Recalling a Witness

Foundations and Applications of Monotonic State

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Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

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Monotonicity in program verification

- Consider a program operating on **set-valued state**

$$\texttt{insert v};\ \texttt{complex\_procedure}();\ \texttt{assert}\ (\texttt{v} \in \texttt{get}())$$

- To prove the assertion (say, in a Floyd-Hoare style logic), we could prove that the code maintains a **stateful invariant**

$$\{\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}\}\ \texttt{complex\_procedure}()\ \{\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}\}$$

  - likely that we have to **carry** $\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}$ **through** the proof of c_p
    - **does not guarantee** that $\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}$ holds at every point in c_p
    - **sensitive** to proving that c_p maintains $\lambda\,\texttt{s}\,.\,\texttt{w} \in \texttt{s}$ for some other $\texttt{w}$

- However, if c_p **does not remove**, then $\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}$ is **stable**, and we would like the program logic to give us $\texttt{v} \in \texttt{get}()$ "**for free**"

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Monotonicity in programming

- **Programming** also relies on **monotonicity**,
  even if you don't realise it!

- Consider ML-style typed **references** `r:ref a`
  - `r` is a **proof of existence** of an `a`-typed value in the heap

- Correctness relies on **monotonicity**!
  1) Allocation **stores** an `a`-typed value in the heap
  2) Writes **don't change type** and there is **no deallocation**
  3) So, given a ref. `r`, it is **guaranteed to point** to an `a`-typed value

- Baked into the memory models of most languages
- We derive them from **global state + general monotonicity**

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Monotonicity is really useful!

- In this talk
  - our **motivating example** and **monotonic counters**
  - **typed references** (`ref t`) and **untyped references** (`uref`)
  - more flexibility with **monotonic references** (`mref t rel`)

- More in the paper
  - temporarily **violating monotonicity** via snapshots
  - two substantial case studies
    - a **secure file-transfer** application
    - Ariadne **state continuity** protocol [Strackx, Piessens 2016]
  - pointers to other works in F* relying on monotonicity for
    - sophisticated **region-based memory models** [`fstar-lang.org`]
    - **crypto** and **TLS verification** [`project-everest.github.io`]

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Key ideas behind our general framework

- We focus on **monotonic programs** and **stable predicates**
    - per verification task, we **choose a preorder** `rel` on states
        - set inclusion, heap inclusion, increasing counter values, ...
    - a stateful program e is **monotonic** (wrt. `rel`) when

$$\forall\, s\, e'\, s'.\ (e, s) \rightsquigarrow^* (e', s') \implies \mathtt{rel}\ s\ s'$$

    - a stateful predicate p is **stable** (wrt. `rel`) when

$$\forall\, s\, s'.\ p\ s\ \wedge\ \mathtt{rel}\ s\ s' \implies p\ s'$$

- **Our solution:** extend Hoare-style program logics (e.g., F*) with
    - a means for turning a p into a **state-independent proposition**
    - a means to **witness** the validity of p s in some state s
    - a means to **recall** the validity of p s′ in any future state s′
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Recap: Ordinary global state in F*

- F* is an ML-like dependently typed language, aimed at verification
- F* supports Hoare-style reasoning about state via the **comp. type**

  $$\mathtt{ST}_{\mathtt{state}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where

  $$\mathtt{pre} : \mathtt{state} \to \mathrm{Type} \qquad \mathtt{post} : \mathtt{state} \to \mathtt{t} \to \mathtt{state} \to \mathrm{Type}$$

- ST is an **abstract** pre-postcondition refinement of

  $$\mathtt{st}\ \mathtt{t} \stackrel{\mathrm{def}}{=} \mathtt{state} \to \mathtt{t} * \mathtt{state}$$

- The global state **actions** have types

$$\mathtt{get} : \mathtt{unit} \to \mathtt{ST}\ \mathtt{state}\ (\mathtt{requires}\ (\lambda\, \_ . \top))\ (\mathtt{ensures}\ (\lambda\, \mathtt{s}_0\ \mathtt{s}\ \mathtt{s}_1 . \mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))$$

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New: Monotonic global state in F*

- We capture monotonic state with a new **computation type**

$$\mathtt{MST}_{\mathtt{state},\mathtt{rel}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where pre and post are typed as in ST

- The **get** action is typed as in ST

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- To ensure **monotonicity**, the **put** action gets a precondition

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- So intuitively, MST is an **abstract** pre-postcondition refinement of

$$\mathtt{mst}\ \mathtt{t} \stackrel{\text{def}}{=} \mathtt{s}_0{:}\mathtt{state} \to \mathtt{t} * \mathtt{s}_1{:}\mathtt{state}\{\mathtt{rel}\ \mathtt{s}_0\ \mathtt{s}_1\}$$

New: Monotonic global state in F*

- We capture monotonic state with a new **computation type**

$$\mathtt{MST}_{\mathtt{state},\mathtt{rel}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where pre and post are typed as in ST

- The **get** action is typed as in ST

$$\begin{aligned}\mathtt{get} : \mathtt{unit} \to \mathtt{MST}\ \mathtt{state}\ &(\mathtt{requires}\ (\lambda\, \_ . \top))\\ &(\mathtt{ensures}\ (\lambda\, \mathtt{s}_0\, \mathtt{s}\, \mathtt{s}_1 \,.\, \mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))\end{aligned}$$

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New: Recalling a Witness

- We introduce a **logical capability** (a **modality** in ongoing work)

$$\texttt{witnessed} : (\texttt{state} \to \texttt{Type}) \to \texttt{Type}$$

  together with a **weakening principle** (**functoriality**)

$$\texttt{wk} : \texttt{p},\texttt{q}{:}(\texttt{state} \to \texttt{Type}) \to \texttt{Lemma}\ (\texttt{requires}\ (\forall\, \texttt{s}\,.\, \texttt{p s} \implies \texttt{q s}))\ (\texttt{ensures}\ (\texttt{witnessed p} \implies \texttt{witnessed q}))$$

- We add a **stateful introduction rule** for `witnessed`

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Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

The motivating example revisited

- Recall the program operating on the **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert } (\mathrm{v} \in \mathrm{get}())$$

- We pick **set inclusion** $\subseteq$ as our preorder `rel` on states
- We **prove the assertion** by inserting a `witness` and `recall`

$$\texttt{insert v; witness } (\lambda\,\mathrm{s}\,.\,\mathrm{v} \in \mathrm{s})\texttt{; c\_p(); recall } (\lambda\,\mathrm{s}\,.\,\mathrm{v} \in \mathrm{s})\texttt{; assert } (\mathrm{v} \in \mathrm{get}())$$

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ML-style typed references (local state)

- First, we define a type of **heaps** as a finite map

$$\texttt{type heap} =$$
$$\quad \mid \texttt{H} : h{:}(\mathbb{N} \to \texttt{cell}) \to ctr{:}\mathbb{N}\{\forall n.\, ctr \le n \implies h\ n = \texttt{Unused}\} \to \texttt{heap}$$

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$$\texttt{type cell} =$$
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- As a result, we can define new **local state effect**

$$\texttt{MLST t pre post} \stackrel{\text{def}}{=} \texttt{MST}_{\texttt{heap,heap\_inclusion}} \texttt{ t pre post}$$

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let contains (H h _) id a =
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- Finally, we define `MLST`'s **actions** using `MST`'s actions
  - `let alloc (a:Type) (v:a) : MLST (ref a) ... = ...`
    - **get** the current heap
    - **create** a fresh ref., and **add** it to the heap
    - **put** the updated heap back
    - **witness** that the created ref. is in the heap
  - `let read (r:ref a) : MLST t ... = ...`
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Adding untyped and monotonic references

- **Untyped references** (`uref`) with strong updates
  - Used `heap` cells are extended with **tags**

    where

    $$| \ \texttt{Used} : \mathsf{a{:}Type} \to \mathsf{v{:}a} \to \texttt{t:tag} \to \texttt{cell}$$

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    $$\mathtt{type}\ \mathtt{tag} \ = \ \mathtt{Typed} : \mathtt{tag} \ | \ \mathtt{Untyped} : \mathtt{tag}$$

  - `urefs` can be extended to also support **deallocation**

- **Monotonic references** (`mref a rel`)
  - Used heap cells are extended with **typed tags**

    $$| \ \mathtt{Used} : \mathtt{a{:}Type} \to \mathtt{v{:}a} \to \mathtt{t{:}tag}\ \mathtt{a} \to \mathtt{cell}$$

    where

    $$\mathtt{type}\ \mathtt{tag}\ \mathtt{a} \ = \ \mathtt{Typed} : \mathtt{rel{:}preorder}\ \mathtt{a} \to \mathtt{tag}\ \mathtt{a} \ | \ \mathtt{Untyped} : \mathtt{tag}\ \mathtt{a}$$

  - `mrefs` provide **more flexibility** with ref.-wise monotonicity

Conclusion

- In conclusion
  - making use of monotonicity is very **useful** in verification
  - using monotonicity can be distilled into a **simple** interface
  - useful for **programming** (refs.) and **verification** (Prj. Everest)

- See the paper for
  - further **examples** and **case studies**
  - **meta-theory** and **correctness results** for `MST`
    - based on an instrumented operational semantics

      $$(\texttt{witness}\ x.\varphi\ ,\ s\ ,\ W)\ \rightsquigarrow\ (\texttt{return}\ ()\ ,\ s\ ,\ W \cup \{x.\varphi\})$$

    - and cut elimination for the `witnessed`-logic
  - first steps towards **monadic reification** for `MST`
    - useful for extrinsic reasoning, e.g., for relational properties
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Thank you!

Questions?