

Comodels as a gateway for interacting with the **external world**

Danel Ahman

(joint work with Andrej Bauer)

Shonan, 26 March 2019

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Computational effects in FP

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- Using **monads** (e.g., as in HASKELL)

```
type St a = String → (a, String)
```

```
f :: St a → St (a, a)
```

```
f c = c >>= (\x → c >>= (\y → return (x, y)))
```

- Using **algebraic effects** and **handlers** (e.g., as in EFF)

```
effect Get : int
```

```
effect Put : int → unit
```

```
(*: int → a*int!{} *)
```

```
let g (c: unit → a!{ Get, Put }) =
```

```
  with st_h handle (perform (Put 42); c ())
```

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```
(*: int → a*int!{} *)
```

```
let g (c: unit → a!{Get, Put}) =
```

```
  with st_h handle (perform (Put 42); c ())
```

- Works well for effects that can be **represented** as pure data!

But what about effects that need access to the **external world**?

External world in FP

- Declare a **signature** of monads or algebraic effects

```
type IO a
```

```
openFile  :: FilePath → IOMode → IO Handle
```

```
hGetLine  :: Handle → IO String
```

```
hClose    :: Handle → IO ()
```

```
effect Read    : string
```

```
effect Raise   : string → empty
```

```
effect RandomInt    : int → int
```

```
effect RandomFloat  : float → float
```

External world in FP

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effect Raise : string → empty
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effect RandomInt    : int → int
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effect RandomFloat : float → float
```

- And then treat them **specially** in the compiler, e.g.,

```
let rec top_handle op =
```

```
  match op with
```

```
  | ...
```

External world in FP

External world in FP



Ohad 🗿 12:17 PM

Can I do file IO (or just O) in Eff?

External world in FP



Ohad 🗨️ 12:17 PM

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Žiga Lukšič 12:18 PM

not currently

External world in FP



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Ohad 8:35 PM

So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
  | "Write" ->
    (match v with
    | V.Tuple vs ->
      let (file_name :: str :: _) = List.map V.to_str vs in
      let file_handle = open_out_gen
        [Open_wronly
         ;Open_append
         ;Open_creat
         ;Open_text
        ] 0o666 file_name in
      Printf.fprintf file_handle "%s" str;
      close_out file_handle;
      top_handle (k V.unit_value)
    )
```

External world in FP



Ohad 12:17 PM

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```

This talk — a principled (co)algebraic approach!

Another issue — **linearity** or lack thereof

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- ```
let f (s:string) =
 let fh = fopen "foo.txt" in
 fwrite fh (s^s);
 fclose fh;
 return fh

let g s =
 let fh = f s in fread fh
```

## Another issue — **linearity** or lack thereof

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```

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- We could resolve this by typing `fh` **linearly** (but `s` **non-linearly**)



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- We could resolve this by typing `fh` **linearly** (but `s` **non-linearly**)
- But what if we wrap `f` in a **handler**?

```
let h = handler  
  | effect (FWrite fh s k) → return fh  
  
let g s = with h handle f ()
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- But what if we wrap `f` in a **handler**?

```
let h = handler
 | effect (FWrite fh s k) → return fh

let g s = with h handle f () (* dangling fh ! *)
```

**So, how could we solve these issues?**

# So, how could we solve these issues?

- We could try using **existing programming mechanisms**, e.g.,
  - **Modules** and **abstraction**, e.g., `System.IO`

```
type IO a

hClose :: Handle → IO ()
```

- **Linear** (and **non-linear**) **types** and **effects**

```
linear type fhandle

effect FClose : (linear fhandle) → unit

linear effect FClose : fhandle → unit
```

- Handlers with **finally clauses**

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- Handlers with **finally clauses**
- **Problem:** They don't really capture the **essence of the problem**

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- Let's look at HASKELL's **IO monad** again
- A common explanation is to think of functions

$$a \rightarrow \text{IO } b$$

as

$$a \rightarrow (\text{RealWorld} \rightarrow (b, \text{RealWorld}))$$

which is the same as

$$(a, \text{RealWorld}) \rightarrow (b, \text{RealWorld})$$



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- With the `System.IO` **module abstraction** ensuring that
  - We **cannot get our hands on** `RealWorld`
  - We have the impression of `RealWorld` **used linearly**
  - We **don't ask more** from `RealWorld` than it can provide

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**But wait a minute!** **RealWorld** looks a lot like a **comodel**!

`hGetLine` : `(Handle, RealWorld) → (String, RealWorld)`

`hClose` : `(Handle, RealWorld) → ((), RealWorld)`

I.e., **IO** is about the **external world** rather than internal effects!

**Important:** co-operations (`hClose`) make a **promise to return**!

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- Intutively, comodels describe **evolution of the world  $W$** 
  - Operational semantics using a tensor of a model and a comodel  
(Plotkin & Power, Abou-Saleh & Pattinson)
  - Stateful runners of effectful programs (Uustalu)
  - Linear state-passing translation (Møgelberg and Staton)
  - Top-level behaviour of alg. effects in  $\text{EFF v2}$  (Bauer & Pretnar)



## Comodels as a gateway to the external world

- ```
let f (s:string) =  
    using IO cohandle  
        let fh = fopen "foo.txt" in  
        fwrite fh (s^s);  
        fclose fh
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 (* in IO *)

Now **external world** explicit, but **dangling** `fh` etc **still possible**

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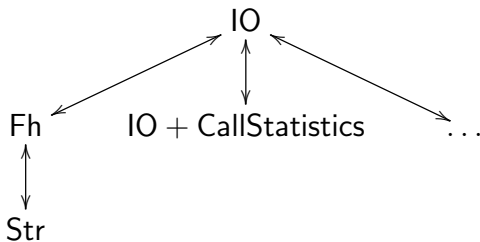
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- **Solution:** **Modular treatment** of **external worlds**

Comodels as a gateway to the external world

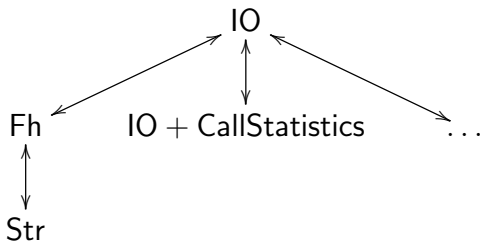
- Examples of **modularity** we might want from comodels



- Fh — “**world** which consists of **exactly one** fh”
- IO \longrightarrow Fh — “call `fopen` with `foo.txt`, store returned `fh`”
- Fh \longrightarrow IO — “call `fclose` with stored `fh`”

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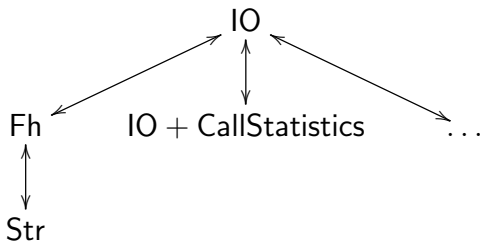
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- Str — “world that is **blissfully unaware** of `fh`”

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- Examples of **modularity** we might want from comodels



- Fh — “**world** which consists of **exactly one** fh”
- IO \longrightarrow Fh — “call `fopen` with `foo.txt`, store returned fh”
- Fh \longrightarrow IO — “call `fclose` with stored fh”
- Str — “world that is **blissfully unaware** of fh”
- Observation:** IO \longleftrightarrow Fh and other \longleftrightarrow look a lot like **lenses**

Comodels as a gateway to the **external world**

Comodels as a gateway to the external world

- Our **general framework** on the file operations example

```
let f (s:string) =  
    using  
        Fh @ (fopen_of_io "foo.txt")  
    cohandle  
        fwrite_of_fh (s^s)  
    finally  
        x @ fh → fclose_of_io fh
```


Comodels as a gateway to the external world

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    fwrite_of_fh (s^s) (* in Fh *)
  finally
    x @ fh → fclose_of_io fh (* in IO *)
```

where

```
Fh = (* W = fhandle *)
{ co_fread _ @ fh → ... ,
  co_fwrite s @ fh → fwrite_of_io s fh ;
  return ((),fh) }

(* co_fread : (unit * W) → (string * W) *)
(* co_fwrite : (string * W) → (unit * W) *)
```

Comodels as a gateway to the **external world**

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- The **modularity aspect** of our general framework

```
let f (s:string) =  
    using Fh @ (fopen_of_io "foo.txt")  
    cohandle  
  
        using Str @ (fread_of_fh ())  
        cohandle  
            write_of_str (s^s)  
        finally  
            _ @ s → fwrite_of_fh s  
  
    finally  
        _ @ fh → fclose_of_io fh
```

where

```
Str = { co_write s @ s' → (* W = string *)  
        return (( ), s'^s) }
```

Comodels as a gateway to the **external world**

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- Comodels can also **extend** the (intermediate) external world

```
let f (s:string) =  
    using Stats @ (fopen_of_io "foo.txt")  
    cohandle  
        fwrite_of_stats (s^s)  
    finally  
        - @ (fh,c) →  
            let fh' = fopen_of_io "stats.txt" in  
            fwrite_of_io fh' c; fclose_of_io fh';  
            fclose_of_io fh
```

where

```
Stats = (* W = fhandle * string *)  
{ co_fread - @ (fh,c) → ... ,  
  co_fwrite s @ (fh,c) → ... ,  
  co_reset - @ (fh,c) → return ((),(fh,0)) }
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Comodels as a gateway to the external world

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Stats = (* W = fhandle * string *)  
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```

- Can also track **nondet./prob. choice results**, etc

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$$\omega ::= \{ \text{op}_1 : A_1 \rightsquigarrow B_1, \dots, \text{op}_n : A_n \rightsquigarrow B_n \}$$

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- **Terms**

$$v ::= x \mid \dots$$

$$c ::= \text{return } v \mid \text{let } x = c_1 \text{ in } c_2 \mid v_1 v_2 \mid \text{op } v \mid \text{using } C @ c_i \text{ cohandle } c \text{ finally } x @ w \rightarrow c_f$$

(comodel op.)

(simple setting, only comodel ops. and no handlers (wait few slides))

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- **Types**

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(simple setting, only comodel ops. and no handlers (wait few slides))

- **Comodels (cohandlers)**

$$C ::= \{ \overline{\text{op}}_1 x @ w \rightarrow c_1, \dots, \overline{\text{op}}_n x @ w \rightarrow c_n \}$$

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- The two central **typing rules** are

$$\Gamma \Vdash D \text{ comodel of } \omega' \text{ with carrier } W_D \qquad \Gamma \Vdash c_i : W_D$$

$$\Gamma \Vdash c : A \qquad \Gamma, x:A, w:W_D \Vdash c_f : B$$

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$$\Gamma \Vdash \text{using } D @ c_i \text{ cohandle } c \text{ finally } x @ w \rightarrow c_f : B$$

and

$$\text{op} : A \rightsquigarrow B \in \omega \quad \Gamma \vdash v : A$$

$$\Gamma \Vdash \text{op } v : B$$

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- **Term interpretation** looks very similar to **alg. effects**:

$$\llbracket \Gamma \vdash v : A \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow \llbracket A \rrbracket \qquad \llbracket \Gamma \stackrel{\omega}{\vdash} c : A \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow T_{\omega} \llbracket A \rrbracket$$

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- **un-cohandled operations wait for a suitable external world!**
- The interesting part is the interpretation of

$$\Gamma \vDash^{\omega} \text{using } D @ c_i \text{ cohandle } c \text{ finally } x @ w \rightarrow c_f : B$$

which is based on the **linear state-passing translation**, i.e.,

$$\frac{\llbracket D \rrbracket \in \text{Comod}_{\omega'}(\text{Kleisli}(T_{\omega}))}{\text{cohandle_with}_{\llbracket D \rrbracket} : T_{\omega'} \llbracket A \rrbracket \longrightarrow \left(\llbracket W_D \rrbracket \rightarrow T_{\omega} (\llbracket A \rrbracket \times \llbracket W_D \rrbracket) \right)}$$

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it is natural to expect that

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- How do **multi-handlers** fit? Co-operating handlers-cohandlers?

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```

```
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```

```
cohandle
```

```
  fwrite_of_d s;
```

```
  fread ()
```

```
finally
```

```
  | x @ w → c_f
```

```
  | throw e → c_do_some_cleanup
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  | op x k → ...
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- **Principle:** Alg. ops. only allowed to appear in co-operations

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Ongoing work

- Iron out all the wrinkles in the **formal details**
- **Algebraic effects** and **(multi-)handlers**
- Clarify the connection with **(effectful) lenses**
- **Combinatorics** of comodels and their lens-like relationships