

Comodels as a gateway for interacting with the **external world**

Danel Ahman

(joint work with Andrej Bauer)

Shonan, 27 March 2019

Comodels as a gateway for interacting with the **external world**

Danel Ahman

(joint work with Andrej Bauer)



Shonan, 27 March 2019

Computational effects in FP

Computational effects in FP

- Using **monads** (as in HASKELL)

```
type St a = String → (a, String)
```

```
f :: St a → St (a, a)
```

```
f c = c >>= (\x → c >>= (\y → return (x, y)))
```

- Using **alg. effects** and **handlers** (as in EFF, FRANK, KOKA)

```
effect Get : int
```

```
effect Put : int → unit
```

```
(*: int → a*int!{} *)
```

```
let g (c: unit → a!{ Get, Put }) =
```

```
  with st_h handle (perform (Put 42); c ())
```

Computational effects in FP

- Using **monads** (as in HASKELL)

```
type St a = String → (a, String)
```

```
f :: St a → St (a, a)
```

```
f c = c >>= (\x → c >>= (\y → return (x, y)))
```

- Using **alg. effects** and **handlers** (as in EFF, FRANK, KOKA)

```
effect Get : int
```

```
effect Put : int → unit
```

```
(*: int → a*int!{} *)
```

```
let g (c: unit → a!{Get, Put}) =
```

```
  with st_h handle (perform (Put 42); c ())
```

- Both are good for **faking comp. effects** in a pure language!

But what about effects that need access to the **external world**?

External world in FP

- Declare a **signature** of monads or algebraic effects

```
type IO a
```

```
openFile  :: FilePath → IOMode → IO Handle
```

```
hGetLine  :: Handle → IO String
```

```
hClose    :: Handle → IO ()
```

```
effect Read  : string
```

```
effect Raise : string → empty
```

```
effect RandomInt    : int → int
```

```
effect RandomFloat : float → float
```

- And then treat them **specially** in the compiler, e.g.,

```
let rec top_handle op =
```

```
  match op with
```

```
  | ...
```

External world in FP

External world in FP



Ohad 🗿 12:17 PM

Can I do file IO (or just O) in Eff?

External world in FP



Ohad 🤖 12:17 PM

Can I do file IO (or just O) in Eff?



Žiga Lukšič 12:18 PM

not currently

External world in FP



Ohad 🤖 12:17 PM

Can I do file IO (or just O) in Eff?



Žiga Lukšič 12:18 PM

not currently



Ohad 🤖 8:35 PM

So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
  | "Write" ->
    (match v with
    | V.Tuple vs ->
      let (file_name :: str :: _) = List.map V.to_str vs in
      let file_handle = open_out_gen
        [Open_wronly
         ;Open_append
         ;Open_creat
         ;Open_text
        ] 0o666 file_name in
      Printf.fprintf file_handle "%s" str;
      close_out file_handle;
      top_handle (k V.unit_value)
    )
```

External world in FP



Ohad 12:17 PM

Can I do file IO (or just O) in Eff?



Žiga Lukšič 12:18 PM

not currently



Ohad 8:35 PM

So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
| "Write" ->
  (match v with
  | V.Tuple vs ->
    let (file_name :: str :: _) = List.map V.to_str vs in
    let file_handle = open_out_gen
      [Open_wronly
       ;Open_append
       ;Open_creat
       ;Open_text
      ] 0o666 file_name in
    Printf.fprintf file_handle "%s" str;
    close_out file_handle;
    top_handle (k V.unit_value)
  )
```

This talk — a principled (co)algebraic approach!

Another issue — **linearity** or lack thereof

Another issue — **linearity** or lack thereof

- ```
let f (s:string) =
 let fh = fopen "foo.txt" in
 fwrite fh (s^s);
 fclose fh;
 return fh

let g s =
 let fh = f s in fread fh
```

## Another issue — **linearity** or lack thereof

- ```
let f (s:string) =  
  let fh = fopen "foo.txt" in  
  fwrite fh (s^s);  
  fclose fh;  
  return fh  
  
let g s =  
  let fh = f s in fread fh  (* fh not open ! *)
```

Another issue — **linearity** or lack thereof

- ```
let f (s:string) =
 let fh = fopen "foo.txt" in
 fwrite fh (s^s);
 fclose fh;
 return fh

let g s =
 let fh = f s in fread fh (* fh not open ! *)
```
- We could resolve this by typing `fh` **linearly** (but `s` **non-linearly**)

## Another issue — **linearity** or lack thereof

- ```
let f (s:string) =  
  let fh = fopen "foo.txt" in  
  fwrite fh (s^s);  
  fclose fh;  
  return fh  
  
let g s =  
  let fh = f s in fread fh  (* fh not open ! *)
```
- We could resolve this by typing `fh` **linearly** (but `s` **non-linearly**)
- But what if we wrap `f` in a **handler**?

```
let h = handler  
  | effect (FWrite fh s k) → return fh  
  
let g s = with h handle f ()
```


Another issue — **linearity** or lack thereof

- ```
let f (s:string) =
 let fh = fopen "foo.txt" in
 fwrite fh (s^s);
 fclose fh;
 return fh

let g s =
 let fh = f s in fread fh (* fh not open ! *)
```
- We could resolve this by typing `fh` **linearly** (but `s` **non-linearly**)
- But what if we wrap `f` in a **handler**?

```
let h = handler
 | effect (FWrite fh s k) → return fh

let g s = with h handle f () (* dangling fh ! *)
```

**So, how could we solve these issues?**

# So, how could we solve these issues?

- We could try using **existing PL techniques**, e.g.,

- **Modules** and **abstraction**, e.g., `System.IO`

```
type IO a
```

```
hClose :: Handle → IO ()
```

- **Linear** (and **non-linear**) **types** and **effects**

```
linear type fhandle
```

```
effect FClose : (linear fhandle) → unit
```

```
linear effect FClose : fhandle → unit
```

- Handlers with **finally clauses**

# So, how could we solve these issues?

- We could try using **existing PL techniques**, e.g.,

- **Modules** and **abstraction**, e.g., `System.IO`

```
type IO a
```

```
hClose :: Handle → IO ()
```

- **Linear** (and **non-linear**) **types** and **effects**

```
linear type fhandle
```

```
effect FClose : (linear fhandle) → unit
```

```
linear effect FClose : fhandle → unit
```

- Handlers with **finally clauses**
- **Problem:** They don't really capture the **essence of the problem**

So, what is that **essence** then?

# So, what is that **essence** then?

- Let's look at HASKELL's **IO monad** again

# So, what is that **essence** then?

- Let's look at HASKELL's **IO monad** again
- A common explanation is to think of functions

$$a \rightarrow \text{IO } b$$

as

$$a \rightarrow (\text{RealWorld} \rightarrow (b, \text{RealWorld}))$$

which is the same as

$$(a, \text{RealWorld}) \rightarrow (b, \text{RealWorld})$$

# So, what is that **essence** then?

- Let's look at `HASKELL`'s **IO monad** again
- A common explanation is to think of functions

$$a \rightarrow \text{IO } b$$

as

$$a \rightarrow (\text{RealWorld} \rightarrow (b, \text{RealWorld}))$$

which is the same as

$$(a, \text{RealWorld}) \rightarrow (b, \text{RealWorld})$$

- With the `System.IO` **module abstraction** ensuring that
  - We **cannot get our hands on** `RealWorld`
  - We have the impression of `RealWorld` **used linearly**
  - We **don't ask more** from `RealWorld` than it can provide



# So, what is that **essence** then?

- Let's look at HASKELL's **IO monad** again
- A common explanation is to think of functions

$$a \rightarrow \text{IO } b$$

as

$$a \rightarrow (\text{RealWorld} \rightarrow (b, \text{RealWorld}))$$

which is the same as

$$(s \rightarrow \text{RealWorld}) \rightarrow (b \rightarrow \text{RealWorld})$$

**But wait a minute!** **RealWorld** looks a lot like a **comodel**!

`hGetLine` : `(Handle, RealWorld) → (String, RealWorld)`

`hClose` : `(Handle, RealWorld) → ((), RealWorld)`

I.e., **IO** is about the **external world** rather than internal effects!

**Important:** co-operations (`hClose`) make a **promise to return**!

**Refresher: what is a comodel?**

# Refresher: what is a comodel?

- A signature  $\Sigma$  is a set of operation symbols  $\text{op} : A \rightsquigarrow B$

# Refresher: what is a comodel?

- A **signature**  $\Sigma$  is a set of operation symbols  $\text{op} : A \rightsquigarrow B$
- A **model/algebra/handler**  $\mathcal{M}$  of  $\Sigma$  is given by

$$\mathcal{M} = \langle M : \text{Set} , \{ \text{op}_{\mathcal{M}} : A \times M^B \longrightarrow M \}_{\text{op} \in \Sigma} \rangle$$

# Refresher: what is a **comodel**?

- A **signature**  $\Sigma$  is a set of operation symbols  $\text{op} : A \rightsquigarrow B$
- A **model/algebra/handler**  $\mathcal{M}$  of  $\Sigma$  is given by

$$\mathcal{M} = \langle M : \text{Set} , \{ \text{op}_{\mathcal{M}} : A \times M^B \longrightarrow M \}_{\text{op} \in \Sigma} \rangle$$

- A **comodel/coalgebra/cohandler**  $\mathcal{W}$  of  $\Sigma$  is given by

$$\mathcal{W} = \langle W : \text{Set} , \{ \overline{\text{op}}_{\mathcal{W}} : A \times W \longrightarrow B \times W \}_{\text{op} \in \Sigma} \rangle$$

- Intutively, comodels describe **evolution of the world**  $W$

# Refresher: what is a **comodel**?

- A **signature**  $\Sigma$  is a set of operation symbols  $\text{op} : A \rightsquigarrow B$
- A **model/algebra/handler**  $\mathcal{M}$  of  $\Sigma$  is given by

$$\mathcal{M} = \langle M : \text{Set} , \{ \text{op}_{\mathcal{M}} : A \times M^B \longrightarrow M \}_{\text{op} \in \Sigma} \rangle$$

- A **comodel/coalgebra/cohandler**  $\mathcal{W}$  of  $\Sigma$  is given by

$$\mathcal{W} = \langle W : \text{Set} , \{ \overline{\text{op}}_{\mathcal{W}} : A \times W \longrightarrow B \times W \}_{\text{op} \in \Sigma} \rangle$$

- Intutively, comodels describe **evolution of the world  $W$** 
  - Operational semantics using a tensor of a model and a comodel  
(Plotkin & Power, Abou-Saleh & Pattinson)
  - Stateful runners of effectful programs (Uustalu)
  - Linear state-passing translation (Møgelberg and Staton)
  - Top-level behaviour of alg. effects in  $\text{EFF v2}$  (Bauer & Pretnar)

## Comodels as a gateway to the external world

- ```
let f (s:string) =  
    using IO cohandle  
        let fh = fopen "foo.txt" in  
        fwrite fh (s^s);  
        fclose fh
```

 (* in IO *)

Now **external world** explicit, but **dangling** `fh` etc **still possible**

Comodels as a gateway to the external world

- ```
let f (s:string) =
 using IO cohandle
 let fh = fopen "foo.txt" in
 fwrite fh (s^s);
 fclose fh
```

 (\* in IO \*)

Now **external world** explicit, but **dangling** `fh` etc **still possible**

- ```
let f (s:string) =  
    using IO cohandle  
        let fh = fopen "foo.txt" in  
        fwrite fh (s^s)  
    finally (fclose fh)
```

 (* in IO *)

Better, but **have to explicitly open and thread through** `fh`

Comodels as a gateway to the external world

- ```
let f (s:string) =
 using IO cohandle
 let fh = fopen "foo.txt" in
 fwrite fh (s^s);
 fclose fh
```

 (\* in IO \*)

Now **external world** explicit, but **dangling** `fh` etc **still possible**

- ```
let f (s:string) =  
    using IO cohandle  
        let fh = fopen "foo.txt" in  
        fwrite fh (s^s)  
    finally (fclose fh)
```

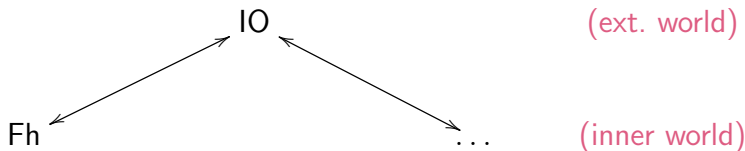
 (* in IO *)

Better, but **have to explicitly open and thread through** `fh`

- **Solution:** **Modular treatment** of **external worlds**

Comodels as a gateway to the external world

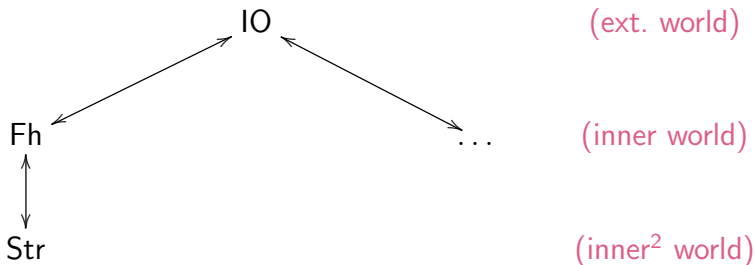
- Examples of **modularity** we might want from comodels



- Fh — “**world** which consists of **exactly one** fh ”
- IO \longrightarrow Fh — “call `fopen` with `foo.txt` , store returned fh ”
- Fh \longrightarrow IO — “call `fclose` with stored fh ”

Comodels as a gateway to the external world

- Examples of **modularity** we might want from comodels



- Fh — “**world** which consists of **exactly one** fh ”
- IO → Fh — “call fopen with foo.txt , store returned fh ”
- Fh → IO — “call fclose with stored fh ”
- Str — “world that is **blissfully unaware** of fh ”

Comodels as a gateway to the external world

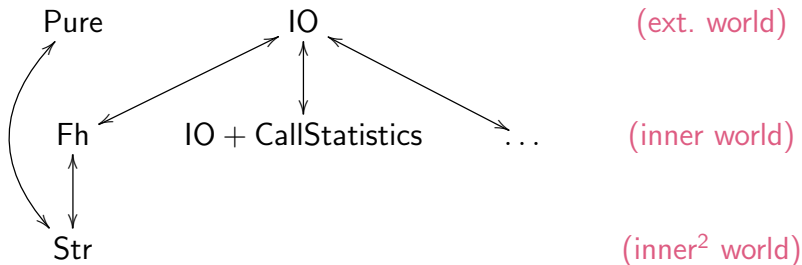
- Examples of **modularity** we might want from comodels



- Fh — “**world** which consists of **exactly one** fh ”
- IO \longrightarrow Fh — “call `fopen` with `foo.txt` , store returned fh ”
- Fh \longrightarrow IO — “call `fclose` with stored fh ”
- Str — “world that is **blissfully unaware** of fh ”

Comodels as a gateway to the external world

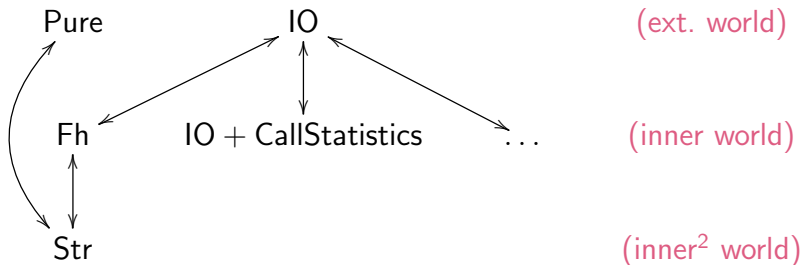
- Examples of **modularity** we might want from comodels



- Fh — “**world** which consists of **exactly one** fh”
- IO \longrightarrow Fh — “call `fopen` with `foo.txt`, store returned fh”
- Fh \longrightarrow IO — “call `fclose` with stored fh”
- Str — “world that is **blissfully unaware** of fh”

Comodels as a gateway to the external world

- Examples of **modularity** we might want from comodels



- Fh** — “**world** which consists of **exactly one** **fh**”
- IO** \longrightarrow **Fh** — “call **fopen** with **foo.txt**, store returned **fh**”
- Fh** \longrightarrow **IO** — “call **fclose** with stored **fh**”
- Str** — “world that is **blissfully unaware** of **fh**”
- Observation:** **IO** \longleftrightarrow **Fh** and other \longleftrightarrow look a lot like **lenses**

Comodels as a gateway to the **external world**

Comodels as a gateway to the external world

- Our **general framework** on the file operations example

```
let f (s:string) =  
    using  
        Fh @ (fopen_of_io "foo.txt")  
    cohandle  
        fwrite_of_fh (s^s)  
    finally  
        x @ fh → fclose_of_io fh
```


Comodels as a gateway to the external world

- Our **general framework** on the file operations example

```
let f (s:string) = (* in IO *)  
  using  
    Fh @ (fopen_of_io "foo.txt") (* in IO *)  
  cohandle  
    fwrite_of_fh (s^s) (* in Fh *)  
  finally  
    x @ fh → fclose_of_io fh (* in IO *)
```

Comodels as a gateway to the external world

- Our **general framework** on the file operations example

```
let f (s:string) = (* in IO *)
  using
    Fh @ (fopen_of_io "foo.txt") (* in IO *)
  cohandle
    fwrite_of_fh (s^s) (* in Fh *)
  finally
    x @ fh → fclose_of_io fh (* in IO *)
```

where

```
Fh = (* W = fhandle *)
{ co_fread _ @ fh → ... ,
  co_fwrite s @ fh → fwrite_of_io s fh ;
  return ((),fh) }

(* co_fread : (unit * W) → (string * W) *)
(* co_fwrite : (string * W) → (unit * W) *)
```

Comodels as a gateway to the **external world**

Comodels as a gateway to the external world

- The **modularity aspect** of our general framework

```
let f (s:string) = (* in IO *)
  using Fh @ (fopen_of_io "foo.txt")
  cohandle

    using Str @ (fread_of_fh ()) (* in Fh *)
    cohandle
      write_of_str (s^s) (* in Str *)
    finally
      _ @ s → fwrite_of_fh s

  finally
    _ @ fh → fclose_of_io fh
```

where

```
Str = { co_write s @ s' → (* W = string *)
        return (( ), s'^s) }
```

Comodels as a gateway to the **external world**

Comodels as a gateway to the external world

- Comodels can also **extend** the (intermediate) external world

```
let f (s:string) = (* in IO *)
  using Stats @ (fopen_of_io "foo.txt")
  cohandle
    fwrite_of_stats (s^s)
  finally
    - @ (fh,c) →
      let fh' = fopen_of_io "stats.txt" in
      fwrite_of_io fh' c; fclose_of_io fh';
      fclose_of_io fh
```

where

```
Stats = (* W = fhandle * nat*)
{ co_fread   - @ (fh,c) → ... ,
  co_fwrite s @ (fh,c) → ... ,
  co_reset   - @ (fh,c) → return ((),(fh,0)) }
```

Comodels as a gateway to the external world

- Comodels can also **extend** the (intermediate) external world

```
let f (s:string) = (* in IO *)
  using Stats @ (fopen_of_io "foo.txt")
  cohandle
    fwrite_of_stats (s^s)
  finally
    - @ (fh,c) →
      let fh' = fopen_of_io "stats.txt" in
      fwrite_of_io fh' c; fclose_of_io fh';
      fclose_of_io fh
```

where

```
Stats = (* W = fhandle * nat*)
{ co_fread    - @ (fh,c) → ... ,
  co_fwrite s @ (fh,c) → ... ,
  co_reset    - @ (fh,c) → return ((),(fh,0)) }
```

- Can also track **nondet./prob. choice results**, etc

Comodels as a gateway to the **external world**

Comodels as a gateway to the external world

- The external world could also be **pure**

```
let f (s:string) = (* in Pure *)
  using Str @ (return "default value")
  cohandle
    ...
    let s = read_of_str () in
    if (s == "foo")
    then (...; write_of_str "bar"; ...)
    else (...)
  finally
    x @ s → return x
```

where

```
Str = (* W = string *)
{ co_read _ @ s → return (s,s) ,
  co_write s @ s' → return ((),s') }
```

So what's happening **more formally**?

So what's happening **more formally?**

- Core calculus for cohandlers (wo/ handlers \Rightarrow wait few slides)

So what's happening **more formally?**

- Core calculus for cohandlers (wo/ handlers \Rightarrow wait few slides)
- **Types**

$$A, B, W ::= b \mid 1 \mid A \times B \mid 0 \mid A + B \mid A \xrightarrow{\Sigma} B$$

So what's happening **more formally**?

- Core calculus for cohandlers (wo/ handlers \Rightarrow wait few slides)
- **Types**

$$A, B, W ::= b \mid 1 \mid A \times B \mid 0 \mid A + B \mid A \xrightarrow{\Sigma} B$$

- Signatures of **worlds**

$$\omega ::= \{ \text{op}_1 : A_1 \rightsquigarrow B_1, \dots, \text{op}_n : A_n \rightsquigarrow B_n \}$$

So what's happening **more formally**?

- Core calculus for cohandlers (wo/ handlers \Rightarrow wait few slides)
- **Types**

$$A, B, W ::= b \mid 1 \mid A \times B \mid 0 \mid A + B \mid A \xrightarrow{\Sigma} B$$

- Signatures of **worlds**

$$\omega ::= \{ \text{op}_1 : A_1 \rightsquigarrow B_1, \dots, \text{op}_n : A_n \rightsquigarrow B_n \}$$

- **Computation terms** (value terms are unsurprising)

$$\begin{aligned} c ::= & \text{ return } v \mid \text{ let } x = c_1 \text{ in } c_2 \mid v_1 v_2 \\ & \mid \widehat{\text{op}} \ v \\ & \mid \text{ using } C @ c_i \text{ cohandle } c \text{ finally } x @ w \rightarrow c_f \end{aligned} \quad \text{(comodel op.)}$$

So what's happening **more formally?**

- Core calculus for cohandlers (wo/ handlers \Rightarrow wait few slides)
- Types**

$$A, B, W ::= b \mid 1 \mid A \times B \mid 0 \mid A + B \mid A \xrightarrow{\Sigma} B$$

- Signatures of **worlds**

$$\omega ::= \{ \text{op}_1 : A_1 \rightsquigarrow B_1, \dots, \text{op}_n : A_n \rightsquigarrow B_n \}$$

- Computation terms** (value terms are unsurprising)

$$\begin{aligned} c ::= & \text{ return } v \mid \text{ let } x = c_1 \text{ in } c_2 \mid v_1 v_2 \\ & \mid \widehat{\text{op}} \ v \hspace{15em} (\text{comodel op.}) \\ & \mid \text{ using } C @ c_i \text{ cohandle } c \text{ finally } x @ w \rightarrow c_f \end{aligned}$$

- Comodels (cohandlers)**

$$C ::= \{ \overline{\text{op}}_1 \ x @ w \rightarrow c_1, \dots, \overline{\text{op}}_n \ x @ w \rightarrow c_n \}$$

So what's happening **more formally**?

So what's happening **more formally?**

- **Typing judgements**

$$\Gamma \vdash v : A$$
$$\Gamma \Vdash c : A$$

So what's happening **more formally?**

- **Typing judgements**

$$\Gamma \vdash v : A$$

$$\Gamma \Vdash c : A$$

- The two central **typing rules** are

$$\Gamma \Vdash D \text{ comodel of } \omega' \text{ with carrier } W_D \quad \Gamma \Vdash c_i : W_D$$

$$\Gamma \Vdash c : A \quad \Gamma, x:A, w:W_D \Vdash c_f : B$$

$$\Gamma \Vdash \text{using } D @ c_i \text{ cohandle } c \text{ finally } x @ w \rightarrow c_f : B$$

So what's happening **more formally?**

- **Typing judgements**

$$\Gamma \vdash v : A \qquad \Gamma \Vdash c : A$$

- The two central **typing rules** are

$$\begin{array}{c} \Gamma \Vdash D \text{ comodel of } \omega' \text{ with carrier } W_D \qquad \Gamma \Vdash c_i : W_D \\ \Gamma \Vdash' c : A \qquad \Gamma, x:A, w:W_D \Vdash c_f : B \\ \hline \Gamma \Vdash \text{using } D @ c_i \text{ cohandle } c \text{ finally } x @ w \rightarrow c_f : B \end{array}$$

and

$$\frac{\text{op} : A \rightsquigarrow B \in \omega \qquad \Gamma \vdash v : A}{\Gamma \Vdash \widehat{\text{op}} v : B}$$

So what's happening **more formally?**

So what's happening **more formally**?

- **Denotational semantics** is heavily inspired by Møgelberg and Staton's **linear state-passing translation**

So what's happening **more formally**?

- **Denotational semantics** is heavily inspired by Møgelberg and Staton's **linear state-passing translation**
- **Term interpretation** looks very similar to **alg. effects**:

$$\llbracket \Gamma \vdash v : A \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow \llbracket A \rrbracket \qquad \llbracket \Gamma \stackrel{\omega}{\vdash} c : A \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow T_{\omega} \llbracket A \rrbracket$$

- **un-cohandled operations wait for a suitable external world!**

So what's happening **more formally**?

- **Denotational semantics** is heavily inspired by Møgelberg and Staton's **linear state-passing translation**
- **Term interpretation** looks very similar to **alg. effects**:

$$\llbracket \Gamma \vdash v : A \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow \llbracket A \rrbracket \qquad \llbracket \Gamma \vDash^{\omega} c : A \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow T_{\omega} \llbracket A \rrbracket$$

- **un-cohandled operations wait for a suitable external world!**
- The interesting part is the interpretation of **cohandling**

$$\Gamma \vDash^{\omega} \text{using } D @ c_i \text{ cohandle } c \text{ finally } x @ w \rightarrow c_f : B$$

which is based on the **linear state-passing translation**, i.e.,

$$\frac{\llbracket D \rrbracket \in \text{Comod}_{\omega'}(\text{Kleisli}(T_{\omega}))}{\text{cohandle_with}_{\llbracket D \rrbracket} : T_{\omega'} \llbracket A \rrbracket \longrightarrow \left(\llbracket W_D \rrbracket \rightarrow T_{\omega} (\llbracket A \rrbracket \times \llbracket W_D \rrbracket) \right)}$$

So what's happening **more formally**?

So what's happening **more formally**?

- Regarding **op. semantics**, e.g., consider confs. $(\overrightarrow{(C, w)}, c)$

So what's happening **more formally**?

- Regarding **op. semantics**, e.g., consider confs. $(\overrightarrow{(C, w)}, c)$
- For example, consider the **big-step evaluation** of **using** D ...

So what's happening **more formally**?

- Regarding **op. semantics**, e.g., consider confs. $((\overrightarrow{C, w}), c)$
- For example, consider the **big-step evaluation** of **using D ...**

$$((\overrightarrow{C, w_0}), (C', w'_0)) , c_i) \Downarrow ((\overrightarrow{C, w_1}), (C', w'_1)) , \text{return } w''_0)$$

$$((\overrightarrow{C, w_1}), (C', w'_1), (D, w''_0)) , c) \Downarrow ((\overrightarrow{C, w_2}), (C', w'_2), (D, w''_1)) , \text{return } v)$$

$$((\overrightarrow{C, w_2}), (C', w'_2)) , c_f[v/x, w''_1/w]) \Downarrow ((\overrightarrow{C, w_3}), (C', w'_3)) , \text{return } v')$$

$$((\overrightarrow{C, w_0}), (C', w'_0)) , \text{using D @ } c_i \text{ cohandle } c \text{ finally } x @ w \rightarrow c_f)$$

$$\Downarrow$$

$$((\overrightarrow{C, w_3}), (C', w'_3)) , \text{return } v')$$

So what's happening **more formally**?

- Regarding **op. semantics**, e.g., consider confs. $((\overrightarrow{C, w}), c)$
- For example, consider the **big-step evaluation** of **using D ...**

$$((\overrightarrow{C, w_0}), (C', w'_0)) , c_i) \Downarrow ((\overrightarrow{C, w_1}), (C', w'_1)) , \text{return } w''_0)$$

$$((\overrightarrow{C, w_1}), (C', w'_1), (D, w''_0)) , c) \Downarrow ((\overrightarrow{C, w_2}), (C', w'_2), (D, w''_1)) , \text{return } v)$$

$$((\overrightarrow{C, w_2}), (C', w'_2)) , c_f[v/x, w''_1/w]) \Downarrow ((\overrightarrow{C, w_3}), (C', w'_3)) , \text{return } v')$$

$$((\overrightarrow{C, w_0}), (C', w'_0)) , \text{using D @ } c_i \text{ cohandle } c \text{ finally } x @ w \rightarrow c_f)$$

$$\Downarrow$$

$$((\overrightarrow{C, w_3}), (C', w'_3)) , \text{return } v')$$

- The interpretation of **operations** uses the **co-operations** of Cs

But what about **alg. effects** and **handlers**?

But what about **alg. effects** and **handlers**?

- **First:** combining this with **standard alg. effects** and **handlers**

But what about **alg. effects** and **handlers**?

- **First:** combining this with **standard alg. effects** and **handlers**
- In the following

```
using C @ c_i  
cohandle c  
finally x @ w  $\rightarrow$  c_f
```

it is natural to want that

- **algebraic operations** (in the sense of E_{FF}) are allowed in `c`,
but they must not be allowed to escape `cohandle`
- to escape, have to use the **co-operations** of the **external world**

But what about **alg. effects** and **handlers**?

- **First:** combining this with **standard alg. effects** and **handlers**
- In the following

```
using C @ c_i  
cohandle c  
finally x @ w → c_f
```

it is natural to want that

- **algebraic operations** (in the sense of EFF) are allowed in `c`,
but they must not be allowed to escape `cohandle`
- to escape, have to use the **co-operations** of the **external world**
- the **continuations of handlers** in `c` are delimited by `cohandle`

But what about **alg. effects** and **handlers**?

- **First:** combining this with **standard alg. effects** and **handlers**
- In the following

```
using C @ c_i  
cohandle c  
finally x @ w → c_f
```

it is natural to want that

- **algebraic operations** (in the sense of E_{FF}) are allowed in `c`,
but they must not be allowed to escape `cohandle`
- to escape, have to use the **co-operations** of the **external world**
- the **continuations of handlers** in `c` are delimited by `cohandle`
- Where do **multi-handlers** fit? Co-operating handlers-cohandlers?

But what about **alg. effects** and **handlers**?

But what about **alg. effects** and **handlers**?

- **Second:** What if the **outer comodel beaks its promise**?
 - E.g., **IO** lost connection to the HDD where “foo.txt” was

But what about **alg. effects** and **handlers**?

- **Second:** What if the **outer comodel beaks its promise**?
 - E.g., **IO** lost connection to the HDD where “foo.txt” was

But what about **alg. effects** and **handlers**?

- **Second:** What if the **outer comodel beaks its promise**?
 - E.g., **IO** lost connection to the HDD where “foo.txt” was
- **Idea:**
 - Use algebraic effects to **communicate downwards**
 - (Algebraic ops. only allowed to appear in co-operations)
 - **finally** acts as a **handler** for **broken promises**

But what about **alg. effects** and **handlers**?

- **Second:** What if the **outer comodel beaks its promise?**
 - E.g., **IO** lost connection to the HDD where “foo.txt” was
- **Idea:**
 - Use algebraic effects to **communicate downwards**
 - (Algebraic ops. only allowed to appear in co-operations)
 - **finally** acts as a **handler** for **broken promises**

using

C @ c_i

cohandle

fwrite_of_d s;

(* co_fwrite throws e *)

fread ()

finally

| x @ w → c_f

| throw e → c_do_some_cleanup

| op x k → ...

Conclusions

Conclusions

- **Comodels** as a gateway for interacting with the **external world**
- We're making them into a **modular programming abstraction**
- **Linearity** by leaving **outer worlds** implicit (via comodel ops.)
- System.IO , KOKA's **initially** & **finally** , PYTHON's **with** , ...

Conclusions

- **Comodels** as a gateway for interacting with the **external world**
- We're making them into a **modular programming abstraction**
- **Linearity** by leaving **outer worlds** implicit (via comodel ops.)
- `System.IO` , KOKA's **initially** & **finally** , PYTHON's **with** , ...

Ongoing work

- **Algebraic effects** and **(multi-)handlers**
- More **examples** and **use cases**
- Clarify the connection with **(effectful) lenses**
- **Combinatorics** of comodels and their lens-like relationships