Leveraging monotonic state in F*

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joint work with

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(Global state +) monotonicity is really useful!

Its essence can be captured very neatly!

Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see POPL'18 paper)
- Monadic reification and reflection (if time permits, or see paper)
- Meta-theory and correctness results (see POPL'18 paper)

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Monotonicity in verification

- Consider a program operating on **set-valued state**

$$\texttt{insert v; complex\_procedure(); } \textbf{assert } (\texttt{v} \in \texttt{get()})$$

- To prove the assertion (say, in a Floyd-Hoare style logic), we could prove that the code maintains a **stateful invariant**

$$\{\lambda\, \texttt{s}\, .\, \texttt{v} \in \texttt{s}\}\ \texttt{complex\_procedure()}\ \{\lambda\, \texttt{s}\, .\, \texttt{v} \in \texttt{s}\}$$

  - likely that we have to **carry** $\lambda\, \texttt{s}\, .\, \texttt{v} \in \texttt{s}$ **through** the proof of c_p
    - **does not guarantee** that $\lambda\, \texttt{s}\, .\, \texttt{v} \in \texttt{s}$ holds at every point in c_p
    - **sensitive** to proving that c_p maintains $\lambda\, \texttt{s}\, .\, \texttt{w} \in \texttt{s}$ for some other $\texttt{w}$

- However, if c_p **never removes**, then $\lambda\, \texttt{s}\, .\, \texttt{v} \in \texttt{s}$ is **stable**, and we would like the program logic to give us $\texttt{v} \in \texttt{get()}$ "**for free**"

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Monotonicity in programming

- **Programming** also relies on **monotonicity**,

  even if you don't realise it!

- Consider ML-style typed **references** `r:ref a`
  - `r` is a **proof of existence** of an `a`-typed value in the heap

- Correctness relies on **monotonicity**!
  1) Allocation **stores** an `a`-typed value in the heap
  2) Writes **don't change type** and there is **no deallocation**
  3) So, given a ref. `r`, it is **guaranteed to point** to an `a`-typed value

- Baked into the memory models of most languages
- We derive them from **global state + general monotonicity**

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  - our **motivating example** and **monotonic counters**
  - **typed references** (`ref t`) and **untyped references** (`uref`)
  - more flexibility with **monotonic references** (`mref t rel`)

- See our POPL 2018 paper for more
  - temporarily **violating monotonicity** via snapshots
  - two substantial case studies in F*
    - a **secure file-transfer** application
    - Ariadne **state continuity** protocol [Strackx, Piessens 2016]
  - pointers to other works in F* relying on monotonicity for
    - sophisticated **region-based memory models** [`fstar-lang.org`]
    - **crypto** and **TLS verification** [`project-everest.github.io`]

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- We make use of **monotonic programs** and **stable predicates**
  - per verification task, we **choose a preorder** `rel` on states
    - set inclusion, heap inclusion, increasing counter values, ...
  - a stateful program e is **monotonic** (wrt. `rel`) when

$$\forall\, s\, e'\, s'.\ (e, s) \rightsquigarrow^{*} (e', s') \implies \mathtt{rel}\ s\ s'$$

  - a stateful predicate `p` is **stable** (wrt. `rel`) when

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- **Our solution:** extend Hoare-style program logics (e.g., F*) with
  - a means to **witness** the validity of `p` s in some state s
  - a means for turning a `p` into a **state-independent proposition**
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Recap: Ordinary global state in F*

- F* supports Hoare-style reasoning about state via the **comp. type**

$$\mathtt{ST}_{\mathtt{state}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where

$$\mathtt{pre} : \mathtt{state} \to \mathtt{Type}_0 \qquad \mathtt{post} : \mathtt{state} \to \mathtt{t} \to \mathtt{state} \to \mathtt{Type}_0$$

- ST is an abstract pre-postcondition refinement of

$$\mathtt{st}\ \mathtt{t} \stackrel{\mathsf{def}}{=} \mathtt{state} \to \mathtt{t} * \mathtt{state}$$

- The global state **actions** have types

$$\mathtt{get} : \mathtt{unit} \to \mathtt{ST}\ \mathtt{state}\ (\mathtt{requires}\ (\lambda\,\_.\top))\ (\mathtt{ensures}\ (\lambda\,\mathtt{s}_0\ \mathtt{s}\ \mathtt{s}_1\,.\,\mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))$$

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New: Monotonic global state in F*

- We capture monotonic state with a new **computational type**

$$\mathtt{MST}_{\mathtt{state},\mathtt{rel}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

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$$\mathtt{MST}_{\mathtt{state},\mathtt{rel}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

- The **get** action is typed as in ST

$$\mathtt{get} : \mathtt{unit} \rightarrow \mathtt{MST}\ \mathtt{state}\ (\mathtt{requires}\ (\lambda\, \_ . \top))$$
$$(\mathtt{ensures}\ (\lambda\, \mathtt{s}_0\, \mathtt{s}\, \mathtt{s}_1 \,.\, \mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))$$

- To ensure **monotonicity**, the **put** action gets a precondition

$$\mathtt{put} : \mathtt{s}{:}\mathtt{state} \rightarrow \mathtt{MST}\ \mathtt{unit}\ (\mathtt{requires}\ (\lambda\, \mathtt{s}_0 \,.\, \mathtt{rel}\ \mathtt{s}_0\ \mathtt{s}))$$
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- So intuitively, MST is an **abstract** pre-postcondition refinement of

$$\mathtt{mst}\ \mathtt{t} \stackrel{\mathrm{def}}{=} \mathtt{s}_0{:}\mathtt{state} \rightarrow \mathtt{t} * \mathtt{s}_1{:}\mathtt{state}\{\mathtt{rel}\ \mathtt{s}_0\ \mathtt{s}_1\}$$

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New: Recalling a Witness

- We extend F* with a **logical capability**

$$\texttt{witnessed} : (\texttt{state} \to \texttt{Type}_0) \to \texttt{Type}_0$$

together with a **weakening principle** (**functoriality**)

$$\begin{aligned}\texttt{wk} : \texttt{p},\texttt{q}{:}(\texttt{state} \to \texttt{Type}_0) \to \texttt{Lemma}\ &(\texttt{requires}\ (\forall\, \texttt{s}\,.\, \texttt{p s} \implies \texttt{q s}))\\ &(\texttt{ensures}\ (\texttt{witnessed p} \implies \texttt{witnessed q}))\end{aligned}$$

- For better intuition, think of it as a **necessity modality**

$$[\![\texttt{witnessed p}]\!](\texttt{s}) \stackrel{\text{def}}{=} \forall\, \texttt{s}'\,.\,\texttt{rel s s}' \implies [\![\texttt{p s}']\!](\texttt{s})$$

- As usual, for natural deduction, need **world-indexed sequents**
- Oh, wait a minute ...

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- ... Hoare-style logics are essentially **world/state-indexed**, so

- we include a **stateful introduction rule** for `witnessed`

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\texttt{witness} \;:\;\; & \texttt{p}{:}(\texttt{state} \to \mathrm{Type}_0) \\
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Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see POPL'18 paper)
- Monadic reification and reflection (if time permits, or see paper)
- Meta-theory and correctness results (see POPL'18 paper)

The motivating example revisited

- Recall the program operating on the **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert } (\mathtt{v} \in \mathtt{get}())$$

  - We pick **set inclusion** $\subseteq$ as our preorder `rel` on states
  - We **prove the assertion** by inserting a `witness` and `recall`

$$\texttt{insert v; witness } (\lambda\, \mathtt{s}\,.\, \mathtt{v} \in \mathtt{s});\ \texttt{c\_p}();\ \texttt{recall } (\lambda\, \mathtt{s}\,.\, \mathtt{v} \in \mathtt{s});\ \texttt{assert } (\mathtt{v} \in \mathtt{get}())$$

  - For **any other** `w`, wrapping

$$\texttt{insert w; [ ]; assert } (\mathtt{w} \in \mathtt{get}())$$

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- **Monotonic counters** are analogous, by picking $\mathbb{N}$ and $\leq$, e.g.,

$$\texttt{create 0; incr(); witness } (\lambda\, \mathtt{c}\,.\, \mathtt{c} > 0);\ \texttt{c\_p}();\ \texttt{recall } (\lambda\, \mathtt{c}\,.\, \mathtt{c} > 0)$$

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ML-style typed references (local state)

- First, we define a type of **heaps** as a finite map

  $\texttt{type heap} =$

  $\quad \mid \texttt{H} : \texttt{h}{:}(\mathbb{N} \to \texttt{cell}) \to \texttt{ctr}{:}\mathbb{N}\{\forall\, \texttt{n} .\, \texttt{ctr} \leq \texttt{n} \implies \texttt{h n} = \texttt{Unused}\} \to \texttt{heap}$

  where

  $\texttt{type cell} =$

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  $\texttt{let heap\_inclusion (H } \texttt{h}_0 \texttt{ \_) (H } \texttt{h}_1 \texttt{ \_)} = \forall\, \texttt{id} .\, \texttt{match } \texttt{h}_0 \texttt{ id} , \texttt{h}_1 \texttt{ id with}$

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- As a result, we can define new **local state effect**

$$\texttt{MLST t pre post} \stackrel{\text{def}}{=} \texttt{MST}_{\texttt{heap},\texttt{heap\_inclusion}}\ \texttt{t pre post}$$

- Next, we define the type of **references** using monotonicity

$$\texttt{abstract type ref a} = \mathrm{id}{:}\mathbb{N}\{\texttt{witnessed}\ (\lambda\,\texttt{h}\,.\,\texttt{contains h id a})\}$$

  where

```
let contains (H h _) id a =
  match h id with
    | Used b _ → a = b
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- Finally, we define `MLST`'s **actions** using `MST`'s actions
  - `let alloc` (a:Type_0) (v:a) : `MLST (ref a) ... = ...`
    - **get** the current heap
    - **create** a fresh ref., and **add** it to the heap
    - **put** the updated heap back
    - **witness** that the created ref. is in the heap
  - `let read` (r:`ref a`) : `MLST t ... = ...`
    - **recall** that the given ref. is in the heap
    - **get** the current heap
    - **select** the given reference from the heap
  - `let write` (r:`ref a`) (v:a) : `MLST unit ... = ...`
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Adding untyped and monotonic references

- **Untyped references** (`uref`) with strong updates
  - Used `heap` cells are extended with **tags**

    where
    $$| \ \texttt{Used} : a{:}\texttt{Type}_0 \to v{:}a \to t{:}\texttt{tag} \to \texttt{cell}$$
    $$\texttt{type tag} \ = \ \texttt{Typed} : \texttt{tag} \ | \ \texttt{Untyped} : \texttt{tag}$$
  - actions corresponding to `uref`s have **weaker types** than for `ref`s

- **Monotonic references** (`mref a rel`)
  - Used `heap` cells are extended with **typed tags**

    where
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    - based on an instrumented operational semantics

$$(\mathtt{witness}\ x.\varphi\ ,\ s\ ,\ W)\ \rightsquigarrow\ (\mathtt{return}\ ()\ ,\ s\ ,\ W \cup \{x.\varphi\})$$

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- Recall from Kenji's talk that in F* an **abstract** ST **computation**

  $$\mathtt{e} : \mathtt{ST}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

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- In our POPL 2018 paper, we support reification and reflection by
  - indexing $\texttt{MST}_{\texttt{state},\texttt{rel},\texttt{b}}$ with a **boolean flag** `b` (reifiable?), and
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  so if `b = true` then `witness` and `recall` are **logically no-ops.**

- This **works** but leads to **duplication** of pre- and postconditions!

- Instead, ongoing work is taking (hybrid) **modal logic** seriously

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