Comodels as a gateway for interacting with the external world

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MSR Redmond, 15 May 2019

A modular programming abstraction for using external resources

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Computational effects in FP

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- Using **monads** (as in HASKELL)

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type St a = String → (a, String)

f :: St a → St (a,a)
f c = c >>= (\x → c >>= (\y → return (x,y)))
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- Using **alg. effects** and **handlers** (as in EFF, FRANK, KOKA)

```
effect Get : int
effect Put : int → unit
                                (*: int → a*int!{} *)
let g (c:unit → a!{Get,Put}) =
  with st_h handle (perform (Put 42); c ())
```

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- Both are good for **faking comp. effects** in a pure language!

  But what about effects that need access to the **external world**?

External world in FP

- Declare a **signature of monads** or **algebraic effects**, e.g.,

```
(* System.IO *)
type IO a
openFile :: FilePath → IOMode → IO Handle
```

```
(* pervasives.eff *)
effect RandomInt   : int → int
effect RandomFloat : float → float
```

- And then **treat them specially** in the compiler, e.g.,

```
(* eff/src/backends/runtime/eval.ml *)
let rec top_handle op =
  match op with
  | ...
```

but . . .

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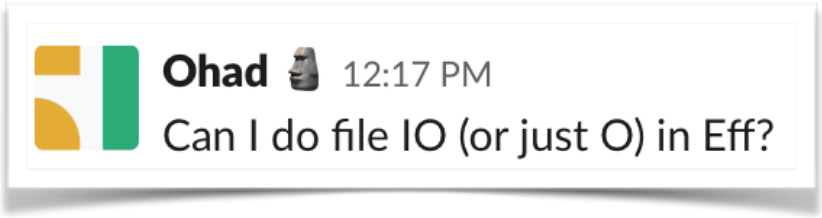

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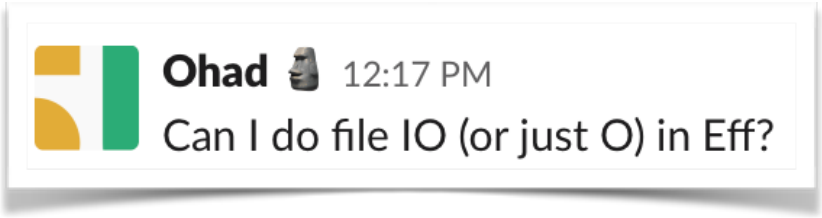

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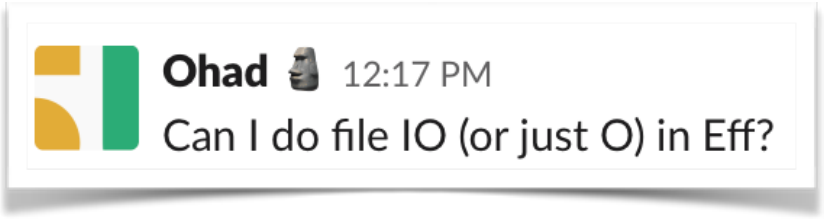

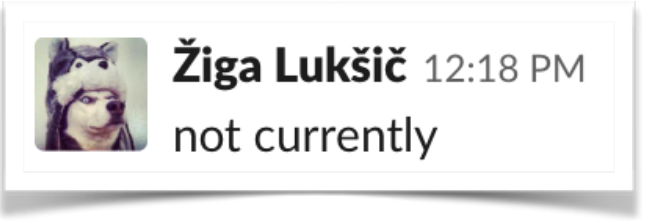

Ohad 8:35 PM

So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
    | "Write" ->
       (match v with
        | V.Tuple vs ->
           let (file_name :: str :: _)  = List.map V.to_str vs in
           let file_handle = open_out_gen
                             [Open_wronly
                             ;Open_append
                             ;Open_creat
                             ;Open_text
                             ] 0o666 file_name in
           Printf.fprintf file_handle "%s" str;
           close_out file_handle;
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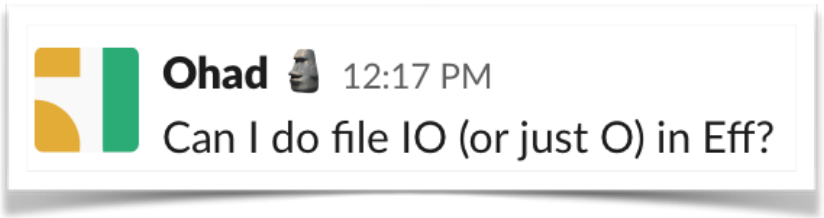

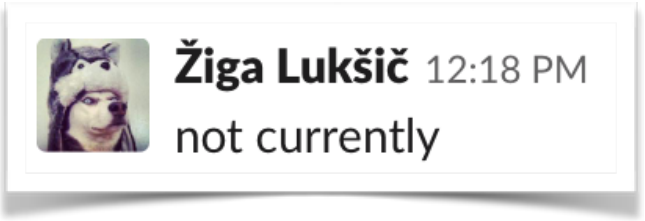

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This talk — a principled modular (co)algebraic approach!

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- ```
  let f (s:string) =
    let fh = fopen "foo.txt" in
    fwrite fh (s^s);
    fclose fh;
    return fh

  let g s =
    let fh = f s in fread fh
  ```
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- Even worse when we wrap `f` in a **handler**?

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let h = handler
        | effect (fwrite fh s k) ↦ return ()

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let h = handler
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  with h handle f ()            (* dangling fh ! *)
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- We could try using **existing PL techniques**, e.g.,
  - **Modules** and **abstraction**, e.g., `System.IO`

```
type IO a

hClose :: Handle → IO ()
```

  - **Linear** (and **non-linear**) **types** and **effects**

```
linear type fhandle

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  - Handlers with **initially** and **finally** **clauses**

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- **Problem:** They don't really capture the **essence of the problem**

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  - Operational semantics using a tensor of a model and a comodel (Plotkin & Power, Abou-Saleh & Pattinson)
  - Stateful runners of effectful programs (Uustalu)
  - Linear state-passing translation (Møgelberg and Staton)
  - Top-level behaviour of alg. effects in Eff v2 (Bauer & Pretnar)

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- With the `System.IO` **module abstraction** ensuring that
  - We **cannot get our hands on** RealWorld (no `get` and `put`)
  - We have the impression of RealWorld **used linearly**
  - We **don't ask more** from RealWorld than it can provide

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But wait a minute! RealWorld looks a lot like a **comodel**!

$$\mathsf{hGetLine} : (\mathsf{Handle}, \mathsf{RealWorld}) \to (\mathsf{String}, \mathsf{RealWorld})$$

$$\mathsf{hClose} : (\mathsf{Handle}, \mathsf{RealWorld}) \to ((), \mathsf{RealWorld})$$

Important: co-operations (hClose) make a **promise to return**!

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      let fh = fopen "foo.txt" in
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- **Our solution: Modular treatment** of **external worlds**

Modular treatment of external worlds

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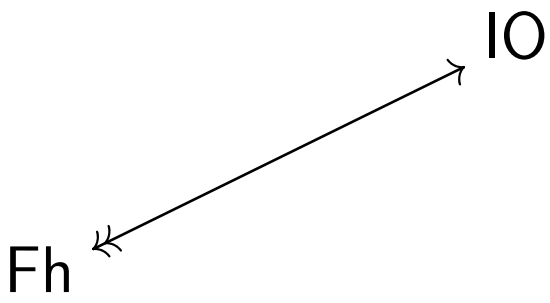

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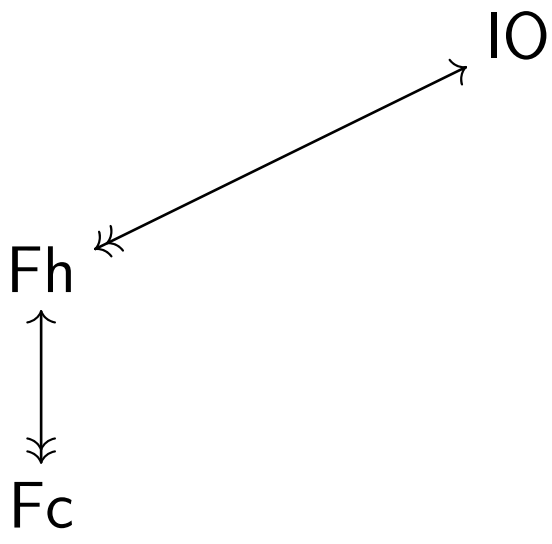

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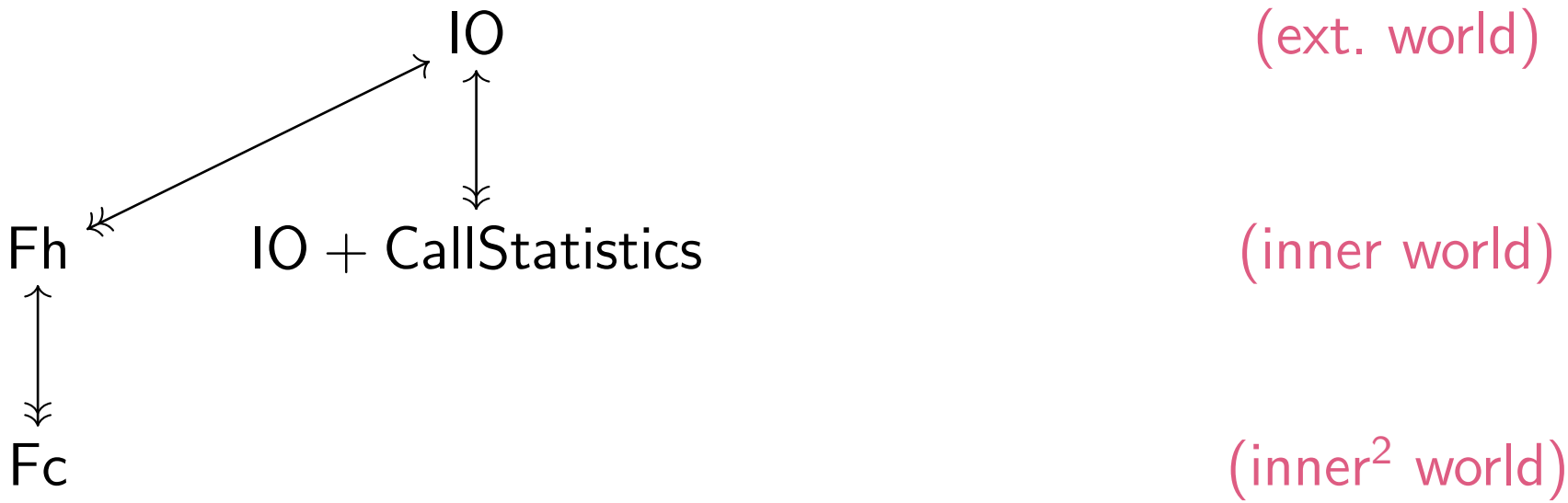

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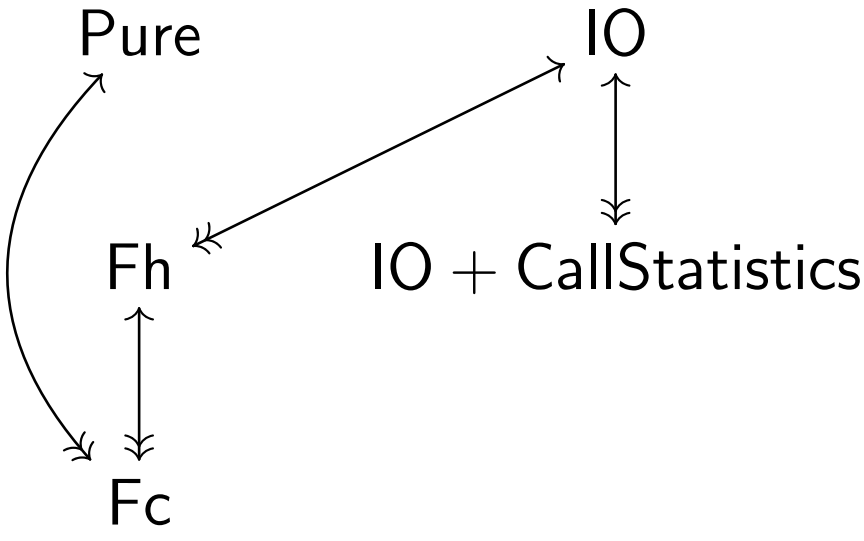

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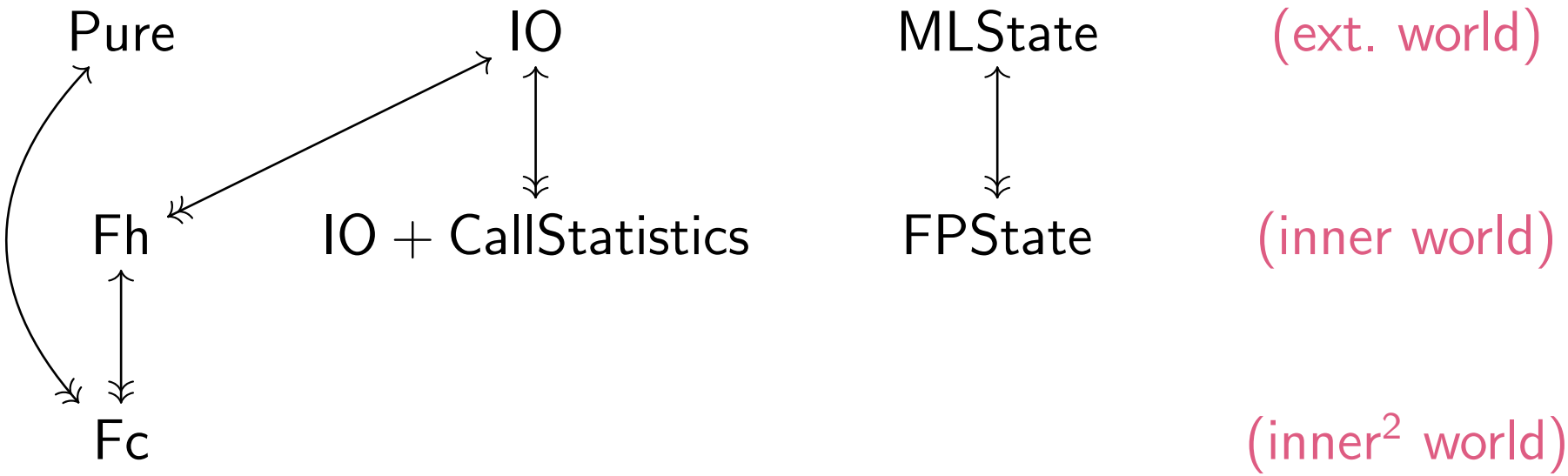

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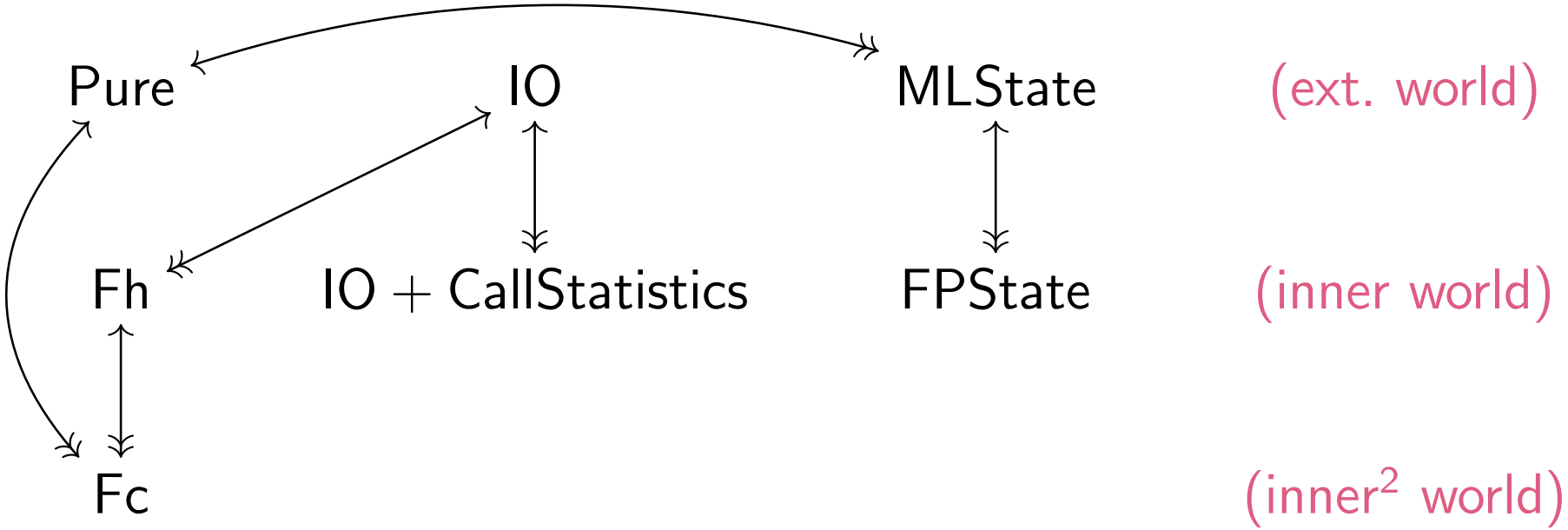

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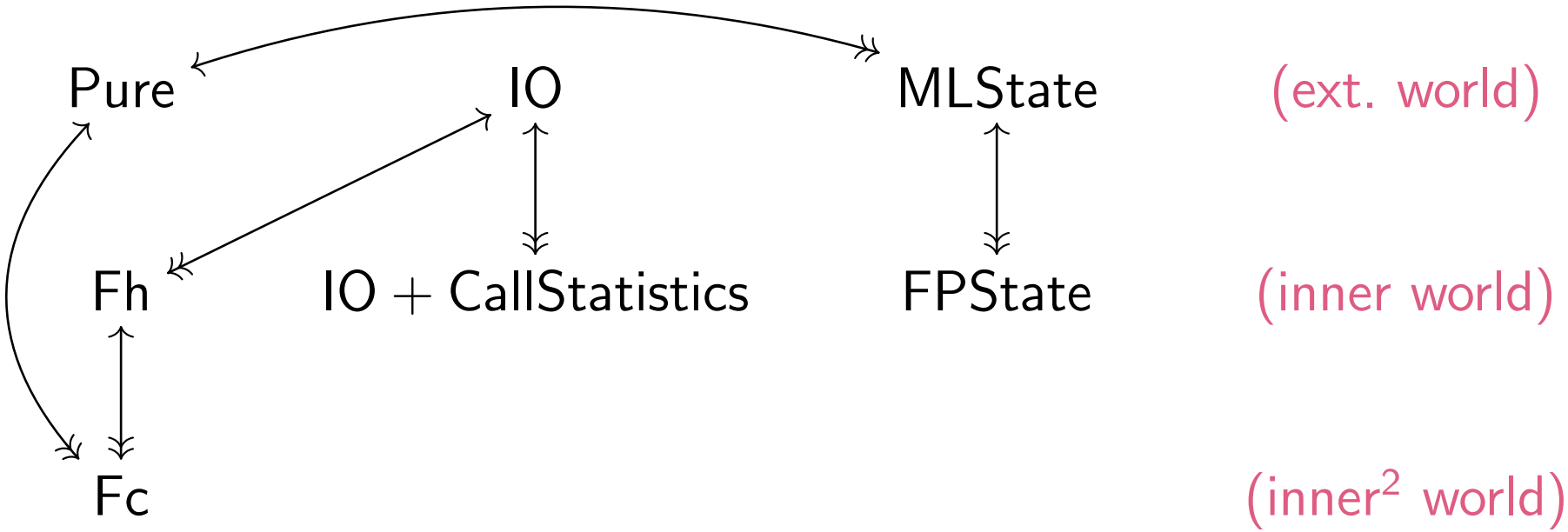

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- **Observation:** IO ⟷↠ Fh and other ⟷↠ look a lot like **lenses**

Comodels as a gateway to the external world

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- **Running a program** on a **comodel** (**using external resources**)

```
using
  C (* : Comodel(Sig,W) *) @ c_init (* : W *)
run
  c (* : A *)
finally @ (w:W) {
  return(x:A) ↦ c_fin(w,x) (* : B *) } (* : B *)
```

- **Comodels** are defined as follows

```
C =
  {
    op (x:A) @ (w:W) ↦ c_op(x,w),   (* : B * W *)
    ...
  }
```

for all **operations** op : $A \rightsquigarrow B$ in a given signature Σ

Focussing on a fragment of the external world

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```
let f (s:string) =
  using
    Fh @ (fopen_of_io "foo.txt")
  run
    fwrite_of_fh (s^s)
  finally @ fh {
    return(x) ↦ fclose_of_io fh }
```

Focussing on a fragment of the external world

```
let f (s:string) =                                  (* in IO *)
  using
    Fh @ (fopen_of_io "foo.txt")                    (* in IO *)
  run
    fwrite_of_fh (s^s)                              (* in Fh *)
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```

where

```
Fh =                                                  (* W = fhandle *)
  { fread  _ @ fh ↦ ...,
    fwrite s @ fh ↦ fwrite_of_io s fh;
                    return ((),fh)        }

          (* fread : (unit * W) → (string * W) *)
        (* fwrite : (string * W) → (unit * W) *)
```

Modular treatment of worlds (IO $\longleftrightarrow$ Fh $\longleftrightarrow$ Fc)

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let f (s:string) =                                  (* in IO *)
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  run

    using Fc @ (fread_of_fh ())                     (* in Fh *)
    run
      fwrite_of_fc (s^s)                            (* in Fc *)
    finally @ s {
      return(_) ↦ fwrite_of_fh s }

  finally @ fh {
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    run
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- **More generally:** comodels allow **transactions** and **sandboxing**

Tracking the world usage (IO $\longleftrightarrow$ IO+Stats)

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```
let f (fh:fhandle) (s:string) =                    (* in IO *)
  using
    IO+Stats @ (return 0)
  run
    fwrite_of_stats fh (s^s)    (* in IO+Stats *)
  finally @ c {
    return(_) ↦
          let fh' = fopen_of_io "stats.txt" in
          fwrite_of_io fh' c; fclose_of_io fh' }
```

where

```
IO+Stats =                                          (* W = nat*)
  { fwrite fh s @ c ↦ fwrite_of_io fh s;
                       return ((),c+1),
    ... }
```

Tracking the world usage (IO $\longleftrightarrow$ IO+Stats)

```
let f (fh:fhandle) (s:string) =                    (* in IO *)
  using
    IO+Stats @ (return 0)
  run
    fwrite_of_stats fh (s^s)    (* in IO+Stats *)
  finally @ c {
    return(_) ↦
          let fh' = fopen_of_io "stats.txt" in
          fwrite_of_io fh' c; fclose_of_io fh' }
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where

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IO+Stats =                                          (* W = nat*)
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- **More generally:** allows to slot in **instrumentation/monitors**

The external world can also be pure (Pure $\longleftrightarrow$ Str)

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```
let f () =                                              (* in Pure *)
  using
    Str @ (return "some default initial value")
  run
    ...
    let s = get () in
    if (s == "foo")
    then (...; set "bar"; ...)
    else (...)
    ...
  finally @ _ {
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```
Str =                                                   (* W = string *)
  { get _ @ s ↦ return (s,s) ,
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```
Str =                                               (* W = string *)
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- Similar to **ambient values** (and **ambient functions**) in Koka

More on ambient values/functions (Pure $\longleftrightarrow$ Amb)

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```
let f (s:string) =
  using                          (* with val amb = ... *)
    Amb @ (return "some default initial value")
  run
    ...
    let amb = get () in
    if (amb == "foo")
    then (...;                   (* with val amb = ... *)
          using Amb @ ... run ... finally ...);
          ...)
    else (...)
    ...
  finally @ _ { return(x) ↦ return x }
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```
Amb = { get _ @ s ↦ return (s,s) }
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- **Amb. functions** by amb. function application as a co-operation

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$$\Sigma \;::=\; \{\, \mathsf{op}_1 : A_1 \rightsquigarrow B_1 \;,\; \ldots \;,\; \mathsf{op}_n : A_n \rightsquigarrow B_n \,\}$$

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- **Computation terms** (value terms are unsurprising)

$$\begin{aligned} c \;::=\;& \mathbf{return}\ v \mid \mathbf{let}\ x = c_1\ \mathbf{in}\ c_2 \mid \mathbf{let\ rec}\ f\ x = c_1\ \mathbf{in}\ c_2 \\ \mid\;& v_1\ v_2 \\ \mid\;& \mathsf{op}\ v\ (x.c) \\ \mid\;& \mathbf{using}\ C\ @\ c_i\ \mathbf{run}\ c\ \mathbf{finally}\ @\ w\ \{\, \mathbf{return}(x) \mapsto c_f \,\} \end{aligned}$$

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- **Comodels (cohandlers)**

$$C \;::=\; \{\, \overline{\mathsf{op}}_1\ x\ @\ w \mapsto c_1 \;,\; \ldots \;,\; \overline{\mathsf{op}}_n\ x\ @\ w \mapsto c_n \,\}$$

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- The two central **typing rules** are

$$\frac{\Gamma \vDash^{\Sigma} \mathsf{C} \text{ comodel of } \Sigma' \text{ with carrier } W_{\mathsf{C}} \qquad \Gamma \vDash^{\Sigma} c_i : W_{\mathsf{C}} \qquad \Gamma \vDash^{\Sigma'} c : A \qquad \Gamma, w : W_{\mathsf{C}}, x : A \vDash^{\Sigma} c_f : B}{\Gamma \vDash^{\Sigma} \textbf{using } C \text{ @ } c_i \textbf{ run } c \textbf{ finally @ } w \{ \textbf{return}(x) \mapsto c_f \} : B}$$

and

$$\frac{\mathsf{op} : A_{\mathsf{op}} \rightsquigarrow B_{\mathsf{op}} \in \Sigma \qquad \Gamma \vdash v : A_{\mathsf{op}} \qquad \Gamma, x : B_{\mathsf{op}} \vDash^{\Sigma} c : A}{\Gamma \vDash^{\Sigma} \mathsf{op}\ v\ (x.c) : A}$$

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- **Term interpretation** looks very similar to **alg. effects**:

$$[\![\Gamma \vdash v : A]\!] : [\![\Gamma]\!] \longrightarrow [\![A]\!] \qquad [\![\Gamma \overset{\Sigma}{\vdash} c : A]\!] : [\![\Gamma]\!] \longrightarrow T_{\Sigma_\perp}\, [\![A]\!]$$

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  - **un-cohandled operations wait for a suitable external world**!

- The interesting part is the **interpretation of using . . . run**

$$\frac{\Gamma \overset{\Sigma}{\vDash} \mathsf{C} \text{ comodel of } \Sigma' \text{ with carrier } W_{\mathsf{C}} \qquad \Gamma \overset{\Sigma}{\vDash} c_i : W_{\mathsf{C}} \qquad \Gamma \overset{\Sigma'}{\vDash} c : A \qquad \Gamma, w : W_{\mathsf{C}}, x : A \overset{\Sigma}{\vDash} c_f : B}{\Gamma \overset{\Sigma}{\vDash} \textbf{using } C \text{ @ } c_i \textbf{ run } c \textbf{ finally @ } w \{ \textbf{return}(x) \mapsto c_f \} : B}$$

which is based on M&S's **linear state-passing translation**, i.e.,

$$\frac{[\![\mathsf{C}]\!] \in \mathrm{Comod}_{\Sigma'_\perp}(\mathrm{Kleisli}(T_{\Sigma_\perp}))}{\mathrm{run\_on}_{[\![\mathsf{C}]\!]} : T_{\Sigma'_\perp}\, [\![A]\!] \longrightarrow \Big([\![W_{\mathsf{C}}]\!] \to T_{\Sigma_\perp}([\![W_{\mathsf{C}}]\!] \times [\![A]\!]) \Big)}$$

Computational behaviour of using . . . run

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- Two semantically valid program equations

$$\begin{array}{l}\textbf{using } C \ @ \ c_i \textbf{ run } (\textbf{return } v) \textbf{ finally } @ \ w \ \{ \ \textbf{return}(x) \mapsto c_f \ \} \\ = \\ \textbf{let } w' = c_i \textbf{ in } c_f[w'/w, v/x]\end{array}$$

$$\begin{array}{l}\textbf{using } C \ @ \ c_i \textbf{ run } (\mathsf{op}\, v\, (y.c)) \textbf{ finally } @ \ w \ \{ \ \textbf{return}(x) \mapsto c_f \ \} \\ = \\ \textbf{let } w' = c_i \textbf{ in } \Big(\\ \quad \textbf{let } z = C_{\overline{\mathsf{op}}}[w'/w, v/x] \textbf{ in } (\\ \quad\quad \textbf{match } z \textbf{ with } \{ \ \langle y', w'' \rangle \mapsto \\ \quad\quad\quad \textbf{using } C \ @ \ (\textbf{return } w'') \\ \quad\quad\quad \textbf{run } (c[y'/y]) \\ \quad\quad\quad \textbf{finally } @ \ w \ \{ \ \textbf{return}(x) \mapsto c_f \ \} \ \}) \Big)\end{array}$$

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- Recall that the **semantics of co-operations**

$$\overline{\mathsf{op}} : [\![A_{\mathsf{op}}]\!] \times [\![W]\!] \longrightarrow T_{\Sigma_\perp}([\![B_{\mathsf{op}}]\!] \times [\![W]\!])$$

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- **Our solution:** Allow the **world** to **raise signals** to talk back

```
C =                                  (* : A x W → T((B x W) + S) *)
   { op x @ w ↦ if b then (...) else (raise s) }
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```

```
using C @ c_init
run c                                                  (* : A ! S   *)
finally @ w {
  return(x) ↦ c_fin(w,x),                              (* : B ! S'  *)
  signal(s) ↦ c_sig(w,s) }                             (* : B ! S'  *)
```

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- **User-raised signals can be** handled locally (exceptional syntax)

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- But **worldly signals cannot be** handled locally, e.g., consider

```
using C @ c_init
run (try x = (raise s) in c unless {(**)...})
finally @ w {
  return(x) ↦ c_fin(w,x),
  signal(s) ↦ c_sig(w,s) }
```

vs

```
using C @ c_init
run (try x = (op v) in c unless {...})
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```
let rec ctr_printer i =
  using Out+Ctr @ (return i)
  run
    while(T) {let j = get_c in print j; incr_c}
  finally @ k {
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- **World-based:** could **store a trace** so as to **replay "old" co-ops**

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- Where do **multi-handlers** fit? Co-operating handlers-cohandlers?

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- Can also be a **basis for FFI**, e.g., in Coop (and future Eff)

$$\frac{f : A \to B \ \in \ \text{OCaml}}{\text{external } f : A \times W_{\text{top-level}} \to B \times W_{\text{top-level}} \ \in \ \text{top-level-comodel}}$$

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- **For the future: interface polymorphism**, **linear typing**, ...