

Interacting with external resources using runners (aka comodels)

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Today's plan

- **Computational effects** and **external resources** in PL
- **Issues with standard approaches** to **external resources**
- **Runners** – a natural model for **top-level runtime**
- **T-runners** – for also modelling **non-top-level runtimes**
- Turning **T**-runners into a **useful programming construct**
- Demonstrate the use of runners through **programming examples**

Computational effects
and
external resources

Computational effects in PL

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- Using **monads** (as in HASKELL)

```
type St a = String → (a,String)
```

```
instance St Monad where
```

```
...
```

```
f :: St a → St (a,a)
```

```
f c = c >>= (\ x → c >>= (\ y → return (x,y)))
```

- Using **alg. effects** and **handlers** (as in EFF, FRANK, KOKA)

```
effect Get : int
```

```
effect Put : int → unit
```

```
let g (c:unit → a!{Get,Put}) : int → a * int ! {} =
```

```
  with st_handler handle (perform (Put 42); c ())
```

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- Both are good for **faking comp. effects** in a pure language!

But what about effects that need access to the **external world**?

External resources in PL

External resources in PL

- Declare a **signature of monads** or **algebraic effects**, e.g.,

```
(* System.IO *)
type IO a
openFile :: FilePath → IOMode → IO Handle
```

```
(* pervasives.eff *)
effect RandomInt : int → int
effect RandomFloat : float → float
```

- And then **treat them specially** in the compiler, e.g., in EFF

```
(* eff/src/backends/runtime/eval.ml *)
let rec top_handle op =
  match op with
  | Value v → v
  | Call (RandomInt, v, k) →
    top_handle (k (Const.of_integer (Random.int (Value.to_int v))))
  | ...
```


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but there are **some issues** with that approach ...

First issue

- Difficult to cover all possible use cases
 - **external resources hard-coded** into the top-level runtime
 - **non-trivial to change** what's available and how it's implemented

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Ohad 8:35 PM

So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
| "Write" ->
  (match v with
  | V.Tuple vs ->
    let (file_name :: str :: _) = List.map V.to_str vs in
    let file_handle = open_out_gen
      [Open_wronly
       ;Open_append
       ;Open_creat
       ;Open_text
       ] 0o666 file_name in
    Printf.fprintf file_handle "%s" str;
    close_out file_handle;
    top_handle (k V.unit_value)
  )
```

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```

This work — a principled modular (co)algebraic approach!

Second issue

- **Lack of linearity** for external resources

```
let f (s:string) =  
  let fh = fopen "foo.txt" in  
  fwrite (fh,s^s);  
  fclose fh;  
  return fh
```

```
let g s =  
  let fh = f s in fread fh
```

(* fh not open any more ! *)

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(* fh not open any more ! *)

- We shall address these kinds of issues **indirectly (!)**,
 - by **not** introducing a linear typing discipline
 - but instead we make it convenient to **hide external resources**
(addressing stronger typing disciplines in the future)

Third issue

- **Excessive generality** of effect handlers

```
let f (s:string) =  
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- But misuse of external resources can also be **purely accidental**

```
let nd_handler =  
  handler { choose () k → return (k true ++ k false) }  
  
let g (s1 s2:string) =  
  let fh = fopen "foo.txt" in  
  let b = choose () in  
  if b then (fwrite (fh,s1^s2)) else (fwrite (fh,s2^s1));  
  fclose fh
```


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let f (s:string) =  
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```

- We shall address these kinds of issues **directly (!!)**,
 - by proposing a **restricted form of handlers** for resources
 - that support **controlled initialisation** and **finalisation**,
 - (and limit how general handlers can be used)

Runners

A natural model of **top-level runtime**

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- Given a **signature**¹ Σ of operation symbols ($A_{\text{op}}, B_{\text{op}}$ are sets)

$$\text{op} : A_{\text{op}} \rightsquigarrow B_{\text{op}}$$

a **runner**² \mathcal{R} for Σ is given by a carrier $|\mathcal{R}|$ and co-operations

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \times |\mathcal{R}| \longrightarrow B_{\text{op}} \times |\mathcal{R}| \right)_{\text{op} \in \Sigma}$$

where we think of $|\mathcal{R}|$ as a set of **runtime configurations**

¹We consider runners for signatures, but the work generalises to alg. theories.

²In the literature also known as **comodels** for Σ (or for an algebraic theory).

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- For example, a natural **runner \mathcal{R} for S -valued state** signature

$$\left\{ \text{get} : \mathbb{1} \rightsquigarrow S \quad , \quad \text{set} : S \rightsquigarrow \mathbb{1} \right\}$$

is given by

$$|\mathcal{R}| \stackrel{\text{def}}{=} S \qquad \overline{\text{get}}_{\mathcal{R}}(\star, s) \stackrel{\text{def}}{=} (s, s) \qquad \overline{\text{set}}_{\mathcal{R}}(s', s) \stackrel{\text{def}}{=} (\star, s')$$

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A natural model of **top-level runtime** ctd.

- Runners/comodels have been used for
 - **operational semantics** using tensors of models and comodels
[Plotkin and Power '08]
 - **top-level implementation of algebraic effects** in EFF
[Bauer and Pretnar '15]and
- **stateful running** of algebraic effects [Uustalu '15]
- **linear-use state-passing translation**
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- **stateful running** of algebraic effects [Uustalu '15]
- **linear-use state-passing translation** [Møgelberg and Staton '11, '14]
- The latter explicitly rely on one-to-one correspondence between
 - **runners** \mathcal{R}
 - **monad morphisms**³ $r : \mathbf{Free}_{\Sigma}(-) \longrightarrow \mathbf{St}_{|\mathcal{R}|}$

³ $\mathbf{Free}_{\Sigma}(X)$ is the free monad ind. defined with leaves $\text{val } x$ and nodes $\text{op}(a, \kappa)$.

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A natural model of **top-level runtime** ctd.

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- But what if this runtime is not ****the**** runtime?
 - hardware vs OSs
 - OSs vs VMs
 - VMs vs sandboxes

but also

- browsers vs web pages
- ...

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- So, runners \mathcal{R} are a natural model of **top-level runtime**
- But what if this runtime is not ****the**** runtime?
 - hardware vs OSs
 - OSs vs VMs
 - VMs vs sandboxes

but also

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 - ...
- Unfortunately, runners, as defined above, are **not readily able to**
 - use **external resources**
 - **signal failure** caused by unavoidable circumstances
- But is there a **useful generalisation** that would achieve this?

Effectful runners for modular top-levels

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- Møgelberg and Staton usefully observed that a **runner** \mathcal{R} is equivalently simply a family of **generic effects** for $\mathbf{St}_{|\mathcal{R}|}$, i.e.,

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow \mathbf{St}_{|\mathcal{R}|} B_{\text{op}} \right)_{\text{op} \in \Sigma}$$

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- Building on this, we define a **T-runner** \mathcal{R} for Σ to be given by

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- The one-to-one correspondence with **monad morphisms**

$$r : \mathbf{Free}_{\Sigma}(-) \longrightarrow \mathbf{T}$$

simply amounts to the **universal property of free models**, i.e.,

$$r_X(\text{val } x) = \eta_X x \qquad r_X(\text{op}(a, \kappa)) = \underbrace{(r_X \circ \kappa)^{\dagger}(\overline{\text{op}}_{\mathcal{R}} a)}_{\text{op}_{\mathcal{M}}(a, r_X \circ \kappa)}$$

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- Observe that κ appears in a **tail call position** on the right!

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- What would be a **useful class of monads T** to use?
- We want a runner to be a bit like a **kernel of an OS**, i.e., to
 - (i) provide management of **(internal) resources**
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- **Algebraically** (and pragmatically), this amounts to taking
 - (i) $\text{getenv} : \mathbb{1} \rightsquigarrow C$ & $\text{setenv} : C \rightsquigarrow \mathbb{1}$
 - (ii) $\text{op} : A_{\text{op}} \rightsquigarrow B_{\text{op}}$ ($\text{op} \in \Sigma'$, for some external Σ')
 - (iii) $\text{kill} : S \rightsquigarrow \mathbb{0}$s.t., (i) satisfy state equations; and (i) commute with (ii) and (iii)

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- The **induced monad** is then isomorphic to

$$\mathbf{T} X \stackrel{\text{def}}{=} C \Rightarrow \mathbf{Free}_{\Sigma'}((X \times C) + S)$$

Effectful runners for modular top-levels ctd.

- The corresponding **T-runners** \mathcal{R} for Σ are then of the form

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- Our solution:** consider signatures Σ with operation symbols

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- With this, our **T-runners** \mathcal{R} for Σ are (with “primitive” excs.)

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow \mathbf{K}_C^{\Sigma' ! E_{\text{op}} \not\leq S} B_{\text{op}} \right)_{\text{op} \in \Sigma}$$

where we call $\mathbf{K}_C^{\Sigma' ! E_{\text{op}} \not\leq S}$ a **kernel monad** (the sum of **T** and excs.)

$$\mathbf{K}_C^{\Sigma' ! E_{\text{op}} \not\leq S} B_{\text{op}} \stackrel{\text{def}}{=} C \Rightarrow \mathbf{Free}_{\Sigma'}(((B_{\text{op}} + E_{\text{op}}) \times C) + S)$$

T-runners as a programming construct
(towards a core calculus for runners)

T-runners as a programming construct

- First, we include **T-runners** for Σ

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in our language **as values**, and **co-ops. as kernel code**, i.e.,

```
let R = runner { op1 x1 → K1 , ... , opn xn → Kn } @ C
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- For instance, we can implement a **write-only file handle** as

```
let RFH = runner {  
  write s → if (length s > maxSize)  
    then (raise WriteSizeExceeded)  
    else (let fh = getenv () in  
      if (isValid fh) then (fwrite (fh,s)) else (kill IOError))  
} @ FileHandle
```

where

$$\Sigma \stackrel{\text{def}}{=} \{ \text{write} : \text{String} \rightsquigarrow 1 ! E \cup \{ \text{WriteSizeExceeded} \} \}$$

$$(\text{fwrite} : \text{FileHandle} \times \text{String} \rightsquigarrow 1 ! E) \in \Sigma' \quad S = \{ \text{IOError} \}$$

Controlled **initialisation** and **finalisation**

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induced by a **T**-runner \mathcal{R} are all **tail-recursive**

- We make use of it to enable programmers to **run user code**:

```
using R @ Minit
run M
finally {return x @ c → Mret , ... raise e @ c → Me ... , ... kill s → Ms ...}
```

where

(a **user monad**)

- M_s are **user code**, modelled using $\mathbf{U}^{\Sigma!E} X \stackrel{\text{def}}{=} \mathbf{Free}_\Sigma(X + E)$

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- M is the user code being **run using the runner** R
- M_{ret} , M_e , M_s **finalise** for return values, exceptions, and signals

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- M_{ret} , M_e , M_s **finalise** for return values, exceptions, and signals
- M_{ret} and M_e **depend on the final state** c , but M_s **does not**

Controlled **initialisation** and **finalisation** ctd.

- For instance, we can define a PYTHON-esque **with construct**

```
with fileName do M
=
using R_FH @ (fopen fileName)
run M
finally {
  return x @ fh → fclose fh; return x ,
  raise WriteSizeExceeded @ fh → fclose fh; return () ,
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  kill IOError → ... }
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- the **file handle is hidden** from M
- M **can only call** `write : String → 1 ! E ∪ {WriteSizeExceeded}`
but **not** (the external operations) `fopen` , `fclose` , and `fwrite`
- `fopen` and `fclose` are **limited to initialisation-finalisation**
- M can itself also catch `WriteSizeExceeded` to **re-try writing**

**A core calculus for
programming with runners**

Core calculus (syntax)

Core calculus (syntax)

- **Ground types** (types of ops. and kernel state)

$$A, B, C ::= B \mid 1 \mid 0 \mid A \times B \mid A + B$$

- **Types**

$$\begin{aligned} X, Y &::= B \mid 1 \mid 0 \mid X \times Y \mid X + Y \\ &\mid X \xrightarrow{\Sigma} Y ! E \\ &\mid X \xrightarrow{\Sigma} Y ! E \Downarrow S @ C \\ &\mid \Sigma \Rightarrow \Sigma' \Downarrow S @ C \end{aligned}$$

- **Values**

$$\Gamma \vdash V : X$$

- **User computations**

$$\Gamma \Vdash M : X ! E$$

- **Kernel computations**

$$\Gamma \Vdash K : X ! E \Downarrow S @ C$$

Core calculus (user computations)

$M, N ::= \text{return } V$

value

| $\text{try } M \text{ with } \{\text{return } x \mapsto N, (\text{raise } e \mapsto N_e)_{e \in E}\}$

exception handler

| $V W$

application

| $\text{match } V \text{ with } \{\langle x, y \rangle \mapsto M\}$

product elimination

| $\text{match } V \text{ with } \{\} X$

empty elimination

| $\text{match } V \text{ with } \{\text{inl } x \mapsto M, \text{inr } y \mapsto N\}$

sum elimination

| $\text{op}_X(V, (x . M), (N_e)_{e \in E_{\text{op}}})$

operation call

| $\text{raise}_X e$

raise exception

| $\text{using } V @ W \text{ run } M \text{ finally } \{$

run

$\text{return } x @ c \mapsto N,$

$(\text{raise } e @ c \mapsto N_e)_{e \in E},$

$(\text{kill } s \mapsto N_s)_{s \in S}\}$

| $\text{kernel } K @ V \text{ finally } \{$

switch to kernel mode

$\text{return } x @ c \mapsto N,$

$(\text{raise } e @ c \mapsto N_e)_{e \in E},$

$(\text{kill } s \mapsto N_s)_{s \in S}\}$

Core calculus (kernel computations)

$K, L ::=$	$\text{return}_C V$	value
	$\text{try } K \text{ with } \{\text{return } x \mapsto L, (\text{raise } e \mapsto L_e)_{e \in E}\}$	exception handler
	$V W$	application
	$\text{match } V \text{ with } \{\langle x, y \rangle \mapsto K\}$	product elimination
	$\text{match } V \text{ with } \{\}_{X@C}$	empty elimination
	$\text{match } V \text{ with } \{\text{inl } x \mapsto K, \text{inr } y \mapsto L\}$	sum elimination
	$\text{op}_{X@C}(V, (x . K), (L_e)_{e \in E_{\text{op}}})$	operation call
	$\text{raise}_{X@C} e$	raise exception
	$\text{kill}_{X@C} s$	send signal
	$\text{getenv}_C(c . K)$	get state
	$\text{setenv}(V, K)$	set state
	$\text{user } M \text{ with } \{\text{return } x \mapsto K, (\text{raise } e \mapsto L_e)_{e \in E}\}$	switch to user mode

Core calculus (type system and eq. theory)

Core calculus (type system and eq. theory)

- For example, the **typing rule for running user comps.** is

$$\begin{array}{c}
 \Gamma \vdash V : \Sigma \Rightarrow \Sigma' \not\downarrow S @ C \quad \Gamma \vdash W : C \\
 \Gamma \Vdash M : X ! E \quad \Gamma, x : X, c : C \Vdash' N_{ret} : Y ! E' \\
 \frac{(\Gamma, c : C \Vdash' N_e : Y ! E')_{e \in E} \quad (\Gamma \Vdash' N_s : Y ! E')_{s \in S}}{\Gamma \Vdash' \text{using } V @ W \text{ run } M \text{ finally } \{ \text{return } x @ c \mapsto N_{ret} , \\
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 \end{array}$$

- and the **main β -equation for running user comps.** is

$$\begin{array}{l}
 \Gamma \Vdash' \text{using } R @ W \text{ run } (\text{op}_X(V, (y.M), (M_e)_{e \in E_{op}})) \text{ finally } F \\
 \equiv \text{kernel } K_{op}[V/x_{op}] @ W \text{ finally } \{ \\
 \quad \text{return } y @ c' \mapsto \text{using } R @ c' \text{ run } M \text{ finally } F , \\
 \quad (\text{raise } e @ c' \mapsto \text{using } R @ c' \text{ run } M_e \text{ finally } F)_{e \in E_{op}} , \\
 \quad (\text{kill } s \mapsto N_s)_{s \in S} \} : Y ! E'
 \end{array}$$

Core calculus (type system and eq. theory)

- The calculus also includes **subtyping**, and **subsumption rules**

$$\frac{\Gamma \vdash V : A \quad A <: B}{\Gamma \vdash V : B}$$

$$\frac{\Gamma \Vdash M : A ! E \quad \Sigma \subseteq \Sigma' \quad A <: B \quad E \subseteq E'}{\Gamma \Vdash' M : B ! E'}$$

$$\frac{\begin{array}{cccc} \Gamma \Vdash K : A ! E \downarrow S @ C & \Sigma \subseteq \Sigma' & & \\ A <: B & E \subseteq E' & S \subseteq S' & C = C' \end{array}}{\Gamma \Vdash' K : B ! E' \downarrow S' @ C'}$$

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- We use $C = C'$ to have (standard) **proof-irrelevant subtyping**
- Otherwise, instead of just $C <: C'$, we would need a **lens** $C' \leftrightarrow C$

Core calculus (semantics)

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- **Monadic semantics**, for concreteness in **Set**, using
 - **user monads** $\mathbf{U}^{\Sigma!E} X \stackrel{\text{def}}{=} \mathbf{Free}_{\Sigma}(X + E)$
 - **kernel monads** $\mathbf{K}_C^{\Sigma!E \not\vdash S} X \stackrel{\text{def}}{=} C \Rightarrow \mathbf{Free}_{\Sigma}(((X + E) \times C) + S)$

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 - **kernel monads** $\mathbf{K}_C^{\Sigma!E \not\downarrow S} X \stackrel{\text{def}}{=} C \Rightarrow \mathbf{Free}_{\Sigma}(((X + E) \times C) + S)$
- (At a high level) the **judgements are interpreted** as

$$\llbracket \Gamma \vdash V : X \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow \llbracket X \rrbracket$$

$$\llbracket \Gamma \models M : X ! E \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow \mathbf{U}^{\Sigma!E} \llbracket X \rrbracket$$

$$\llbracket \Gamma \models K : X ! E \not\downarrow S @ C \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow \mathbf{K}_{\llbracket C \rrbracket}^{\Sigma!E \not\downarrow S} \llbracket X \rrbracket$$

Core calculus (semantics ctd.)

- However, to prove **coherence** of the semantics (**subtyping!**), we actually give the semantics in the **subset fibration**

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- For instance, **kernel computations** are interpreted as

$$\begin{array}{ccc}
 \llbracket \Gamma \rrbracket & \xrightarrow{\llbracket \Gamma \models K : X ! E \not\leq S @ C \rrbracket} & \mathbf{K}_{\llbracket C \rrbracket}^{\Sigma ! E \not\leq S} \llbracket X \rrbracket \\
 \downarrow \subseteq & & \downarrow \subseteq \\
 \llbracket \Gamma^s \rrbracket & \xrightarrow{\llbracket \Gamma^s \vdash K : X^s @ C \rrbracket} & \mathbf{K}_{\llbracket C \rrbracket}^{\mathcal{O} ! E \not\leq S + \{\perp\}} \llbracket X^s \rrbracket
 \end{array}$$

where $\Gamma^s \vdash K : X^s @ C$ is a **skeletal kernel typing judgement**

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- No essential obstacles to extending to **Sub(Cpo)** and beyond

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- However, to prove **coherence** of the semantics (**subtyping!**), we actually give the semantics in the **subset fibration**
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 \end{array}$$

where $\Gamma^s \vdash K : X^s @ C$ is a **skeletal kernel typing judgement**

- No essential obstacles to extending to **Sub(Cpo)** and beyond
- **Ground type restriction** on C needed to stay within **Sub(...)**
 - Otherwise, analogously to subtyping, we'd need **lenses** instead

Implementing runners

Experimenting with the **theory in practice**

Experimenting with the theory in practice

- A **small experimental language** COOP⁴
 - Implements the core calculus with few extras
 - The interpreter is directly based on the denotational semantics
 - Top-level containers for running external (OCaml) code

⁴coop [/ku:p/] – a cage where small animals are kept, especially chickens

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- A **HASKELL library** HASKELL-COOP
 - A shallow-embedding of the core calculus in HASKELL
 - Uses one of the Freer monad implementations underneath
 - Again, the operational aspects implement the denot. semantics
 - Top-level containers for arbitrary HASKELL monads
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 - Examples make use of HASKELL's features (GADTs, ...)
- Both still need some finishing touches, but will be public soon

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Runners in action

Runners can be **vertically nested**

Runners can be vertically nested

- ```
using RFH @ (fopen fileName)
run (
 using RFC @ (return "")
 run M
 finally {
 return x @ str → write str; return x ,
 raise WriteSizeExceeded @ str → write str; raise WriteSizeExceeded }
)
finally {
 return x @ fh → ... , raise e @ fh → ... , kill IOError → ... }
```

where the **file contents runner** (with  $\Sigma' = \{\}$ ) is defined as

```
let RFC = runner {
 write str' → let str = getenv () in
 if (length (str^str') > max) then (raise WriteSizeExceeded)
 else (setenv (str^str'))
} @ String
```

# Vertical nesting for instrumentation

# Vertical nesting for instrumentation

- ```
using RSniffer @ (return 0)
run M
finally {
  return x @ c →
    let fh = fopen "nsa.txt" in fwrite (fh,toStr c); fclose fh; return x }
```

where the **instrumenting runner** is defined as

```
let RSniffer = runner {
  ... ,
  op a → let c = getenv () in
    setenv (c + 1);
    op a ,
  ...
} @ Nat
```

(* forwards op outwards *)

- The runner R_{Sniffer} implements the same sig. Σ that M is using
- As a result, the runner R_{Sniffer} is **invisible** from M 's viewpoint

Vertical nesting for **active monitoring**

Vertical nesting for **active monitoring**

- First, we define a runner for **integer-valued ML-style state** as

type IntHeap = (Nat \rightarrow (Int + 1)) \times Nat

type Ref = Nat

```
let RIntState = runner {  
  alloc x  $\rightarrow$  let h = getenv () in                                     (* alloc : Int  $\rightsquigarrow$  Ref ! {} *)  
    let (r,h') = heapAlloc h x in  
    setenv h';  
    return r ,  
  
  deref r  $\rightarrow$  let h = getenv () in                                     (* deref : Ref  $\rightsquigarrow$  Int ! {} *)  
    match (heapSel h r) with  
    | inl x  $\rightarrow$  return x  
    | inr ()  $\rightarrow$  kill ReferenceDoesNotExist ,  
  
  assign r y  $\rightarrow$  let h = getenv () in                                 (* assign : Ref  $\times$  Int  $\rightsquigarrow$  1 ! {} *)  
    match (heapUpd h r y) with  
    | inl h'  $\rightarrow$  setenv h'  
    | inr ()  $\rightarrow$  kill ReferenceDoesNotExist  
}
```

@ IntHeap

Vertical nesting for **active monitoring** ctd.

- Next we define a runner for **monotonicity layer** on top of R_{IntState}

Vertical nesting for **active monitoring** ctd.

- Next we define a runner for **monotonicity layer** on top of R_{IntState}

type MonMemory = Ref $\rightarrow ((\text{Int} \rightarrow \text{Int} \rightarrow \text{Bool}) + 1)$

```
let RMonState = runner {  
  mAlloc x rel  $\rightarrow$  let r = alloc x in (* : Int  $\times$  Ord  $\rightsquigarrow$  Ref ! {} *)  
    let m = getenv () in  
    setenv (memAdd m r rel);  
    return r,  
  
  mDeref r  $\rightarrow$  deref r , (* monDeref : Ref  $\rightsquigarrow$  Int ! {} *)  
  
  mAssign r y  $\rightarrow$  let x = deref r in (* : Ref  $\times$  Int  $\rightsquigarrow$  1 ! {MV} *)  
    let m = getenv () in  
    match (memSel m r) with  
    | inl rel  $\rightarrow$  if (rel x y)  
        then (assign r y)  
        else (raise MonotonicityViolation)  
    | inr  $\rightarrow$  kill PreorderDoesNotExist  
}  
@ MonMemory
```

Vertical nesting for **active monitoring** ctd.

- We can then perform **runtime monotonicity verification** as

Vertical nesting for **active monitoring** ctd.

- We can then perform **runtime monotonicity verification** as

```
using R_IntState @ ((fun _ → inr ()), 0)    (* init. empty ML-style heap *)
run (

  using R_MonState @ (fun _ → inr ())      (* init. empty preorders memory *)
  run (

    let r = mAlloc 0 (≤) in
    mAssign r 1;
    mAssign r 0;      (* R_MonState raises MonotonicityViolation exception *)
    mAssign r 2

  )
  finally { ..., raise MonotonicityViolation @ m → ..., ... }

)
finally { ... }
```

Runners can also be **horizontally paired**

Runners can also be horizontally paired

- Given runners for Σ and Σ'

```
let R1 = runner { ... , op1i x → K1i , ... } @ C1  
let R2 = runner { ... , op2j x → K2j , ... } @ C2
```

we can **pair them** to get a runner for $\Sigma + \Sigma'$

```
let R = runner { ... ,  
  op1i x → let (c,c') = getenv () in  
    user (kernel (K1i x) @ c finally {  
      return y @ c'' → return (inl (inl y,c'')),  
      raise e @ c'' → return (inl (inr e,c'')), (* e ∈ Eop1i *)  
      kill s → return (inr s) } (* s ∈ S1 *)  
    finally {  
      return (inl (inl y,c'')) → setenv (c'',c'); return y,  
      return (inl (inr e,c'')) → setenv (c'',c'); raise e,  
      return (inr s) → kill s },  
  ... ,  
  op2j x → ..., (* analogously to above, just on 2nd comp. of state *)  
  ... } @ C1 × C2
```

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      return (inr s) → kill s },  
  ... ,  
  op2j x → ..., (* analogously to above, just on 2nd comp. of state *)  
  ... } @ C1 × C2
```

- For instance, this way we can build a runner for **IO and state**

Other examples (in HASKELL)

Other examples (in HASKELL)

- More general forms of **(ML-style) state** (for general Ref A)
 - if the host language allows it, we use GADTs, etc for safety
 - some examples extract a footprint from a larger memory
- **Combinations** of different effects and runners
 - in particular the combination of IO and state
 - good use case for both vertical and horizontal composition
- KOKA-style **ambient values** and **ambient functions**
 - **ambient values** are essentially **mutable variables/parameters**
 - **ambient functions** are **applied in their lexical context**
 - a runner that treats **amb. fun. application as a co-operation**
 - amb. funs. are stored in a context-depth-sensitive heap
 - the appl. co-operation restores the heap to the lexical context

Other examples (ambient functions)

```
module Control.Runner.Ambients

...

ambCoOps :: Amb a -> Kernel sig AmbHeap a
ambCoOps (Bind f) =
  do h <- getEnv;
    (f,h') <- return (ambHeapAlloc h f);
    setEnv h';
    return f
ambCoOps (Apply f x) =
  do h <- getEnv;
    (f,d) <- return (ambHeapSel h f (depth h));
    user
      (run
        ambRunner
          (return (h {depth = d}))
          (f x)
          ambFinaliser)
    return
ambCoOps (Rebind f g) =
  do h <- getEnv;
    setEnv (ambHeapUpd h f g)

ambRunner :: Runner '[Amb] sig AmbHeap
ambRunner = mkRunner ambCoOps
```

```
module AmbientsTests where

import Control.Runner
import Control.Runner.Ambients

ambFun :: AmbVal Int -> Int -> AmbEff Int
ambFun x y =
  do x <- getVal x;
    return (x + y)

test1 :: AmbEff Int
test1 =
  withAmbVal
    (4 :: Int)
    (\ x ->
      withAmbFun
        (ambFun x)
        (\ f ->
          do rebindVal x 2;
            applyFun f 1))

test2 = ambTopLevel test1
```

Wrapping up

- **Runners** are a natural model of **top-level runtime**
- We propose **T-runners** to also model **non-top-level runtimes**
- We have turned **T-runners** into a **(practical ?) programming construct**, that supports controlled initialisation and finalisation
- I showed you some **combinators** and **programming examples**
- Two **implementations** in the works, COOP & HASKELL-COOP
- **Ongoing** and **future**: lenses in subtyping and semantics, cat. of runners, handlers, case studies, refinement typing, compilation, ...

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Thank you!



Core calculus (semantics ctd.)

$$\begin{aligned} \llbracket \Gamma \models' \text{ using } V @ W \text{ run } M \text{ finally } \{ \text{ return } x @ c \mapsto N_{ret} , \\ (\text{ raise } e @ c \mapsto N_e)_{e \in E} , \\ (\text{ kill } s \mapsto N_s)_{s \in S} \} : Y ! E' \rrbracket_\gamma \stackrel{\text{def}}{=} \dots \end{aligned}$$

- $\llbracket V \rrbracket_\gamma = \mathcal{R} = \left(\overline{\text{op}}_{\mathcal{R}} : \llbracket A_{\text{op}} \rrbracket \longrightarrow \mathbf{K}_{\llbracket C \rrbracket}^{\Sigma'!E_{\text{op}} \not\leq S} \llbracket B_{\text{op}} \rrbracket \right)_{\text{op} \in \Sigma}$
- $\llbracket W \rrbracket_\gamma \in \llbracket C \rrbracket$
- $\llbracket M \rrbracket_\gamma \in \mathbf{U}^{\Sigma!E} \llbracket A \rrbracket$
- $\llbracket \text{ return } x @ c \mapsto N_{ret} \rrbracket_\gamma \in \llbracket A \rrbracket \times \llbracket C \rrbracket \longrightarrow \mathbf{U}^{\Sigma'!E'} \llbracket B \rrbracket$
- $\llbracket (\text{ raise } e @ c \mapsto N_e)_{e \in E} \rrbracket_\gamma \in E \times \llbracket C \rrbracket \longrightarrow \mathbf{U}^{\Sigma'!E'} \llbracket B \rrbracket$
- $\llbracket (\text{ kill } s \mapsto N_s)_{s \in S} \rrbracket_\gamma \in S \longrightarrow \mathbf{U}^{\Sigma'!E'} \llbracket B \rrbracket$
- allowing us to use the **free model property** to get

$$\mathbf{U}^{\Sigma!E} \llbracket A \rrbracket \xrightarrow{r_{\llbracket A \rrbracket} + E} \mathbf{K}_{\llbracket C \rrbracket}^{\Sigma'!E \not\leq S} \llbracket A \rrbracket \xrightarrow{(\lambda \llbracket N_{ret} \rrbracket_\gamma)^\dagger} \llbracket C \rrbracket \Rightarrow \mathbf{U}^{\Sigma'!E'} \llbracket B \rrbracket$$

and then apply the resulting composite to $\llbracket M \rrbracket_\gamma$ and $\llbracket W \rrbracket_\gamma$