consider a program operating on **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert } (\texttt{v} \in \texttt{get()})$$

- To prove the assertion (say, in a Floyd-Hoare style logic), we could prove that the code maintains a **stateful invariant**

$$\{\lambda\, \texttt{s} . \texttt{v} \in \texttt{s}\}\ \texttt{complex\_procedure()}\ \{\lambda\, \texttt{s} . \texttt{v} \in \texttt{s}\}$$

  - likely that we have to **carry** $\lambda\, \texttt{s} . \texttt{v} \in \texttt{s}$ **through** the proof of c_p
    - **does not guarantee** that $\lambda\, \texttt{s} . \texttt{v} \in \texttt{s}$ holds at every point in c_p
    - **sensitive** to proving that c_p maintains $\lambda\, \texttt{s} . \texttt{w} \in \texttt{s}$ for some other w

- However, if c_p **never removes**, then $\lambda\, \texttt{s} . \texttt{v} \in \texttt{s}$ is **stable**, and we would like the program logic to give us $\texttt{v} \in \texttt{get()}$ "**for free**"

Monotonicity in program verification

- Consider a program operating on **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert } (\texttt{v} \in \texttt{get()})$$

- To prove the assertion (say, in a Floyd-Hoare style logic), we could prove that the code maintains a **stateful invariant**

$$\{\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}\}\ \texttt{complex\_procedure()}\ \{\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}\}$$

  - likely that we have to **carry** $\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}$ **through** the proof of c_p
    - **does not guarantee** that $\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}$ holds at every point in c_p
    - **sensitive** to proving that c_p maintains $\lambda\,\texttt{s}\,.\,\texttt{w} \in \texttt{s}$ for some other w

- However, if c_p **never removes**, then $\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}$ is **stable**, and we would like the program logic to give us $\texttt{v} \in \texttt{get()}$ "**for free**"

Monotonicity in programming

- **Programming** also relies on **monotonicity**,
  even if you don't realise it!

- Consider ML-style typed **references** `r:ref a`
  - `r` is a **proof of existence** of an `a`-typed value in the heap

- Correctness relies on **monotonicity**!
  1) Allocation **stores** an `a`-typed value in the heap
  2) Writes **don't change type** and there is **no deallocation**
  3) So, given a ref. `r`, it is **guaranteed to point** to an `a`-typed value

- Baked into the memory models of most languages
- We derive them from **global state + general monotonicity**

Monotonicity in programming

- **Programming** also relies on **monotonicity**,

  even if you don't realise it!

- Consider ML-style typed **references** `r:ref a`
  - `r` is a **proof of existence** of an `a`-typed value in the heap

- Correctness relies on **monotonicity**!
  1) Allocation **stores** an `a`-typed value in the heap
  2) Writes **don't change type** and there is **no deallocation**
  3) So, given a ref. `r`, it is **guaranteed to point** to an `a`-typed value

- Baked into the memory models of most languages

- We derive them from **global state** + **general monotonicity**

Monotonicity in programming

- **Programming** also relies on **monotonicity**,

  even if you don't realise it!

- Consider ML-style typed **references** `r:ref a`
  - `r` is a **proof of existence** of an `a`-typed value in the heap

- Correctness relies on **monotonicity**!
  1) Allocation **stores** an `a`-typed value in the heap
  2) Writes **don't change type** and there is **no deallocation**
  3) So, given a ref. `r`, it is **guaranteed to point** to an `a`-typed value

- Baked into the memory models of most languages

- We derive them from **global state** + **general monotonicity**

Monotonicity in programming

- **Programming** also relies on **monotonicity**,

  even if you don't realise it!

- Consider ML-style typed **references** `r:ref a`
  - `r` is a **proof of existence** of an `a`-typed value in the heap

- Correctness relies on **monotonicity**!
  1) Allocation **stores** an `a`-typed value in the heap
  2) Writes **don't change type** and there is **no deallocation**
  3) So, given a ref. `r`, it is **guaranteed to point** to an `a`-typed value

- Baked into the memory models of most languages
- We derive them from **global state** + **general monotonicity**

Monotonicity is really useful!

- In this talk
  - our **motivating example** and **monotonic counters**
  - **typed references** (`ref t`) and **untyped references** (`uref`)
  - more flexibility with **monotonic references** (`mref t rel`)
- More in the paper
  - temporarily **violating monotonicity** via snapshots
  - two substantial case studies
    - a **secure file-transfer** application
    - Ariadne **state continuity** protocol [Strackx, Piessens 2016]
  - pointers to other works in F* relying on monotonicity for
    - sophisticated **region-based memory models** [`fstar-lang.org`]
    - **crypto** and **TLS verification** [`project-everest.github.io`]

Monotonicity is really useful!

- In this talk
  - our **motivating example** and **monotonic counters**
  - **typed references** (`ref t`) and **untyped references** (`uref`)
  - more flexibility with **monotonic references** (`mref t rel`)
- More in the paper
  - temporarily **violating monotonicity** via snapshots
  - two substantial case studies
    - a **secure file-transfer** application
    - Ariadne **state continuity** protocol [Strackx, Piessens 2016]
  - pointers to other works in F* relying on monotonicity for
    - sophisticated **region-based memory models** [`fstar-lang.org`]
    - **crypto** and **TLS verification** [`project-everest.github.io`]

Monotonicity is really useful!

- In this talk
  - our **motivating example** and **monotonic counters**
  - **typed references** (`ref t`) and **untyped references** (`uref`)
  - more flexibility with **monotonic references** (`mref t rel`)

- More in the paper
  - temporarily **violating monotonicity** via snapshots
  - two substantial case studies
    - a **secure file-transfer** application
    - Ariadne **state continuity** protocol [Strackx, Piessens 2016]
  - pointers to other works in F* relying on monotonicity for
    - sophisticated **region-based memory models** [`fstar-lang.org`]
    - **crypto** and **TLS verification** [`project-everest.github.io`]

Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

Key ideas behind our general framework

- We focus on **monotonic programs** and **stable predicates**
  - per verification task, we **choose a preorder** rel on states
    - set inclusion, heap inclusion, increasing counter values, ...
  - a stateful program e is **monotonic** (wrt. rel) when

$$\forall\, s\, e'\, s'.\ (e, s) \leadsto^* (e', s') \implies \mathtt{rel}\ s\ s'$$

  - a stateful predicate p is **stable** (wrt. rel) when

$$\forall\, s\, s'.\ \mathtt{p}\ s\ \wedge\ \mathtt{rel}\ s\ s' \implies \mathtt{p}\ s'$$

- **Our solution:** extend Hoare-style program logics (e.g., F*) with
  - a means to **witness** the validity of p s in some state s
  - a means for turning a p into a **state-independent proposition**
  - a means to **recall** the validity of p s′ in any future state s′
- Provides a **unifying account** of the existing *ad hoc* uses in F*

Key ideas behind our general framework

- We focus on **monotonic programs** and **stable predicates**
  - per verification task, we **choose a preorder** `rel` on states
    - set inclusion, heap inclusion, increasing counter values, ...
  - a stateful program e is **monotonic** (wrt. `rel`) when

$$\forall\, \mathtt{s}\ \mathtt{e}'\ \mathtt{s}'.\ (\mathtt{e},\mathtt{s}) \rightsquigarrow^{*} (\mathtt{e}',\mathtt{s}') \implies \mathtt{rel}\ \mathtt{s}\ \mathtt{s}'$$

  - a stateful predicate `p` is **stable** (wrt. `rel`) when

$$\forall\, \mathtt{s}\ \mathtt{s}'.\ \mathtt{p}\ \mathtt{s}\ \wedge\ \mathtt{rel}\ \mathtt{s}\ \mathtt{s}' \implies \mathtt{p}\ \mathtt{s}'$$

- **Our solution:** extend Hoare-style program logics (e.g., F*) with
  - a means to **witness** the validity of `p` s in some state s
  - a means for turning a `p` into a **state-independent proposition**
  - a means to **recall** the validity of `p` s′ in any future state s′
- Provides a **unifying account** of the existing *ad hoc* uses in F*

Key ideas behind our general framework

- We focus on **monotonic programs** and **stable predicates**
  - per verification task, we **choose a preorder** `rel` on states
    - set inclusion, heap inclusion, increasing counter values, ...
  - a stateful program e is **monotonic** (wrt. `rel`) when

$$\forall\, \mathrm{s}\, \mathrm{e}'\, \mathrm{s}'.\ (\mathrm{e}, \mathrm{s}) \leadsto^* (\mathrm{e}', \mathrm{s}') \implies \mathtt{rel}\ \mathrm{s}\ \mathrm{s}'$$

  - a stateful predicate p is **stable** (wrt. `rel`) when

$$\forall\, \mathrm{s}\, \mathrm{s}'.\ \mathtt{p}\ \mathrm{s}\ \wedge\ \mathtt{rel}\ \mathrm{s}\ \mathrm{s}' \implies \mathtt{p}\ \mathrm{s}'$$

- **Our solution:** extend Hoare-style program logics (e.g., F*) with
  - a means to **witness** the validity of p s in some state s
  - a means for turning a p into a **state-independent proposition**
  - a means to **recall** the validity of p s′ in any future state s′
- Provides a **unifying account** of the existing *ad hoc* uses in F*

Key ideas behind our general framework

- We focus on **monotonic programs** and **stable predicates**
  - per verification task, we **choose a preorder** `rel` on states
    - set inclusion, heap inclusion, increasing counter values, ...
  - a stateful program e is **monotonic** (wrt. `rel`) when

$$\forall\, \mathtt{s}\; \mathtt{e}'\; \mathtt{s}'.\ (\mathtt{e}, \mathtt{s}) \rightsquigarrow^* (\mathtt{e}', \mathtt{s}') \implies \mathtt{rel}\ \mathtt{s}\ \mathtt{s}'$$

  - a stateful predicate p is **stable** (wrt. `rel`) when

$$\forall\, \mathtt{s}\; \mathtt{s}'.\ \mathtt{p}\ \mathtt{s}\ \wedge\ \mathtt{rel}\ \mathtt{s}\ \mathtt{s}' \implies \mathtt{p}\ \mathtt{s}'$$

- **Our solution:** extend Hoare-style program logics (e.g., F*) with
  - a means to **witness** the validity of p s in some state s
  - a means for turning a p into a **state-independent proposition**
  - a means to **recall** the validity of p s′ in any future state s′
- Provides a **unifying account** of the existing *ad hoc* uses in F*

Key ideas behind our general framework

- We focus on **monotonic programs** and **stable predicates**
  - per verification task, we **choose a preorder** `rel` on states
    - set inclusion, heap inclusion, increasing counter values, ...
  - a stateful program e is **monotonic** (wrt. `rel`) when

$$\forall\, \mathsf{s}\, \mathsf{e}'\, \mathsf{s}'.\ (\mathsf{e}, \mathsf{s}) \rightsquigarrow^* (\mathsf{e}', \mathsf{s}') \implies \mathtt{rel}\ \mathtt{s}\ \mathtt{s}'$$

  - a stateful predicate `p` is **stable** (wrt. `rel`) when

$$\forall\, \mathsf{s}\, \mathsf{s}'.\ \mathtt{p}\ \mathtt{s}\ \wedge\ \mathtt{rel}\ \mathtt{s}\ \mathtt{s}' \implies \mathtt{p}\ \mathtt{s}'$$

- **Our solution:** extend Hoare-style program logics (e.g., F*) with
  - a means to **witness** the validity of `p` s in some state s
  - a means for turning a `p` into a **state-independent proposition**
  - a means to **recall** the validity of `p` s′ in any future state s′
- Provides a **unifying account** of the existing *ad hoc* uses in F*

Key ideas behind our general framework

- We focus on **monotonic programs** and **stable predicates**
  - per verification task, we **choose a preorder** `rel` on states
    - set inclusion, heap inclusion, increasing counter values, ...
  - a stateful program e is **monotonic** (wrt. `rel`) when

$$\forall\, \mathtt{s}\, \mathtt{e}'\, \mathtt{s}'.\ (\mathtt{e}, \mathtt{s}) \rightsquigarrow^* (\mathtt{e}', \mathtt{s}') \implies \mathtt{rel}\ \mathtt{s}\ \mathtt{s}'$$

  - a stateful predicate `p` is **stable** (wrt. `rel`) when

$$\forall\, \mathtt{s}\, \mathtt{s}'.\ \mathtt{p}\ \mathtt{s}\ \wedge\ \mathtt{rel}\ \mathtt{s}\ \mathtt{s}' \implies \mathtt{p}\ \mathtt{s}'$$

- **Our solution:** extend Hoare-style program logics (e.g., F*) with
  - a means to **witness** the validity of `p` s in some state s
  - a means for turning a `p` into a **state-independent proposition**
  - a means to **recall** the validity of `p` s′ in any future state s′

- Provides a **unifying account** of the existing *ad hoc* uses in F*

Key ideas behind our general framework

- We focus on **monotonic programs** and **stable predicates**
  - per verification task, we **choose a preorder** `rel` on states
    - set inclusion, heap inclusion, increasing counter values, ...
  - a stateful program e is **monotonic** (wrt. `rel`) when

$$\forall\, \mathtt{s}\, \mathtt{e}'\, \mathtt{s}'.\ (\mathtt{e}, \mathtt{s}) \rightsquigarrow^* (\mathtt{e}', \mathtt{s}') \implies \mathtt{rel}\ \mathtt{s}\ \mathtt{s}'$$

  - a stateful predicate `p` is **stable** (wrt. `rel`) when

$$\forall\, \mathtt{s}\, \mathtt{s}'.\ \mathtt{p}\ \mathtt{s}\ \wedge\ \mathtt{rel}\ \mathtt{s}\ \mathtt{s}' \implies \mathtt{p}\ \mathtt{s}'$$

- **Our solution:** extend Hoare-style program logics (e.g., F*) with
  - a means to **witness** the validity of `p` s in some state s
  - a means for turning a `p` into a **state-independent proposition**
  - a means to **recall** the validity of `p` s′ in any future state s′
- Provides a **unifying account** of the existing *ad hoc* uses in F*

Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

Recap: Ordinary global state in F*

- F* is an ML-like dependently typed language, aimed at verification
- F* supports Hoare-style reasoning about state via the **comp. type**

$$\mathrm{ST}_{\mathtt{state}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where

$$\mathtt{pre} : \mathtt{state} \to \mathrm{Type} \qquad\qquad \mathtt{post} : \mathtt{state} \to \mathtt{t} \to \mathtt{state} \to \mathrm{Type}$$

- ST is an abstract pre-postcondition refinement of

$$\mathtt{st}\ \mathtt{t} \stackrel{\mathsf{def}}{=} \mathtt{state} \to \mathtt{t} * \mathtt{state}$$

- The global state **actions** have types

$$\mathtt{get} : \mathtt{unit} \to \mathrm{ST}\ \mathtt{state}\ (\mathtt{requires}\ (\lambda\, \_ . \top))\ (\mathtt{ensures}\ (\lambda\, \mathtt{s}_0\, \mathtt{s}\, \mathtt{s}_1 . \mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))$$

$$\mathtt{put} : \mathtt{s}{:}\mathtt{state} \to \mathrm{ST}\ \mathtt{unit}\ (\mathtt{requires}\ (\lambda\, \_ . \top))\ (\mathtt{ensures}\ (\lambda\, \_\, \_\, \mathtt{s}_1 . \mathtt{s}_1 = \mathtt{s}))$$

- **Refs.** and **local state** will be defined in F* using **monotonicity**

Recap: Ordinary global state in F*

- F* is an ML-like dependently typed language, aimed at verification
- F* supports Hoare-style reasoning about state via the **comp. type**

$$\mathtt{ST}_{\mathtt{state}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where

$$\mathtt{pre} : \mathtt{state} \to \mathtt{Type} \qquad \mathtt{post} : \mathtt{state} \to \mathtt{t} \to \mathtt{state} \to \mathtt{Type}$$

- $\mathtt{ST}$ is an abstract pre-postcondition refinement of

$$\mathtt{st}\ \mathtt{t} \stackrel{\mathsf{def}}{=} \mathtt{state} \to \mathtt{t} * \mathtt{state}$$

- The global state **actions** have types

$$\mathtt{get} : \mathtt{unit} \to \mathtt{ST}\ \mathtt{state}\ (\mathtt{requires}\ (\lambda\, \_ .\top))\ (\mathtt{ensures}\ (\lambda\, \mathtt{s}_0\ \mathtt{s}\ \mathtt{s}_1 .\ \mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))$$

$$\mathtt{put} : \mathtt{s}{:}\mathtt{state} \to \mathtt{ST}\ \mathtt{unit}\ (\mathtt{requires}\ (\lambda\, \_ .\top))\ (\mathtt{ensures}\ (\lambda\, \_\ \_\ \mathtt{s}_1 .\ \mathtt{s}_1 = \mathtt{s}))$$

- **Refs.** and **local state** will be defined in F* using **monotonicity**

Recap: Ordinary global state in F*

- F* is an ML-like dependently typed language, aimed at verification
- F* supports Hoare-style reasoning about state via the **comp. type**

$$\mathtt{ST}_{\mathtt{state}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where

$$\mathtt{pre} : \mathtt{state} \to \mathtt{Type} \qquad\qquad \mathtt{post} : \mathtt{state} \to \mathtt{t} \to \mathtt{state} \to \mathtt{Type}$$

- ST is an abstract pre-postcondition refinement of

$$\mathtt{st}\ \mathtt{t} \stackrel{\text{def}}{=} \mathtt{state} \to \mathtt{t} * \mathtt{state}$$

- The global state **actions** have types

$$\mathtt{get} : \mathtt{unit} \to \mathtt{ST}\ \mathtt{state}\ (\mathtt{requires}\ (\lambda\,\_.\top))\ (\mathtt{ensures}\ (\lambda\,\mathtt{s}_0\ \mathtt{s}\ \mathtt{s}_1\,.\,\mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))$$

$$\mathtt{put} : \mathtt{s}{:}\mathtt{state} \to \mathtt{ST}\ \mathtt{unit}\ (\mathtt{requires}\ (\lambda\,\_.\top))\ (\mathtt{ensures}\ (\lambda\,\_\,\_\,\mathtt{s}_1\,.\,\mathtt{s}_1 = \mathtt{s}))$$

- **Refs.** and **local state** will be defined in F* using **monotonicity**

Recap: Ordinary global state in F*

- F* is an ML-like dependently typed language, aimed at verification
- F* supports Hoare-style reasoning about state via the **comp. type**

$$\mathtt{ST}_{\mathtt{state}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where

$$\mathtt{pre} : \mathtt{state} \to \mathtt{Type} \qquad \mathtt{post} : \mathtt{state} \to \mathtt{t} \to \mathtt{state} \to \mathtt{Type}$$

- $\mathtt{ST}$ is an abstract pre-postcondition refinement of

$$\mathtt{st}\ \mathtt{t} \stackrel{\text{def}}{=} \mathtt{state} \to \mathtt{t} * \mathtt{state}$$

- The global state **actions** have types

$$\mathtt{get} : \mathtt{unit} \to \mathtt{ST}\ \mathtt{state}\ (\mathtt{requires}\ (\lambda\_.\top))\ (\mathtt{ensures}\ (\lambda\, \mathtt{s}_0\, \mathtt{s}\, \mathtt{s}_1\, .\, \mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))$$

$$\mathtt{put} : \mathtt{s}{:}\mathtt{state} \to \mathtt{ST}\ \mathtt{unit}\ (\mathtt{requires}\ (\lambda\_.\top))\ (\mathtt{ensures}\ (\lambda\,\_\,\_\,\mathtt{s}_1\, .\, \mathtt{s}_1 = \mathtt{s}))$$

- **Refs.** and **local state** will be defined in F* using **monotonicity**

New: Monotonic global state in F*

- We capture monotonic state with a new **computational type**

$$\mathtt{MST}_{\mathtt{state},\mathtt{rel}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where pre and post are typed as in ST

- The **get** action is typed as in ST

$$\begin{aligned}\mathtt{get} : \mathtt{unit} \to \mathtt{MST}\ \mathtt{state}\ &(\mathtt{requires}\ (\lambda\,\_\,.\top))\\ &(\mathtt{ensures}\ (\lambda\,\mathtt{s}_0\,\mathtt{s}\,\mathtt{s}_1\,.\,\mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))\end{aligned}$$

- To ensure **monotonicity**, the **put** action gets a precondition

$$\begin{aligned}\mathtt{put} : \mathtt{s}{:}\mathtt{state} \to \mathtt{MST}\ \mathtt{unit}\ &(\mathtt{requires}\ (\lambda\,\mathtt{s}_0\,.\,\mathtt{rel}\ \mathtt{s}_0\ \mathtt{s}))\\ &(\mathtt{ensures}\ (\lambda\,\_\,\_\,\mathtt{s}_1\,.\,\mathtt{s}_1 = \mathtt{s}))\end{aligned}$$

- So intuitively, MST is an **abstract** pre-postcondition refinement of

$$\mathtt{mst}\ \mathtt{t} \stackrel{\text{def}}{=} \mathtt{s}_0{:}\mathtt{state} \to \mathtt{t} * \mathtt{s}_1{:}\mathtt{state}\{\mathtt{rel}\ \mathtt{s}_0\ \mathtt{s}_1\}$$

New: Monotonic global state in F*

- We capture monotonic state with a new **computational type**

$$\mathtt{MST}_{\mathtt{state},\mathtt{rel}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where $\mathtt{pre}$ and $\mathtt{post}$ are typed as in $\mathtt{ST}$

- The **get** action is typed as in $\mathtt{ST}$

$$\begin{aligned}\mathtt{get} : \mathtt{unit} \rightarrow \mathtt{MST}\ \mathtt{state}\ & (\mathtt{requires}\ (\lambda\ \_ . \top))\\ & (\mathtt{ensures}\ (\lambda\, \mathtt{s}_0\, \mathtt{s}\, \mathtt{s}_1 . \mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))\end{aligned}$$

- To ensure **monotonicity**, the **put** action gets a precondition

$$\begin{aligned}\mathtt{put} : \mathtt{s}{:}\mathtt{state} \rightarrow \mathtt{MST}\ \mathtt{unit}\ & (\mathtt{requires}\ (\lambda\, \mathtt{s}_0 . \mathtt{rel}\ \mathtt{s}_0\ \mathtt{s}))\\ & (\mathtt{ensures}\ (\lambda\ \_\ \_\ \mathtt{s}_1 . \mathtt{s}_1 = \mathtt{s}))\end{aligned}$$

- So intuitively, $\mathtt{MST}$ is an **abstract** pre-postcondition refinement of

$$\mathtt{mst}\ \mathtt{t}\ \stackrel{\text{def}}{=}\ \mathtt{s}_0{:}\mathtt{state} \rightarrow \mathtt{t} * \mathtt{s}_1{:}\mathtt{state}\{\mathtt{rel}\ \mathtt{s}_0\ \mathtt{s}_1\}$$

New: Monotonic global state in F*

- We capture monotonic state with a new **computational type**

$$\texttt{MST}_{\texttt{state},\texttt{rel}}\ \texttt{t}\ (\texttt{requires}\ \texttt{pre})\ (\texttt{ensures}\ \texttt{post})$$

  where pre and post are typed as in ST

- The **get** action is typed as in ST

$$\begin{aligned}\texttt{get} : \texttt{unit} \to \texttt{MST}\ \texttt{state}\ & (\texttt{requires}\ (\lambda\ \_ . \top)) \\ & (\texttt{ensures}\ (\lambda\, \texttt{s}_0\ \texttt{s}\ \texttt{s}_1 \,.\, \texttt{s}_0 = \texttt{s} = \texttt{s}_1))\end{aligned}$$

- To ensure **monotonicity**, the **put** action gets a precondition

$$\begin{aligned}\texttt{put} : \texttt{s}{:}\texttt{state} \to \texttt{MST}\ \texttt{unit}\ & (\texttt{requires}\ (\lambda\, \texttt{s}_0 \,.\, \texttt{rel}\ \texttt{s}_0\ \texttt{s})) \\ & (\texttt{ensures}\ (\lambda\ \_\ \_\ \texttt{s}_1 \,.\, \texttt{s}_1 = \texttt{s}))\end{aligned}$$

- So intuitively, MST is an **abstract** pre-postcondition refinement of

$$\texttt{mst}\ \texttt{t} \stackrel{\text{def}}{=} \texttt{s}_0{:}\texttt{state} \to \texttt{t} * \texttt{s}_1{:}\texttt{state}\{\texttt{rel}\ \texttt{s}_0\ \texttt{s}_1\}$$

New: Monotonic global state in F*

- We capture monotonic state with a new **computational type**

$$\mathtt{MST}_{\mathtt{state},\mathtt{rel}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where pre and post are typed as in ST

- The **get** action is typed as in ST

$$\mathtt{get} : \mathtt{unit} \to \mathtt{MST}\ \mathtt{state}\ (\mathtt{requires}\ (\lambda\, \_ . \top))\ (\mathtt{ensures}\ (\lambda\, \mathtt{s}_0\, \mathtt{s}\, \mathtt{s}_1 \,.\, \mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))$$

- To ensure **monotonicity**, the **put** action gets a precondition

$$\mathtt{put} : \mathtt{s}{:}\mathtt{state} \to \mathtt{MST}\ \mathtt{unit}\ (\mathtt{requires}\ (\lambda\, \mathtt{s}_0 \,.\, \mathtt{rel}\ \mathtt{s}_0\ \mathtt{s}))\ (\mathtt{ensures}\ (\lambda\, \_\, \_\, \mathtt{s}_1 \,.\, \mathtt{s}_1 = \mathtt{s}))$$

- So intuitively, MST is an **abstract** pre-postcondition refinement of

$$\mathtt{mst}\ \mathtt{t} \stackrel{\text{def}}{=} \mathtt{s}_0{:}\mathtt{state} \to \mathtt{t} * \mathtt{s}_1{:}\mathtt{state}\{\mathtt{rel}\ \mathtt{s}_0\ \mathtt{s}_1\}$$

New: Monotonic global state in F*

- We capture monotonic state with a new **computational type**

$$\mathtt{MST}_{\mathtt{state},\mathtt{rel}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where pre and post are typed as in ST

- The **get** action is typed as in ST

$$\begin{aligned}\mathtt{get} : \mathtt{unit} \to \mathtt{MST}\ \mathtt{state}\ &(\mathtt{requires}\ (\lambda\,\_\,.\top))\\ &(\mathtt{ensures}\ (\lambda\,\mathtt{s}_0\,\mathtt{s}\,\mathtt{s}_1\,.\,\mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))\end{aligned}$$

- To ensure **monotonicity**, the **put** action gets a precondition

$$\begin{aligned}\mathtt{put} : \mathtt{s}{:}\mathtt{state} \to \mathtt{MST}\ \mathtt{unit}\ &(\mathtt{requires}\ (\lambda\,\mathtt{s}_0\,.\,\mathtt{rel}\ \mathtt{s}_0\ \mathtt{s}))\\ &(\mathtt{ensures}\ (\lambda\,\_\,\_\,\mathtt{s}_1\,.\,\mathtt{s}_1 = \mathtt{s}))\end{aligned}$$

- So intuitively, MST is an **abstract** pre-postcondition refinement of

$$\mathtt{mst}\ \mathtt{t} \stackrel{\mathsf{def}}{=} \mathtt{s}_0{:}\mathtt{state} \to \mathtt{t} * \mathtt{s}_1{:}\mathtt{state}\{\mathtt{rel}\ \mathtt{s}_0\ \mathtt{s}_1\}$$

New: Recalling a Witness

- We introduce a **logical capability** (a **modality** in ongoing work)

$$\texttt{witnessed} : (\texttt{state} \to \mathrm{Type}) \to \mathrm{Type}$$

  together with a **weakening principle** (**functoriality**)

$$\texttt{wk} : \mathrm{p,q}{:}(\texttt{state} \to \mathrm{Type}) \to \texttt{Lemma}\ (\texttt{requires}\ (\forall\, \mathrm{s}\,.\, \mathrm{p\ s} \implies \mathrm{q\ s}))$$
$$(\texttt{ensures}\ (\texttt{witnessed}\ \mathrm{p} \implies \texttt{witnessed}\ \mathrm{q}))$$

- We add a **stateful introduction rule** for witnessed

$$\texttt{witness} : \mathrm{p}{:}(\texttt{state} \to \mathrm{Type}) \to \mathrm{MST}\ \texttt{unit}\ (\texttt{requires}\ (\lambda\, \mathrm{s_0}\,.\, \mathrm{p\ s_0} \wedge \texttt{stable}\ \mathrm{p}))$$
$$(\texttt{ensures}\ (\lambda\, \mathrm{s_0}\ \_\ \mathrm{s_1}\,.\, \mathrm{s_0} = \mathrm{s_1} \wedge \texttt{witnessed}\ \mathrm{p}))$$

- We add a **stateful elimination rule** for witnessed

$$\texttt{recall} : \mathrm{p}{:}(\texttt{state} \to \mathrm{Type}) \to \mathrm{MST}\ \texttt{unit}\ (\texttt{requires}\ (\lambda\, \_\,.\, \texttt{witnessed}\ \mathrm{p}))$$
$$(\texttt{ensures}\ (\lambda\, \mathrm{s_0}\ \_\ \mathrm{s_1}\,.\, \mathrm{s_0} = \mathrm{s_1} \wedge \mathrm{p\ s_1}))$$

New: Recalling a Witness

- We introduce a **logical capability** (a **modality** in ongoing work)

$$\mathtt{witnessed} : (\mathtt{state} \rightarrow \mathtt{Type}) \rightarrow \mathtt{Type}$$

  together with a **weakening principle** (**functoriality**)

$$\begin{aligned}\mathtt{wk} : \mathrm{p,q}{:}(\mathtt{state} \rightarrow \mathtt{Type}) \rightarrow \mathtt{Lemma}\ &(\mathtt{requires}\ (\forall\, \mathtt{s}\, .\, \mathtt{p\ s} \implies \mathtt{q\ s}))\\ &(\mathtt{ensures}\ (\mathtt{witnessed}\ \mathrm{p} \implies \mathtt{witnessed}\ \mathrm{q}))\end{aligned}$$

- We add a **stateful introduction rule** for `witnessed`

$$\begin{aligned}\mathtt{witness} : \mathrm{p}{:}(\mathtt{state} \rightarrow \mathtt{Type}) \rightarrow \mathrm{MST\ unit}\ &(\mathtt{requires}\ (\lambda\, \mathtt{s}_0\, .\, \mathtt{p}\ \mathtt{s}_0 \wedge \mathtt{stable}\ \mathtt{p}))\\ &(\mathtt{ensures}\ (\lambda\, \mathtt{s}_0\ \_\ \mathtt{s}_1\, .\, \mathtt{s}_0 = \mathtt{s}_1 \wedge\\ &\qquad \mathtt{witnessed}\ \mathtt{p}))\end{aligned}$$

- We add a **stateful elimination rule** for `witnessed`

$$\begin{aligned}\mathtt{recall} : \mathrm{p}{:}(\mathtt{state} \rightarrow \mathtt{Type}) \rightarrow \mathrm{MST\ unit}\ &(\mathtt{requires}\ (\lambda\, \_\, .\, \mathtt{witnessed}\ \mathtt{p}))\\ &(\mathtt{ensures}\ (\lambda\, \mathtt{s}_0\ \_\ \mathtt{s}_1\, .\, \mathtt{s}_0 = \mathtt{s}_1 \wedge \mathtt{p}\ \mathtt{s}_1))\end{aligned}$$

New: Recalling a Witness

- We introduce a **logical capability** (a **modality** in ongoing work)

$$\texttt{witnessed} : (\texttt{state} \rightarrow \texttt{Type}) \rightarrow \texttt{Type}$$

  together with a **weakening principle** (**functoriality**)

$$\begin{aligned}\texttt{wk} : \texttt{p,q}{:}(\texttt{state} \rightarrow \texttt{Type}) \rightarrow \texttt{Lemma}\ &(\texttt{requires}\ (\forall\, \texttt{s}\,.\, \texttt{p s} \implies \texttt{q s}))\\ &(\texttt{ensures}\ (\texttt{witnessed p} \implies \texttt{witnessed q}))\end{aligned}$$

- We add a **stateful introduction rule** for `witnessed`

$$\begin{aligned}\texttt{witness} : \texttt{p}{:}(\texttt{state} \rightarrow \texttt{Type}) \rightarrow \texttt{MST unit}\ &(\texttt{requires}\ (\lambda\, \texttt{s}_0\,.\, \texttt{p s}_0 \wedge \texttt{stable p}))\\ &(\texttt{ensures}\ (\lambda\, \texttt{s}_0\ \_\ \texttt{s}_1\,.\, \texttt{s}_0 = \texttt{s}_1 \wedge \\ &\qquad \texttt{witnessed p}))\end{aligned}$$

- We add a **stateful elimination rule** for `witnessed`

$$\begin{aligned}\texttt{recall} : \texttt{p}{:}(\texttt{state} \rightarrow \texttt{Type}) \rightarrow \texttt{MST unit}\ &(\texttt{requires}\ (\lambda\, \_\,.\, \texttt{witnessed p}))\\ &(\texttt{ensures}\ (\lambda\, \texttt{s}_0\ \_\ \texttt{s}_1\,.\, \texttt{s}_0 = \texttt{s}_1 \wedge \texttt{p s}_1))\end{aligned}$$

New: Recalling a Witness

- We introduce a **logical capability** (a **modality** in ongoing work)

$$\texttt{witnessed} : (\texttt{state} \to \texttt{Type}) \to \texttt{Type}$$

  together with a **weakening principle** (**functoriality**)

$$\begin{aligned}\texttt{wk} : \texttt{p},\texttt{q}{:}(\texttt{state} \to \texttt{Type}) \to \texttt{Lemma}\ &(\texttt{requires}\ (\forall\, \texttt{s}\,.\,\texttt{p s} \implies \texttt{q s}))\\ &(\texttt{ensures}\ (\texttt{witnessed p} \implies \texttt{witnessed q}))\end{aligned}$$

- We add a **stateful introduction rule** for `witnessed`

$$\begin{aligned}\texttt{witness} : \texttt{p}{:}(\texttt{state} \to \texttt{Type}) \to \texttt{MST unit}\ &(\texttt{requires}\ (\lambda\, \texttt{s}_0\,.\,\texttt{p}\ \texttt{s}_0 \wedge \texttt{stable p}))\\ &(\texttt{ensures}\ (\lambda\, \texttt{s}_0\ \_\ \texttt{s}_1\,.\,\texttt{s}_0 = \texttt{s}_1 \wedge\\ &\qquad \texttt{witnessed p}))\end{aligned}$$

- We add a **stateful elimination rule** for `witnessed`

$$\begin{aligned}\texttt{recall} : \texttt{p}{:}(\texttt{state} \to \texttt{Type}) \to \texttt{MST unit}\ &(\texttt{requires}\ (\lambda\, \_\,.\,\texttt{witnessed p}))\\ &(\texttt{ensures}\ (\lambda\, \texttt{s}_0\ \_\ \texttt{s}_1\,.\,\texttt{s}_0 = \texttt{s}_1 \wedge \texttt{p}\ \texttt{s}_1))\end{aligned}$$

Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

The motivating example revisited

- Recall the program operating on the **set-valued state**

$$\texttt{insert v};\ \texttt{complex\_procedure}();\ \texttt{assert}\ (\mathtt{v} \in \mathtt{get}())$$

  - We pick **set inclusion** $\subseteq$ as our preorder `rel` on states
  - We **prove the assertion** by inserting a `witness` and `recall`

$$\texttt{insert v};\ \texttt{witness}\ (\lambda\,\mathtt{s}\,.\,\mathtt{v} \in \mathtt{s});\ \mathtt{c\_p}();\ \texttt{recall}\ (\lambda\,\mathtt{s}\,.\,\mathtt{v} \in \mathtt{s});\ \texttt{assert}\ (\mathtt{v} \in \mathtt{get}())$$

  - For **any other** `w`, wrapping

$$\texttt{insert w};\ [\ \ ];\ \texttt{assert}\ (\mathtt{w} \in \mathtt{get}())$$

    around the program is handled **similarly easily** by

$$\texttt{insert w};\ \texttt{witness}\ (\lambda\,\mathtt{s}\,.\,\mathtt{w} \in \mathtt{s});\ [\ \ ];\ \texttt{recall}\ (\lambda\,\mathtt{s}\,.\,\mathtt{w} \in \mathtt{s});\ \texttt{assert}\ (\mathtt{w} \in \mathtt{get}())$$

- **Monotonic counters** are analogous, by picking $\mathbb{N}$ and $\leq$, e.g.,

$$\texttt{create 0};\ \texttt{incr}();\ \texttt{witness}\ (\lambda\,\mathtt{c}\,.\,\mathtt{c} > 0);\ \mathtt{c\_p}();\ \texttt{recall}\ (\lambda\,\mathtt{c}\,.\,\mathtt{c} > 0)$$

The motivating example revisited

- Recall the program operating on the **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert } (\texttt{v} \in \texttt{get()})$$

  - We pick **set inclusion** $\subseteq$ as our preorder `rel` on states
  - We **prove the assertion** by inserting a `witness` and `recall`

$$\texttt{insert v; witness } (\lambda\, \texttt{s} \,.\, \texttt{v} \in \texttt{s})\texttt{; c\_p(); recall } (\lambda\, \texttt{s} \,.\, \texttt{v} \in \texttt{s})\texttt{; assert } (\texttt{v} \in \texttt{get()})$$

  - For **any other** `w`, wrapping

$$\texttt{insert w; [ ]; assert } (\texttt{w} \in \texttt{get()})$$

  around the program is handled **similarly easily** by

$$\texttt{insert w; witness } (\lambda\, \texttt{s} \,.\, \texttt{w} \in \texttt{s})\texttt{; [ ]; recall } (\lambda\, \texttt{s} \,.\, \texttt{w} \in \texttt{s})\texttt{; assert } (\texttt{w} \in \texttt{get()})$$

- **Monotonic counters** are analogous, by picking $\mathbb{N}$ and $\leq$, e.g.,

$$\texttt{create 0; incr(); witness } (\lambda\, \texttt{c} \,.\, \texttt{c} > 0)\texttt{; c\_p(); recall } (\lambda\, \texttt{c} \,.\, \texttt{c} > 0)$$

The motivating example revisited

- Recall the program operating on the **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert } (\mathrm{v} \in \mathrm{get}())$$

  - We pick **set inclusion** $\subseteq$ as our preorder `rel` on states
  - We **prove the assertion** by inserting a `witness` and `recall`

$$\texttt{insert v; witness } (\lambda\, \mathrm{s}\,.\, \mathrm{v} \in \mathrm{s});\ \texttt{c\_p}();\ \texttt{recall } (\lambda\, \mathrm{s}\,.\, \mathrm{v} \in \mathrm{s});\ \texttt{assert } (\mathrm{v} \in \mathrm{get}())$$

  - For **any other** `w`, wrapping

$$\texttt{insert w; [ ]; assert } (\mathrm{w} \in \mathrm{get}())$$

  around the program is handled **similarly easily** by

$$\texttt{insert w; witness } (\lambda\, \mathrm{s}\,.\, \mathrm{w} \in \mathrm{s});\ [\ ];\ \texttt{recall } (\lambda\, \mathrm{s}\,.\, \mathrm{w} \in \mathrm{s});\ \texttt{assert } (\mathrm{w} \in \mathrm{get}())$$

- **Monotonic counters** are analogous, by picking $\mathbb{N}$ and $\leq$, e.g.,

$$\texttt{create 0; incr(); witness } (\lambda\, \mathrm{c}\,.\, \mathrm{c} > 0);\ \texttt{c\_p}();\ \texttt{recall } (\lambda\, \mathrm{c}\,.\, \mathrm{c} > 0)$$

The motivating example revisited

- Recall the program operating on the **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert } (\mathtt{v} \in \mathtt{get}())$$

  - We pick **set inclusion** $\subseteq$ as our preorder `rel` on states
  - We **prove the assertion** by inserting a `witness` and `recall`

$$\texttt{insert v; witness } (\lambda\, \mathtt{s}\,.\, \mathtt{v} \in \mathtt{s});\ \texttt{c\_p}();\ \texttt{recall } (\lambda\, \mathtt{s}\,.\, \mathtt{v} \in \mathtt{s});\ \texttt{assert } (\mathtt{v} \in \mathtt{get}())$$

  - For **any other** `w`, wrapping

$$\texttt{insert w; [ ]; assert } (\mathtt{w} \in \mathtt{get}())$$

    around the program is handled **similarly easily** by

$$\texttt{insert w; witness } (\lambda\, \mathtt{s}\,.\, \mathtt{w} \in \mathtt{s});\ [\ \ ];\ \texttt{recall } (\lambda\, \mathtt{s}\,.\, \mathtt{w} \in \mathtt{s});\ \texttt{assert } (\mathtt{w} \in \mathtt{get}())$$

- **Monotonic counters** are analogous, by picking $\mathbb{N}$ and $\leq$, e.g.,

$$\texttt{create 0; incr(); witness } (\lambda\, \mathtt{c}\,.\, \mathtt{c} > 0);\ \texttt{c\_p}();\ \texttt{recall } (\lambda\, \mathtt{c}\,.\, \mathtt{c} > 0)$$

The motivating example revisited

- Recall the program operating on the **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert } (\texttt{v} \in \texttt{get}())$$

  - We pick **set inclusion** $\subseteq$ as our preorder `rel` on states
  - We **prove the assertion** by inserting a `witness` and `recall`

$$\texttt{insert v; witness } (\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s})\texttt{; c\_p(); recall } (\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s})\texttt{; assert } (\texttt{v} \in \texttt{get}())$$

  - For **any other** `w`, wrapping

$$\texttt{insert w; [ ]; assert } (\texttt{w} \in \texttt{get}())$$

    around the program is handled **similarly easily** by

$$\texttt{insert w; witness } (\lambda\,\texttt{s}\,.\,\texttt{w} \in \texttt{s})\texttt{; [ ]; recall } (\lambda\,\texttt{s}\,.\,\texttt{w} \in \texttt{s})\texttt{; assert } (\texttt{w} \in \texttt{get}())$$

- **Monotonic counters** are analogous, by picking $\mathbb{N}$ and $\leq$, e.g.,

$$\texttt{create 0; incr(); witness } (\lambda\,\texttt{c}\,.\,\texttt{c} > 0)\texttt{; c\_p(); recall } (\lambda\,\texttt{c}\,.\,\texttt{c} > 0)$$

ML-style typed references (local state)

- First, we define a type of **heaps** as a finite map

  $\mathtt{type\ heap} =$

  $\quad \mid \mathtt{H} : \mathtt{h}{:}(\mathbb{N} \to \mathtt{cell}) \to \mathtt{ctr}{:}\mathbb{N}\{\forall\,\mathtt{n}\,.\,\mathtt{ctr} \leq \mathtt{n} \implies \mathtt{h\ n} = \mathtt{Unused}\} \to \mathtt{heap}$

  where

  $\mathtt{type\ cell} =$

  $\quad \mid \mathtt{Unused} : \mathtt{cell}$

  $\quad \mid \mathtt{Used} : \mathtt{a}{:}\mathrm{Type} \to \mathtt{v}{:}\mathtt{a} \to \mathtt{cell}$

- Next, we define a **preorder** on heaps (**heap inclusion**)

  $\mathtt{let\ heap\_inclusion\ (H\ h_0\ \_)\ (H\ h_1\ \_)} = \forall\,\mathtt{id}\,.\,\mathtt{match\ h_0\ id\,,h_1\ id\ with}$

  $\quad \mid \mathtt{Used\ a\ \_\,,Used\ b\ \_} \ \to\ \mathtt{a} = \mathtt{b}$

  $\quad \mid \mathtt{Unused\,,Used\ \_\ \_} \ \to\ \top$

  $\quad \mid \mathtt{Unused\,,Unused} \ \to\ \top$

  $\quad \mid \mathtt{Used\ \_\ \_\,,Unused} \ \to\ \bot$

ML-style typed references (local state)

- First, we define a type of **heaps** as a finite map

$$
\begin{array}{l}
\texttt{type heap} = \\
\quad \mid \texttt{H} : \texttt{h}{:}(\mathbb{N} \to \texttt{cell}) \to \texttt{ctr}{:}\mathbb{N}\{\forall\, \texttt{n} \,.\, \texttt{ctr} \le \texttt{n} \implies \texttt{h n} = \texttt{Unused}\} \to \texttt{heap}
\end{array}
$$

  where

$$
\begin{array}{l}
\texttt{type cell} = \\
\quad \mid \texttt{Unused} : \texttt{cell} \\
\quad \mid \texttt{Used} : \texttt{a}{:}\texttt{Type} \to \texttt{v}{:}\texttt{a} \to \texttt{cell}
\end{array}
$$

- Next, we define a **preorder** on heaps (**heap inclusion**)

$$
\begin{array}{l}
\texttt{let heap\_inclusion (H } h_0\ \_\texttt{) (H } h_1\ \_\texttt{)} = \forall\, \texttt{id} \,.\, \texttt{match } h_0 \texttt{ id}, h_1 \texttt{ id with} \\
\quad \mid \texttt{Used a \_, Used b \_} \ \to\ \texttt{a} = \texttt{b} \\
\quad \mid \texttt{Unused, Used \_ \_} \ \to\ \top \\
\quad \mid \texttt{Unused, Unused} \ \to\ \top \\
\quad \mid \texttt{Used \_ \_, Unused} \ \to\ \bot
\end{array}
$$

ML-style typed references (local state)

- First, we define a type of **heaps** as a finite map

  $\texttt{type heap} =$

  $\quad | \; \texttt{H} : \texttt{h}{:}(\mathbb{N} \to \texttt{cell}) \to \texttt{ctr}{:}\mathbb{N}\{\forall\, \texttt{n}\,.\, \texttt{ctr} \leq \texttt{n} \implies \texttt{h n} = \texttt{Unused}\} \to \texttt{heap}$

  where

  $\texttt{type cell} =$

  $\quad | \; \texttt{Unused} : \texttt{cell}$

  $\quad | \; \texttt{Used} : \texttt{a}{:}\mathrm{Type} \to \texttt{v}{:}\texttt{a} \to \texttt{cell}$

- Next, we define a **preorder** on heaps (**heap inclusion**)

  $\texttt{let heap\_inclusion} \; (\texttt{H}\; \texttt{h}_0 \; \_) \; (\texttt{H}\; \texttt{h}_1 \; \_) = \forall\, \texttt{id}\,.\, \texttt{match}\; \texttt{h}_0 \; \texttt{id}\,, \texttt{h}_1 \; \texttt{id} \; \texttt{with}$

  $\quad | \; \texttt{Used a} \; \_ , \texttt{Used b} \; \_ \;\to\; \texttt{a} = \texttt{b}$

  $\quad | \; \texttt{Unused}, \texttt{Used} \; \_ \; \_ \;\to\; \top$

  $\quad | \; \texttt{Unused}, \texttt{Unused} \;\to\; \top$

  $\quad | \; \texttt{Used} \; \_ \; \_ , \texttt{Unused} \;\to\; \bot$

ML-style typed references (local state)

- As a result, we can define new **local state effect**

$$\texttt{MLST t pre post} \stackrel{\text{def}}{=} \texttt{MST}_{\texttt{heap,heap\_inclusion}} \texttt{ t pre post}$$

- Next, we define the type of **references** using monotonicity

```
abstract type ref a = id:ℕ{witnessed (λ h . contains h id a)}
```

where

```
let contains (H h _) id a =
  match h id with
    | Used b _ → a = b
    | Unused → ⊥
```

- Important: `contains` is **stable** wrt. `heap_inclusion`

ML-style typed references (local state)

- As a result, we can define new **local state effect**

$$\texttt{MLST}\ \texttt{t pre post} \stackrel{\text{def}}{=} \texttt{MST}_{\texttt{heap},\texttt{heap\_inclusion}}\ \texttt{t pre post}$$

- Next, we define the type of **references** using monotonicity

$$\texttt{abstract type ref a} = \texttt{id}{:}\mathbb{N}\{\texttt{witnessed}\ (\lambda\,\texttt{h}\,.\,\texttt{contains h id a})\}$$

where

```
let contains (H h _) id a =
  match h id with
    | Used b _ → a = b
    | Unused → ⊥
```

- Important: `contains` is **stable** wrt. `heap_inclusion`

ML-style typed references (local state)

- As a result, we can define new **local state effect**

$$\mathtt{MLST}\ \mathtt{t}\ \mathtt{pre}\ \mathtt{post} \stackrel{\text{def}}{=} \mathtt{MST}_{\mathtt{heap},\mathtt{heap\_inclusion}}\ \mathtt{t}\ \mathtt{pre}\ \mathtt{post}$$

- Next, we define the type of **references** using monotonicity

  $\mathtt{abstract\ type\ ref\ a} = \mathtt{id}{:}\mathbb{N}\{\mathtt{witnessed}\ (\lambda\,\mathtt{h}\,.\,\mathtt{contains\ h\ id\ a})\}$

  where

```
let contains (H h _) id a =
  match h id with
    | Used b _  →  a = b
    | Unused  →  ⊥
```

- Important: contains is **stable** wrt. heap_inclusion

ML-style typed references (local state)

- As a result, we can define new **local state effect**

$$\mathtt{MLST}\ \mathtt{t}\ \mathtt{pre}\ \mathtt{post} \stackrel{\text{def}}{=} \mathtt{MST}_{\mathtt{heap},\mathtt{heap\_inclusion}}\ \mathtt{t}\ \mathtt{pre}\ \mathtt{post}$$

- Next, we define the type of **references** using monotonicity

$$\mathtt{abstract\ type\ ref\ a} = \mathtt{id}{:}\mathbb{N}\{\mathtt{witnessed}\ (\lambda\,\mathtt{h}\,.\,\mathtt{contains\ h\ id\ a})\}$$

  where

```
let contains (H h _) id a =
  match h id with
    | Used b _ → a = b
    | Unused → ⊥
```

- Important: `contains` is **stable** wrt. `heap_inclusion`

ML-style typed references (local state)

- Finally, we define `MLST`'s **actions** using `MST`'s actions
  - `let alloc (a:Type) (v:a) : MLST (ref a) ... = ...`
    - **get** the current heap
    - **create** a fresh ref., and **add** it to the heap
    - **put** the updated heap back
    - **witness** that the created ref. is in the heap
  - `let read (r:ref a) : MLST t ... = ...`
    - **recall** that the given ref. is in the heap
    - **get** the current heap
    - **select** the given reference from the heap
  - `let write (r:ref a) (v:a) : MLST unit ... = ...`
    - **recall** that the given ref. is in the heap
    - **get** the current heap
    - **update** the heap with the given value at the given ref.
    - **put** the updated heap back

ML-style typed references (local state)

- Finally, we define `MLST`'s **actions** using `MST`'s actions
  - `let alloc (a:Type) (v:a) : MLST (ref a) ... = ...`
    - **get** the current heap
    - **create** a fresh ref., and **add** it to the heap
    - **put** the updated heap back
    - **witness** that the created ref. is in the heap
  - `let read (r:ref a) : MLST t ... = ...`
    - **recall** that the given ref. is in the heap
    - **get** the current heap
    - **select** the given reference from the heap
  - `let write (r:ref a) (v:a) : MLST unit ... = ...`
    - **recall** that the given ref. is in the heap
    - **get** the current heap
    - **update** the heap with the given value at the given ref.
    - **put** the updated heap back

Adding untyped and monotonic references

- **Untyped references** (uref) with strong updates
  - Used heap cells are extended with **tags**

    where

    $$| \ \texttt{Used} : \texttt{a:Type} \to \texttt{v:a} \to \texttt{t:tag} \to \texttt{cell}$$

    $$\texttt{type tag} = \texttt{Typed} : \texttt{tag} \mid \texttt{Untyped} : \texttt{tag}$$

  - urefs can be extended to also support **deallocation**

- **Monotonic references** (mref a rel)
  - Used heap cells are extended with **typed tags**

    where

    $$| \ \texttt{Used} : \texttt{a:Type} \to \texttt{v:a} \to \texttt{t:tag a} \to \texttt{cell}$$

    $$\texttt{type tag a} = \texttt{Typed} : \texttt{rel:preorder a} \to \texttt{tag a} \mid \texttt{Untyped} : \texttt{tag a}$$

  - mrefs provide **more flexibility** with ref.-wise monotonicity

Adding untyped and monotonic references

- **Untyped references** (`uref`) with strong updates
  - `Used heap` cells are extended with **tags**

    $$\mid \ \texttt{Used} : \texttt{a:Type} \rightarrow \texttt{v:a} \rightarrow \texttt{t:tag} \rightarrow \texttt{cell}$$

    where

    $$\texttt{type tag} \ = \ \texttt{Typed} : \texttt{tag} \mid \texttt{Untyped} : \texttt{tag}$$

  - `urefs` can be extended to also support **deallocation**

- **Monotonic references** (`mref a rel`)
  - `Used heap` cells are extended with **typed tags**

    $$\mid \ \texttt{Used} : \texttt{a:Type} \rightarrow \texttt{v:a} \rightarrow \texttt{t:tag a} \rightarrow \texttt{cell}$$

    where

    $$\texttt{type tag a} \ = \ \texttt{Typed} : \texttt{rel:preorder a} \rightarrow \texttt{tag a} \mid \texttt{Untyped} : \texttt{tag a}$$

  - `mrefs` provide **more flexibility** with ref.-wise monotonicity

Adding untyped and monotonic references

- **Untyped references** (`uref`) with strong updates
  - Used heap cells are extended with **tags**

    $$\mid \ \mathtt{Used} : \mathrm{a{:}Type} \to \mathtt{v{:}a} \to \mathtt{t{:}tag} \to \mathtt{cell}$$

    where

    $$\mathtt{type\ tag} \ = \ \mathtt{Typed} : \mathtt{tag} \ \mid \ \mathtt{Untyped} : \mathtt{tag}$$

  - `urefs` can be extended to also support **deallocation**

- **Monotonic references** (`mref a rel`)
  - Used heap cells are extended with **typed tags**

    $$\mid \ \mathtt{Used} : \mathrm{a{:}Type} \to \mathtt{v{:}a} \to \mathtt{t{:}tag\ a} \to \mathtt{cell}$$

    where

    $$\mathtt{type\ tag\ a} \ = \ \mathtt{Typed} : \mathtt{rel{:}preorder\ a} \to \mathtt{tag\ a} \ \mid \ \mathtt{Untyped} : \mathtt{tag\ a}$$

  - `mrefs` provide **more flexibility** with ref.-wise monotonicity

Conclusion

- Monotonicity
  - can be distilled into a **simple** and **general** framework
  - is **useful** for **programming** (refs.) and **verification** (Prj. Everest)

- See the paper for
  - further **examples** and **case studies**
  - **meta-theory** and **correctness results** for `MST`
    - based on an instrumented operational semantics

$$(\mathtt{witness}\ x.\varphi\ ,\ s\ ,\ W) \rightsquigarrow (\mathtt{return}\ ()\ ,\ s\ ,\ W \cup \{x.\varphi\})$$

    - and cut elimination for the `witnessed`-logic
  - first steps towards **monadic reification** for `MST`
    - useful for extrinsic reasoning, e.g., for relational properties
    - but have to be careful when breaking abstraction

Conclusion

- Monotonicity
  - can be distilled into a **simple** and **general** framework
  - is **useful** for **programming** (refs.) and **verification** (Prj. Everest)

- See the paper for
  - further **examples** and **case studies**
  - **meta-theory** and **correctness results** for `MST`
    - based on an instrumented operational semantics

$$(\mathtt{witness}\ x.\varphi\,,\ s\,,\ W)\ \rightsquigarrow\ (\mathtt{return}\ ()\,,\ s\,,\ W \cup \{x.\varphi\})$$

    - and cut elimination for the `witnessed`-logic
  - first steps towards **monadic reification** for `MST`
    - useful for extrinsic reasoning, e.g., for relational properties
    - but have to be careful when breaking abstraction

Thank you!

Interested in doing an F* internship?

Get in touch with the F* team!

`www.fstar-lang.org`