

Comodels as a gateway for interacting with the **external world**

Danel Ahman

(joint work with Andrej Bauer)

Shonan, 27 March 2019

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Computational effects in FP

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- Using **monads** (as in HASKELL)

```
type St a = String → (a, String)
```

```
f :: St a → St (a, a)
```

```
f c = c >>= (\x → c >>= (\y → return (x, y)))
```

- Using **alg. effects** and **handlers** (as in EFF, FRANK, KOKA)

```
effect Get : int
```

```
effect Put : int → unit
```

```
(*: int → a*int!{ } *)
```

```
let g (c: unit → a!{ Get, Put }) =
```

```
  with st_h handle (perform (Put 42); c ())
```

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```

```
  with st_h handle (perform (Put 42); c ())
```

- Both are good for **faking comp. effects** in a pure language!

But what about effects that need access to the **external world**?

External world in FP

- Declare a **signature** of monads or algebraic effects

```
type IO a
```

```
openFile  :: FilePath → IOMode → IO Handle
```

```
hGetLine  :: Handle → IO String
```

```
hClose    :: Handle → IO ()
```

```
effect Read  : string
```

```
effect Raise : string → empty
```

```
effect RandomInt    : int → int
```

```
effect RandomFloat : float → float
```

- And then treat them **specially** in the compiler, e.g.,

```
let rec top_handle op =
```

```
    match op with
```

```
    | ...
```

External world in FP

External world in FP



Ohad 🤖 12:17 PM

Can I do file IO (or just O) in Eff?

External world in FP



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Žiga Lukšič 12:18 PM

not currently

External world in FP



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Ohad 8:35 PM

So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
| "Write" ->
  (match v with
  | V.Tuple vs ->
    let (file_name :: str :: _) = List.map V.to_str vs in
    let file_handle = open_out_gen
      [Open_wronly
       ;Open_append
       ;Open_creat
       ;Open_text
      ] 0o666 file_name in
    Printf.fprintf file_handle "%s" str;
    close_out file_handle;
    top_handle (k V.unit_value)
  )
```

External world in FP



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```

This talk — a principled (co)algebraic approach!

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- ```
let f (s:string) =
 let fh = fopen "foo.txt" in
 fwrite fh (s^s);
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 return fh

let g s =
 let fh = f s in fread fh
```

## Another issue — **linearity** or lack thereof

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- Even worse when we wrap `f` in a **handler**?

```
let h = handler
 | effect (FWrite fh s k) → return ()

let g' s =
 with h handle f ()
```

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- Even worse when we wrap `f` in a **handler**?

```
let h = handler  
  | effect (FWrite fh s k) → return ()  
  
let g' s =  
  with h handle f ()          (* dangling fh ! *)
```


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- We could try using **existing PL techniques**, e.g.,

- **Modules** and **abstraction**, e.g., `System.IO`

```
type IO a
```

```
hClose :: Handle → IO ()
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- **Linear** (and **non-linear**) **types** and **effects**

```
linear type fhandle
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effect FClose : (linear fhandle) → unit
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linear effect FClose : fhandle → unit
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- Handlers with **finally clauses**

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- Handlers with **finally clauses**
- **Problem:** They don't really capture the **essence of the problem**

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- A common explanation is to think of functions

$$a \rightarrow \text{IO } b$$

as

$$a \rightarrow (\text{RealWorld} \rightarrow (b, \text{RealWorld}))$$

which is the same as

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- With the `System.IO` **module abstraction** ensuring that
 - We **cannot get our hands on** `RealWorld`
 - We have the impression of `RealWorld` **used linearly**
 - We **don't ask more** from `RealWorld` than it can provide

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But wait a minute! **RealWorld** looks a lot like a **comodel**!

`hGetLine` : $(\text{Handle}, \text{RealWorld}) \rightarrow (\text{String}, \text{RealWorld})$

`hClose` : $(\text{Handle}, \text{RealWorld}) \rightarrow ((), \text{RealWorld})$

Important: co-operations (`hClose`) make a **promise to return**!

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- Intutively, comodels describe **evolution of the world W**
 - Operational semantics using a tensor of a model and a comodel
(Plotkin & Power, Abou-Saleh & Pattinson)
 - Stateful runners of effectful programs (Uustalu)
 - Linear state-passing translation (Møgelberg and Staton)
 - Top-level behaviour of alg. effects in EFF v2 (Bauer & Pretnar)

Comodels as a gateway to the **external world**

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- ```
let f (s:string) =
 using IO cohandle
 let fh = fopen "foo.txt" in
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 (\* in IO \*)

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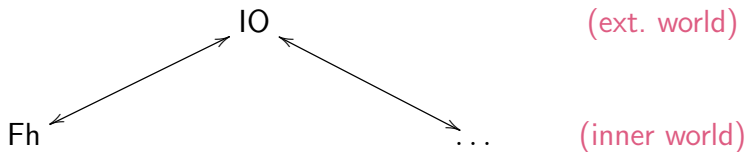
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- **Solution:** **Modular treatment** of **external worlds**

## Comodels as a gateway to the external world

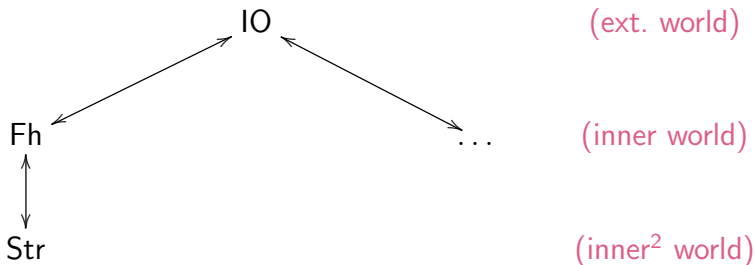
- Examples of **modularity** we might want from comodels



- Fh — “**world** which consists of **exactly one** fh ”
- IO  $\longrightarrow$  Fh — “call `fopen` with `foo.txt` , store returned fh ”
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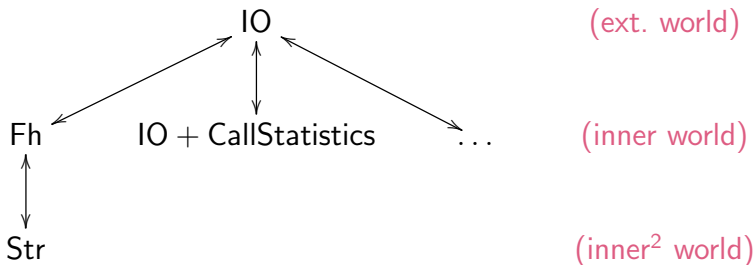
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- IO → Fh — “call fopen with foo.txt, store returned fh”
- Fh → IO — “call fclose with stored fh”
- Str — “world that is **blissfully unaware** of fh”

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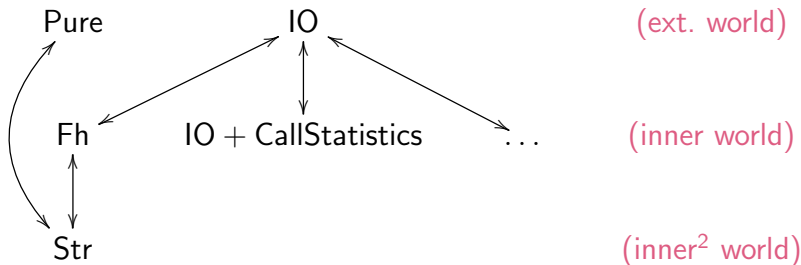
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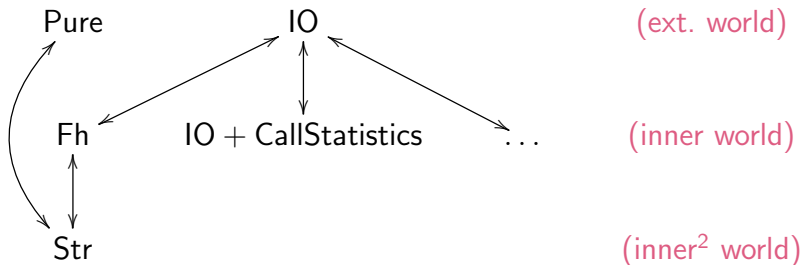
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# Comodels as a gateway to the external world

- Examples of **modularity** we might want from comodels



- Fh** — “**world** which consists of **exactly one** **fh**”
- IO**  $\longrightarrow$  **Fh** — “call **fopen** with **foo.txt**, store returned **fh**”
- Fh**  $\longrightarrow$  **IO** — “call **fclose** with stored **fh**”
- Str** — “world that is **blissfully unaware** of **fh**”
- Observation:** **IO**  $\longleftrightarrow$  **Fh** and other  $\longleftrightarrow$  look a lot like **lenses**

**Comodels** as a gateway to the **external world**

## Comodels as a gateway to the external world

- Our **general framework** on the file operations example

```
let f (s:string) =
 using
 Fh @ (fopen_of_io "foo.txt")
 cohandle
 fwrite_of_fh (s^s)
 finally
 x @ fh → fclose_of_io fh
```



## Comodels as a gateway to the external world

- Our **general framework** on the file operations example

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let f (s:string) = (* in IO *)
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 Fh @ (fopen_of_io "foo.txt") (* in IO *)
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```

where

```
Fh = (* W = fhandle *)
{ co_fread _ @ fh → ... ,
 co_fwrite s @ fh → fwrite_of_io s fh ;
 return ((),fh) }

(* co_fread : (unit * W) → (string * W) *)
(* co_fwrite : (string * W) → (unit * W) *)
```

**Comodels** as a gateway to the **external world**

## Comodels as a gateway to the external world

- The **modularity aspect** of our general framework

```
let f (s:string) = (* in IO *)
 using Fh @ (fopen_of_io "foo.txt")
 cohandle

 using Str @ (fread_of_fh ()) (* in Fh *)
 cohandle
 write_of_str (s^s) (* in Str *)
 finally
 _ @ s → fwrite_of_fh s

 finally
 _ @ fh → fclose_of_io fh
```

where

```
Str = { co_write s @ s' → (* W = string *)
 return ((), s'^s) }
```

**Comodels** as a gateway to the **external world**

## Comodels as a gateway to the external world

- Comodels can also **extend** the (intermediate) external world

```
let f (s:string) = (* in IO *)
 using Stats @ (fopen_of_io "foo.txt")
 cohandle
 fwrite_of_stats (s^s)
 finally
 - @ (fh,c) →
 let fh' = fopen_of_io "stats.txt" in
 fwrite_of_io fh' c; fclose_of_io fh';
 fclose_of_io fh
```

where

```
Stats = (* W = fhandle * nat*)
{ co_fread - @ (fh,c) → ... ,
 co_fwrite s @ (fh,c) → ... ,
 co_reset - @ (fh,c) → return ((),(fh,0)) }
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where

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Stats = (* W = fhandle * nat*)
{ co_fread - @ (fh,c) → ... ,
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```

- Can also track **nondet./prob. choice results**, etc

**Comodels** as a gateway to the **external world**



## Comodels as a gateway to the external world

- The external world could also be **pure**

```
let f (s:string) = (* in Pure *)
 using Str @ (return "default value")
 cohandle
 ...
 let s = read_of_str () in
 if (s == "foo")
 then (...; write_of_str "bar"; ...)
 else (...)
 finally
 x @ s → return x
```

where

```
Str = (* W = string *)
 { co_read _ @ s → return (s,s) ,
 co_write s @ s' → return ((),s') }
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$$\omega ::= \{ \text{op}_1 : A_1 \rightsquigarrow B_1, \dots, \text{op}_n : A_n \rightsquigarrow B_n \}$$

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- **Computation terms** (value terms are unsurprising)

$$\begin{aligned} c ::= & \text{ return } v \mid \text{ let } x = c_1 \text{ in } c_2 \mid v_1 v_2 \\ & \mid \widehat{\text{op}} \ v \\ & \mid \text{ using } C @ c_i \text{ cohandle } c \text{ finally } x @ w \rightarrow c_f \end{aligned} \quad \text{(comodel op.)}$$

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- Comodels (cohandlers)**

$$C ::= \{ \overline{\text{op}}_1 \ x @ w \rightarrow c_1, \dots, \overline{\text{op}}_n \ x @ w \rightarrow c_n \}$$

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- **Typing judgements**

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- The two central **typing rules** are

$$\Gamma \Vdash D \text{ comodel of } \omega' \text{ with carrier } W_D \quad \Gamma \Vdash c_i : W_D$$

$$\Gamma \Vdash c : A \quad \Gamma, x:A, w:W_D \Vdash c_f : B$$

---

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and

$$\text{op} : A \rightsquigarrow B \in \omega \quad \Gamma \vdash v : A$$

---

$$\Gamma \Vdash \hat{\text{op}} v : B$$

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- **Term interpretation** looks very similar to **alg. effects**:

$$\llbracket \Gamma \vdash v : A \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow \llbracket A \rrbracket \qquad \llbracket \Gamma \stackrel{\omega}{\vdash} c : A \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow T_{\omega} \llbracket A \rrbracket$$

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- **un-cohandled operations wait for a suitable external world!**
- The interesting part is the interpretation of **cohandling**

$$\Gamma \vDash^{\omega} \text{using } D @ c_i \text{ cohandle } c \text{ finally } x @ w \rightarrow c_f : B$$

which is based on the **linear state-passing translation**, i.e.,

$$\frac{\llbracket D \rrbracket \in \text{Comod}_{\omega'}(\text{Kleisli}(T_{\omega}))}{\text{cohandle\_with}_{\llbracket D \rrbracket} : T_{\omega'} \llbracket A \rrbracket \longrightarrow \left( \llbracket W_D \rrbracket \rightarrow T_{\omega} (\llbracket A \rrbracket \times \llbracket W_D \rrbracket) \right)}$$

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- The interpretation of **operations** uses the **co-operations** of Cs

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- Where do **multi-handlers** fit? Co-operating handlers-cohandlers?

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**using**

D @ c\_i

**cohandle**

fwrite\_of\_d s;

(\* co\_fwrite throws e \*)

fread ()

**finally**

| x @ w → c\_f

| throw e → c\_do\_some\_cleanup

| op x k → ...

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## Some ongoing work

- Interaction with **algebraic effects** and **(multi-)handlers**
- Clarify the connection with **(effectful) lenses**
- **Combinatorics** of comodels and their lens-like relationships