

# **Comodels** as a gateway for interacting with the **external world**

Danel Ahman

(joint work with Andrej Bauer)

Shonan, 26 March 2019

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# Computational effects in FP

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- Using **monads** (e.g., as in HASKELL)

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type St a = String → (a, String)
```

```
f :: St a → St (a, a)
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f c = c >>= (\x → c >>= (\y → return (x, y)))
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- Using **algebraic effects** and **handlers** (e.g., as in EFF)

```
effect Get : int
```

```
effect Put : int → unit
```

```
(*: int → a*int!{} *)
```

```
let g (c: unit → a!{ Get, Put }) =
```

```
  with st_h handle (perform (Put 42); c ())
```

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let g (c: unit → a!{Get, Put}) =
```

```
  with st_h handle (perform (Put 42); c ())
```

- Works well for effects that can be **represented** as pure data!

But what about effects that need access to the **external world**?

# External world in FP

- Declare a **signature** of monads or algebraic effects

```
type IO a
```

```
openFile  :: FilePath → IOMode → IO Handle
```

```
hGetLine  :: Handle → IO String
```

```
hClose    :: Handle → IO ()
```

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```
effect Read    : string
```

```
effect Raise   : string → empty
```

```
effect RandomInt    : int → int
```

```
effect RandomFloat  : float → float
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effect RandomInt    : int → int
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```
effect RandomFloat : float → float
```

- And then treat them **specially** in the compiler, e.g.,

```
let rec top_handle op =
```

```
    match op with
```

```
    | ...
```

# External world in FP

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**Ohad** 🗿 12:17 PM

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**Žiga Lukšič** 12:18 PM

not currently

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So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
  | "Write" ->
    (match v with
    | V.Tuple vs ->
      let (file_name :: str :: _) = List.map V.to_str vs in
      let file_handle = open_out_gen
        [Open_wronly
         ;Open_append
         ;Open_creat
         ;Open_text
        ] 0o666 file_name in
      Printf.fprintf file_handle "%s" str;
      close_out file_handle;
      top_handle (k V.unit_value)
    )
```

# External world in FP



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```

**This talk — a principled (co)algebraic approach!**

Another issue — **linearity** or lack thereof

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- ```
let f (s:string) =  
    let fh = fopen "foo.txt" in  
    fwrite fh (s^s);  
    fclose fh;  
    return fh  
  
let g s =  
    let fh = f s in fread fh
```



## Another issue — **linearity** or lack thereof

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- But what if we wrap `f` in a **handler**?

```
let h = handler  
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- We could resolve this by typing `fh` **linearly** (but `s` **non-linearly**)
- But what if we wrap `f` in a **handler**?

```
let h = handler  
  | effect (FWrite fh s k) → return s  
  
let g s = with h handle f ()  (* dangling fh ! *)
```

**So, how could we solve these issues?**

# So, how could we solve these issues?

- We could try using **existing programming mechanisms**, e.g.,
  - **Modules** and **abstraction**, e.g., `System.IO`

```
type IO a

hClose :: Handle → IO ()
```

- **Linear** (and **non-linear**) **types** and **effects**

```
linear type fhandle

effect FClose : (linear fhandle) → unit

linear effect FClose : fhandle → unit
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- Handlers with **finally clauses**

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linear effect FClose : fhandle → unit
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- Handlers with **finally clauses**
- **Problem:** They don't really capture the **essence of the problem**

So, what is that **essence** then?



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- Let's look at HASKELL's **IO monad** again
- A common explanation is to think of functions

$$a \rightarrow \text{IO } b$$

as

$$a \rightarrow (\text{RealWorld} \rightarrow (b, \text{RealWorld}))$$

which is the same as

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- With the `System.IO` **module abstraction** ensuring that
  - We can't get our hands on **RealWorld** — it's **not material**
  - The **RealWorld** is **affected linearly**
  - We **don't ask more** from **RealWorld** than it can provide

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**But wait a minute!** **RealWorld** looks a lot like a **comodel**!

`hGetLine` :  $(\text{Handle}, \text{RealWorld}) \rightarrow (\text{String}, \text{RealWorld})$

`hClose` :  $(\text{Handle}, \text{RealWorld}) \rightarrow ((), \text{RealWorld})$

I.e., **IO** is about the **external world** rather than internal effects!

**Refresher: what is a comodel?**

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$\text{op} : A \rightsquigarrow B$  (in univ. alg.,  $|A|$ -many  $\text{op} : |B|$ )

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- Intuitively, **comodels describe a notion of state/world**, e.g.,
  - Operational semantics using a tensor of a model and a comodel  
(Plotkin & Power, Abou-Saleh & Pattinson)
  - Stateful runners of effectful programs (Uustalu)
  - Default top-level behaviour of alg. effects (Bauer & Pretnar)

## Comodels as a gateway to the external world

- ```
let f (s:string) =  
    using IO cohandle  
        let fh = fopen "foo.txt" in  
        fwrite fh (s^s);  
        fclose fh
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 (\* in IO \*)

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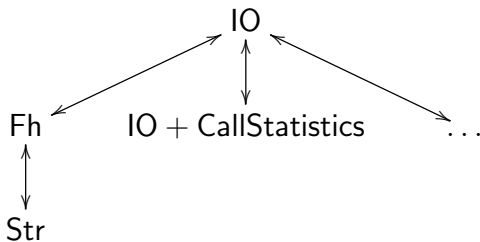
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- Better, but **have to explicitly open and thread through** `fh`
- **Solution:** **Modular treatment** of **external worlds**

# Comodels as a gateway to the external world

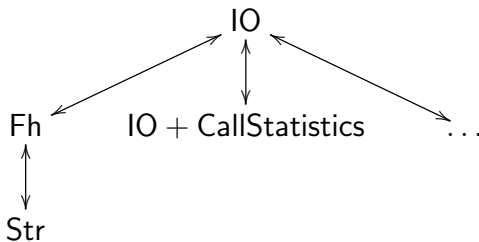
- Examples of **modularity** we might want from comodels



- Fh — “**world** which consists of **exactly one** fh”
- IO  $\longrightarrow$  Fh — “call `fopen` with `foo.txt`, store returned `fh`”
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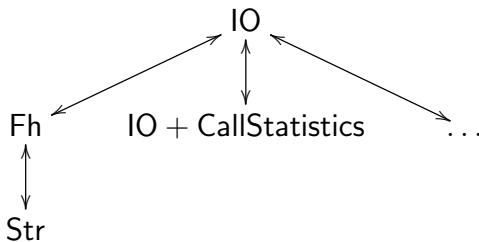


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- Str — “world that is **blissfully unaware** of `fh`”



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- Fh — “**world** which consists of **exactly one** fh”
- IO  $\longrightarrow$  Fh — “call `fopen` with `foo.txt`, store returned `fh`”
- Fh  $\longrightarrow$  IO — “call `fclose` with stored `fh`”
- Str — “world that is **blissfully unaware** of `fh`”
- Observation:** IO  $\longleftrightarrow$  Fh and other  $\longleftrightarrow$  look a lot like **lenses**

**Comodels** as a gateway to the **external world**

## Comodels as a gateway to the external world

- Our **general framework** on the file operations example

```
let f (s:string) =  
    using  
        Fh @ (fopen_of_io "foo.txt")  
    cohandle  
        fwrite_of_fh (s^s)  
    finally  
        x @ fh → fclose_of_io fh
```

## Comodels as a gateway to the external world

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    fwrite_of_fh (s^s) (* in Fh *)
  finally
    x @ fh → fclose_of_io fh (* in IO *)
```

where

```
Fh = (* W = fhandle *)
{ co_fread _ @ fh → ... ,
  co_fwrite s @ fh → fwrite_of_io s fh ;
  return ((),fh) }

(* co_fread : (unit * W) → (string * W) *)
(* co_fwrite : (string * W) → (unit * W) *)
```

**Comodels** as a gateway to the **external world**

## Comodels as a gateway to the external world

- The **modularity aspect** of our general framework

```
let f (s:string) =  
  using Fh @ (fopen_of_io "foo.txt")  
  cohandle  
  
    using Str @ (fread_of_fh ())  
    cohandle  
      write_of_str (s^s)  
    finally  
      _ @ s → fwrite_of_fh s  
  
  finally  
    _ @ fh → fclose_of_io fh
```

where

```
Str = { co_write s @ s' → (* W = string *)  
        return (( ), s'^s) }
```

**Comodels** as a gateway to the **external world**



## Comodels as a gateway to the external world

- Comodels can also **extend** the (intermediate) external world

```
let f (s:string) =  
    using Stats @ (fopen_of_io "foo.txt")  
    cohandle  
        fwrite_of_stats (s^s)  
    finally  
        - @ (fh,c) →  
            let fh' = fopen_of_io "stats.txt" in  
            fwrite_of_io fh' c; fclose_of_io fh';  
            fclose_of_io fh
```

where

```
Stats = { co_fread    - @ (fh,c) → ... ,  
          co_fwrite  s @ (fh,c) → ... ,  
          co_reset   - @ (fh,c) → return (fh,0) }  
(* W = fhandle * string*)
```

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    cohandle  
        fwrite_of_stats (s^s)  
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where

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Stats = { co_fread    - @ (fh,c) → ...,  
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(* W = fhandle * string*)
```

- Can also track **nondet./prob. choice results**, etc

So what's happening **more formally?**

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- **Types**

$$A, B, W ::= b \mid 1 \mid A \times B \mid 0 \mid A + B \mid A \xrightarrow{\Sigma} B$$

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- **Terms**

$$v ::= x \mid \dots$$

$$c ::= \text{return } v \mid \text{let } x = c_1 \text{ in } c_2 \mid v_1 v_2 \mid$$

(comodel op.)

$$\text{op } v \mid$$
$$\text{using } C @ c_i \text{ cohandle } c \text{ finally } x @ w \rightarrow c_f$$

(simple setting, only comodel ops. and no handlers (wait few slides))

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(simple setting, only comodel ops. and no handlers (wait few slides))

- **Comodels (cohandlers)**

$$C ::= \{ \overline{\text{op}}_1 x @ w \rightarrow c_1, \dots, \overline{\text{op}}_n x @ w \rightarrow c_n \}$$

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- **Typing judgements**

$$\Gamma \vdash v : A$$
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$$\Gamma \vdash v : A \qquad \Gamma \vdash^{\Sigma} c : A$$

- The two central **typing rules** are

$$\Gamma \vdash^{\Sigma} D \text{ comodel of } \Sigma' \text{ with carrier } W_D \qquad \Gamma \vdash^{\Sigma} c_i : W_D$$

$$\Gamma \vdash^{\Sigma'} c : A \qquad \Gamma, x:A, w:W_D \vdash^{\Sigma} c_f : B$$

---

$$\Gamma \vdash^{\Sigma} \text{ using } D @ c_i \text{ cohandle } c \text{ finally } x @ w \rightarrow c_f : B$$

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and

$$\frac{\text{op} : A \rightsquigarrow B \in \Sigma \qquad \Gamma \vdash v : A}{\Gamma \vdash^{\Sigma} \text{op } v : B}$$

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- **Denotational semantics** is heavily inspired by Møgelberg and Staton's **linear state-passing translation**
- **Term interpretation** looks very similar to **alg. effects**:

$$\llbracket \Gamma \vdash v : A \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow \llbracket A \rrbracket \qquad \llbracket \Gamma \overset{\Sigma}{\vdash} c : A \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow T_{\Sigma} \llbracket A \rrbracket$$

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- **un-cohandled operations** wait for a suitable **external world**!
- The interesting part is the interpretation of

$$\Gamma \stackrel{\Sigma}{\vdash} \text{using } D @ c_i \text{ cohandle } c \text{ finally } x @ w \rightarrow c_f : B$$

which is based on the **linear state-passing translation**

$$\frac{\llbracket D \rrbracket \in \text{Comodel}(\Sigma')}{\text{cohandle\_with}_{\llbracket D \rrbracket} : T_{\Sigma'} \llbracket A \rrbracket \longrightarrow (\llbracket W_D \rrbracket \rightarrow \llbracket A \rrbracket \times \llbracket W_D \rrbracket)}$$

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- The interpretation of **operations** uses the **co-operations** of Cs
  - In fact, is parametric in the semantics of (outer) co-operations

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it is natural that

- **Algebraic operations** (in the sense of  $\text{EFF}$ ) allowed in `c`,  
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- The **continuations of handlers** in `c` are delimited by `cohandle`
- How do **multi-handlers** fit here? Interacting handler-cohandler?

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  fread ()
  finally @ w →
    | return x → c_f
    | throw e → c_do_some_cleanup
    | op x k → ...
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- **Important:** **finally** **does not (!)** jump back into **cohandle**
- Algebraic operations only allowed to appear in co-operations

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- We're making them into a **modular programming abstraction**
- **Linearity** by leaving **outer worlds** implicit (via comodel ops.)
- System.IO , KOKA's **initially** & **finally** , PYTHON's **with** , ...

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## Ongoing and future work

- Work out all the **formal details** of what I have shown you today
- **Algebraic effects** and **(multi-)handlers**
- More **examples** and **use cases**
- Clarify the connection with **(effectful) lenses**
- **Combinatorics** of comodels and their lens-like relationships