Recalling a Witness

Foundations and Applications of Monotonic State

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joint work with

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Outline

- Monotonic state and program verification by example
- Key ideas behind our interface for monotonic state
- Adding monotonic state to F*
- Examples of monotonic state at work
- A glimpse of the meta-theory

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Monotonic state and program verification

- Consider a program operating on **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert } (\texttt{v} \in \texttt{get()})$$

- To prove the assertion (say, in a Floyd-Hoare style logic), we could prove that the code maintains a **stateful invariant**

$$\{\lambda\, \texttt{s} \,.\, \texttt{v} \in \texttt{s}\}\ \texttt{complex\_procedure()}\ \{\lambda\, \texttt{s} \,.\, \texttt{v} \in \texttt{s}\}$$

  - likely that we have to **carry** $\lambda\, \texttt{s} \,.\, \texttt{v} \in \texttt{s}$ **through** the proof of `c_p`
    - **sensitive** to proving that `c_p` maintains $\lambda\, \texttt{s} \,.\, \texttt{w} \in \texttt{s}$ for some other `w`
    - **does not guarantee** that $\lambda\, \texttt{s} \,.\, \texttt{v} \in \texttt{s}$ holds at every point in `c_p`

- However, if `c_p` **only inserts**, then $\lambda\, \texttt{s} \,.\, \texttt{v} \in \texttt{s}$ is **stable**, and we would like the program logic to give us $\texttt{v} \in \texttt{get()}$ "**for free**"

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Monotonicity is really useful!

- To come later in this talk
  - reasoning about **monotonic counters**
  - implementing **typed** (`ref t`) and **untyped references** (`uref`)
  - more flexibility with **monotonic references** (`mref t rel`)
- For other examples of the usefulness of monotonicity,

  Recalling a Witness:
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  which includes
  - a secure **file-transfer** application
  - pointers to works using monotonicity in **crypto** and **TLS verif.**
  - Ariadne **state continuity** protocol [Strackx, Piessens 2016]

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Overview of our solution

- We focus on **monotonic** programs and **stable** predicates
  - per verification task, we choose a **preorder** `rel` on states
    - set inclusion, heap inclusion, increasing counters, ...
  - a program $\mathtt{e}$ is **monotonic** (wrt. `rel`) when

$$(\mathtt{s}, \mathtt{e}) \rightsquigarrow^* (\mathtt{s}', \mathtt{e}') \implies \mathtt{rel}\ \mathtt{s}\ \mathtt{s}'$$

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$$\forall\, \mathtt{s}\, \mathtt{s}'.\ \mathtt{p}\ \mathtt{s} \ \wedge\ \mathtt{rel}\ \mathtt{s}\ \mathtt{s}' \implies \mathtt{p}\ \mathtt{s}'$$

- **Our solution:** extend Hoare-style program logics (e.g., F*) with
  - means for turning a $\mathtt{p}$ into a **state-independent proposition**
  - operation to **witness** the validity of $\mathtt{p}\ \mathtt{s}$ in some state $\mathtt{s}$
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Reasoning about ordinary state in F*

- An ML-like dependently typed language, aimed at verification
- F* supports Hoare-style reasoning about state via the **comp. type**

$$\texttt{ST t (requires pre) (ensures post)}$$

where

$\texttt{t : Type}$ $\quad$ $\texttt{pre : state} \to \texttt{Type}$ $\quad$ $\texttt{post : state} \to \texttt{t} \to \texttt{state} \to \texttt{Type}$

(formally, this type is derived from a WP calculus for state)

- The **get** and **put** actions are typed as follows

$$\texttt{get} : \texttt{unit} \to \texttt{ST state (requires } (\lambda \_ . \top)) \texttt{ (ensures } (\lambda\, \texttt{s}_0\, \texttt{s}\, \texttt{s}_1 . \texttt{s}_0 = \texttt{s} = \texttt{s}_1))$$

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- We capture monotonic state with a new **computation type**

$$\texttt{MST}\ \texttt{rel}\ \texttt{t}\ (\texttt{requires}\ \texttt{pre})\ (\texttt{ensures}\ \texttt{post})$$

  where `t`, `pre`, and `post` are typed as in `ST`

- The **get** action is typed as in `ST`

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- We introduce a **logical capability**

$$\texttt{witnessed} : \texttt{pred state} \to \texttt{Type}$$

  together with a **weakening** principle

$$\texttt{wk} : \texttt{p,q:pred state} \to \texttt{Lemma}\ (\texttt{requires}\ (\forall\, \texttt{s}\,.\,\texttt{p s} \implies \texttt{q s}))\ (\texttt{ensures}\ (\texttt{witnessed p} \implies \texttt{witnessed q}))$$

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The motivating example revisited

- Recall the program operating on **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert } (\texttt{v} \in \texttt{get()})$$

  - We pick **set inclusion** $\subseteq$ as our preorder on states
  - We **prove the assertion** by adding a `witness` and a `recall`

$$\texttt{insert v; witness } (\lambda\, \texttt{s}\,.\,\texttt{v} \in \texttt{s})\texttt{; c\_p(); recall } (\lambda\, \texttt{s}\,.\,\texttt{v} \in \texttt{s})\texttt{; assert } (\texttt{v} \in \texttt{get()})$$

  - For any other `w`, wrapping

$$\texttt{insert w; [ ]; assert } (\texttt{w} \in \texttt{get()})$$

    around the program is handled similarly easily

- **Monotonic counters** are analogous, with $\mathbb{N}$ and $\leq$

$$\texttt{create 0; incr(); witness } (\lambda\, \texttt{c}\,.\,\texttt{c} > 0)\texttt{; c\_p(); recall } (\lambda\, \texttt{c}\,.\,\texttt{c} > 0)$$

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- **Monotonic counters** are analogous, with $\mathbb{N}$ and $\leq$

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References: both typed and untyped

- We define **local state** using global state + monotonicity
- We define **heaps** as maps

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type heap =
  | H : h:(ℕ → cell) → ctr:ℕ{∀ n . ctr ≤ n ⟹ h n = Unused} → heap
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  type cell = Unused : cell | Used : a:Type → v:a → t:tag → cell
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let rel (H h0 _) (H h1 _) = ∀ id . match h0 id, h1 id with
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- **Typed references** are defined as

$$\texttt{abstract type ref t} = \texttt{id}{:}\mathbb{N}\{\texttt{witnessed}\ (\lambda \texttt{h}\,.\,\texttt{has\_used\_typed id t h})\}$$

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- The state actions for **typed references** use **witness** and **recall**
  - `let alloc t (v:t) : MST (ref t) ... = ...`
    - **get** the current heap (using global state `get`)
    - **create** a fresh ref., and **add** it to the heap
    - **put** the updated heap back (using global state `put`)
    - **witness** that the created ref. is in the heap
  - `let read t (r:ref t) : MST t ... = ...`
    - **recall** that the given ref. is in the heap
    - **get** the current heap (using global state `get`)
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  - `let write t (r:ref t) (v:t) : MST unit ... = ...`
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    - **get** the current heap (using global state `get`)
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Monotonic references: more flexibility

- The heap now associates a **local preorder** with each reference

$$\texttt{type tag a} = \texttt{Typed} : \texttt{rel:preorder a} \to \texttt{tag a} \mid \texttt{Untyped} : \texttt{live:bool} \to \texttt{tag a}$$

- The **global preorder** is a point-wise lifting of the individual ones

$$\begin{aligned}&\texttt{let rel } (\texttt{H h}_0\ \_)\ (\texttt{H h}_1\ \_) = \forall\, \texttt{id}.\, \texttt{match h}_0\ \texttt{id}, \texttt{h}_1\ \texttt{id with}\\ &\quad \mid \texttt{Used a}_0\ \texttt{v}_0\ (\texttt{Typed rel}_0),\\ &\quad\ \texttt{Used a}_1\ \texttt{v}_1\ (\texttt{Typed rel}_1) \to \texttt{a}_0 = \texttt{a}_1 \wedge \texttt{rel}_0 = \texttt{rel}_1 \wedge \texttt{rel}_0\ \texttt{v}_0\ \texttt{v}_1\\ &\quad \mid \ldots\end{aligned}$$

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- State actions
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Outline

- Monotonic state and program verification by example
- Key ideas behind our interface for monotonic state
- Adding monotonic state to F*
- Examples of monotonic state at work
- A glimpse of the meta-theory

A glimpse of the meta-theory

- We formalize `MST` in a small dependently typed CBV calculus

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\begin{aligned}
t &::= \mathsf{state} \mid x{:}t_1 \to \mathbf{Tot}\ t_2 \mid x{:}t_1 \to \mathbf{MST}\ t_2\ (s.\varphi_{\mathsf{pre}})\ (s.y.s'.\varphi_{\mathsf{post}}) \mid \ldots \\
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- **Consistency** and **props. of the logic** via seq. calc. and cut-adm.

- **Operational semantics** on configurations (e, σ, W)

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(\mathsf{witness}\ s.\varphi, \sigma, W) &\rightsquigarrow (\mathsf{return}\ (), \sigma, W \cup \{s.\varphi\}) \\
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- **Total correctness** via progress, preservation, and SN

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$$

$$
\Longrightarrow
$$

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(e, \sigma, W) \rightsquigarrow^* (\mathsf{return}\ v, \sigma', W') \qquad \vdash v : t
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&\quad \Longrightarrow \\
&\quad (e, \sigma, W) \rightsquigarrow^* (\text{return}\ v, \sigma', W') \quad \vdash v : t \\
&\quad\quad W \subseteq W' \quad \text{witnessed}\ W' \vdash \text{rel}\ \sigma\ \sigma' \\
&\quad\quad \text{witnessed}\ W' \vdash \varphi_{\text{post}}[\sigma/s, v/x, \sigma'/s']
\end{aligned}
$$

A glimpse of the meta-theory

- We formalize `MST` in a small dependently typed CBV calculus

$$
\begin{aligned}
t &::= \mathsf{state} \mid x{:}t_1 \to \mathbf{Tot}\ t_2 \mid x{:}t_1 \to \mathbf{MST}\ t_2\ (s.\varphi_{\mathsf{pre}})\ (s.y.s'.\varphi_{\mathsf{post}}) \mid \ldots \\
e &::= \mathsf{get} \mid \mathsf{put}\ v \mid \mathsf{witness}\ s.\varphi \mid \mathsf{recall}\ s.\varphi \mid \ldots \\
\varphi &::= \mathsf{rel}\ v_1\ v_2 \mid \mathsf{witnessed}\ s.\varphi \mid \ldots
\end{aligned}
$$

  - **Consistency** and **props. of the logic** via seq. calc. and cut-adm.
  - **Operational semantics** on configurations (e, σ, W)

$$
\begin{aligned}
(\mathsf{witness}\ s.\varphi, \sigma, W) &\rightsquigarrow (\mathsf{return}\ (), \sigma, W \cup \{s.\varphi\}) \\
(\mathsf{recall}\ s.\varphi, \sigma, W) &\rightsquigarrow (\mathsf{return}\ (), \sigma, W)
\end{aligned}
$$

  - **Total correctness** via progress, preservation, and SN

$$
\begin{array}{l}
\vdash e : \mathbf{MST}\ t\ (s.\varphi_{\mathsf{pre}})\ (s.x.s'.\varphi_{\mathsf{post}}) \\
\quad \mathsf{witnessed}\ W \vdash \varphi_{\mathsf{pre}}[\sigma/s]
\end{array}
\quad \Longrightarrow \quad
\begin{array}{l}
(e, \sigma, W) \rightsquigarrow^* (\mathsf{return}\ v, \sigma', W') \quad \vdash v : t \\
\quad W \subseteq W' \quad \mathsf{witnessed}\ W' \vdash \mathsf{rel}\ \sigma\ \sigma' \\
\quad \mathsf{witnessed}\ W' \vdash \varphi_{\mathsf{post}}[\sigma/s, v/x, \sigma'/s']
\end{array}
$$

Conclusion

- In conclusion
  - making use of monotonicity is quite useful in verification
  - using monotonicity can be distilled into a simple interface
  - useful for both programming (refs.) and verification (crypto,TLS)

- Not in this talk (see the draft paper on arXiv)
  - temporarily **escaping the preorder** via snapshots
  - **revealing the representation** via selective monadic reification

- Future work
  - extending F* with indexed effects
  - combining preorders (e.g., ala graded monads)
  - modal aspects of `witnessed p`
  - connections with other works, e.g., Iris and [Pilkiewicz,Pottier'11]

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Thank you!

Questions?

Recalling a Witness:
Foundations and Applications of Monotonic State

(arXiv:1707.02466)