Comodels as a gateway for interacting with the external world

Danel Ahman

(joint work with Andrej Bauer)

Shonan, 26 March 2019

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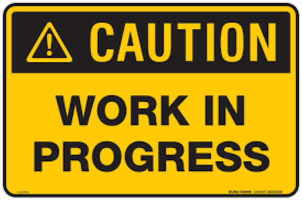

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Computational effects in FP

Computational effects in FP

- Using **monads** (e.g., as in Haskell)

```
type St a = String → (a,String)

f :: St a → St (a,a)
f c = c >>= (\x → c >>= (\y → return (x,y)))
```

- Using **algebraic effects** and **handlers** (e.g., as in Eff)

```
effect Get : int
effect Put : int → unit
                                        (*: int → a*int!{} *)
let g (c:unit → a!{Get,Put}) =
  with st_h handle (perform (Put 42); c ())
```

Computational effects in FP

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type St a = String → (a,String)

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effect Get : int
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                                   (*: int → a*int!{} *)
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```

- Works well for effects that can be **represented** as pure data!

  But what about effects that need access to the **external world**?

External world in FP

- Declare a **signature** of monads or algebraic effects

```
type IO a

openFile  :: FilePath → IOMode → IO Handle
hGetLine  :: Handle → IO String
hClose    :: Handle → IO ()
```

```
effect Read  : string
effect Raise : string → empty

effect RandomInt   : int → int
effect RandomFloat : float → float
```

External world in FP

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```

```
effect Read  : string
effect Raise : string → empty

effect RandomInt   : int → int
effect RandomFloat : float → float
```

- And then treat them **specially** in the compiler, e.g.,

```
let rec top_handle op =
  match op with
  | ...
```

External world in FP

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Ohad 12:17 PM
Can I do file IO (or just O) in Eff?

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So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
    | "Write" ->
      (match v with
       | V.Tuple vs ->
         let (file_name :: str :: _)  = List.map V.to_str vs in
         let file_handle = open_out_gen
                             [Open_wronly
                             ;Open_append
                             ;Open_creat
                             ;Open_text
                             ] 0o666 file_name in
         Printf.fprintf file_handle "%s" str;
         close_out file_handle;
         top_handle (k V.unit_value)
      )
```

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This talk — a principled (co)algebraic approach!

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- ```
  let f (s:string) =
    let fh = fopen "foo.txt" in
    fwrite fh (s^s);
    fclose fh;
    return fh

  let g s =
    let fh = f s in fread fh
  ```
  ```

Another issue — linearity or lack thereof

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- We could resolve this by typing `fh` **linearly** (but `s` **non-linearly**)

- But what if we wrap `f` in a **handler**?

```
let h = handler
        | effect (FWrite fh s k) → return fh

let g s = with h handle f ()
```
```

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- But what if we wrap `f` in a **handler**?

  ```
  let h = handler
          | effect (FWrite fh s k) → return fh

  let g s = with h handle f () (* dangling fh ! *)
  ```
  ```

So, how could we solve these issues?

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- We could try using **existing programming mechanisms**, e.g.,
  - **Modules** and **abstraction**, e.g., `System.IO`

    ```
    type IO a

    hClose :: Handle → IO ()
    ```

  - **Linear** (and **non-linear**) **types** and **effects**

    ```
    linear type fhandle

    effect FClose : (linear fhandle) → unit

    linear effect FClose : fhandle → unit
    ```

  - Handlers with `finally` **clauses**

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    ```

  - Handlers with `finally` **clauses**
- **Problem:** They don't really capture the **essence of the problem**

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- A common explanation is to think of functions

$$a \to \mathsf{IO}\ b$$

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- With the `System.IO` **module abstraction** ensuring that
  - We can't get our hands on RealWorld — it's **not material**
  - The RealWorld is **affected linearly**
  - We **don't ask more** from RealWorld than it can provide

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But wait a minute! RealWorld looks a lot like a **comodel**!

$$\mathsf{hGetLine} : (\mathsf{Handle}, \mathsf{RealWorld}) \to (\mathsf{String}, \mathsf{RealWorld})$$

$$\mathsf{hClose} : (\mathsf{Handle}, \mathsf{RealWorld}) \to ((), \mathsf{RealWorld})$$

I.e., **IO** is about the **external world** rather than internal effects!

Refresher: what is a comodel?

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- A **comodel**/**coalgebra**/**<u>cohandler</u>** $\mathcal{W}$ of Σ is given by

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- Intutively, **comodels describe a notion of state/world**, e.g.,
  - Operational semantics using a tensor of a model and a comodel (Plotkin & Power, Abou-Saleh & Pattinson)
  - Stateful runners of effectful programs (Uustalu)
  - Default top-level behaviour of alg. effects (Bauer & Pretnar)

Comodels as a gateway to the external world

- ```
  let f (s:string) =
    using IO cohandle
      let fh = fopen "foo.txt" in
      fwrite fh (s^s);
      fclose fh                                    (* in IO *)
  ```

  Now **external world** explicit, but **dangling** `fh` etc **still possible**
  ```

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  Better, but **have to explicitly open** and **thread through** `fh`

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- **Solution: Modular treatment** of **external worlds**

Comodels as a gateway to the external world

- Examples of **modularity** we might want from comodels

$$\begin{tikzcd} & \mathsf{IO} \arrow[dl, leftrightarrow] \arrow[d, leftrightarrow] \arrow[dr, leftrightarrow] & \\ \mathsf{Fh} \arrow[d, leftrightarrow] & \mathsf{IO} + \mathsf{CallStatistics} & \ldots \\ \mathsf{Str} & & \end{tikzcd}$$

- Fh — "**world** which consists of **exactly one** `fh`"
- IO $\longrightarrow$ Fh — "call `fopen` with `foo.txt`, store returned `fh`"
- Fh $\longrightarrow$ IO — "call `fclose` with stored `fh`"

Comodels as a gateway to the external world

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Comodels as a gateway to the **external world**

- Examples of **modularity** we might want from comodels

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- Fh — "**world** which consists of **exactly one** `fh`"
- $\mathsf{IO} \longrightarrow \mathsf{Fh}$ — "call `fopen` with `foo.txt`, store returned `fh`"
- $\mathsf{Fh} \longrightarrow \mathsf{IO}$ — "call `fclose` with stored `fh`"
- Str — "world that is **blissfully unaware** of `fh`"
- **Observation:** $\mathsf{IO} \longleftrightarrow \mathsf{Fh}$ and other $\longleftrightarrow$ look a lot like **lenses**

Comodels as a gateway to the external world

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- Our **general framework** on the file operations example

```
let f (s:string) =
  using
    Fh @ (fopen_of_io "foo.txt")
  cohandle
    fwrite_of_fh (s^s)
  finally
    x @ fh → fclose_of_io fh
```

Comodels as a gateway to the external world

- Our **general framework** on the file operations example

```
let f (s:string) =                        (* in IO *)
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  using
    Fh @ (fopen_of_io "foo.txt")            (* in IO *)
  cohandle
    fwrite_of_fh (s^s)                      (* in Fh *)
  finally
    x @ fh → fclose_of_io fh                (* in IO *)
```

where

```
Fh =                                   (* W = fhandle *)
 { co_fread   _ @ fh → ... ,
   co_fwrite s @ fh → fwrite_of_io s fh;
                      return ((),fh)        }

    (* co_fread  : (unit * W) → (string * W) *)
   (* co_fwrite : (string * W) → (unit * W) *)
```

Comodels as a gateway to the external world

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- The **modularity aspect** of our general framework

```
let f (s:string) =
  using Fh @ (fopen_of_io "foo.txt")
  cohandle

    using Str @ (fread_of_fh ())
    cohandle
      write_of_str (s^s)
    finally
      _ @ s → fwrite_of_fh s

  finally
    _ @ fh → fclose_of_io fh
```

where

```
Str = { co_write s @ s' →        (* W = string *)
                return ((),s'^s) }
```

Comodels as a gateway to the external world

Comodels as a gateway to the external world

- Comodels can also **extend** the (intermediate) external world

```
let f (s:string) =
  using Stats @ (fopen_of_io "foo.txt")
  cohandle
    fwrite_of_stats (s^s)
  finally
    _ @ (fh,c) →
          let fh' = fopen_of_io "stats.txt" in
          fwrite_of_io fh' c; fclose_of_io fh';
          fclose_of_io fh
```

where

```
Stats =                         (* W = fhandle * string*)
  { co_fread   _ @ (fh,c) → ...,
    co_fwrite  s @ (fh,c) → ...,
    co_reset   _ @ (fh,c) → return ((),(fh,0)) }
```

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where

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Stats =                        (* W = fhandle * string*)
  { co_fread   _ @ (fh,c) → ...,
    co_fwrite  s @ (fh,c) → ...,
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```

- Can also track **nondet./prob. choice results**, etc

So what's happening **more formally**?

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- **Types**

$$A, B, W \;::=\; \mathsf{b} \mid 1 \mid A \times B \mid 0 \mid A + B \mid A \xrightarrow{\Sigma} B$$

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$$\Sigma \;::=\; \{\, \mathsf{op}_1 : A_1 \rightsquigarrow B_1 \,,\; \ldots \;,\; \mathsf{op}_n : A_n \rightsquigarrow B_n \,\}$$

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- **Terms**

$$v \ ::= \ x \mid \ldots$$

$$\begin{aligned} c \ ::= \ & \textbf{return}\ v \mid \textbf{let}\ x = c_1\ \textbf{in}\ c_2 \mid v_1 v_2 \mid \\ & \mathsf{op}\ v \mid \qquad\qquad\qquad\qquad \text{(comodel op.)} \\ & \textbf{using}\ C\ @\ c_i\ \textbf{cohandle}\ c\ \textbf{finally}\ x\ @\ w \to c_f \end{aligned}$$

(simple setting, only comodel ops. and no handlers (wait few slides))

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- **Comodels (cohandlers)**

$$C \ ::= \ \{ \ \overline{\mathsf{op}}_1 \ x \ @ \ w \to c_1 \ , \ \ldots \ , \ \overline{\mathsf{op}}_n \ x \ @ \ w \to c_n \ \}$$

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- **Typing judgements**

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- **Typing judgements**

$$\Gamma \vdash v : A \qquad\qquad \Gamma \models^{\Sigma} c : A$$

- The two central **typing rules** are

$$\frac{\Gamma \models^{\Sigma} \mathsf{D} \text{ comodel of } \Sigma' \text{ with carrier } W_{\mathsf{D}} \qquad \Gamma \models^{\Sigma} c_i : W_{\mathsf{D}} \qquad \Gamma \models^{\Sigma'} c : A \qquad \Gamma, x : A, w : W_{\mathsf{D}} \models^{\Sigma} c_f : B}{\Gamma \models^{\Sigma} \textbf{using } \mathsf{D} \mathbin{@} c_i \textbf{ cohandle } c \textbf{ finally } x \mathbin{@} w \to c_f : B}$$

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and

$$\frac{\mathsf{op} : A \rightsquigarrow B \in \Sigma \qquad \Gamma \vdash v : A}{\Gamma \vDash^{\Sigma} \mathsf{op}\ v : B}$$

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- **Term interpretation** looks very similar to **alg. effects**:

$$[\![\Gamma \vdash v : A]\!] : [\![\Gamma]\!] \longrightarrow [\![A]\!] \qquad [\![\Gamma \overset{\Sigma}{\vdash} c : A]\!] : [\![\Gamma]\!] \longrightarrow T_{\Sigma}\,[\![A]\!]$$

  - **un-cohandled operations** wait for a suitable **external world**!

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  - **un-cohandled operations** wait for a suitable **external world**!

- The interesting part is the interpretation of

$$\Gamma \overset{\Sigma}{\vDash} \textbf{using}\ \mathrm{D} \mathbin{@} c_i\ \textbf{cohandle}\ c\ \textbf{finally}\ x \mathbin{@} w \to c_f : B$$

  which is based on the **linear state-passing translation**

$$\frac{[\![\mathrm{D}]\!] \in \mathrm{Comodel}(\Sigma')}{\mathrm{cohandle\_with}_{[\![\mathrm{D}]\!]} : T_{\Sigma'}\,[\![A]\!] \longrightarrow ([\![W_{\mathrm{D}}]\!] \to [\![A]\!] \times [\![W_{\mathrm{D}}]\!])}$$

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- Regarding **op. semantics**, e.g., consider confs. $\left(\overrightarrow{(\mathsf{C}, w)} \,,\; c \right)$

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(\ (\overrightarrow{(\mathsf{C}, w_2)}, (\mathsf{C}', w_2'))\ ,\ c_f[v/x, w_1''/w]\ ) \Downarrow (\ (\overrightarrow{(\mathsf{C}, w_3)}, (\mathsf{C}', w_3'))\ ,\ \mathbf{return}\ v'\ )
\end{array}
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- The interpretation of **operations** uses the **co-operations** of Cs
  - In fact, is parametric in the semantics of (outer) co-operations

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finally x @ w → c_f
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it is natural that

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- How do **multi-handlers** fit here? Interacting handlers-cohandlers?

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using
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finally @ w →
  | return x → c_f
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  - **Important:** **finally** **does not (!)** jump back into **cohandle**
  - Algebraic operations only allowed to appear in co-operations

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- Of course also **initialisation** might **break the promise**

```
using
  C
initially @ c_i
  | op x k → ...
  | ...
cohandle
  c
finally @ w →
  | return x → c_f
  | op x k → ...
  | ...
```

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- **Linearity** by leaving **outer worlds** implicit (via comodel ops.)
- `System.IO`, KOKA's **initially** & **finally**, PYTHON's **with**, ...

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- `System.IO`, Koka's `initially` & `finally`, Python's `with`, ...

Ongoing and future work

- Work out all the **formal details** of what I have shown you today
- **Algebraic effects** and **(multi-)handlers**
- More **examples** and **use cases**
- Clarify the connection with **(effectful) lenses**
- **Combinatorics** of comodels and their lens-like relationships