

Embracing monotonicity in



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based on a joint POPL 2018 paper with

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Software Science Departmental Seminar, TUT

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Outline

- * F* overview
- Monotonic state by example
- Key ideas behind our general extension to Hoare-style logics
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- Glimpse of meta-theory and correctness results
- More examples of monotonic state at work (see our paper)
- Monadic reification and reflection (see our paper)

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 - a **functional programming language**
 - ML, OCaml, F#, Haskell, ...
 - extracted to OCaml or F#; subset compiled to efficient C code
 - an **interactive proof assistant**
 - Agda, Coq, Lean, Isabelle/HOL, ...
 - interactive modes for Emacs and Atom
 - a **semi-automated verifier** of imperative programs
 - Dafny, Why3, FramaC, ...
 - Z3-based SMT-automation; tactics and metaprogramming (WIP)
- **Application-driven development**
 - Project Everest [project-everest.github.io]
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module Talk

// Dependent (inductive) types

type vector 'a : nat -> Type =
  | Nil : vector 'a 0
  | Cons : #n:nat -> 'a -> vector 'a n -> vector 'a (n + 1)

// Dependently typed (recursive, total) functions

val append : #a:Type -> #n:nat -> #m:nat -> vector a n -> vector a m -> Tot (vector a (n + m))
let rec append #a #n #m xs ys =
  match xs with
  | Nil -> ys
  | Cons #n x xs' -> Cons x (append xs' ys)

// Refinement types

let in_range_index (min:nat) (max:nat) = i:nat{min <= i /\ i <= max}

val lkp : #a:Type -> #n:nat -> vector a n -> in_range_index 1 n -> Tot a
let rec lkp #a #n xs i =
  match xs with
  | Cons x xs' -> if i = 1 then x else lkp xs' (i - 1)
|
// First-class predicates (for which Type0 behaves like (classical) Prop)

type is_prefix_of (#a:Type) (#n:nat) (#m:nat) (xs:vector a n) (zs:vector a m{n <= m}) : Type0 =
  forall (i:nat) . (1 <= i /\ i <= n) ==> lkp xs i == lkp zs i

// Extrinsic reasoning (using separate lemmas)

val lemma : #a:Type -> #n:nat -> #m:nat -> xs:vector a n -> ys:vector a m -> Lemma (requires (True))
  (ensures (xs `is_prefix_of` (append xs ys)))

let rec lemma #a #n #m xs ys =
  match xs with
  | Nil -> ()
  | Cons x xs' -> lemma xs' ys

// Intrinsic reasoning (making lemmas part of definitions, e.g., using Hoare-style pre- and postconditions)

val take : #a:Type -> #n:nat -> zs:vector a n -> m:nat -> Pure (vector a m) (requires (m <= n))
  (ensures (fun xs -> xs `is_prefix_of` zs))

let rec take #a #n zs m =
  if m > 0 then match zs with
  | Cons z zs' -> let m':nat = m - 1 in Cons z (take zs' m')
  else Nil

```

F* – not just a pure programming language

- Tot, Lemma, Pure, ... are just some **effects** amongst many
 - Tot t
 - Lemma (requires $\text{pre}_{\text{Lemma}}$) (ensures $\text{post}_{\text{Lemma}}$)
 - Pure t (requires pre_{Pure}) (ensures $\text{post}_{\text{Pure}}$)
 - Div t (requires pre_{Div}) (ensures post_{Div})
 - Exc t (requires pre_{Exc}) (ensures post_{Exc})
 - ST t (requires pre_{ST}) (ensures post_{ST})
 - ...
- **Monad morphs.** $\text{Pure} \rightsquigarrow \{\text{Div}, \text{Exc}, \text{ST}\}; \text{Exc} \rightsquigarrow \text{STExc}; \dots$
- Systematically derived from **WP-calculi** (see POPL'17 paper)

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Monotonicity in program verification

- Consider a program operating on **set-valued state**

`insert v; complex_procedure(); assert (v ∈ get())`

- To prove the assertion (say, in a Floyd-Hoare style logic), we could prove that the code maintains a **stateful invariant**

$\{\lambda s.v \in s\}$ `complex_procedure()` $\{\lambda s.v \in s\}$

- likely that we have to **carry** $\lambda s.v \in s$ **through** the proof of `c_p`
- does not guarantee** that $\lambda s.v \in s$ holds at every point in `c_p`
- sensitive** to proving that `c_p` maintains $\lambda s.w \in s$ for some `w`
- However, if `c_p` **never removes**, then $\lambda s.v \in s$ is **stable**, and we would like the program logic to give us `v ∈ get()` “for free”

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Monotonicity in programming

- **Programming** also relies on **monotonicity**, even if you don't realise it!
- Consider ML-style typed references $r:\text{ref } a$
 - r is a **proof of existence** of an a -typed value in the heap
- Correctness relies on **monotonicity**!
 - 1) Allocation **stores** an a -typed value in the heap
 - 2) Writes **don't change type** and there is **no deallocation**
 - 3) So, given a ref. r , it is **guaranteed to point** to an a -typed value
- Baked into the memory models of most languages
- We derive them from **global state** + **general monotonicity**

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Monotonicity is really useful!

- In this talk, we will see how monotonicity gives us
 - our **motivating example** and **monotonic counters**
 - **typed references** (`ref t`) and **untyped references** (`uref`)
 - more flexibility with **monotonic references** (`mref t rel`)
- See our POPL 2018 paper for more
 - temporarily **violating monotonicity** via snapshots
 - two substantial case studies in F^*
 - a **secure file-transfer** application
 - Ariadne **state continuity** protocol [Strackx, Piessens 2016]
 - pointers to other works in F^* relying on monotonicity for
 - sophisticated **region-based memory models** [fstar-lang.org]
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Key ideas behind our general framework

- Based on **monotonic programs** and **stable predicates**
 - per verification task, we **choose a preorder rel** on states
 - set inclusion, heap inclusion, increasing counter values, ...
 - a stateful program e is **monotonic** (wrt. rel) when
$$\forall s e' s'. (e, s) \rightsquigarrow^* (e', s') \implies \text{rel } s s'$$
 - a stateful predicate p is **stable** (wrt. rel) when
$$\forall s s'. p \ s \wedge \text{rel } s s' \implies p \ s'$$
- **Our solution:** extend Hoare-style program logics (e.g., F^*) with
 - a means to **witness** the validity of $p \ s$ in some state s
 - a means for turning a p into a **state-independent proposition**
 - a means to **recall** the validity of $p \ s'$ in any future state s'
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Recap: Ordinary global state in F*

- F* supports Hoare-style reasoning about state via the **comp. type**

$$\text{ST}_{\text{state}}\ t\ (\text{requires}\ \text{pre})\ (\text{ensures}\ \text{post})$$

where

$$\text{pre} : \text{state} \rightarrow \text{Type} \qquad \text{post} : \text{state} \rightarrow t \rightarrow \text{state} \rightarrow \text{Type}$$

- ST is an abstract pre-postcondition refinement of

$$\text{st}\ t \stackrel{\text{def}}{=} \text{state} \rightarrow t * \text{state}$$

- The global state **actions** have types

$$\text{get} : \text{unit} \rightarrow \text{ST}\ \text{state}\ (\text{requires}\ (\lambda_.\top))\ (\text{ensures}\ (\lambda\ s_0\ s\ s_1.\ s_0 = s = s_1))$$
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New: Monotonic global state in F*

- We capture monotonic state with a new **computational type**

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- To ensure **monotonicity**, the **put** action gets a precondition

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$\text{put} : s : \text{state} \rightarrow \text{MST} \ \text{unit} \ (\text{requires} \ (\lambda s_0 . \text{rel} \ s_0 \ s))$
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$\text{mst} \ t \stackrel{\text{def}}{=} s_0 : \text{state} \rightarrow t * s_1 : \text{state} \{ \text{rel} \ s_0 \ s_1 \}$

New: Monotonic global state in F*

- We capture monotonic state with a new **computational type**

$\text{MST}_{\text{state}, \text{rel}} \, t \, (\text{requires} \, \text{pre}) \, (\text{ensures} \, \text{post})$

- The **get** action is typed as in **ST**

$\text{get} : \text{unit} \rightarrow \text{MST} \, \text{state} \, (\text{requires} \, (\lambda _ . \top))$
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New: Recalling a Witness

- We extend F^* with a **logical capability**

$\text{witnessed} : (\text{state} \rightarrow \text{Type}) \rightarrow \text{Type}$

together with a **weakening principle** (functoriality)

$\text{wk} : p, q : (\text{state} \rightarrow \text{Type}) \rightarrow \text{Lemma} (\text{requires } (\forall s. p\ s \implies q\ s))$
 $(\text{ensures } (\text{witnessed } p \implies \text{witnessed } q))$

- Intuitively, think of it as a **necessity modality**

$$\begin{aligned} \llbracket \text{witnessed } p \rrbracket (s) &\stackrel{\text{def}}{=} p \text{ 'stable_from' } s \\ &\stackrel{\text{def}}{=} \forall s'. \text{rel } s\ s' \implies \llbracket p\ s' \rrbracket (s) \end{aligned}$$

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- ... Hoare-style logics are essentially **world/state-indexed**, so
- we include a **stateful introduction rule** for witnessed

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witness : p:(state  $\rightarrow$  Type0)  
          $\rightarrow$  MST unit (requires ( $\lambda s_0. p$  'stable_from'  $s_0$ ))  
                     (ensures ( $\lambda s_0 - s_1. s_0 = s_1 \wedge$  witnessed  $p$ )))
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- and a **stateful elimination rule** for witnessed

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Outline

- * F^* overview
- Monotonic state by example
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- Accommodating monotonic state in F^*
- Some examples of monotonic state at work
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- More examples of monotonic state at work (see our paper)
- Monadic reification and reflection (see our paper)

The motivating example revisited

- Recall the program operating on the **set-valued state**

```
insert v; complex_procedure(); assert (v ∈ get())
```

- We pick **set inclusion** \subseteq as our preorder rel on states
- We **prove the assertion** by inserting a witness and recall

```
insert v; witness ( $\lambda s. v \in s$ ); c_p(); recall ( $\lambda s. v \in s$ ); assert (v ∈ get())
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- For any other w, wrapping

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around the program is handled **similarly easily** by

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create 0; incr(); witness ( $\lambda c. c > 0$ ); c_p(); recall ( $\lambda c. c > 0$ )
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ML-style typed references (local state)

- First, we define a type of **heaps** as a finite map

```
type heap =
```

```
| H : h : (N → cell) → ctr : N { ∀ n. ctr ≤ n ⇒ h n = Unused } → heap
```

where

```
type cell =
```

```
| Unused : cell
```

```
| Used : a : Type → v : a → cell
```

- Next, we define a **preorder** on heaps (**heap inclusion**)

```
let heap_inclusion (H h0 _) (H h1 _) = ∀ id. match h0 id, h1 id with
```

```
| Used a _, Used b _ → a = b
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- As a result, we can define new **local state effect**

$$\text{MLST } t \text{ pre post} \stackrel{\text{def}}{=} \text{MST}_{\text{heap, heap_inclusion}} t \text{ pre post}$$

- Next, we define the type of **references** using monotonicity

```
abstract type ref a = id:N{witnessed ( $\lambda h$ . contains h id a)}
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let contains (H h _) id a =  
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- Important: contains is **stable** wrt. heap_inclusion

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- Finally, we define **MLST**'s **actions** using **MST**'s actions

- `let alloc (a:Type) (v:a) : MLST (ref a) ... = ...`
 - get the current heap
 - create a fresh ref., and add it to the heap
 - put the updated heap back
 - witness that the created ref. is in the heap
- `let read (r:ref a) : MLST a (req. (\top)) (ens. (...)) = ...`
 - recall that the given ref. is in the heap
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 - select the given reference from the heap
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Adding untyped and monotonic references

- Untyped references (`uref`) with strong updates

- Used heap cells are extended with **tags**

where
$$\text{Used} : a:\text{Type} \rightarrow v:a \rightarrow \text{t:tag} \rightarrow \text{cell}$$

$$\text{type tag} = \text{Typed} : \text{tag} \mid \text{Untyped} : \text{tag}$$

- actions corresponding to urefs have **weaker types** than for refs

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- `mrefs` provide **more flexibility** with ref.-wise monotonicity

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- Glimpse of meta-theory and correctness results
- More examples of monotonic state at work (see our paper)
- Monadic reification and reflection (see our paper)

Glimpse of meta-theory

- A small **dependently typed** λ -calculus with **Tot** and **MST** effects
- Using an **instrumented operational semantics**, where

$$\begin{aligned}(\text{witness } p, s, W) &\rightsquigarrow (\text{return } (), s, W \cup \{p\}) \\(\text{recall } p, s, W) &\rightsquigarrow (\text{return } (), s, W)\end{aligned}$$

- **Strong normalisation** shown via type-erasure and TT-lifting
- Hoare-style **total correctness** via SN, progress, and preservation

if $\vdash e : \text{MST } t \text{ } \textit{pre post}$ and

$\vdash (s, W) \text{ wf}$ and witnessed $W \vdash \textit{pre } s$

then $(e, s, W) \rightsquigarrow^* (\text{return } v, s', W')$ and $\vdash v : t$ and

witnessed $W' \vdash \textit{rel } s \ s'$ and $W \subseteq W'$ and

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Conclusion

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 - can be distilled into a **simple** and **general** framework
 - is **useful** for **programming** (refs.) and **verification** (Prj. Everest)
- See our POPL 2018 paper for
 - further **examples** and **case studies**
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- Ongoing: taking the **modality** aspect of witnessed seriously
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Thank you for your attention!

Questions?

D. Ahman, C. Fournet, C. Hrițcu, K. Maillard, A. Rastogi, N. Swamy.

Recalling a Witness: Foundations and Applications of Monotonic State

Proc. ACM Program. Lang., volume 2, issue POPL, article 65, 2018.

Appendix: Mon. reification and reflection

- In F^* every **abstract ST computation**

$$e : \text{ST } t \text{ (requires pre) (ensures post)}$$

can be **reified** into its **underlying Pure representation**

$$\text{reify } e : s_0 : \text{state} \rightarrow \text{Pure } (t * \text{state}) \text{ (requires (pre } s_0)) \\ \text{(ensures } (\lambda (x, s_1). \text{post } s_0 \ x \ s_1))$$

and vice versa using **reflection** (see our POPL 2017 paper)

- Useful for **extrinsic reasoning**, e.g., for relational properties
- We also need it for **MST**!

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- We cannot simply turn an **abstract MST computation**

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$$\text{recall} : p : (\text{state} \rightarrow \text{Type}) \rightarrow \text{MST unit (requires } (\lambda _. \text{witnessed } p))$$
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which we would like to **reduce** as

$$\text{reify (recall } p) \rightsquigarrow \lambda s_0. \text{return } ((), s_0)$$

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 - indexing $\text{MST}_{\text{state}, \text{rel}, \mathbf{b}}$ with a **boolean flag** \mathbf{b} (reifiable?), and
 - **guarding** the pre-postconditions of witness and recall with \mathbf{b}so if $\mathbf{b} = \text{true}$ then witness and recall are **logically no-ops**.
- This works but leads to **duplication** of pre- and postconditions!
- Instead, ongoing work is taking (hybrid) **modal logic** seriously

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