Comodels as a gateway for interacting with the external world

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MSR Redmond, 15 May 2019

A modular programming abstraction for using external resources

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Computational effects in FP

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- Using **monads** (as in Haskell)

```haskell
type St a = String → (a, String)

f :: St a → St (a,a)
f c = c >>= (\x → c >>= (\y → return (x,y)))
```

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- Using **alg. effects** and **handlers** (as in EFF, FRANK, KOKA)

```
effect Get : int
effect Put : int → unit
                                   (*: int → a*int!{} *)
let g (c:unit → a!{Get,Put}) =
  with st_h handle (perform (Put 42); c ())
```

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```

- Both are good for **faking comp. effects** in a pure language!

  But what about effects that need access to the **external world**?

External world in FP

- Declare a **signature of monads** or **algebraic effects**, e.g.,

```
(* System.IO *)
type IO a
openFile :: FilePath → IOMode → IO Handle
```

```
(* pervasives.eff *)
effect RandomInt   : int → int
effect RandomFloat : float → float
```

- And then **treat them specially** in the compiler, e.g.,

```
(* eff/src/backends/runtime/eval.ml *)
let rec top_handle op =
  match op with
  | ...
```

but . . .

An issue — difficult to cover all use cases

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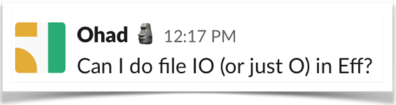

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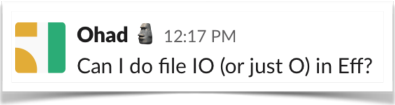

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not currently

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So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
    | "Write" ->
      (match v with
       | V.Tuple vs ->
         let (file_name :: str :: _)  = List.map V.to_str vs in
         let file_handle = open_out_gen
                             [Open_wronly
                             ;Open_append
                             ;Open_creat
                             ;Open_text
                             ] 0o666 file_name in
         Printf.fprintf file_handle "%s" str;
         close_out file_handle;
         top_handle (k V.unit_value)
      )
```

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This talk — a principled modular (co)algebraic approach!

A bigger issue — linearity or lack thereof

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- ```
  let f (s:string) =
    let fh = fopen "foo.txt" in
    fwrite fh (s^s);
    fclose fh;
    return fh

  let g s =
    let fh = f s in fread fh
  ```
  ```

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- Even worse when we wrap `f` in a **handler**?

  ```
  let h = handler
          | effect (fwrite fh s k) ↦ return ()

  let g' s =
    with h handle f ()
  ```
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- Even worse when we wrap `f` in a **handler**?

  ```
  let h = handler
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  let g' s =
    with h handle f ()          (* dangling fh ! *)
  ```
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- We could try using **existing PL techniques**, e.g.,
  - **Modules** and **abstraction**, e.g., `System.IO`

    ```
    type IO a

    hClose :: Handle → IO ()
    ```

  - **Linear** (and **non-linear**) **types** and **effects**

    ```
    linear type fhandle

    effect FClose : (linear fhandle) → unit

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  - Handlers with `initially` and `finally` **clauses**

So, how could we solve these issues?

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  - Handlers with **initially** and **finally** **clauses**
- **Problem:** They don't really capture the **essence of the problem**

Algebraic digression: What's a **comodel**?

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- Intutively, comodels describe **evolution of worlds** $w_1, w_2, w_3, \ldots$

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- Intutively, comodels describe **evolution of worlds** $w_1, w_2, w_3, \ldots$
  - Operational semantics using a tensor of a model and a comodel (Plotkin & Power, Abou-Saleh & Pattinson)
  - Stateful runners of effectful programs (Uustalu)
  - Linear state-passing translation (Møgelberg and Staton)
  - Top-level behaviour of alg. effects in Eff v2 (Bauer & Pretnar)

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  $$\mathsf{a} \to \mathsf{IO\ b}$$

  as

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- With the `System.IO` **module abstraction** ensuring that
  - We **cannot get our hands on** RealWorld (no `get` and `put`)
  - We have the impression of RealWorld **used linearly**
  - We **don't ask more** from RealWorld than it can provide

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But wait a minute! RealWorld looks a lot like a **comodel**!

$$\text{hGetLine} : (\text{Handle}, \text{RealWorld}) \to (\text{String}, \text{RealWorld})$$

$$\text{hClose} \quad : (\text{Handle}, \text{RealWorld}) \to ((), \text{RealWorld})$$

Important: co-operations (hClose) make a **promise to return**!

Towards a general programming abstraction

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- ```
  let f (s:string) =          (* in top level world *)
    using IO run
      let fh = fopen "foo.txt" in
      fwrite fh (s^s);
      fclose fh                   (* in IO world *)
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  Now **external world** explicit, but **dangling** `fh` etc **still possible**
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- **Our solution: Modular treatment of external worlds**

Modular treatment of **external worlds**

- **For example**

$$\begin{tikzcd} & \mathsf{IO} & & & (\text{ext. world}) \\ \mathsf{Fh} \arrow[ur, <->] & & & & (\text{inner world}) \end{tikzcd}$$

- Fh — "**world** which consists of **exactly one** `fh`"
- IO $\longrightarrow$ Fh — "call `fopen` with `foo.txt`, store returned `fh`"
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\mathsf{Pure} \arrow[dd, bend right, two heads, tail reversed] & \mathsf{IO} \arrow[dl, two heads, tail reversed] \arrow[d, two heads, tail reversed] & \mathsf{MLState} \arrow[d, two heads, tail reversed] & \text{(ext. world)} \\
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- **Observation:** IO $\leftarrow\!\!\!\twoheadrightarrow$ Fh and other $\leftarrow\!\!\!\twoheadrightarrow$ look a lot like **lenses**

Comodels as a gateway to the external world

Comodels as a gateway to the external world

- **Running a program** on a **comodel** (**using external resources**)

```
using
  C (* : Comodel(Sig,W) *) @ c_init (* : W *)
run
  c (* : A *)
finally @ (w:W) {
  return(x:A) ↦ c_fin(w,x) (* : B *) } (* : B *)
```

- **Comodels** are defined as follows

```
C =
  {
    op (x:A) @ (w:W) ↦ c_op(x,w),   (* : B * W *)
    ...
  }
```

for all **operations** $\mathsf{op} : A \rightsquigarrow B$ in a given signature Σ

Focussing on a fragment of the external world

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```
let f (s:string) =
  using
    Fh @ (fopen_of_io "foo.txt")
  run
    fwrite_of_fh (s^s)
  finally @ fh {
    return(x) ↦ fclose_of_io fh }
```

Focussing on a fragment of the external world

```
let f (s:string) =                              (* in IO *)
  using
    Fh @ (fopen_of_io "foo.txt")                (* in IO *)
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  finally @ fh {
    return(x) ↦ fclose_of_io fh }         (* in IO *)
```

where

```
Fh =                                   (* W = fhandle *)
  { fread  _ @ fh ↦ ...,
    fwrite s @ fh ↦ fwrite_of_io s fh;
                    return ((),fh)      }

         (* fread  : (unit * W) → (string * W) *)
        (* fwrite : (string * W) → (unit * W) *)
```

Modular treatment of worlds (IO $\longleftrightarrow$ Fh $\longleftrightarrow$ Fc)

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```
let f (s:string) =                              (* in IO *)
  using Fh @ (fopen_of_io "foo.txt")
  run

    using Fc @ (fread_of_fh ())                 (* in Fh *)
    run
      fwrite_of_fc (s^s)                        (* in Fc *)
    finally @ s {
      return(_) ↦ fwrite_of_fh s }

  finally @ fh {
    return(_) ↦ fclose_of_io fh }
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Fc = { fwrite s @ s' ↦ return ((),s'^s) }
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Modular treatment of worlds (IO $\longleftrightarrow$ Fh $\longleftrightarrow$ Fc)

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    finally @ s {
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where

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- **More generally:** comodels allow **transactions** and **sandboxing**

Tracking the world usage (IO $\longleftrightarrow$ IO+Stats)

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```
let f (fh:fhandle) (s:string) =          (* in IO *)
  using
    IO+Stats @ (return 0)
  run
    fwrite_of_stats fh (s^s)    (* in IO+Stats *)
  finally @ c {
    return(_) ↦
          let fh' = fopen_of_io "stats.txt" in
          fwrite_of_io fh' c; fclose_of_io fh' }
```

where

```
IO+Stats =                                  (* W = nat*)
  { fwrite fh s @ c ↦ fwrite_of_io fh s;
                      return ((),c+1),
    ... }
```

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where

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IO+Stats =                                (* W = nat*)
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    ... }
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- **More generally:** allows to slot in **instrumentation/monitors**

The external world can also be pure ($\mathsf{Pure} \longleftrightarrow \mathsf{Str}$)

The external world can also be pure (Pure $\longleftrightarrow$ Str)

```
let f (s:string) =                              (* in Pure *)
  using
    Str @ (return "some default initial value")
  run
    ...
    let s = get () in
    if (s == "foo")
    then (...; set "bar"; ...)
    else (...)
    ...
  finally @ _ {
    return(x) ↦ return x }
```

```
Str =                                           (* W = string *)
  { get _ @ s ↦ return (s,s) ,
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```
Str =                                       (* W = string *)
  { get _ @ s ↦ return (s,s) ,
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```

- Similar to **ambient values** (and **ambient functions**) in KOKA

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- **Types**

$$A, B, W \ ::= \ \mathsf{b} \mid 1 \mid A \times B \mid 0 \mid A + B \mid A \xrightarrow{\Sigma} B$$

So what's happening formally?

- Core calculus for cohandlers (wo/ handlers $\Rightarrow$ wait a few slides)
- **Types**

$$A, B, W \;::=\; \mathsf{b} \mid 1 \mid A \times B \mid 0 \mid A + B \mid A \xrightarrow{\Sigma} B$$

- **Interfaces** (**signatures**) of **external worlds**

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So what's happening formally?

- Core calculus for cohandlers (wo/ handlers $\Rightarrow$ wait a few slides)
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- **Comodels (cohandlers)**

$$C \;::=\; \{\, \overline{\mathsf{op}}_1\ x\ @\ w \mapsto c_1 \,,\; \ldots \;,\; \overline{\mathsf{op}}_n\ x\ @\ w \mapsto c_n \,\}$$

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- The two central **typing rules** are

$$\boxed{\dfrac{\begin{array}{c}\Gamma \models^{\Sigma} \mathsf{C} \text{ comodel of } \Sigma' \text{ with carrier } W_{\mathsf{C}} \\ \Gamma \models^{\Sigma} c_i : W_{\mathsf{C}} \qquad \Gamma \models^{\Sigma'} c : A \qquad \Gamma, w \mathbin{:} W_{\mathsf{C}}, x \mathbin{:} A \models^{\Sigma} c_f : B\end{array}}{\Gamma \models^{\Sigma} \textbf{using } C \mathbin{@} c_i \textbf{ run } c \textbf{ finally } @\, w \;\{\, \textbf{return}(x) \mapsto c_f \,\} : B}}$$

and

$$\boxed{\dfrac{\mathrm{op} : A_{\mathrm{op}} \rightsquigarrow B_{\mathrm{op}} \in \Sigma \qquad \Gamma \vdash v : A_{\mathrm{op}} \qquad \Gamma, x \mathbin{:} B_{\mathrm{op}} \models^{\Sigma} c : A}{\Gamma \models^{\Sigma} \mathrm{op}\ v\ (x.c) : A}}$$

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$$[\![\Gamma \vdash v : A]\!] : [\![\Gamma]\!] \longrightarrow [\![A]\!] \qquad [\![\Gamma \overset{\Sigma}{\models} c : A]\!] : [\![\Gamma]\!] \longrightarrow T_{\Sigma_\perp}\,[\![A]\!]$$

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- The interesting part is the **interpretation of using ... run**

$$\frac{\begin{array}{c}\Gamma \mathrel{\overset{\Sigma}{\vDash}} \mathsf{C} \text{ comodel of } \Sigma' \text{ with carrier } W_{\mathsf{C}} \\ \Gamma \mathrel{\overset{\Sigma}{\vDash}} c_i : W_{\mathsf{C}} \qquad \Gamma \mathrel{\overset{\Sigma'}{\vDash}} c : A \qquad \Gamma, w : W_{\mathsf{C}}, x : A \mathrel{\overset{\Sigma}{\vDash}} c_f : B\end{array}}{\Gamma \mathrel{\overset{\Sigma}{\vDash}} \textbf{using } C \mathbin{@} c_i \textbf{ run } c \textbf{ finally } @ \, w \; \{ \, \textbf{return}(x) \mapsto c_f \, \} : B}$$

which is based on M&S's **linear state-passing translation**, i.e.,

$$\frac{[\![\mathsf{C}]\!] \in \mathrm{Comod}_{\Sigma'_\perp}(\mathrm{Kleisli}(T_{\Sigma_\perp}))}{\mathrm{run\_on}_{[\![\mathsf{C}]\!]} : T_{\Sigma'_\perp} [\![A]\!] \longrightarrow \Big([\![W_{\mathsf{C}}]\!] \to T_{\Sigma_\perp}([\![W_{\mathsf{C}}]\!] \times [\![A]\!])\Big)}$$

Computational behaviour of using . . . run

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- Two semantically valid program equations

$$
\begin{array}{l}
\textbf{using}\ C\ @\ c_i\ \textbf{run}\ (\textbf{return}\, v)\ \textbf{finally}\ @\ w\ \{\ \textbf{return}(x) \mapsto c_f\ \} \\
= \\
\textbf{let}\ w' = c_i\ \textbf{in}\ c_f[w'/w, v/x]
\end{array}
$$

$$
\begin{array}{l}
\textbf{using}\ C\ @\ c_i\ \textbf{run}\ (\mathsf{op}\, v\, (y.c))\ \textbf{finally}\ @\ w\ \{\ \textbf{return}(x) \mapsto c_f\ \} \\
= \\
\textbf{let}\ w' = c_i\ \textbf{in}\ \Big(\\
\quad \textbf{let}\ z = C_{\overline{\mathsf{op}}}[w'/w, v/x]\ \textbf{in}\ (\\
\qquad \textbf{match}\ z\ \textbf{with}\ \{\ \langle y', w'' \rangle \mapsto \\
\qquad\quad \textbf{using}\ C\ @\ (\textbf{return}\, w'') \\
\qquad\quad \textbf{run}\ (c[y'/y]) \\
\qquad\quad \textbf{finally}\ @\ w\ \{\ \textbf{return}(x) \mapsto c_f\ \}\ \})\Big)
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$$

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- Recall that the **semantics of co-operations**

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- **Our solution:** Allow the **world** to **raise signals** to talk back

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C =                    (* : A * W → T((B * W) + S) *)
  { op x @ w ↦ if b then (...) else (raise s) }
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```
using C @ c_init
run c                                          (* : A ! S  *)
finally @ w {
  return(x) ↦ c_fin(w,x),                      (* : B ! S' *)
  signal(s) ↦ c_sig(w,s) }                     (* : B ! S' *)
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- But **worldly signals cannot be** handled locally, e.g., consider

```
using C @ c_init
run (try x = (raise s) in c unless {(**)...})
finally @ w {
  return(x) ↦ c_fin(w,x),
  signal(s) ↦ c_sig(w,s) }
```

vs

```
using C @ c_init
run (try x = (op v) in c unless {...})
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let rec ctr_printer i =
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  finally @ k {
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- **World-based:** could **store a trace** so as to **replay "old" co-ops**

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- Where do **multi-handlers** fit? Co-operating handlers-cohandlers?

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- `System.IO`, KOKA's **initially** & **finally**, PYTHON's **with**, ...
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- Two prototypes: **library** in Haskell, and **toy language** Coop
- Can also be a **basis for FFI**, e.g., in Coop (and future Eff)

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- **For the future: interface polymorphism, linear typing, ...**