Embracing monotonicity in

Danel Ahman @ INRIA Paris

joint work with

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(based on our POPL 2018 paper)

ICE-TCS Seminar

January 29, 2018

Outline

- F*
- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

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F* [fstar-lang.org]

- **F*** is
  - a **functional programming language**
    - ML, OCaml, F#, Haskell, ...
    - extracted to OCaml or F#; subset compiled to efficient C code
  - an **interactive proof assistant**
    - Agda, Coq, Lean, Isabelle/HOL, ...
    - interactive modes for Emacs and Atom
  - a **semi-automated verifier** of imperative programs
    - Dafny, Why3, FramaC, ...
    - Z3-based SMT-automation; tactics and metaprogramming (WIP)
- **Application-driven** development
  - Project Everest [project-everest.github.io]
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F* – a prog. lang./proof assistant/verifier

```
module Talk

// Inductive types

type vector 'a : nat -> Type =
  | Nil : vector 'a 0
  | Cons : #n:nat -> 'a -> vector 'a n -> vector 'a (n + 1)

// Dependently typed recursive functions

val append : #a:Type -> #n:nat -> #m:nat -> vector a n -> vector a m -> Tot (vector a (n + m))
let rec append #a #n #m xs ys =
  match xs with
  | Nil -> ys
  | Cons #n x xs -> Cons x (append xs ys)

// Refinement types (nat is defined as z:int{z >= 0})

val lkp : #a:Type -> #n:nat -> vector a n -> i:nat{0 < i /\ i <= n} -> a
let rec lkp #a #n xs i =
  match xs with
  | Cons x xs -> if i = n then x else lkp xs i

// First-class predicates (for which Type0 behaves like (classical) Prop)

type is_prefix_of (#a:Type) (#n:nat) (#m:nat{n <= m}) (xs:vector a n) (zs:vector a m) : Type0 =
  forall (i:nat) . (0 < i /\ i <= n) ==> lkp xs i == lkp zs i

type is_suffix_of (#a:Type) (#n:nat) (#m:nat{n <= m}) (ys:vector a n) (zs:vector a m) : Type0 =
  forall (i:nat) . (0 < i /\ i <= n) ==> lkp ys i == lkp zs (m - n + i)

// Extrinsic reasoning (using separate lemmas)

val lemma : #a:Type -> #n:nat -> #m:nat -> xs:vector a n -> ys:vector a m -> Lemma (requires (True))
                                                                                 (ensures  (xs `is_prefix_of` (append xs ys)))
let lemma #a #n #m xs ys =
  match xs with
  | Nil -> ()
  | Cons x xs -> admit () // need to call an appropriate sub-lemma here

// Intrinsic reasoning (making lemmas part of definitions)

val append' : #a:Type -> #n:nat -> #m:nat -> xs:vector a n -> ys:vector a m -> Pure (vector a (n + m)) (requires (True))
                                                                                                       (ensures  (fun zs -> xs `is_prefix_of` zs
                                                                                                                          /\ ys `is_suffix_of` zs))
```

F* – not just a pure programming language

- `Pure`, `Lemma`, ... are just some **effects** amongst many
  - `Tot t`
  - `Pure t (requires pre) (ensures post)`
  - `Lemma (requires pre) (ensures post)`
  - `Div t (requires pre) (ensures post)`
  - `Exc t (requires` $\texttt{pre}_{\texttt{Exc}}$`) (ensures` $\texttt{post}_{\texttt{Exc}}$`)`
  - `ST t (requires` $\texttt{pre}_{\texttt{ST}}$`) (ensures` $\texttt{post}_{\texttt{ST}}$`)`
  - ...
- Some connected by **monad morphisms**
- Most derived from **WP-calculi** (see our POPL'17 paper)

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Monotonicity in verification

- Consider a program operating on **set-valued state**

$$\texttt{insert v; complex\_procedure(); } \textbf{assert } (\texttt{v} \in \texttt{get()})$$

- To prove the assertion (say, in a Floyd-Hoare style logic), we could prove that the code maintains a **stateful invariant**

$$\{\lambda\, \texttt{s}\,.\,\texttt{v} \in \texttt{s}\}\ \texttt{complex\_procedure()}\ \{\lambda\, \texttt{s}\,.\,\texttt{v} \in \texttt{s}\}$$

  - likely that we have to **carry** $\lambda\, \texttt{s}\,.\,\texttt{v} \in \texttt{s}$ **through** the proof of c_p
    - **does not guarantee** that $\lambda\, \texttt{s}\,.\,\texttt{v} \in \texttt{s}$ holds at every point in c_p
    - **sensitive** to proving that c_p maintains $\lambda\, \texttt{s}\,.\,\texttt{w} \in \texttt{s}$ for some other $\texttt{w}$

- However, if c_p **never removes**, then $\lambda\, \texttt{s}\,.\,\texttt{v} \in \texttt{s}$ is **stable**, and we would like the program logic to give us $\texttt{v} \in \texttt{get()}$ "**for free**"

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- **Programming** also relies on **monotonicity**,
  even if you don't realise it!

- Consider ML-style typed **references** `r:ref a`
  - `r` is a **proof of existence** of an `a`-typed value in the heap

- Correctness relies on **monotonicity**!
  1) Allocation **stores** an `a`-typed value in the heap
  2) Writes **don't change type** and there is **no deallocation**
  3) So, given a ref. `r`, it is **guaranteed to point** to an `a`-typed value

- Baked into the memory models of most languages
- We derive them from **global state** + **general monotonicity**

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Monotonicity is really useful!

- In this talk
  - our **motivating example** and **monotonic counters**
  - **typed references** (`ref t`) and **untyped references** (`uref`)
  - more flexibility with **monotonic references** (`mref t rel`)
- See our POPL 2018 paper for more
  - temporarily **violating monotonicity** via snapshots
  - two substantial case studies in F*
    - a **secure file-transfer** application
    - Ariadne **state continuity** protocol [Strackx, Piessens 2016]
  - pointers to other works in F* relying on monotonicity for
    - sophisticated **region-based memory models** [`fstar-lang.org`]
    - **crypto** and **TLS verification** [`project-everest.github.io`]

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Key ideas behind our general framework

- We make use of **monotonic programs** and **stable predicates**
  - per verification task, we **choose a preorder** `rel` on states
    - set inclusion, heap inclusion, increasing counter values, ...
  - a stateful program e is **monotonic** (wrt. `rel`) when

$$\forall \mathtt{s}\,\mathtt{e}'\,\mathtt{s}'.\ (\mathtt{e},\mathtt{s}) \leadsto^* (\mathtt{e}',\mathtt{s}') \implies \mathtt{rel}\ \mathtt{s}\ \mathtt{s}'$$

  - a stateful predicate p is **stable** (wrt. `rel`) when

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- **Our solution:** extend Hoare-style program logics (e.g., F*) with
  - a means to **witness** the validity of p s in some state s
  - a means for turning a p into a **state-independent proposition**
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Recap: Ordinary global state in F*

- F* supports Hoare-style reasoning about state via the **comp. type**

$$\mathtt{ST}_{\mathtt{state}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where

$$\mathtt{pre} : \mathtt{state} \to \mathrm{Type} \qquad \mathtt{post} : \mathtt{state} \to \mathtt{t} \to \mathtt{state} \to \mathrm{Type}$$

- $\mathtt{ST}$ is an abstract pre-postcondition refinement of

$$\mathtt{st}\ \mathtt{t} \stackrel{\text{def}}{=} \mathtt{state} \to \mathtt{t} * \mathtt{state}$$

- The global state **actions** have types

$$\mathtt{get} : \mathtt{unit} \to \mathtt{ST}\ \mathtt{state}\ (\mathtt{requires}\ (\lambda\,\_\,.\top))\ (\mathtt{ensures}\ (\lambda\,\mathtt{s}_0\ \mathtt{s}\ \mathtt{s}_1\,.\,\mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))$$

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$$\mathtt{get} : \mathtt{unit} \rightarrow \mathtt{ST}\ \mathtt{state}\ (\mathtt{requires}\ (\lambda\, \_ . \top))\ (\mathtt{ensures}\ (\lambda\, \mathtt{s}_0\ \mathtt{s}\ \mathtt{s}_1 . \mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))$$

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- **Refs.** and **local state** are defined in F* using **monotonicity**

New: Monotonic global state in F*

- We capture monotonic state with a new **computational type**

$$\mathtt{MST}_{\mathtt{state},\mathtt{rel}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

- The **get** action is typed as in ST

$$\mathtt{get} : \mathtt{unit} \rightarrow \mathtt{MST}\ \mathtt{state}\ (\mathtt{requires}\ (\lambda\_.\top))$$
$$(\mathtt{ensures}\ (\lambda\, s_0\, s\, s_1\, .\, s_0 = s = s_1))$$

- To ensure **monotonicity**, the **put** action gets a precondition

$$\mathtt{put} : s{:}\mathtt{state} \rightarrow \mathtt{MST}\ \mathtt{unit}\ (\mathtt{requires}\ (\lambda\, s_0\, .\, \mathtt{rel}\ s_0\ s))$$
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- So intuitively, MST is an **abstract** pre-postcondition refinement of

$$\mathtt{mst}\ \mathtt{t}\ \stackrel{\mathrm{def}}{=}\ s_0{:}\mathtt{state} \rightarrow \mathtt{t} * s_1{:}\mathtt{state}\{\mathtt{rel}\ s_0\ s_1\}$$

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New: Recalling a Witness

- We extend F* with a **logical capability**

$$\texttt{witnessed} : (\texttt{state} \to \mathrm{Type}) \to \mathrm{Type}$$

  together with a **weakening principle (functoriality)**

$$\texttt{wk} : \mathrm{p,q}{:}(\texttt{state} \to \mathrm{Type}) \to \texttt{Lemma}\ (\texttt{requires}\ (\forall\, \mathrm{s}\,.\,\mathrm{p\ s} \implies \mathrm{q\ s}))\\ (\texttt{ensures}\ (\texttt{witnessed}\ \mathrm{p} \implies \texttt{witnessed}\ \mathrm{q}))$$

- Intuitively, think of it as a **necessity modality**

$$[\![\texttt{witnessed}\ \mathrm{p}]\!](\mathrm{s}) \stackrel{\mathrm{def}}{=} \forall\, \mathrm{s'}\,.\,\texttt{rel}\ \mathrm{s}\ \mathrm{s'} \implies [\![\mathrm{p}\ \mathrm{s'}]\!](\mathrm{s})$$

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- … Hoare-style logics are essentially **world/state-indexed**, so

- we include a **stateful introduction rule** for `witnessed`

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\mathtt{witness} \;:\; & \mathtt{p}{:}(\mathtt{state} \to \mathtt{Type}_0) \\
& \to \mathsf{MST}\ \mathtt{unit}\ (\mathtt{requires}\ (\lambda\, \mathtt{s}_0\, .\, \mathtt{p}\ \text{`}\mathtt{stable\_from}\text{'}\ \mathtt{s}_0)) \\
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$$

- and a **stateful elimination rule** for `witnessed`

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Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

The motivating example revisited

- Recall the program operating on the **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert } (\mathtt{v} \in \mathtt{get}())$$

  - We pick **set inclusion** $\subseteq$ as our preorder `rel` on states
  - We **prove the assertion** by inserting a `witness` and `recall`

$$\texttt{insert v; witness } (\lambda\, \mathtt{s}\,.\, \mathtt{v} \in \mathtt{s})\texttt{; c\_p(); recall } (\lambda\, \mathtt{s}\,.\, \mathtt{v} \in \mathtt{s})\texttt{; assert } (\mathtt{v} \in \mathtt{get}())$$

  - For **any other** `w`, wrapping

$$\texttt{insert w; [ ]; assert } (\mathtt{w} \in \mathtt{get}())$$

  around the program is handled **similarly easily** by

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- **Monotonic counters** are analogous, by picking $\mathbb{N}$ and $\leq$, e.g.,

$$\texttt{create 0; incr(); witness } (\lambda\, \mathtt{c}\,.\, \mathtt{c} > 0)\texttt{; c\_p(); recall } (\lambda\, \mathtt{c}\,.\, \mathtt{c} > 0)$$

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ML-style typed references (local state)

- First, we define a type of **heaps** as a finite map

```
type heap =
  | H : h:(ℕ → cell) → ctr:ℕ{∀n. ctr ≤ n ⟹ h n = Unused} → heap
```

  where

```
type cell =
  | Unused : cell
  | Used : a:Type → v:a → cell
```

- Next, we define a **preorder** on heaps (**heap inclusion**)

```
let heap_inclusion (H h₀ _) (H h₁ _) = ∀id. match h₀ id, h₁ id with
  | Used a _, Used b _ → a = b
  | Unused, Used _ _ → ⊤
  | Unused, Unused → ⊤
  | Used _ _, Unused → ⊥
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- As a result, we can define new **local state effect**

$$\mathtt{MLST}\ \mathtt{t}\ \mathtt{pre}\ \mathtt{post} \stackrel{\text{def}}{=} \mathtt{MST}_{\mathtt{heap},\mathtt{heap\_inclusion}}\ \mathtt{t}\ \mathtt{pre}\ \mathtt{post}$$

- Next, we define the type of **references** using monotonicity

$$\mathtt{abstract\ type\ ref\ a} = \mathtt{id}{:}\mathbb{N}\{\mathtt{witnessed}\ (\lambda\,\mathtt{h}\,.\,\mathtt{contains}\ \mathtt{h}\ \mathtt{id}\ \mathtt{a})\}$$

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```
let contains (H h _) id a =
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- Important: contains is **stable** wrt. heap_inclusion

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ML-style typed references (local state)

- As a result, we can define new **local state effect**

$$\texttt{MLST}\ \texttt{t}\ \texttt{pre}\ \texttt{post} \stackrel{\text{def}}{=} \texttt{MST}_{\texttt{heap},\texttt{heap\_inclusion}}\ \texttt{t}\ \texttt{pre}\ \texttt{post}$$

- Next, we define the type of **references** using monotonicity

$$\texttt{abstract type ref a} = \texttt{id}:\mathbb{N}\{\texttt{witnessed}\ (\lambda\, \texttt{h}\,.\, \texttt{contains h id a})\}$$

  where

```
let contains (H h _) id a =
  match h id with
    | Used b _  →  a = b
    | Unused  →  ⊥
```

- Important: `contains` is **stable** wrt. `heap_inclusion`

ML-style typed references (local state)

- Finally, we define `MLST`'s **actions** using `MST`'s actions
  - `let alloc` (a:Type) (v:a) : `MLST` (`ref a`) `...` = `...`
    - **get** the current heap
    - **create** a fresh ref., and **add** it to the heap
    - **put** the updated heap back
    - **witness** that the created ref. is in the heap
  - `let read` (r:`ref a`) : `MLST t ...` = `...`
    - **recall** that the given ref. is in the heap
    - **get** the current heap
    - **select** the given reference from the heap
  - `let write` (r:`ref a`) (v:a) : `MLST unit ...` = `...`
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Adding untyped and monotonic references

- **Untyped references** (`uref`) with strong updates
  - Used `heap` cells are extended with **tags**

    where

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  - actions corresponding to `uref`s have **weaker types** than for `ref`s

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  - `mref`s provide **more flexibility** with ref.-wise monotonicity

- Further, all three can be extended with **manually managed** refs.

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Conclusion

- Monotonicity
  - can be distilled into a **simple** and **general** framework
  - is **useful** for **programming** (refs.) and **verification** (Prj. Everest)

- See our POPL 2018 paper for
  - further **examples** and **case studies**
  - **meta-theory** and **total correctness** for `MST`
    - based on an instrumented operational semantics

$$(\texttt{witness}\ x.\varphi\ ,\ s\ ,\ W) \rightsquigarrow (\texttt{return}\ ()\ ,\ s\ ,\ W \cup \{x.\varphi\})$$

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  - first steps towards **monadic reification** for `MST`
    - useful for extrinsic reasoning, e.g., for relational properties
    - but have to be careful when breaking abstraction

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Thank you for your attention!

Questions?

Appendix: Mon. reification and reflection

- In F* every **abstract** ST **computation**

$$\texttt{e} : \texttt{ST}\ \texttt{t}\ (\texttt{requires}\ \texttt{pre})\ (\texttt{ensures}\ \texttt{post})$$

can be **reified** into its **underlying** Pure **representation**

$$\texttt{reify}\ \texttt{e} : \texttt{s}_0\texttt{:state} \to \texttt{Pure}\ (\texttt{t} * \texttt{state})\ (\texttt{requires}\ (\texttt{pre}\ \texttt{s}_0))\ (\texttt{ensures}\ (\lambda\ (\texttt{x}, \texttt{s}_1)\ .\ \texttt{post}\ \texttt{s}_0\ \texttt{x}\ \texttt{s}_1))$$

and vice versa using **reflection** (see our POPL 2017 paper)

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- We cannot simply turn an **abstract** MST **computation**

$$\mathtt{e} : \mathtt{MST}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  into a **state-passing function**

$$\mathtt{s_0}{:}\mathtt{state} \rightarrow \mathtt{Pure}\ (\mathtt{t} * \mathtt{s_1}{:}\mathtt{state}\{\mathtt{rel}\ \mathtt{s_0}\ \mathtt{s_1}\})\ (\mathtt{req.}\ (\mathtt{pre}\ \mathtt{s_0}))$$
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- For example, consider the **recalling** action

$$\mathtt{recall} : \mathtt{p}{:}(\mathtt{state} \rightarrow \mathtt{Type}) \rightarrow \mathtt{MST}\ \mathtt{unit}\ (\mathtt{requires}\ (\lambda\,\_\,.\,\mathtt{witnessed}\ \mathtt{p}))$$
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$$\mathtt{reify}\ (\mathtt{recall}\ \mathtt{p}) \rightsquigarrow \lambda\,\mathtt{s_0}\,.\,\mathtt{return}\ ((), \mathtt{s_0})$$

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- In our POPL 2018 paper, we support reification and reflection by
  - indexing $\mathtt{MST}_{\mathtt{state},\mathtt{rel},\mathtt{b}}$ with a **boolean flag** $\mathtt{b}$ (reifiable?), and
  - **guarding** the pre-postconditions of `witness` and `recall` with $\mathtt{b}$

  so if $\mathtt{b} = \mathtt{true}$ then `witness` and `recall` are **logically no-ops.**

- This **works** but leads to **duplication** of pre- and postconditions!

- Instead, ongoing work is taking (hybrid) **modal logic** seriously

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