Runners in action

Danel Ahman

(joint work with Andrej Bauer)

University of Ljubljana, Slovenia

07.11.2019

Today's plan

- **Computational effects** and **external resources** in PL
- **Issues with standard approaches** to **external resources**
- **Runners** – a natural model for **top-level runtime**
- **T-runners** – for also modelling **non-top-level runtimes**
- Turn **T**-runners into a **useful programming construct**
- Demonstrate the use of runners through **programming examples**

Computational effects

and

external resources

Computational effects in PL

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- Using **monads** (as in Haskell)

```
type St a = String → (a,String)
instance St Monad where
  ...

f :: St a → St (a,a)
f c = c >>= (\ x → c >>= (\ y → return (x,y)))
```

- Using **alg. effects** and **handlers** (as in Eff, Frank, Koka)

```
effect Get : unit → int
effect Put : int → unit

let g (c:unit → a!{Get,Put}) : int → a * int ! {} =
  with st_handler handle (perform (Put 42); c ())
```

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```

- Good for **simulating comp. effects** in a pure language!

  But what about effects that need access to the **external world**?

External resources in PL

External resources in PL

- Declare a **signature of monads** or **algebraic effects**, e.g.,

```
(* System.IO *)
type IO a
openFile :: FilePath → IOMode → IO Handle
```

```
(* pervasives.eff *)
effect RandomInt : int → int
effect RandomFloat : float → float
```

- And then **treat them specially** in the compiler, e.g., in Eff

```
(* eff/src/backends/runtime/eval.ml *)
let rec top_handle op =
  match op with
  | Value v → v
  | Call (RandomInt, v, k) →
      top_handle (k (Const.of_integer (Random.int (Value.to_int v))))
  | ...
```

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```

but there are **some issues** with that approach . . .

First issue

- Difficult to cover all possible use cases
  - **external resources hard-coded** into the top-level runtime
  - **non-trivial to change** what's available and how it's implemented

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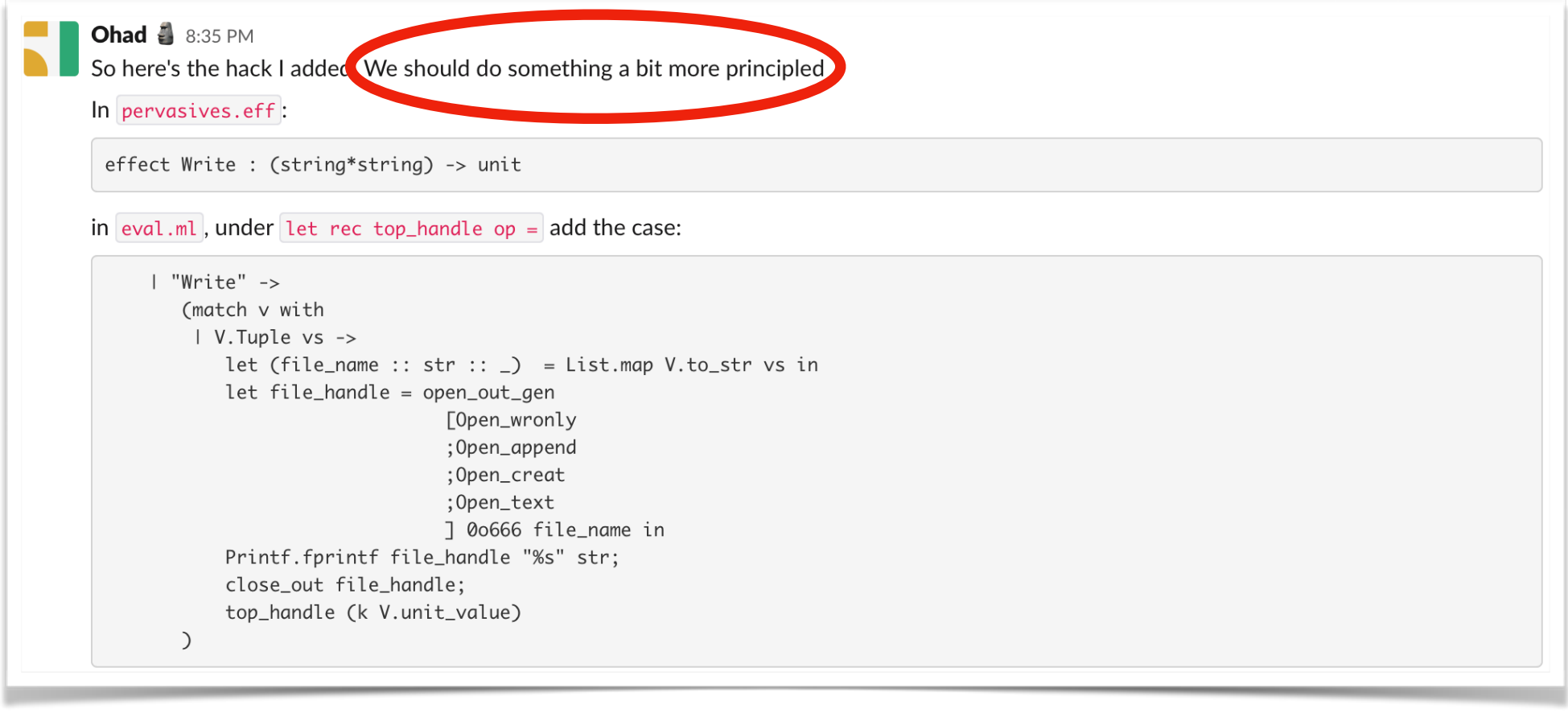

Ohad 8:35 PM

So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
    | "Write" ->
       (match v with
        | V.Tuple vs ->
           let (file_name :: str :: _)  = List.map V.to_str vs in
           let file_handle = open_out_gen
                             [Open_wronly
                             ;Open_append
                             ;Open_creat
                             ;Open_text
                             ] 0o666 file_name in
           Printf.fprintf file_handle "%s" str;
           close_out file_handle;
           top_handle (k V.unit_value)
       )
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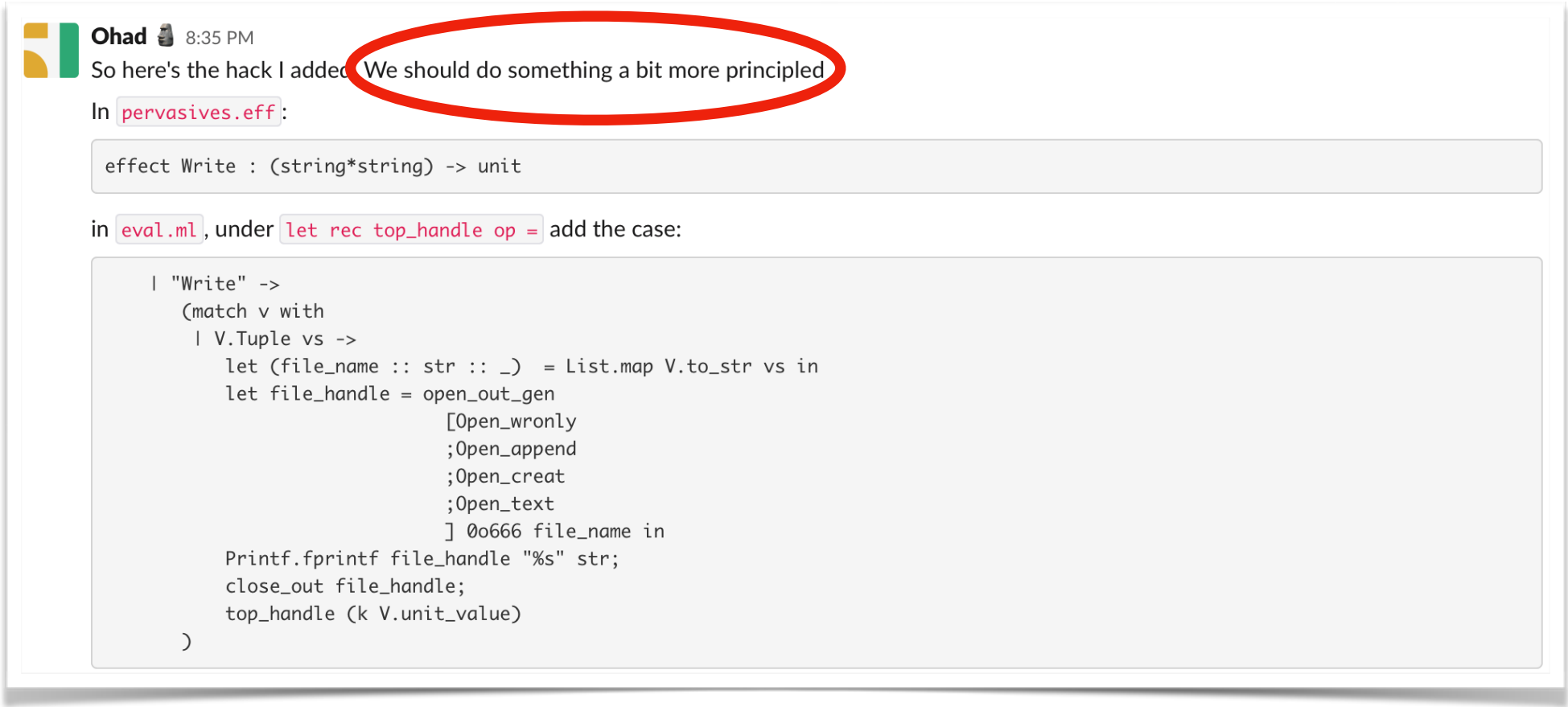

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```

This talk — a principled modular (co)algebraic approach!

Second issue

- **Lack of linearity** for external resources

```
let f (s:string) =
  let fh = fopen "foo.txt" in
  fwrite (fh,s^s);
  fclose fh;
  return fh

let g s =
  let fh = f s in fread fh                    (* fh not open any more ! *)
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```

- We shall address these kinds of issues **indirectly (!)**:
  - by **not** introducing a linear typing discipline
  - instead we make it convenient to **hide external resources**

    (addressing stronger typing disciplines in the future)

Third issue

- **Excessive generality** of effect handlers

```
let f (s:string) =
  let fh = fopen "foo.txt" in
  fwrite (fh,s^s);
  fclose fh

let h = handler { fwrite (fh,s) k → return () }
```

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  let fh = fopen "foo.txt" in
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let h = handler { fwrite (fh,s) k → return () }
```

- But misuse of external resources can also be **purely accidental**

```
let g (s1 s2:string) =
  let fh = fopen "foo.txt" in
  let b = choose () in
  if b then (fwrite (fh,s1^s2)) else (fwrite (fh,s2^s1));
  fclose fh

let nd_handler =
  handler { choose () k → return (k true ++ k false) }
```

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let f (s:string) =
  let fh = fopen "foo.txt" in
  fwrite (fh,s^s);
  fclose fh

let h = handler { fwrite (fh,s) k → return () }
```

- We shall address these kinds of issues **directly (!!)**,
  - by proposing a **restricted form of handlers** for resources
  - that support **controlled initialisation** and **finalisation**,
  - (and limit how general handlers can be used)

Runners

A natural model of top-level runtime

A natural model of top-level runtime

- Given a **signature**[1] Σ of operation symbols ($A_{\mathsf{op}}, B_{\mathsf{op}}$ are sets)

$$\mathsf{op} : A_{\mathsf{op}} \rightsquigarrow B_{\mathsf{op}}$$

a **runner**[2] $\mathcal{R}$ for Σ is given by a carrier $|\mathcal{R}|$ and co-operations

$$\left(\overline{\mathsf{op}}_{\mathcal{R}} : A_{\mathsf{op}} \times |\mathcal{R}| \longrightarrow B_{\mathsf{op}} \times |\mathcal{R}|\right)_{\mathsf{op} \in \Sigma}$$

where we think of $|\mathcal{R}|$ as a set of **runtime configurations**

[1] We consider runners for signatures, but the work generalises to alg. theories.

[2] In the literature also known as **comodels** for Σ (or for an algebraic theory).

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- For example, a natural **runner $\mathcal{R}$ for S-valued state** signature

$$\left\{ \quad \mathsf{get} : \mathbb{1} \rightsquigarrow S \quad , \quad \mathsf{set} : S \rightsquigarrow \mathbb{1} \quad \right\}$$

  is given by

$$|\mathcal{R}| \stackrel{\text{def}}{=} S \qquad \overline{\mathsf{get}}_{\mathcal{R}}\,(\star, s) \stackrel{\text{def}}{=} (s, s) \qquad \overline{\mathsf{set}}_{\mathcal{R}}\,(s', s) \stackrel{\text{def}}{=} (\star, s')$$

[1] We consider runners for signatures, but the work generalises to alg. theories.

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A natural model of top-level runtime ctd.

- Runners/comodels have been used for
  - **operational semantics** using tensors of models and comodels [Plotkin and Power '08]
  - **top-level implementation of algebraic effects** in Eff [Bauer and Pretnar '15]

    and
  - **stateful running** of algebraic effects [Uustalu '15]
  - **linear-use state-passing translation** [Møgelberg and Staton '11, '14]

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- The latter explicitly rely on one-to-one correspondence between
  - **runners** $\mathcal{R}$
  - **monad morphisms**[3] $\mathsf{r} : \mathbf{Free}_\Sigma(-) \longrightarrow \mathbf{St}_{|\mathcal{R}|}$

[3] $\mathbf{Free}_\Sigma(X)$ is the free monad ind. defined with leaves $\mathrm{val}\, x$ and nodes $\mathrm{op}(a, \kappa)$.

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- So, runners $\mathcal{R}$ are a natural model of **top-level runtime**

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- So, runners $\mathcal{R}$ are a natural model of **top-level runtime**
- But what if this runtime is not ****the**** runtime?
  - hardware vs OSs
  - OSs vs VMs
  - VMs vs sandboxes

  but also
  - browsers vs web pages
  - ...

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- But what if this runtime is not ****the**** runtime?
  - hardware vs OSs
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  - ...
- Unfortunately, runners, as defined above, are **not readily able to**
  - use **external resources**
  - **signal failure** caused by unavoidable circumstances
- But is there a **useful generalisation** that would achieve this?

Effectful runners for modular top-levels

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- Møgelberg and Staton usefully observed that a **runner** $\mathcal{R}$ is equivalently simply a family of **generic effects** for $\mathbf{St}_{|\mathcal{R}|}$, i.e.,

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- The one-to-one correspondence with **monad morphisms**

$$\mathsf{r} : \mathbf{Free}_{\Sigma}(-) \longrightarrow \mathbf{T}$$

simply amounts to the **universal property of free models**, i.e.,

$$\mathsf{r}_X\,(\mathsf{val}\,x) = \eta_X\,x \qquad \mathsf{r}_X\,(\mathsf{op}(a,\kappa)) = \underbrace{(\mathsf{r}_X \circ \kappa)^{\dagger}(\overline{\mathsf{op}}_{\mathcal{R}}\; a)}_{\mathsf{op}_{\mathcal{M}}(a,\mathsf{r}_X\circ\kappa)}$$

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- Observe that κ appears in a **tail call position** on the right!

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- What would be a **useful class of monads** **T** to use?

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- We want a runner to be a bit like a **kernel of an OS**, i.e., to
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  - **(i)** $\mathrm{getenv} : \mathbb{1} \rightsquigarrow C \quad \& \quad \mathrm{setenv} : C \rightsquigarrow \mathbb{1}$
  - **(ii)** $\mathrm{op} : A_{\mathrm{op}} \rightsquigarrow B_{\mathrm{op}}$ $\quad (\mathrm{op} \in \Sigma', \text{ for some external } \Sigma')$
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  s.t., **(i)** satisfy state equations; and **(i)** commute with **(ii)** and **(iii)**
- The **induced monad** is then isomorphic to

$$\mathbf{T}\,X \quad \stackrel{\text{def}}{=} \quad C \Rightarrow \mathbf{Free}_{\Sigma'}\big(X \times C + S\big)$$

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- The corresponding **T-runners** $\mathcal{R}$ for Σ are then of the form

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- With this, our **T-runners** $\mathcal{R}$ for Σ are (with "primitive" excs.)

$$\Big(\overline{\mathsf{op}}_{\mathcal{R}} : A_{\mathsf{op}} \longrightarrow \mathbf{K}_{\Sigma',E_{\mathsf{op}},S,C}\, B_{\mathsf{op}}\Big)_{\mathsf{op}\in\Sigma}$$

  where we call $\mathbf{K}_{\Sigma,E,S,C}$ a **kernel monad** (the sum of $\mathbf{T}$ and excs.)

$$\mathbf{K}_{\Sigma',E_{\mathsf{op}},S,C}\, B_{\mathsf{op}} \quad \stackrel{\text{def}}{=} \quad C \Rightarrow \mathbf{Free}_{\Sigma'}\big((B_{\mathsf{op}} + E_{\mathsf{op}}) \times C + S\big)$$

T-runners as a programming construct

(towards a core calculus for runners)

T-runners as a programming construct

- First, we include **T-runners** for Σ

$$\left(\overline{\mathsf{op}}_{\mathcal{R}} : A_{\mathsf{op}} \longrightarrow \mathbf{K}_{\Sigma', E_{\mathsf{op}}, S, C}\, B_{\mathsf{op}}\right)_{\mathsf{op} \in \Sigma}$$

  in our language **as values**, and **co-ops. as kernel code**, i.e.,

$$\textbf{let}\ \mathsf{R} = \{\ \mathsf{op}_1\ \mathsf{x}_1 \to \mathsf{K}_1\ \ ,\ \ \ldots\ \ ,\ \ \mathsf{op}_n\ \mathsf{x}_n \to \mathsf{K}_n\ \ \}_C$$

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```
let R = { op1 x1 → K1 , ... , opn xn → Kn }C
```

- For instance, we can provide **write-only file access** as

```
let RFH = {
  write s → if (length s > maxSize)
            then (raise WriteSizeExceeded)
            else  (let fh = getenv () in
                     if (isValid fh) then (fwrite (fh,s)) else (kill IOError))
}FileHandle
```

where

$$\Sigma \stackrel{\text{def}}{=} \{\ \mathsf{write} : \mathsf{String} \rightsquigarrow 1\ !\ E \cup \{\mathsf{WriteSizeExceeded}\}\ \}$$

$$\left(\mathsf{fwrite} : \mathsf{FileHandle} \times \mathsf{String} \rightsquigarrow 1\ !\ E\right) \in \Sigma' \qquad S = \{\ \mathsf{IOError}\ \}$$

Controlled initialisation and finalisation

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- Recall that the components r_X of the monad morphism

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  induced by a $\mathbf{T}$-runner $\mathcal{R}$ are all **tail-recursive**

Controlled initialisation and finalisation

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  induced by a $\mathbf{T}$-runner $\mathcal{R}$ are all **tail-recursive**

- We make use of it to **run user code**:

```
using R @ M_init
run M
finally {return x @ c → M_ret , ... raise e @ c → M_e ... , ... kill s → M_s ...}
```

  where (**user monads**)

  - Ms are **user code**, modelled using $\mathbf{U}_{\Sigma,E}\, X \stackrel{\text{def}}{=} \mathbf{Free}_\Sigma(X + E)$

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  - M is the user code being **run using the runner** R
  - M_{ret}, M_e, M_s **finalise** for return values, exceptions, and signals

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  induced by a $\mathbf{T}$-runner $\mathcal{R}$ are all **tail-recursive**

- We make use of it to **run user code**:

  using R **@** $\mathsf{M}_{\mathsf{init}}$
  run M
  finally {**return** x **@** c $\to$ $\mathsf{M}_{\mathsf{ret}}$, ... **raise** e **@** c $\to$ M_{e} ... , ... **kill** s $\to$ M_{s} ...}

  where

  (**user monads**)

  - Ms are **user code**, modelled using $\mathbf{U}_{\Sigma,E}\, X \stackrel{\text{def}}{=} \mathbf{Free}_\Sigma(X + E)$
  - $\mathsf{M}_{\mathsf{init}}$ produces the **initial kernel state**
  - M is the user code being **run using the runner** R
  - $\mathsf{M}_{\mathsf{ret}}$, M_{e}, M_{s} **finalise** for return values, exceptions, and signals
  - $\mathsf{M}_{\mathsf{ret}}$ and M_{e} **depend on the final state** c, but M_{s} **does not**

Controlled initialisation and finalisation ctd.

- For instance, we can define a PYTHON-esque **with construct**

```
with fileName do M
=
using R_FH @ (fopen fileName)
run M
finally {
  return x @ fh → fclose fh; return x ,
  raise WriteSizeExceeded @ fh → fclose fh; return () ,
  raise e @ fh → fclose fh; raise e , (* other exceptions in E are re-raised *)
  kill IOError → ... }
```

Controlled initialisation and finalisation ctd.

- For instance, we can define a PYTHON-esque **with construct**

```
with fileName do M
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using R_FH @ (fopen fileName)
run M
finally {
  return x @ fh → fclose fh; return x ,
  raise WriteSizeExceeded @ fh → fclose fh; return () ,
  raise e @ fh → fclose fh; raise e , (* other exceptions in E are re-raised *)
  kill IOError → ... }
```

  - the **file handle is hidden** from `M`
  - `M` **can only call** write : String $\rightsquigarrow$ 1 ! $E \cup \{\text{WriteSizeExceeded}\}$ but **not** (the external operations) `fopen` , `fclose` , and `fwrite`
  - `fopen` and `fclose` are **limited to initialisation-finalisation**
  - `M` can itself also catch WriteSizeExceeded to **re-try writing**

A core calculus for programming with runners

Core calculus (types and judgements)

Core calculus (types and judgements)

- **Ground types** (for types of operations and kernel state)

$$A, B, C \quad ::= \quad \mathsf{B} \mid 1 \mid 0 \mid A \times B \mid A + B$$

- **Types**

$$\begin{aligned} X, Y \quad ::= \quad & \mathsf{B} \mid 1 \mid 0 \mid X \times Y \mid X + Y \\ \mid \quad & X \to Y \,!\, (\Sigma, E) \\ \mid \quad & X \to Y \lightning (\Sigma, E, S, C) \\ \mid \quad & \Sigma \Rightarrow (\Sigma', S, C) \end{aligned}$$

- **Values**

$$\Gamma \vdash V : X \qquad \Gamma \vdash V \equiv W : X$$

- **User computations**

$$\Gamma \vdash M : X \,!\, (\Sigma, E) \qquad \Gamma \vdash M \equiv N : X \,!\, (\Sigma, E)$$

- **Kernel computations**

$$\Gamma \vdash K : X \lightning (\Sigma, E, S, C) \qquad \Gamma \vdash K \equiv L : X \lightning (\Sigma, E, S, C)$$

Core calculus (user computations)

$$
\begin{array}{lll}
M, N & ::= & \mathsf{return}\ V & \text{value} \\
& \mid & \mathsf{try}\ M\ \mathsf{with}\ \{\mathsf{return}\ x \mapsto N, (\mathsf{raise}\ e \mapsto N_e)_{e \in E}\} & \text{exception handler} \\
& \mid & V\ W & \text{application} \\
& \mid & \mathsf{match}\ V\ \mathsf{with}\ \{\langle x, y\rangle \mapsto M\} & \text{product elimination} \\
& \mid & \mathsf{match}\ V\ \mathsf{with}\ \{\}_X & \text{empty elimination} \\
& \mid & \mathsf{match}\ V\ \mathsf{with}\ \{\mathsf{inl}\ x \mapsto M, \mathsf{inr}\ y \mapsto N\} & \text{sum elimination} \\
& \mid & \mathsf{op}_X(V, (x\,.\,M), (N_e)_{e \in E_{\mathsf{op}}}) & \text{operation call} \\
& \mid & \mathsf{raise}_X\ e & \text{raise exception} \\
& \mid & \mathsf{using}\ V\ @\ W\ \mathsf{run}\ M\ \mathsf{finally}\ \{ & \text{run} \\
& & \quad \mathsf{return}\ x\ @\ c \mapsto N, & \\
& & \quad (\mathsf{raise}\ e\ @\ c \mapsto N_e)_{e \in E}, & \\
& & \quad (\mathsf{kill}\ s \mapsto N_s)_{s \in S}\} & \\
& \mid & \mathsf{kernel}\ K\ @\ V\ \mathsf{finally}\ \{ & \text{switch to kernel mode} \\
& & \quad \mathsf{return}\ x\ @\ c \mapsto N, & \\
& & \quad (\mathsf{raise}\ e\ @\ c \mapsto N_e)_{e \in E}, & \\
& & \quad (\mathsf{kill}\ s \mapsto N_s)_{s \in S}\} &
\end{array}
$$

Core calculus (kernel computations)

K, L	$::= \mathsf{return}_C\, V$	value
	$\mid \mathsf{try}\ K\ \mathsf{with}\ \{\mathsf{return}\ x \mapsto L, (\mathsf{raise}\ e \mapsto L_e)_{e \in E}\}$	exception handler
	$\mid V\, W$	application
	$\mid \mathsf{match}\ V\ \mathsf{with}\ \{\langle x, y\rangle \mapsto K\}$	product elimination
	$\mid \mathsf{match}\ V\ \mathsf{with}\ \{\}_{X@C}$	empty elimination
	$\mid \mathsf{match}\ V\ \mathsf{with}\ \{\mathsf{inl}\ x \mapsto K, \mathsf{inr}\ y \mapsto L\}$	sum elimination
	$\mid \mathsf{op}_{X@C}(V, (x\,.\,K), (L_e)_{e \in E_{\mathsf{op}}})$	operation call
	$\mid \mathsf{raise}_{X@C}\, e$	raise exception
	$\mid \mathsf{kill}_{X@C}\, s$	send signal
	$\mid \mathsf{getenv}_C(c\,.\,K)$	get state
	$\mid \mathsf{setenv}(V, K)$	set state
	$\mid \mathsf{user}\ M\ \mathsf{with}\ \{\mathsf{return}\ x \mapsto K, (\mathsf{raise}\ e \mapsto L_e)_{e \in E}\}$	switch to user mode

Core calculus (type system)

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- For example, the **typing rule for runners** is

$$\frac{\Sigma = \{\, \mathsf{op}_1 \,, \ldots \,, \mathsf{op}_n \,\} \qquad \big(\; \Gamma, x_i : A_{\mathsf{op}_i} \vdash K_i : B_{\mathsf{op}_i} \not\downarrow (\Sigma', E_{\mathsf{op}_i}, S, C) \;\big)_{1 \leqslant i \leqslant n}}{\Gamma \vdash \{\, \mathsf{op}_1\, x_1 \mapsto K_1 \,, \ldots \,, \mathsf{op}_n\, x_n \mapsto K_n \,\}_C : \Sigma \Rightarrow (\Sigma', S, C)}$$

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- and the **typing rule for running user comps.** is

$$\frac{\begin{array}{c}\Gamma \vdash V : \Sigma \Rightarrow (\Sigma', S, C) \qquad \Gamma \vdash W : C \qquad \Gamma \vdash M : X\,!\,(\Sigma, E) \\ \Gamma, x : X, c : C \vdash N_{ret} : Y\,!\,(\Sigma', E') \qquad \big(\, \Gamma, c : C \vdash N_e : Y\,!\,(\Sigma', E') \,\big)_{e \in E} \\ \big(\, \Gamma \vdash N_s : Y\,!\,(\Sigma', E') \,\big)_{s \in S}\end{array}}{\begin{array}{l}\Gamma \vdash \textbf{using}\ V\ @\ W\ \textbf{run}\ M\ \textbf{finally}\ \{\ \textbf{return}\ x\ @\ c \mapsto N_{ret}\ , \\ \qquad\qquad \big(\textbf{raise}\ e\ @\ c \mapsto N_e\big)_{e \in E}\ , \\ \qquad\qquad \big(\textbf{kill}\ s \mapsto N_s\big)_{s \in S}\ \} : Y\,!\,(\Sigma', E')\end{array}}$$

Core calculus (equational theory)

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- For example, the β-**equations for running user comps.** are

$$\Gamma \vdash \textbf{using } V \textbf{ @ } W \textbf{ run } (\textbf{return } V') \textbf{ finally } F \equiv N_{ret}[V'/x, W/c] : Y \,!\, (\Sigma', E')$$

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$$\Gamma \vdash \textbf{using } R \textbf{ @ } W \textbf{ run } \left(\mathrm{op}_X\,(V, (y.M), (M_e)_{e \in E_{\mathrm{op}}})\right) \textbf{ finally } F$$

$$\equiv \textbf{kernel } K_{op}[V/x_{op}] \textbf{ @ } W \textbf{ finally } \{$$

$$\textbf{return } y \textbf{ @ } c' \mapsto \textbf{using } R \textbf{ @ } c' \textbf{ run } M \textbf{ finally } F\ ,$$

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and the β-**equation for signal handling** is

$$\Gamma \vdash \textbf{kernel } (\textbf{kill}_{X @ C}\ s) \textbf{ @ } W \textbf{ finally } F \equiv N_s : Y\,!\,(\Sigma', E')$$

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and kernel comp. equations include **kernel theory equations**

Core calculus (subtyping)

- The calculus also includes **subtyping**, and **subsumption rules**

$$\frac{\Gamma \vdash V : A \qquad A <: B}{\Gamma \vdash V : B}$$

$$\frac{\Gamma \vdash M : A\,!\,(\Sigma, E) \qquad \Sigma \subseteq \Sigma' \qquad A <: B \qquad E \subseteq E'}{\Gamma \vdash M : B\,!\,(\Sigma', E')}$$

$$\frac{\begin{array}{c}\Gamma \vdash K : A \lightning (\Sigma, E, S, C) \qquad \Sigma \subseteq \Sigma' \\ A <: B \qquad E \subseteq E' \qquad S \subseteq S' \qquad C \equiv C'\end{array}}{\Gamma \vdash K : B \lightning (\Sigma', E', S', C')}$$

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- We use $C \equiv C'$ to have (standard) **proof-irrelevant subtyping**
- Otherwise, instead of just $C <: C'$, we would need a **lens** $C' \leftrightarrow C$

Core calculus (semantics)

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- **Monadic semantics**, for concreteness in **Set**, using
  - **user monads** $\mathbf{U}_{\Sigma,E}\,X \stackrel{\text{def}}{=} \mathbf{Free}_{\Sigma}(X + E)$
  - **kernel monads** $\mathbf{K}_{\Sigma,E,S,C}\,X \stackrel{\text{def}}{=} C \Rightarrow \mathbf{Free}_{\Sigma}\big((X + E) \times C + S\big)$

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- (At a high level) the **judgements are interpreted** as

$$[\![\Gamma \vdash V : X]\!] : [\![\Gamma]\!] \longrightarrow [\![X]\!]$$

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- **Theorem:** The semantics is coherent (**subtyping!**) and sound.

Core calculus (semantics ctd.)

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- For instance, **kernel computations** are interpreted as

$$\begin{array}{ccc} [\![\Gamma]\!] & \xrightarrow{[\![\Gamma \vdash K : X \not\downarrow (\Sigma,E,S,C)]\!]} & \mathbf{K}_{\Sigma,E,S,[\![C]\!]}[\![X]\!] \\ \subseteq\downarrow & & \downarrow\subseteq \\ [\![\![\Gamma^s]\!]\!] & \xrightarrow[[\![\![\Gamma^s \vdash K : X^s \not\downarrow C]\!]\!]]{} & \mathbf{K}_{\mathcal{O}+\{\lightning\},\mathcal{E},\mathcal{S},[\![\![C]\!]\!]}[\![\![X^s]\!]\!] \end{array}$$

  where $\Gamma^s \vdash K : X^s \not\downarrow C$ is a **skeletal kernel typing judgement**
  and use the extra op. $\lightning : 1 \rightsquigarrow 0 \mid \{\}$ to model **runtime errors**

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- No essential obstacles to extending to $\mathbf{Sub}(\mathbf{Cpo})$ and beyond
- **Ground type restriction** on C simplifies the sem. ($[\![\![C]\!]\!] = [\![C]\!]$)

Core calculus (semantics ctd.)

$$[\![\Gamma \vdash \textbf{using } V \textbf{ @ } W \textbf{ run } M \textbf{ finally } \{ \textbf{return } x \textbf{ @ } c \mapsto N_{ret} \ , \ \big(\textbf{raise } e \textbf{ @ } c \mapsto N_e\big)_{e \in E} \ , \ \big(\textbf{kill } s \mapsto N_s\big)_{s \in S} \ \} : Y \,!\, (\Sigma', E')]\!]_\gamma \stackrel{\text{def}}{=} \ \ldots$$

- $[\![V]\!]_\gamma \ = \ \mathcal{R} \ = \ \Big(\overline{\mathsf{op}}_{\mathcal{R}} : [\![A_{\mathsf{op}}]\!] \longrightarrow \mathbf{K}_{\Sigma', E_{\mathsf{op}}, S, [\![C]\!]}\, [\![B_{\mathsf{op}}]\!]\Big)_{\mathsf{op} \in \Sigma}$
- $[\![\textbf{return } \mathsf{x} \textbf{ @ } \mathsf{c} \to N_{ret}]\!]_\gamma \ \in \ [\![X]\!] \times [\![C]\!] \longrightarrow \mathbf{U}_{\Sigma', E'}\, [\![Y]\!]$
- $[\![(\textbf{raise } \mathsf{e} \textbf{ @ } \mathsf{c} \to N_e)_{e \in E}]\!]_\gamma \ \in \ E \times [\![C]\!] \longrightarrow \mathbf{U}_{\Sigma', E'}\, [\![Y]\!]$
- $[\![(\textbf{kill } \mathsf{s} \to N_s)_{s \in S}]\!]_\gamma \ \in \ S \longrightarrow \mathbf{U}_{\Sigma', E'}\, [\![Y]\!]$
- allowing us to use the **free model property** to get

$$\mathbf{U}_{\Sigma, E}\, [\![X]\!] \xrightarrow{\mathsf{r}_{[\![X]\!] + E}} \mathbf{K}_{\Sigma', E, S, [\![C]\!]}\, [\![X]\!] \xrightarrow{(\lambda [\![N_{ret}]\!]_\gamma)^\ddagger} [\![C]\!] \Rightarrow \mathbf{U}_{\Sigma', E'}\, [\![Y]\!]$$

- and then apply the resulting composite map to

$$[\![M]\!]_\gamma \ \in \ \mathbf{U}_{\Sigma, E}\, [\![X]\!] \qquad \text{and} \qquad [\![W]\!]_\gamma \ \in \ [\![C]\!]$$

Core calculus (finalisation)

$$\Gamma \vdash \textbf{using}\ V\ @\ W\ \textbf{run}\ M\ \textbf{finally}\ \{\ \textbf{return}\ x\ @\ c \mapsto N_{ret}\ ,\ \left(\textbf{raise}\ e\ @\ c \mapsto N_e\right)_{e \in E}\ ,\ \left(\textbf{kill}\ s \mapsto N_s\right)_{s \in S}\ \} : Y\ !\ (\Sigma', E')$$

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- The above finalisation clauses determine a **finalisation maps**

$$\phi_\gamma : (\llbracket X \rrbracket + E) \times \llbracket C \rrbracket + S \longrightarrow \mathbf{U}_{\Sigma', E'} \llbracket Y \rrbracket$$

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- The above finalisation clauses determine a **finalisation maps**

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- **Theorem (finalisation):**
  - for every environment $\gamma \in \llbracket \Gamma \rrbracket$,
  - there exists a tree $t \in \mathbf{Free}_{\Sigma'}((\llbracket X \rrbracket + E) \times \llbracket C \rrbracket + S)$,
  - such that

$$\llbracket \Gamma \vdash \textbf{using } V \textbf{ @ } W \textbf{ run } M \textbf{ finally } F : Y \,!\, (\Sigma', E') \rrbracket_\gamma = \phi^\dagger_\gamma \, t$$

Implementing runners

Experimenting with the theory in practice

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- A **small experimental language** COOP[4]
  - Implements the core calculus with few extras
  - The interpreter is directly based on the equational theory
  - Top-level containers for running external (OCaml) code
  - `https://github.com/andrejbauer/coop`

[4] coop [/ku:p/] – a cage where small animals are kept, especially chickens

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  - `https://github.com/andrejbauer/coop`
- A HASKELL **library** HASKELL-COOP
  - A shallow-embedding of the core calculus in HASKELL
  - Uses one of the Freer monad implementations underneath
  - Operational aspects implement the denotational semantics
  - Top-level containers for arbitrary HASKELL monads
  - Examples make use of HASKELL's features (GADTs, ...)
  - `https://github.com/danelahman/haskell-coop`

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Runners in action

Runners can be vertically nested

Runners can be vertically nested

- ```
  using R_FH @ (fopen fileName)
  run (
    using R_FC @ (return "")
    run M
    finally {
      return x @ str → write str; return x ,
      raise WriteSizeExceeded @ str → write str; raise WriteSizeExceeded }
  )
  finally {
    return x @ fh → ... , raise e @ fh → ... , kill IOError → ... }
  ```

where the **file contents runner** (with $\Sigma' = \{\}$) is defined as

```
let R_FC = {
  write str' → let str = getenv () in
                 if (length (str^str') > max) then (raise WriteSizeExceeded)
                                              else (setenv (str^str'))
}String
```
```

Vertical nesting for instrumentation

Vertical nesting for instrumentation

- ```
  using R_Cost @ (return 0)
  run M
  finally {
    return x @ c → report_cost c; return x ,
    raise e @ c → report_cost c; raise e }
  ```

  where the **cost model runner** is defined as

  ```
  let R_Cost = {
    ... ,
    op a → let c = getenv () in
           setenv (c + 1);
           op a ,                          (* forwards op outwards *)
    ...
  }_Nat
  ```

- The runner R_{Cost} implements the same sig. Σ that `M` is using
- As a result, the runner R_{Cost} is **invisible** from `M`'s viewpoint
  ```

Vertical nesting for active monitoring

Vertical nesting for active monitoring

- First, we define a runner for **integer-valued ML-style state** as

```
type IntHeap = (Nat → (Int + 1)) × Nat                    type Ref = Nat

let R_IntState = {
  alloc x → let h = getenv () in                  (* alloc : Int ⇝ Ref ! {} *)
            let (r,h') = heapAlloc h x in
            setenv h';
            return r ,

  deref r → let h = getenv () in                  (* deref : Ref ⇝ Int ! {} *)
            match (heapSel h r) with
            | inl x → return x
            | inr () → kill ReferenceDoesNotExist ,

  assign r y → let h = getenv () in          (* assign : Ref × Int ⇝ 1 ! {} *)
               match (heapUpd h r y) with
               | inl h' → setenv h'
               | inr () → kill ReferenceDoesNotExist
}IntHeap
```

Vertical nesting for active monitoring ctd.

- Next, we define $F^\star$-style **monotonic state** on top of $\mathsf{R}_{\mathsf{IntState}}$

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- Next, we define $F^\star$-style **monotonic state** on top of $\mathsf{R}_{\mathsf{IntState}}$

```
type MonMemory = Ref → (Ord + 1)        type Ord = Int → Int → Bool

let R_MonState = {
  mAlloc x rel → let r = alloc x in                (* : Int × Ord ⇝ Ref ! {} *)
                 let m = getenv () in
                 setenv (memAdd m r rel);
                 return r,

  mDeref r → deref r ,                             (* monDeref : Ref ⇝ Int ! {} *)

  mAssign r y → let x = deref r in                 (* : Ref × Int ⇝ 1 ! {MV} *)
                let m = getenv () in
                match (memSel m r) with
                | inl rel → if (rel x y)
                            then (assign r y)
                            else (raise MonotonicityViolation)
                | inr → kill PreorderDoesNotExist
}MonMemory
```

Vertical nesting for active monitoring ctd.

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Vertical nesting for active monitoring ctd.

- We can then perform **runtime monotonicity verification** as

```
using R_IntState @ ((fun _ → inr ()) , 0)      (* init. empty ML−style heap *)
run (

  using R_MonState @ (fun _ → inr ())      (* init. empty preorders memory *)
  run (

    let r = mAlloc 0 (⩽) in
    mAssign r 1;
    mAssign r 0;        (* R_MonState raises MonotonicityViolation exception *)
    mAssign r 2

  )
  finally { ... , raise MonotonicityViolation @ m → ... , ... }

)
finally { ... }
```

Runners can also be horizontally paired

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- Given runners for

$$\textbf{let } R_1 = \{ \ldots \ , \ op_{1i}\ x_{1i} \to K_{1i} \ , \ \ldots \}_{C_1} \qquad (* : \Sigma_1 \Rightarrow (\Sigma_1', S_1, C_1)\ *)$$
$$\textbf{let } R_2 = \{ \ldots \ , \ op_{2j}\ x_{2j} \to K_{2j} \ , \ \ldots \}_{C_2} \qquad (* : \Sigma_2 \Rightarrow (\Sigma_2', S_2, C_2)\ *)$$

we can **pair them** to get the runner

```
let R = { ... ,                                   (* : Σ1 + Σ2 ⇒ (Σ1' + Σ2', S1 + S2, C1 × C2) *)
  op1i x1i  → let (c,c') = getenv () in
              user (kernel (K1i x1i) @ c finally {
                      return y @ c'' → return (inl (inl y,c'')),
                      raise e @ c'' → return (inl (inr e,c'')),     (* e ∈ Eop1i *)
                      kill s → return (inr s) }                       (* s ∈ S1 *)
              finally {
                return (inl (inl y,c'')) → setenv (c'',c'); return y,
                return (inl (inr e,c'')) → setenv (c'',c'); raise e,
                return (inr s) → kill s },
  ... ,
  op2j x2j → ..., ... }C1×C2
```

Runners can also be horizontally paired

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$$\textbf{let } R_1 = \{ \ldots \ , \ op_{1i}\ x_{1i} \to K_{1i} \ , \ \ldots \}_{C_1} \qquad (* : \Sigma_1 \Rightarrow (\Sigma_1', S_1, C_1)\ *)$$
$$\textbf{let } R_2 = \{ \ldots \ , \ op_{2j}\ x_{2j} \to K_{2j} \ , \ \ldots \}_{C_2} \qquad (* : \Sigma_2 \Rightarrow (\Sigma_2', S_2, C_2)\ *)$$

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  ... ,
  op2j x2j → ..., ... }C1×C2
```

- For instance, this way we can build a runner for IO **and** state

Other examples

Other examples

- More general forms of **(ML-style) state** (for general `Ref A`)
  - if the host language allows it, we use GADTs, etc for safety
  - some examples extract a footprint from a larger memory
- **Combinations** of different effects and runners
  - in particular the combination of IO and state
  - good use case for both vertical and horizontal composition
- KOKA-style **ambient values** and **ambient functions**
  - **ambient values** are essentially **mutable variables/parameters**
  - **ambient functions** are **applied in their lexical context**
  - a runner that treats **amb. fun. application as a co-operation**
  - amb. funs. are stored in a context-depth-sensitive heap
  - the appl. co-operation restores the heap to the lexical context

Other examples (ambient functions)

```
module Control.Runner.Ambients

...

ambCoOps :: Amb a -> Kernel sig AmbHeap a
ambCoOps (Bind f) =
  do h <- getEnv;
     (f,h') <- return (ambHeapAlloc h f);
     setEnv h';
     return f
ambCoOps (Apply f x) =
  do h <- getEnv;
     (f,d) <- return (ambHeapSel h f (depth h));
     user
       (run
          ambRunner
          (return (h {depth = d}))
          (f x)
          ambFinaliser)
       return
ambCoOps (Rebind f g) =
  do h <- getEnv;
     setEnv (ambHeapUpd h f g)

ambRunner :: Runner '[Amb] sig AmbHeap
ambRunner = mkRunner ambCoOps
```

```
module AmbientsTests where

import Control.Runner
import Control.Runner.Ambients

ambFun :: AmbVal Int -> Int -> AmbEff Int
ambFun x y =
  do x <- getVal x;
     return (x + y)

test1 :: AmbEff Int
test1 =
  withAmbVal
    (4 :: Int)
    (\ x ->
      withAmbFun
        (ambFun x)
        (\ f ->
          do rebindVal x 2;
             applyFun f 1))

test2 = ambTopLevel test1
```

Wrapping up

- **Runners** are a natural model of **top-level runtime**
- We propose **T-runners** to also model **non-top-level runtimes**
- We have turned **T**-runners into a **(practical ?) programming construct**, that supports controlled initialisation and finalisation
- I showed you some **combinators** and **programming examples**
- Two **implementations**, Coop & Haskell-Coop
- **Ongoing** and **future:** lenses in subtyping and semantics, cat. of runners, handlers, case studies, refinement typing, compilation, ...

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No 834146.

This material is based upon work supported by the Air Force Office of Scientific Research under award number FA9550-17-1-0326.