

Leveraging monotonic state in F^*

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joint work with

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Global state + monotonicity is really useful!

Its essence can be captured very neatly!

Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F^*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see POPL'18 paper)
- First steps in mon. reification and reflection (see POPL'18 paper)
- Meta-theory and correctness results (see POPL'18 paper)

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Monotonicity in verification

- Consider a program operating on **set-valued state**

```
insert v; complex_procedure(); assert (v ∈ get())
```

- To prove the assertion (say, in a Floyd-Hoare style logic), we could prove that the code maintains a **stateful invariant**

$$\{\lambda s. v \in s\} \text{ complex_procedure() } \{\lambda s. v \in s\}$$

- likely that we have to **carry** $\lambda s. v \in s$ **through** the proof of `c_p`
 - does not guarantee** that $\lambda s. v \in s$ holds at every point in `c_p`
 - sensitive** to proving that `c_p` maintains $\lambda s. w \in s$ for some other `w`
- However, if `c_p` **never removes**, then $\lambda s. v \in s$ is **stable**, and we would like the program logic to give us `v ∈ get()` “for free”

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Monotonicity in programming

- **Programming** also relies on **monotonicity**, even if you don't realise it!
- Consider ML-style typed references $r:\text{ref } a$
 - r is a **proof of existence** of an a -typed value in the heap
- Correctness relies on **monotonicity**!
 - 1) Allocation **stores** an a -typed value in the heap
 - 2) Writes **don't change type** and there is **no deallocation**
 - 3) So, given a ref. r , it is **guaranteed to point** to an a -typed value
- Baked into the memory models of most languages
- We derive them from **global state** + **general monotonicity**

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Monotonicity is really useful!

- In this talk
 - our **motivating example** and **monotonic counters**
 - **typed references** (`ref t`) and **untyped references** (`uref`)
 - more flexibility with **monotonic references** (`mref t rel`)
- See our POPL 2018 paper for more
 - temporarily **violating monotonicity** via snapshots
 - two substantial case studies in F^*
 - a **secure file-transfer** application
 - Ariadne **state continuity** protocol [Strackx, Piessens 2016]
 - pointers to other works in F^* relying on monotonicity for
 - sophisticated **region-based memory models** [fstar-lang.org]
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Key ideas behind our general framework

- We make use of **monotonic programs** and **stable predicates**
 - per verification task, we **choose a preorder rel** on states
 - set inclusion, heap inclusion, increasing counter values, ...

- a stateful program e is **monotonic** (wrt. rel) when

$$\forall s e' s'. (e, s) \rightsquigarrow^* (e', s') \implies \text{rel } s s'$$

- a stateful predicate p is **stable** (wrt. rel) when

$$\forall s s'. p s \wedge \text{rel } s s' \implies p s'$$

- **Our solution:** extend Hoare-style program logics (e.g., F^*) with
 - a means to **witness** the validity of $p s$ in some state s
 - a means for turning a p into a **state-independent proposition**
 - a means to **recall** the validity of $p s'$ in any future state s'
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Recap: Ordinary global state in F*

- F* supports Hoare-style reasoning about state via the **comp. type**

$$\text{ST}_{\text{state}}\ t\ (\text{requires}\ \text{pre})\ (\text{ensures}\ \text{post})$$

where

$$\text{pre} : \text{state} \rightarrow \text{Type}_0 \qquad \text{post} : \text{state} \rightarrow t \rightarrow \text{state} \rightarrow \text{Type}_0$$

- ST is an abstract pre-postcondition refinement of

$$\text{st}\ t \stackrel{\text{def}}{=} \text{state} \rightarrow t * \text{state}$$

- The global state **actions** have types

$$\text{get} : \text{unit} \rightarrow \text{ST}\ \text{state}\ (\text{requires}\ (\lambda _.\top))\ (\text{ensures}\ (\lambda\ s_0\ s\ s_1.\ s_0 = s = s_1))$$
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New: Monotonic global state in F*

- We capture monotonic state with a new **computational type**

$\text{MST}_{\text{state}, \text{rel}}\ t\ (\text{requires}\ \text{pre})\ (\text{ensures}\ \text{post})$

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New: Recalling a Witness

- We extend F^* with a **logical capability**

$\text{witnessed} : (\text{state} \rightarrow \text{Type}_0) \rightarrow \text{Type}_0$

together with a **weakening principle** (functoriality)

$\text{wk} : p, q : (\text{state} \rightarrow \text{Type}_0) \rightarrow \text{Lemma} \left(\begin{array}{l} \text{requires } (\forall s. p\ s \implies q\ s) \\ \text{ensures } (\text{witnessed } p \implies \text{witnessed } q) \end{array} \right)$

- Intuitively, a lot like the **necessity modality** \Box

$$\llbracket \text{witnessed } p \rrbracket(s) \stackrel{\text{def}}{=} \forall s'. \text{rel } s\ s' \implies \llbracket p\ s' \rrbracket(s)$$

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- Oh, wait a minute ...

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- We extend F^* with a **logical capability**

$$\text{witnessed} : (\text{state} \rightarrow \text{Type}_0) \rightarrow \text{Type}_0$$

together with a **weakening principle (functoriality)**

$$\text{wk} : p, q : (\text{state} \rightarrow \text{Type}_0) \rightarrow \text{Lemma} \left(\text{requires} \left(\forall s. p \ s \implies q \ s \right) \right. \\ \left. \left(\text{ensures} \left(\text{witnessed } p \implies \text{witnessed } q \right) \right) \right)$$

- Intuitively, a lot like the **necessity modality** \Box

$$\llbracket \text{witnessed } p \rrbracket (s) \stackrel{\text{def}}{=} \forall s'. \text{rel } s \ s' \implies \llbracket p \ s' \rrbracket (s)$$

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- ... Hoare-style logics are essentially **world/state-indexed**, so
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witness : p:(state  $\rightarrow$  Type0)  
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Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F^*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see POPL'18 paper)
- First steps in mon. reification and reflection (see POPL'18 paper)
- Meta-theory and correctness results (see POPL'18 paper)

The motivating example revisited

- Recall the program operating on the **set-valued state**

```
insert v; complex_procedure(); assert (v ∈ get())
```

- We pick **set inclusion** \subseteq as our preorder rel on states
- We **prove the assertion** by inserting a witness and recall

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insert v; witness ( $\lambda s. v \in s$ ); c_p(); recall ( $\lambda s. v \in s$ ); assert (v ∈ get())
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- For any other w, wrapping

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around the program is handled **similarly easily** by

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ML-style typed references (local state)

- First, we define a type of **heaps** as a finite map

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type heap =
```

```
| H : h : (N → cell) → ctr : N { ∀ n. ctr ≤ n ⇒ h n = Unused } → heap
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where

```
type cell =
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```
| Unused : cell
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```
| Used : a : Type0 → v : a → cell
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- Next, we define a **preorder** on heaps (**heap inclusion**)

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let heap_inclusion (H h0 _) (H h1 _) = ∀ id. match h0 id, h1 id with
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- As a result, we can define new **local state effect**

$$\text{MLST } t \text{ pre post} \stackrel{\text{def}}{=} \text{MST}_{\text{heap, heap_inclusion}} t \text{ pre post}$$

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abstract type ref a = id:N{witnessed (λh . contains h id a)}

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- Finally, we define **MLST**'s **actions** using **MST**'s actions

- `let alloc (a:Type0) (v:a) : MLST (ref a) ... = ...`
 - get the current heap
 - create a fresh ref., and add it to the heap
 - put the updated heap back
 - witness that the created ref. is in the heap
- `let read (r:ref a) : MLST t ... = ...`
 - recall that the given ref. is in the heap
 - get the current heap
 - select the given reference from the heap
- `let write (r:ref a) (v:a) : MLST unit ... = ...`
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Adding untyped and monotonic references

- Untyped references (`uref`) with strong updates

- Used heap cells are extended with **tags**

where
$$\text{Used} : a:\text{Type}_0 \rightarrow v:a \rightarrow \text{t:tag} \rightarrow \text{cell}$$

$$\text{type tag} = \text{Typed} : \text{tag} \mid \text{Untyped} : \text{tag}$$

- actions corresponding to urefs have **weaker types** than for refs

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Conclusion

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 - can be distilled into a **simple** and **general** framework
 - is **useful** for **programming** (refs.) and **verification** (Prj. Everest)
- See our POPL 2018 paper for
 - further **examples** and **case studies**
 - **meta-theory** and **total correctness** for MST
 - based on an instrumented operational semantics
$$(\text{witness } x.\varphi, s, W) \rightsquigarrow (\text{return } (), s, W \cup \{x.\varphi\})$$
 - and cut elimination for the witnessed-logic
 - first steps towards **monadic reification** for MST
 - useful for extrinsic reasoning, e.g., for relational properties
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Appendix: Mon. reification and reflection

- Recall from Kenji's talk that in F^* an **abstract ST computation**

$$e : \text{ST } t \text{ (requires pre) (ensures post)}$$

can be **reified** into its **underlying Pure representation**

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and vice versa using **reflection** (see our POPL 2017 paper)

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- We cannot simply turn an **abstract MST computation**

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- For example, consider the **recalling** action

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which we would like to **reduce** as

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- In our POPL 2018 paper, we support reification and reflection by
 - indexing $\text{MST}_{\text{state}, \text{rel}, \mathbf{b}}$ with a **boolean flag** \mathbf{b} (reifiable?), and
 - **guarding** the pre-postconditions of witness and recall with \mathbf{b}so if $\mathbf{b} = \text{true}$ then witness and recall are **logically no-ops**.
- This **works** but leads to **duplication** of pre- and postconditions!
- Instead, ongoing work is taking (hybrid) **modal logic** seriously

$$\mathbf{s}_0:\text{state} \rightarrow \text{Pure } (t * \mathbf{s}_1:\text{state}\{\text{rel } \mathbf{s}_0 \ \mathbf{s}_1\}) \ (\text{req. } (\text{pre } \mathbf{s}_0 \ @ \ \mathbf{s}_0)) \\ (\text{ens. } (\lambda (x, \mathbf{s}_1). \text{post } \mathbf{s}_0 \ x \ \mathbf{s}_1 \ @ \ \mathbf{s}_1))$$

where $@$ is the **standard translation** of modal logic

Appendix: Mon. reification and reflection

- In our POPL 2018 paper, we support reification and reflection by
 - indexing $\text{MST}_{\text{state}, \text{rel}, \mathbf{b}}$ with a **boolean flag** \mathbf{b} (reifiable?), and
 - **guarding** the pre-postconditions of witness and recall with \mathbf{b}so if $\mathbf{b} = \text{true}$ then witness and recall are **logically no-ops**.
- This **works** but leads to **duplication** of pre- and postconditions!

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