Comodels as a gateway for interacting with the external world

Danel Ahman

(joint work with Andrej Bauer)

Shonan, 26 March 2019

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Computational effects in FP

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- Using **monads** (e.g., as in HASKELL)

```
type St a = String → (a, String)

f :: St a → St (a,a)
f c = c >>= (\x → c >>= (\y → return (x,y)))
```

- Using **algebraic effects** and **handlers** (e.g., as in EFF)

```
effect Get : int
effect Put : int → unit
                                (*: int → a*int!{} *)
let g (c:unit → a!{Get,Put}) =
  with st_h handle (perform (Put 42); c ())
```

Computational effects in FP

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                                (*: int → a*int!{} *)
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  with st_h handle (perform (Put 42); c ())
```

- Works well for effects that can be **represented** as pure data!

  But what about effects that need access to the **external world**?

External world in FP

- Declare a **signature** of monads or algebraic effects

```
type IO a

openFile  :: FilePath → IOMode → IO Handle
hGetLine  :: Handle → IO String
hClose    :: Handle → IO ()
```

```
effect Read   : string
effect Raise  : string → empty

effect RandomInt    : int → int
effect RandomFloat  : float → float
```

External world in FP

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effect Read   : string
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effect RandomInt    : int → int
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- And then treat them **specially** in the compiler, e.g.,

```
let rec top_handle op =
  match op with
  | ...
```

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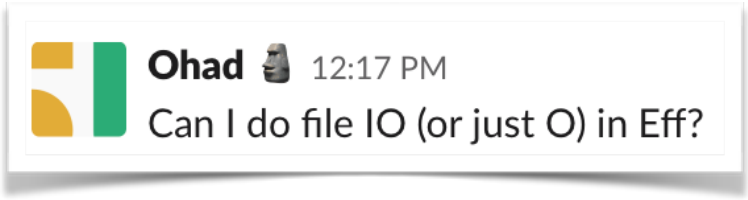

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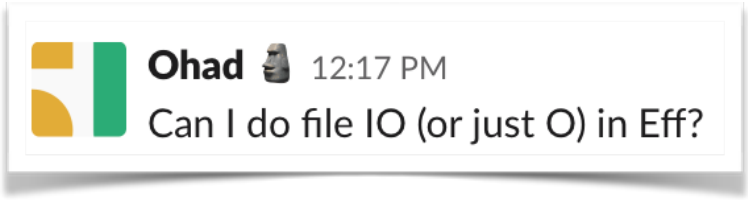

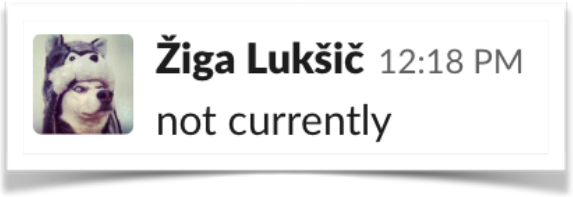

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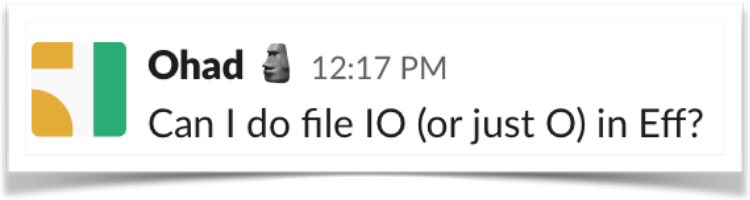
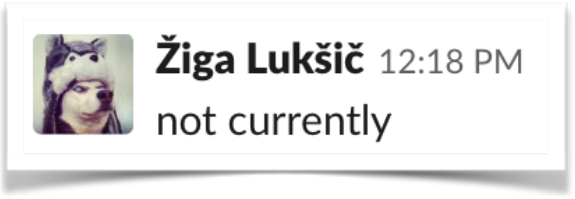




Ohad 8:35 PM

So here's the hack I added. We should do something a bit more principled

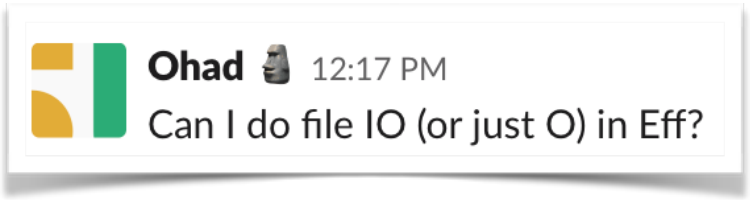
In `pervasives.eff`:

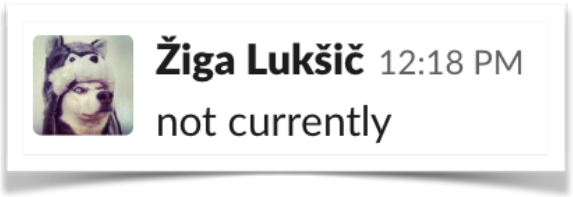
```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
    | "Write" ->
       (match v with
        | V.Tuple vs ->
           let (file_name :: str :: _)  = List.map V.to_str vs in
           let file_handle = open_out_gen
                             [Open_wronly
                             ;Open_append
                             ;Open_creat
                             ;Open_text
                             ] 0o666 file_name in
           Printf.fprintf file_handle "%s" str;
           close_out file_handle;
           top_handle (k V.unit_value)
       )
```

External world in FP





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This talk — a principled (co)algebraic approach!

Another issue — linearity or lack thereof

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- ```
  let f (s:string) =
    let fh = fopen "foo.txt" in
    fwrite fh (s^s);
    fclose fh;
    return fh

  let g s =
    let fh = f s in fread fh
  ```
  ```

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- We could resolve this by typing `fh` **linearly** (but `s` **non-linearly**)

- But what if we wrap `f` in a **handler**?

  ```
  let h = handler
          | effect (FWrite fh s k) → return fh

  let g s = with h handle f ()
  ```
  ```

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- But what if we wrap `f` in a **handler**?

  ```
  let h = handler
          | effect (FWrite fh s k) → return fh

  let g s = with h handle f () (* dangling fh ! *)
  ```
  ```

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- We could try using **existing programming mechanisms**, e.g.,
  - **Modules** and **abstraction**, e.g., `System.IO`

```
type IO a

hClose :: Handle → IO ()
```

  - **Linear** (and **non-linear**) **types** and **effects**

```
linear type fhandle

effect FClose : (linear fhandle) → unit

linear effect FClose : fhandle → unit
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  - Handlers with **finally** **clauses**

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  - Handlers with **finally clauses**

- **Problem:** They don't really capture the **essence of the problem**

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$$\mathsf{a} \to \mathsf{IO\ b}$$

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which is the same as

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- With the `System.IO` **module abstraction** ensuring that
  - We **cannot get our hands on** RealWorld
  - We have the impression of RealWorld **used linearly**
  - We **don't ask more** from RealWorld than it can provide

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But wait a minute! RealWorld looks a lot like a **comodel**!

hGetLine : (Handle, RealWorld) → (String, RealWorld)

hClose : (Handle, RealWorld) → ((), RealWorld)

I.e., **IO** is about the **external world** rather than internal effects!

Important: co-operations (hClose) make a **promise to return**!

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- Intutively, comodels describe **evolution of the world** W
  - Operational semantics using a tensor of a model and a comodel (Plotkin & Power, Abou-Saleh & Pattinson)
  - Stateful runners of effectful programs (Uustalu)
  - Linear state-passing translation (Møgelberg and Staton)
  - Top-level behaviour of alg. effects in Eff v2 (Bauer & Pretnar)

Comodels as a gateway to the external world

- ```
  let f (s:string) =
    using IO cohandle
      let fh = fopen "foo.txt" in
      fwrite fh (s^s);
      fclose fh                    (* in IO *)
  ```

  Now **external world** explicit, but **dangling** `fh` etc **still possible**
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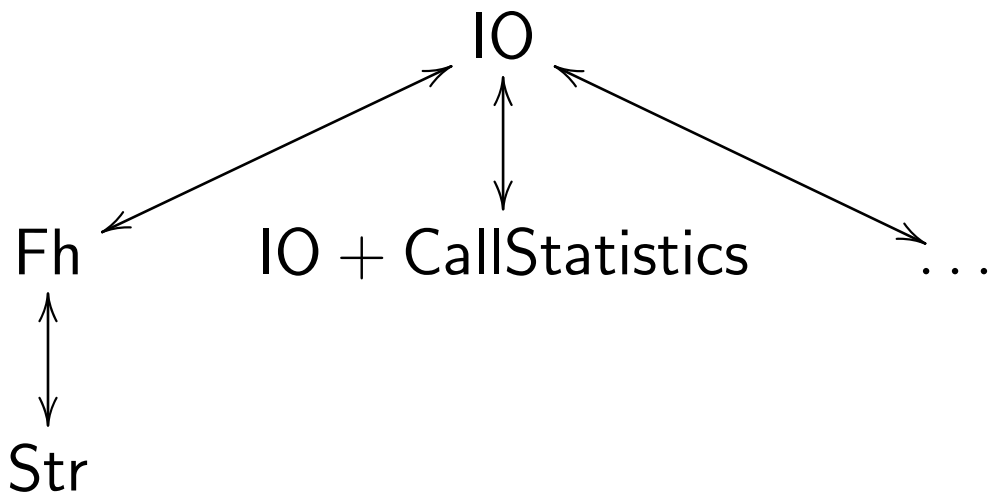
- **Solution: Modular treatment** of **external worlds**

Comodels as a gateway to the **external world**

- Examples of **modularity** we might want from comodels



- Fh — "**world** which consists of **exactly one** `fh`"
- IO $\longrightarrow$ Fh — "call `fopen` with `foo.txt`, store returned `fh`"
- Fh $\longrightarrow$ IO — "call `fclose` with stored `fh`"

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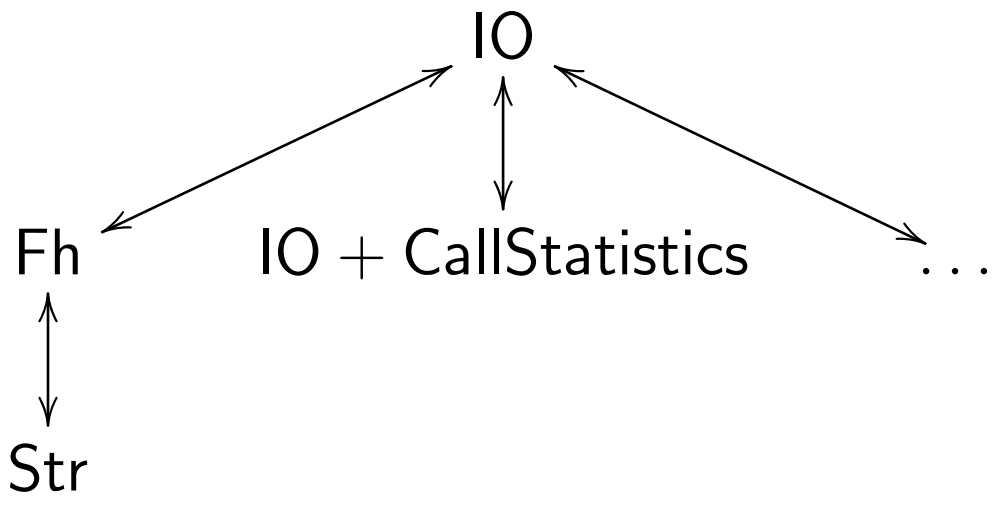

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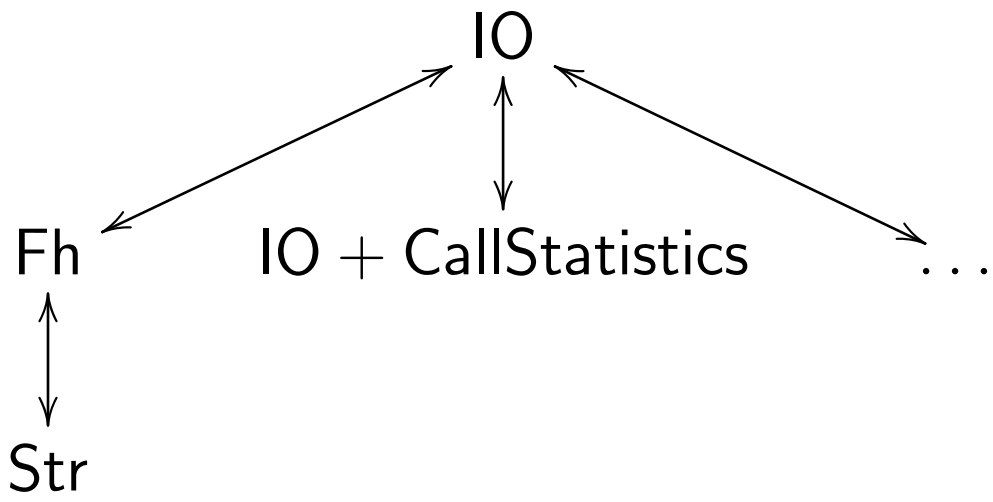

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- Fh $\longrightarrow$ IO — "call `fclose` with stored `fh`"
- Str — "world that is **blissfully unaware** of `fh`"
- **Observation:** IO $\longleftrightarrow$ Fh and other $\longleftrightarrow$ look a lot like **lenses**

Comodels as a gateway to the external world

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- Our **general framework** on the file operations example

```
let f (s:string) =
  using
    Fh @ (fopen_of_io "foo.txt")
  cohandle
    fwrite_of_fh (s^s)
  finally
    x @ fh → fclose_of_io fh
```

Comodels as a gateway to the external world

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  using
    Fh @ (fopen_of_io "foo.txt")                (* in IO *)
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    fwrite_of_fh (s^s)                              (* in Fh *)
  finally
    x @ fh → fclose_of_io fh                        (* in IO *)
```

where

```
Fh =                                                (* W = fhandle *)
 { co_fread   _ @ fh → ...,
   co_fwrite  s @ fh → fwrite_of_io s fh;
                       return ((),fh)          }

    (* co_fread  : (unit * W) → (string * W) *)
   (* co_fwrite : (string * W) → (unit * W) *)
```

Comodels as a gateway to the external world

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- The **modularity aspect** of our general framework

```
let f (s:string) =
  using Fh @ (fopen_of_io "foo.txt")
  cohandle

    using Str @ (fread_of_fh ())
    cohandle
      write_of_str (s^s)
    finally
      _ @ s → fwrite_of_fh s

  finally
    _ @ fh → fclose_of_io fh
```

where

```
Str = { co_write s @ s' →          (* W = string *)
            return ((),s'^s) }
```

Comodels as a gateway to the external world

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- Comodels can also **extend** the (intermediate) external world

```
let f (s:string) =
  using Stats @ (fopen_of_io "foo.txt")
  cohandle
    fwrite_of_stats (s^s)
  finally
    _ @ (fh,c) →
          let fh' = fopen_of_io "stats.txt" in
          fwrite_of_io fh' c; fclose_of_io fh';
          fclose_of_io fh
```

where

```
Stats =                              (* W = fhandle * string *)
  { co_fread   _ @ (fh,c) → ...,
    co_fwrite  s @ (fh,c) → ...,
    co_reset   _ @ (fh,c) → return ((),(fh,0)) }
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- Can also track **nondet./prob. choice results**, etc

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$$\omega \;::=\; \{\, \mathsf{op}_1 : A_1 \rightsquigarrow B_1 \;,\; \ldots \;,\; \mathsf{op}_n : A_n \rightsquigarrow B_n \,\}$$

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- **Terms**

$$v \;::=\; x \mid \ldots$$

$$c \;::=\; \mathbf{return}\ v \mid \mathbf{let}\ x = c_1\ \mathbf{in}\ c_2 \mid v_1 v_2 \mid$$
$$\mathrm{op}\ v \mid \qquad \text{(comodel op.)}$$
$$\mathbf{using}\ C\ @\ c_i\ \mathbf{cohandle}\ c\ \mathbf{finally}\ x\ @\ w \to c_f$$

(simple setting, only comodel ops. and no handlers (wait few slides))

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- **Comodels (cohandlers)**

$$C \;::=\; \{\, \overline{\mathrm{op}}_1\ x\ @\ w \to c_1 \;,\; \ldots \;,\; \overline{\mathrm{op}}_n\ x\ @\ w \to c_n \,\}$$

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- The two central **typing rules** are

$$\frac{\Gamma \vDash^{\omega} \mathsf{D} \text{ comodel of } \omega' \text{ with carrier } W_{\mathsf{D}} \qquad \Gamma \vDash^{\omega} c_i : W_{\mathsf{D}} \qquad \Gamma \vDash^{\omega'} c : A \qquad \Gamma, x : A, w : W_{\mathsf{D}} \vDash^{\omega} c_f : B}{\Gamma \vDash^{\omega} \textbf{using } \mathsf{D} \textbf{ @ } c_i \textbf{ cohandle } c \textbf{ finally } x \textbf{ @ } w \to c_f : B}$$

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- The two central **typing rules** are

$$\frac{\Gamma \vDash^{\omega} \mathsf{D} \text{ comodel of } \omega' \text{ with carrier } W_{\mathsf{D}} \qquad \Gamma \vDash^{\omega} c_i : W_{\mathsf{D}} \qquad \Gamma \vDash^{\omega'} c : A \qquad \Gamma, x : A, w : W_{\mathsf{D}} \vDash^{\omega} c_f : B}{\Gamma \vDash^{\omega} \textbf{using } \mathsf{D} \textbf{ @ } c_i \textbf{ cohandle } c \textbf{ finally } x \textbf{ @ } w \to c_f : B}$$

and

$$\frac{\mathrm{op} : A \rightsquigarrow B \in \omega \qquad \Gamma \vdash v : A}{\Gamma \vDash^{\omega} \mathrm{op}\ v : B}$$

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- The interesting part is the interpretation of

$$\Gamma \vdash^{\omega} \textbf{using } \mathsf{D} \mathbin{@} c_i \textbf{ cohandle } c \textbf{ finally } x \mathbin{@} w \to c_f : B$$

  which is based on the **linear state-passing translation**, i.e.,

$$\frac{[\![\mathsf{D}]\!] \in \mathsf{Comod}_{\omega'}(\mathsf{Kleisli}(T_{\omega}))}{\mathsf{cohandle\_with}_{[\![\mathsf{D}]\!]} : T_{\omega'}\,[\![A]\!] \longrightarrow \Big([\![W_{\mathsf{D}}]\!] \to T_{\omega}\,([\![A]\!] \times [\![W_{\mathsf{D}}]\!])\Big)}$$

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- The interpretation of **operations** uses the **co-operations** of Cs

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  it is natural to expect that

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- How do **multi-handlers** fit? Co-operating handlers-cohandlers?

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using
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  | throw e → c_do_some_cleanup
  | op x k → ...
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- **Principle:** Alg. ops. only allowed to appear in co-operations

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Ongoing work

- Iron out all the wrinkles in the **formal details**
- **Algebraic effects** and **(multi-)handlers**
- Clarify the connection with **(effectful) lenses**
- **Combinatorics** of comodels and their lens-like relationships
- **Prototype** in Eff