Fibred Computational Effects

(thesis overview)

Danel Ahman

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informatics
lfcs | Laboratory for Foundations of Computer Science
ARCHIMEDES
Haridus- ja Teadusministeerium
Microsoft Research

Overall aims

Investigate language design principles for combining

- refinement types $(\langle \mathsf{op} \rangle(\tau_1, \ldots, \tau_n), \ldots)$
- dependent types $(\Pi, \Sigma, V =_A W, \ldots)$
- computational effects (state, I/O, probability, recursion, ...)

The goal of this work was to establish that

- refinement types and comp. effects admit a natural combination
- dependent types and comp. effects admit a natural combination
- this combination covers a wide range of computational effects

The work was driven by two guiding questions

- Effectful programs in types?
- Type-dependency in seq. composition?

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- Type-dependency in seq. composition? $\Rightarrow$ **via comp. Σ-type**

Thesis structure

- **Chapter 1:** Introduction and related work
- **Chapter 2:** Preliminaries of models of effects and dep. types
- **Chapter 3:** The core language eMLTT
- **Chapter 4:** Categorical models of eMLTT
- **Chapter 5:** Interpretation of eMLTT, soundness, completeness
- **Chapter 6:** An extension of eMLTT with algebraic effects
- **Chapter 7:** An extension of eMLTT with handlers
- **Chapter 8:** Conclusion and future work
- **Appendices:** Details of longer proofs

Chapter 3: The core language eMLTT

Values and computations

- clear distinction, at the level of types and terms (CBPV, EEC)
- both kinds of types are allowed to depend only on values
- selective dependency on computations is possible via thunks

Value and computation types

$$A, B \quad ::= \quad \ldots \mid \Sigma x{:}A.B \mid \Pi x{:}A.B \mid V =_A W \mid U\,\underline{C} \mid \underline{C} \multimap \underline{D}$$

$$\underline{C}, \underline{D} \quad ::= \quad FA \mid \Sigma x{:}A.\underline{C} \mid \Pi x{:}A.\underline{C}$$

Value, computation, and homomorphism terms

$$V, W \quad ::= \quad x \mid \ldots \mid \mathtt{thunk}\, M \mid \lambda z{:}\underline{C}.K$$

$$M, N \quad ::= \quad \mathtt{force}_{\underline{C}}\, V \mid \mathtt{return}\, V \mid M \;\mathtt{to}\; x{:}A \;\mathtt{in}_{\underline{C}}\; N$$
$$\mid \quad \langle V, M\rangle \quad \mid M \;\mathtt{to}\; (x{:}A, z{:}\underline{C}) \;\mathtt{in}_{\underline{D}}\; K \mid \ldots$$

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Chapter 4: Fibred adjunction models

A natural combination of

- fibrational models of dependent types
- adjunction-based models of computational effects

as depicted in

Also in Chapter 4

- structures for modelling eMLTT's value and comp. types
- examples of such models, based on models of EEC, families of sets fibration, fibred EM-resolution, and continuous families fibration

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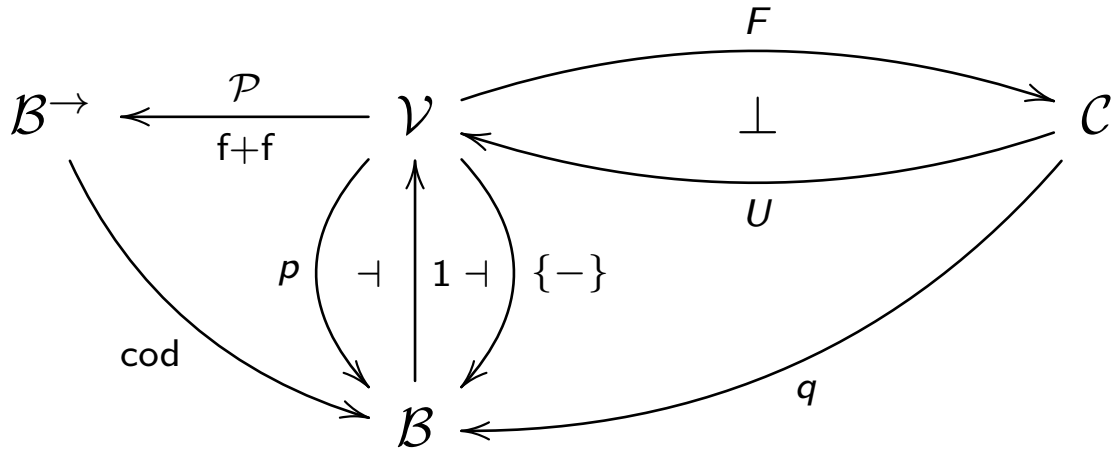

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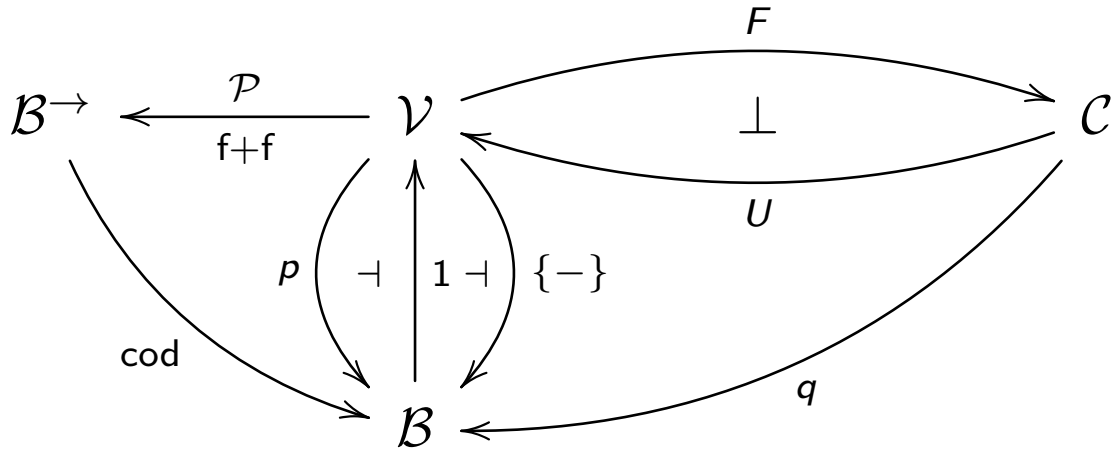

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Chapter 5: Interpretation of eMLTT

Interpretation $[\![-]\!]$

- an a priori partial interpretation function that, if defined, maps
  - contexts to objects of $\mathcal{B}$
  - value types to objects of $\mathcal{V}$ and computation types to objects of $\mathcal{C}$
  - value and computation terms to global elements in the fibres of $\mathcal{V}$
  - homomorphism terms to vertical morphisms in $\mathcal{C}$

Soundness

- weakening is interpreted as reindexing along $\pi_{[\![\Gamma;A]\!]}$
- substitution is interpreted as reindexing along $\mathsf{s}([\![\Gamma;V]\!])$
- $[\![-]\!]$ is defined on well-formed syntax
- $[\![-]\!]$ identifies definitionally equal contexts, types, and terms

Completeness

- constructing a classifying model from the syntax of eMLTT

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Chapter 6: eMLTT with algebraic effects

Fibred effect theories $\mathcal{T}_{\mathbf{eff}} = (\mathcal{S}_{\mathbf{eff}}, \mathcal{E}_{\mathbf{eff}})$

- dependently typed operation symbols $\mathrm{op} : (x : I) \longrightarrow O$
- effect terms $T ::= w(V) \mid \mathrm{op}_V(y.T) \mid \ldots$
- equations $\Gamma \mid \Delta \vdash T_1 = T_2$
- examples
  - global state, I/O, exceptions, ...
  - global state with location-dependent store types
  - dependently typed update monads

Algebraic effects in eMLTT$_{\mathcal{T}_{\mathbf{eff}}}$

- algebraic operations as comp. terms $M ::= \ldots \mid \mathrm{op}^C_V(y.M)$
- equations are included via translation $(\!| \Gamma \mid \Delta \vdash T |\!)_{A, \vec{V_i}, \vec{V'_j}, \overrightarrow{W_{\mathrm{op}}}}$
- general algebraicity equation (for $\Gamma \mid z : \underline{C} \vDash_{\mathrm{h}} K : \underline{D}$)

$$\Gamma \vDash_{\mathrm{c}} K[\mathrm{op}^C_V(y.M)/z] = \mathrm{op}^D_V(y.K[M/z]) : \underline{D}$$

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Chapter 7: eMLTT$_{\mathcal{T}_{\mathsf{eff}}}$ with handlers

Problem with the conventional term-level def. of handlers

- as handling denotes a homomorphism, natural to include

$$K \texttt{ handled with } \{\mathsf{op}_x(x') \mapsto N_{\mathsf{op}}\}_{\mathsf{op} \in \mathcal{S}_{\mathsf{eff}}} \texttt{ to } y : A \texttt{ in } N_{\mathsf{ret}}$$

- but then can prove unsound equations such as

$$\Gamma \vdash_{\mathsf{c}} \mathtt{write}^{F1}_{\mathtt{true}}(\mathtt{return} \star) = \mathtt{write}^{F1}_{\mathtt{false}}(\mathtt{return} \star) : F1$$

Handlers in eMLTT$^{\mathcal{H}}_{\mathcal{T}_{\mathsf{eff}}}$

- user-defined algebra type $\underline{C} ::= \ldots \mid \langle A, \{V_{\mathsf{op}}\}_{\mathsf{op} \in \mathcal{S}_{\mathsf{eff}}} \rangle$
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This extension enables to

- derive the conventional presentation of handlers and handling
- define predicates $\Gamma \vdash_{\mathsf{v}} V : UFA \to \mathsf{VU}$ on effectful computations

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