

# Runners in action

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# Today's plan

- **Computational effects** and **external resources** in PL
- **Issues with standard approaches** to **external resources**
- **Runners** – a natural model for **top-level runtime**
- **T-runners** – for also modelling **non-top-level runtimes**
- Turn **T**-runners into a **useful programming construct**
- Demonstrate the use of runners through **programming examples**

**Computational effects**  
**and**  
**external resources**

# Computational effects in PL

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- Using **monads** (as in HASKELL)

```
type St a = String → (a,String)
```

```
instance St Monad where
```

```
...
```

```
f :: St a → St (a,a)
```

```
f c = c >>= (\ x → c >>= (\ y → return (x,y)))
```

- Using **alg. effects** and **handlers** (as in EFF, FRANK, KOKA)

```
effect Get : unit → int
```

```
effect Put : int → unit
```

```
let g (c:unit → a!{Get,Put}) : int → a * int ! {} =
```

```
  with st_handler handle (perform (Put 42); c ())
```

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```

- Good for **simulating comp. effects** in a pure language!

But what about effects that need access to the **external world**?

# External resources in PL

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- Declare a **signature of monads** or **algebraic effects**, e.g.,

```
(* System.IO *)
type IO a
openFile :: FilePath → IOMode → IO Handle
```

```
(* pervasives.eff *)
effect RandomInt : int → int
effect RandomFloat : float → float
```

- And then **treat them specially** in the compiler, e.g., in EFF

```
(* eff/src/backends/runtime/eval.ml *)
let rec top_handle op =
  match op with
  | Value v → v
  | Call (RandomInt, v, k) →
    top_handle (k (Const.of_integer (Random.int (Value.to_int v))))
  | ...
```



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but there are **some issues** with that approach ...

# First issue

- Difficult to cover all possible use cases
  - **external resources hard-coded** into the top-level runtime
  - **non-trivial to change** what's available and how it's implemented

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So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
| "Write" ->
  (match v with
  | V.Tuple vs ->
    let (file_name :: str :: _) = List.map V.to_str vs in
    let file_handle = open_out_gen
      [Open_wronly
       ;Open_append
       ;Open_creat
       ;Open_text
       ] 0o666 file_name in
    Printf.fprintf file_handle "%s" str;
    close_out file_handle;
    top_handle (k V.unit_value)
  )
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**This talk — a principled modular (co)algebraic approach!**

# Second issue

- **Lack of linearity** for external resources

```
let f (s:string) =  
  let fh = fopen "foo.txt" in  
  fwrite (fh,s^s);  
  fclose fh;  
  return fh
```

```
let g s =  
  let fh = f s in fread fh
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(\* fh not open any more ! \*)

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(\* fh not open any more ! \*)

- We shall address these kinds of issues **indirectly (!)**:
  - by **not** introducing a linear typing discipline
  - instead we make it convenient to **hide external resources**  
(addressing stronger typing disciplines in the future)

# Third issue

- **Excessive generality** of effect handlers

```
let f (s:string) =  
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```

- But misuse of external resources can also be **purely accidental**

```
let g (s1 s2:string) =  
  let fh = fopen "foo.txt" in  
  let b = choose () in  
  if b then (fwrite (fh,s1^s2)) else (fwrite (fh,s2^s1));  
  fclose fh  
  
let nd_handler =  
  handler { choose () k → return (k true ++ k false) }
```



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```

- We shall address these kinds of issues **directly (!!)**,
  - by proposing a **restricted form of handlers** for resources
  - that support **controlled initialisation** and **finalisation**,
  - (and limit how general handlers can be used)

# Runners

# A natural model of **top-level runtime**

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- Given a **signature**<sup>1</sup>  $\Sigma$  of operation symbols ( $A_{\text{op}}, B_{\text{op}}$  are sets)

$$\text{op} : A_{\text{op}} \rightsquigarrow B_{\text{op}}$$

a **runner**<sup>2</sup>  $\mathcal{R}$  for  $\Sigma$  is given by a carrier  $|\mathcal{R}|$  and co-operations

$$\left( \overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \times |\mathcal{R}| \longrightarrow B_{\text{op}} \times |\mathcal{R}| \right)_{\text{op} \in \Sigma}$$

where we think of  $|\mathcal{R}|$  as a set of **runtime configurations**

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- For example, a natural **runner  $\mathcal{R}$  for  $S$ -valued state** signature

$$\left\{ \text{get} : \mathbb{1} \rightsquigarrow S \quad , \quad \text{set} : S \rightsquigarrow \mathbb{1} \right\}$$

is given by

$$|\mathcal{R}| \stackrel{\text{def}}{=} S \qquad \overline{\text{get}}_{\mathcal{R}}(\star, s) \stackrel{\text{def}}{=} (s, s) \qquad \overline{\text{set}}_{\mathcal{R}}(s', s) \stackrel{\text{def}}{=} (\star, s')$$

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# A natural model of **top-level runtime** ctd.

- Runners/comodels have been used for
  - **operational semantics** using tensors of models and comodels  
[Plotkin and Power '08]
  - **top-level implementation of algebraic effects** in  $\text{EFF}$   
[Bauer and Pretnar '15]and
- **stateful running** of algebraic effects [Uustalu '15]
- **linear-use state-passing translation**  
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- **stateful running** of algebraic effects [Uustalu '15]
- **linear-use state-passing translation** [Møgelberg and Staton '11, '14]
- The latter explicitly rely on one-to-one correspondence between
  - **runners**  $\mathcal{R}$
  - **monad morphisms**<sup>3</sup>  $r : \mathbf{Free}_{\Sigma}(-) \longrightarrow \mathbf{St}_{|\mathcal{R}|}$

---

<sup>3</sup> $\mathbf{Free}_{\Sigma}(X)$  is the free monad ind. defined with leaves  $\text{val } x$  and nodes  $\text{op}(a, \kappa)$ .

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- But what if this runtime is not **\*\*the\*\*** runtime?
  - hardware vs OSs
  - OSs vs VMs
  - VMs vs sandboxes

but also

- browsers vs web pages
- ...

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- But what if this runtime is not **\*\*the\*\*** runtime?
  - hardware vs OSs
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- Unfortunately, runners, as defined above, are **not readily able to**
  - use **external resources**
  - **signal failure** caused by unavoidable circumstances
- But is there a **useful generalisation** that would achieve this?

# Effectful runners for modular top-levels

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- Møgelberg and Staton usefully observed that a **runner**  $\mathcal{R}$  is equivalently simply a family of **generic effects** for  $\mathbf{St}_{|\mathcal{R}|}$ , i.e.,

$$\left( \overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow \mathbf{St}_{|\mathcal{R}|} B_{\text{op}} \right)_{\text{op} \in \Sigma}$$

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- Building on this, we define a **T-runner**  $\mathcal{R}$  for  $\Sigma$  to be given by

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- The one-to-one correspondence with **monad morphisms**

$$r : \mathbf{Free}_{\Sigma}(-) \longrightarrow \mathbf{T}$$

simply amounts to the **universal property of free models**, i.e.,

$$r_X(\text{val } x) = \eta_X x \qquad r_X(\text{op}(a, \kappa)) = \underbrace{(r_X \circ \kappa)^{\dagger}(\overline{\text{op}}_{\mathcal{R}} a)}_{\text{op}_{\mathcal{M}}(a, r_X \circ \kappa)}$$

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- Observe that  $\kappa$  appears in a **tail call position** on the right!

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  - (i)  $\text{getenv} : \mathbb{1} \rightsquigarrow C$     &     $\text{setenv} : C \rightsquigarrow \mathbb{1}$
  - (ii)  $\text{op} : A_{\text{op}} \rightsquigarrow B_{\text{op}}$     ( $\text{op} \in \Sigma'$ , for some external  $\Sigma'$ )
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- The **induced monad** is then isomorphic to

$$\mathbf{T} X \stackrel{\text{def}}{=} C \Rightarrow \mathbf{Free}_{\Sigma'}(X \times C + S)$$

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- The corresponding **T-runners**  $\mathcal{R}$  for  $\Sigma$  are then of the form

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- With this, our **T-runners**  $\mathcal{R}$  for  $\Sigma$  are (with “primitive” excs.)

$$\left( \overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow \mathbf{K}_{\Sigma', E_{\text{op}}, S, C} B_{\text{op}} \right)_{\text{op} \in \Sigma}$$

where we call  $\mathbf{K}_{\Sigma, E, S, C}$  a **kernel monad** (the sum of **T** and excs.)

$$\mathbf{K}_{\Sigma', E_{\text{op}}, S, C} B_{\text{op}} \stackrel{\text{def}}{=} C \Rightarrow \mathbf{Free}_{\Sigma'}((B_{\text{op}} + E_{\text{op}}) \times C + S)$$



**T-runners as a programming construct**  
(towards a core calculus for runners)

# T-runners as a programming construct

- First, we include **T-runners** for  $\Sigma$

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in our language **as values**, and **co-ops. as kernel code**, i.e.,

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let R = { op1 x1 → K1 , ... , opn xn → Kn }C
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- For instance, we can provide **write-only file access** as

```
let RFH = {  
  write s → if (length s > maxSize)  
    then (raise WriteSizeExceeded)  
    else (let fh = getenv () in  
      if (isValid fh) then (fwrite (fh,s)) else (kill IOError))  
}FileHandle
```

where

$$\Sigma \stackrel{\text{def}}{=} \{ \text{write} : \text{String} \rightsquigarrow 1 ! E \cup \{ \text{WriteSizeExceeded} \} \}$$

$$(\text{fwrite} : \text{FileHandle} \times \text{String} \rightsquigarrow 1 ! E) \in \Sigma' \quad S = \{ \text{IOError} \}$$

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- We make use of it to **run user code**:

```
using R @ Minit
run M
finally {return x @ c → Mret , ... raise e @ c → Me ... , ... kill s → Ms ...}
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where

(**user monads**)

- $M_s$  are **user code**, modelled using  $\mathbf{U}_{\Sigma,E} X \stackrel{\text{def}}{=} \mathbf{Free}_\Sigma(X + E)$

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- $M_{\text{init}}$  produces the **initial kernel state**
- $M$  is the user code being **run using the runner**  $R$
- $M_{\text{ret}}$ ,  $M_e$ ,  $M_s$  **finalise** for return values, exceptions, and signals

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- $M_{\text{init}}$  produces the **initial kernel state**
- $M$  is the user code being **run using the runner**  $R$
- $M_{\text{ret}}$ ,  $M_e$ ,  $M_s$  **finalise** for return values, exceptions, and signals
- $M_{\text{ret}}$  and  $M_e$  **depend on the final state**  $c$ , but  $M_s$  **does not**



# Controlled **initialisation** and **finalisation** ctd.

- For instance, we can define a PYTHON-esque **with construct**

```
with fileName do M
=
using R_FH @ (fopen fileName)
run M
finally {
  return x @ fh → fclose fh; return x ,
  raise WriteSizeExceeded @ fh → fclose fh; return () ,
  raise e @ fh → fclose fh; raise e , (* other exceptions in E are re-raised *)
  kill IOError → ... }
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  kill IOError → ... }
```

- the **file handle is hidden** from M
- M **can only call** write : String  $\rightsquigarrow$  1 ! E  $\cup$  {WriteSizeExceeded}  
but **not** (the external operations) **fopen** , **fclose** , and **fwrite**
- fopen** and **fclose** are **limited to initialisation-finalisation**
- M can itself also catch WriteSizeExceeded to **re-try writing**

**A core calculus for  
programming with runners**

# Core calculus (types and judgements)

# Core calculus (types and judgements)

- **Ground types** (for types of operations and kernel state)

$$A, B, C ::= B \mid 1 \mid 0 \mid A \times B \mid A + B$$

- **Types**

$$\begin{aligned} X, Y ::= & B \mid 1 \mid 0 \mid X \times Y \mid X + Y \\ & \mid X \rightarrow Y! (\Sigma, E) \\ & \mid X \rightarrow Y \downarrow (\Sigma, E, S, C) \\ & \mid \Sigma \Rightarrow (\Sigma', S, C) \end{aligned}$$

- **Values**

$$\Gamma \vdash V : X \qquad \Gamma \vdash V \equiv W : X$$

- **User computations**

$$\Gamma \vdash M : X! (\Sigma, E) \qquad \Gamma \vdash M \equiv N : X! (\Sigma, E)$$

- **Kernel computations**

$$\Gamma \vdash K : X \downarrow (\Sigma, E, S, C) \qquad \Gamma \vdash K \equiv L : X \downarrow (\Sigma, E, S, C)$$

# Core calculus (user computations)

$M, N ::= \text{return } V$

value

|  $\text{try } M \text{ with } \{\text{return } x \mapsto N, (\text{raise } e \mapsto N_e)_{e \in E}\}$

exception handler

|  $V W$

application

|  $\text{match } V \text{ with } \{\langle x, y \rangle \mapsto M\}$

product elimination

|  $\text{match } V \text{ with } \{\} X$

empty elimination

|  $\text{match } V \text{ with } \{\text{inl } x \mapsto M, \text{inr } y \mapsto N\}$

sum elimination

|  $\text{op}_X(V, (x . M), (N_e)_{e \in E_{\text{op}}})$

operation call

|  $\text{raise}_X e$

raise exception

|  $\text{using } V @ W \text{ run } M \text{ finally } \{$

run

$\text{return } x @ c \mapsto N,$

$(\text{raise } e @ c \mapsto N_e)_{e \in E},$

$(\text{kill } s \mapsto N_s)_{s \in S}\}$

|  $\text{kernel } K @ V \text{ finally } \{$

switch to kernel mode

$\text{return } x @ c \mapsto N,$

$(\text{raise } e @ c \mapsto N_e)_{e \in E},$

$(\text{kill } s \mapsto N_s)_{s \in S}\}$

# Core calculus (kernel computations)

$K, L ::=$	$\text{return}_C V$	value
	$\text{try } K \text{ with } \{\text{return } x \mapsto L, (\text{raise } e \mapsto L_e)_{e \in E}\}$	exception handler
	$V W$	application
	$\text{match } V \text{ with } \{\langle x, y \rangle \mapsto K\}$	product elimination
	$\text{match } V \text{ with } \{\}_{X@C}$	empty elimination
	$\text{match } V \text{ with } \{\text{inl } x \mapsto K, \text{inr } y \mapsto L\}$	sum elimination
	$\text{op}_{X@C}(V, (x . K), (L_e)_{e \in E_{\text{op}}})$	operation call
	$\text{raise}_{X@C} e$	raise exception
	$\text{kill}_{X@C} s$	send signal
	$\text{getenv}_C(c . K)$	get state
	$\text{setenv}(V, K)$	set state
	$\text{user } M \text{ with } \{\text{return } x \mapsto K, (\text{raise } e \mapsto L_e)_{e \in E}\}$	switch to user mode

# Core calculus (type system)



# Core calculus (type system)

- For example, the **typing rule for runners** is

$$\frac{\Sigma = \{ \text{op}_1, \dots, \text{op}_n \} \quad \left( \Gamma, x_i : A_{\text{op}_i} \vdash K_i : B_{\text{op}_i} \not\Downarrow (\Sigma', E_{\text{op}_i}, S, C) \right)_{1 \leq i \leq n}}{\Gamma \vdash \{ \text{op}_1 x_1 \mapsto K_1, \dots, \text{op}_n x_n \mapsto K_n \}_C : \Sigma \Rightarrow (\Sigma', S, C)}$$

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- and the **typing rule for running user comps.** is

$$\frac{\begin{array}{l} \Gamma \vdash V : \Sigma \Rightarrow (\Sigma', S, C) \quad \Gamma \vdash W : C \quad \Gamma \vdash M : X!(\Sigma, E) \\ \Gamma, x : X, c : C \vdash N_{\text{ret}} : Y!(\Sigma', E') \quad \left( \Gamma, c : C \vdash N_e : Y!(\Sigma', E') \right)_{e \in E} \\ \left( \Gamma \vdash N_s : Y!(\Sigma', E') \right)_{s \in S} \end{array}}{\Gamma \vdash \text{using } V @ W \text{ run } M \text{ finally } \{ \text{return } x @ c \mapsto N_{\text{ret}} , \\ \text{(raise } e @ c \mapsto N_e)_{e \in E} , \\ \text{(kill } s \mapsto N_s)_{s \in S} \} : Y!(\Sigma', E')}$$

# Core calculus (equational theory)

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- For example, the  $\beta$ -equations for running user comps. are

$$\Gamma \vdash \text{using } V @ W \text{ run } (\text{return } V') \text{ finally } F \equiv N_{\text{ret}}[V'/x, W/c] : Y! (\Sigma', E')$$

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$$\Gamma \vdash \text{using } R @ W \text{ run } (\text{op}_X (V, (y.M), (M_e)_{e \in E_{op}})) \text{ finally } F$$

$$\begin{aligned} &\equiv \text{kernel } K_{op}[V/x_{op}] @ W \text{ finally } \{ \\ &\quad \text{return } y @ c' \mapsto \text{using } R @ c' \text{ run } M \text{ finally } F, \\ &\quad (\text{raise } e @ c' \mapsto \text{using } R @ c' \text{ run } M_e \text{ finally } F)_{e \in E_{op}}, \\ &\quad (\text{kill } s \mapsto N_s)_{s \in S} \} : Y! (\Sigma', E') \end{aligned}$$

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and kernel comp. equations include **kernel theory equations**



# Core calculus (subtyping)

- The calculus also includes **subtyping**, and **subsumption rules**

$$\frac{\Gamma \vdash V : A \quad A <: B}{\Gamma \vdash V : B}$$

$$\frac{\Gamma \vdash M : A! (\Sigma, E) \quad \Sigma \subseteq \Sigma' \quad A <: B \quad E \subseteq E'}{\Gamma \vdash M : B! (\Sigma', E')}$$

$$\frac{\begin{array}{cccc} \Gamma \vdash K : A \downarrow (\Sigma, E, S, C) & \Sigma \subseteq \Sigma' & & \\ A <: B & E \subseteq E' & S \subseteq S' & C \equiv C' \end{array}}{\Gamma \vdash K : B \downarrow (\Sigma', E', S', C')}$$

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- We use  $C \equiv C'$  to have (standard) **proof-irrelevant subtyping**
- Otherwise, instead of just  $C <: C'$ , we would need a **lens**  $C' \leftrightarrow C$

# Core calculus (semantics)

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- **Monadic semantics**, for concreteness in **Set**, using
  - **user monads**  $\mathbf{U}_{\Sigma, E} X \stackrel{\text{def}}{=} \mathbf{Free}_{\Sigma}(X + E)$
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- (At a high level) the **judgements are interpreted** as

$$\llbracket \Gamma \vdash V : X \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow \llbracket X \rrbracket$$

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- **Theorem:** The semantics is coherent (**subtyping!**) and sound.

## Core calculus (semantics ctd.)

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- For instance, **kernel computations** are interpreted as

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 \llbracket \Gamma \rrbracket & \xrightarrow{\llbracket \Gamma \vdash K : X \Downarrow (\Sigma, E, S, C) \rrbracket} & \mathbf{K}_{\Sigma, E, S, \llbracket C \rrbracket} \llbracket X \rrbracket \\
 \subseteq \downarrow & & \downarrow \subseteq \\
 \llbracket \Gamma^s \rrbracket & \xrightarrow{\llbracket \Gamma^s \vdash K : X^s \Downarrow C \rrbracket} & \mathbf{K}_{\mathcal{O} + \{\perp\}, \mathcal{E}, S, \llbracket C \rrbracket} \llbracket X^s \rrbracket
 \end{array}$$

where  $\Gamma^s \vdash K : X^s \Downarrow C$  is a **skeletal kernel typing judgement** and use the extra op.  $\perp : 1 \rightsquigarrow 0 ! \{ \}$  to model **runtime errors**



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- No essential obstacles to extending to **Sub(Cpo)** and beyond
- Ground type restriction** on  $C$  simplifies the sem. ( $\llbracket C \rrbracket = \llbracket C \rrbracket$ )

# Core calculus (semantics ctd.)

$$\begin{aligned} \llbracket \Gamma \vdash \text{using } V @ W \text{ run } M \text{ finally } \{ & \text{return } x @ c \mapsto N_{ret} , \\ & (\text{raise } e @ c \mapsto N_e)_{e \in E} , \\ & (\text{kill } s \mapsto N_s)_{s \in S} \} : Y! (\Sigma', E') \rrbracket_\gamma \stackrel{\text{def}}{=} \dots \end{aligned}$$

- $\llbracket V \rrbracket_\gamma = \mathcal{R} = \left( \overline{\text{op}}_{\mathcal{R}} : \llbracket A_{\text{op}} \rrbracket \longrightarrow \mathbf{K}_{\Sigma', E_{\text{op}}, S, \llbracket C \rrbracket} \llbracket B_{\text{op}} \rrbracket \right)_{\text{op} \in \Sigma}$
- $\llbracket \text{return } x @ c \mapsto N_{ret} \rrbracket_\gamma \in \llbracket X \rrbracket \times \llbracket C \rrbracket \longrightarrow \mathbf{U}_{\Sigma', E'} \llbracket Y \rrbracket$
- $\llbracket (\text{raise } e @ c \mapsto N_e)_{e \in E} \rrbracket_\gamma \in E \times \llbracket C \rrbracket \longrightarrow \mathbf{U}_{\Sigma', E'} \llbracket Y \rrbracket$
- $\llbracket (\text{kill } s \mapsto N_s)_{s \in S} \rrbracket_\gamma \in S \longrightarrow \mathbf{U}_{\Sigma', E'} \llbracket Y \rrbracket$
- allowing us to use the **free model property** to get

$$\mathbf{U}_{\Sigma, E} \llbracket X \rrbracket \xrightarrow{r_{\llbracket X \rrbracket} + E} \mathbf{K}_{\Sigma', E, S, \llbracket C \rrbracket} \llbracket X \rrbracket \xrightarrow{(\lambda \llbracket N_{ret} \rrbracket_\gamma)^\dagger} \llbracket C \rrbracket \Rightarrow \mathbf{U}_{\Sigma', E'} \llbracket Y \rrbracket$$

- and then apply the resulting composite map to

$$\llbracket M \rrbracket_\gamma \in \mathbf{U}_{\Sigma, E} \llbracket X \rrbracket \quad \text{and} \quad \llbracket W \rrbracket_\gamma \in \llbracket C \rrbracket$$

# Core calculus (finalisation)

$$\Gamma \vdash \text{using } V @ W \text{ run } M \text{ finally } \left\{ \begin{array}{l} \text{return } x @ c \mapsto N_{ret} , \\ (\text{raise } e @ c \mapsto N_e)_{e \in E} , \\ (\text{kill } s \mapsto N_s)_{s \in S} \end{array} \right\} : Y ! (\Sigma', E')$$

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- The above finalisation clauses determine a **finalisation maps**

$$\phi_\gamma : (\llbracket X \rrbracket + E) \times \llbracket C \rrbracket + S \longrightarrow \mathbf{U}_{\Sigma', E'} \llbracket Y \rrbracket$$

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$$\phi_\gamma : (\llbracket X \rrbracket + E) \times \llbracket C \rrbracket + S \longrightarrow \mathbf{U}_{\Sigma', E'} \llbracket Y \rrbracket$$

- Theorem (finalisation):**

- for every environment  $\gamma \in \llbracket \Gamma \rrbracket$ ,
- there exists a tree  $t \in \mathbf{Free}_{\Sigma'}((\llbracket X \rrbracket + E) \times \llbracket C \rrbracket + S)$ ,
- such that

$$\llbracket \Gamma \vdash \text{using } V @ W \text{ run } M \text{ finally } F : Y ! (\Sigma', E') \rrbracket_\gamma = \phi_\gamma^\dagger t$$

## **Implementing runners**

**Experimenting with the theory in practice**



# Experimenting with the theory in practice

- A **small experimental language** COOP<sup>4</sup>
  - Implements the core calculus with few extras
  - The interpreter is directly based on the equational theory
  - Top-level containers for running external (OCaml) code
  - <https://github.com/andrejbauer/coop>

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<sup>4</sup>coop [/ku:p/] – a cage where small animals are kept, especially chickens

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  - Top-level containers for running external (OCaml) code
  - <https://github.com/andrejbauer/coop>
- A **HASKELL library** HASKELL-COOP
  - A shallow-embedding of the core calculus in HASKELL
  - Uses one of the Freer monad implementations underneath
  - Operational aspects implement the denotational semantics
  - Top-level containers for arbitrary HASKELL monads
  - Examples make use of HASKELL's features (GADTs, ...)
  - <https://github.com/danelahman/haskell-coop>

---

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**Runners in action**

Runners can be **vertically nested**

# Runners can be **vertically nested**

- ```
using RFH @ (fopen fileName)
run (
  using RFC @ (return "")
  run M
  finally {
    return x @ str → write str; return x ,
    raise WriteSizeExceeded @ str → write str; raise WriteSizeExceeded }
)
finally {
  return x @ fh → ... , raise e @ fh → ... , kill IOError → ... }
```

where the **file contents runner** (with  $\Sigma' = \{\}$ ) is defined as

```
let RFC = {
  write strl → let str = getenv () in
    if (length (str^strl) > max) then (raise WriteSizeExceeded)
    else (setenv (str^strl))
}String
```

# Vertical nesting for instrumentation

# Vertical nesting for instrumentation

- ```
using RCost @ (return 0)
run M
finally {
  return x @ c → report_cost c; return x ,
  raise e @ c → report_cost c; raise e }
```

where the **cost model runner** is defined as

```
let RCost = {
  ... ,
  op a → let c = getenv () in
    setenv (c + 1);
    op a ,
  ...
} Nat
```

(\* forwards op outwards \*)

- The runner  $R_{\text{Cost}}$  implements the same sig.  $\Sigma$  that  $M$  is using
- As a result, the runner  $R_{\text{Cost}}$  is **invisible** from  $M$ 's viewpoint

# Vertical nesting for **active monitoring**



# Vertical nesting for **active monitoring**

- First, we define a runner for **integer-valued ML-style state** as

**type** IntHeap = (Nat  $\rightarrow$  (Int + 1))  $\times$  Nat

**type** Ref = Nat

```
let RIntState = {  
  alloc x  $\rightarrow$  let h = getenv () in                                     (* alloc : Int  $\rightsquigarrow$  Ref ! {} *)  
    let (r, h') = heapAlloc h x in  
    setenv h';  
    return r ,  
  
  deref r  $\rightarrow$  let h = getenv () in                                     (* deref : Ref  $\rightsquigarrow$  Int ! {} *)  
    match (heapSel h r) with  
    | inl x  $\rightarrow$  return x  
    | inr ()  $\rightarrow$  kill ReferenceDoesNotExist ,  
  
  assign r y  $\rightarrow$  let h = getenv () in                                     (* assign : Ref  $\times$  Int  $\rightsquigarrow$  1 ! {} *)  
    match (heapUpd h r y) with  
    | inl h'  $\rightarrow$  setenv h'  
    | inr ()  $\rightarrow$  kill ReferenceDoesNotExist  
}  
  
}IntHeap
```

## Vertical nesting for **active monitoring** ctd.

- Next, we define  $F^*$ -style **monotonic state** on top of  $R_{\text{IntState}}$

# Vertical nesting for active monitoring ctd.

- Next, we define  $F^*$ -style **monotonic state** on top of  $R_{\text{IntState}}$

**type** MonMemory = Ref  $\rightarrow$  (Ord + 1)      **type** Ord = Int  $\rightarrow$  Int  $\rightarrow$  Bool

```
let RMonState = {  
  mAlloc x rel  $\rightarrow$  let r = alloc x in                               (* : Int  $\times$  Ord  $\rightsquigarrow$  Ref ! {} *)  
    let m = getenv () in  
    setenv (memAdd m r rel);  
    return r,  
  
  mDeref r  $\rightarrow$  deref r ,                                           (* monDeref : Ref  $\rightsquigarrow$  Int ! {} *)  
  
  mAssign r y  $\rightarrow$  let x = deref r in                                  (* : Ref  $\times$  Int  $\rightsquigarrow$  1 ! {MV} *)  
    let m = getenv () in  
    match (memSel m r) with  
    | inl rel  $\rightarrow$  if (rel x y)  
        then (assign r y)  
        else (raise MonotonicityViolation)  
    | inr  $\rightarrow$  kill PreorderDoesNotExist  
}  
} MonMemory
```

## Vertical nesting for **active monitoring** ctd.

- We can then perform **runtime monotonicity verification** as

# Vertical nesting for **active monitoring** ctd.

- We can then perform **runtime monotonicity verification** as

```
using RIntState @ ((fun _ → inr ()), 0)      (* init. empty ML-style heap *)
run (

  using RMonState @ (fun _ → inr ())          (* init. empty preorders memory *)
  run (

    let r = mAlloc 0 (≤) in
    mAssign r 1;
    mAssign r 0;      (* RMonState raises MonotonicityViolation exception *)
    mAssign r 2

  )
  finally { ..., raise MonotonicityViolation @ m → ..., ... }

)
finally { ... }
```

Runners can also be **horizontally paired**

# Runners can also be horizontally paired

- Given runners for

**let**  $R_1 = \{ \dots, \text{op}_{1i} \ x_{1i} \rightarrow K_{1i}, \dots \}_{C_1}$   $(* : \Sigma_1 \Rightarrow (\Sigma'_1, S_1, C_1) *)$   
**let**  $R_2 = \{ \dots, \text{op}_{2j} \ x_{2j} \rightarrow K_{2j}, \dots \}_{C_2}$   $(* : \Sigma_2 \Rightarrow (\Sigma'_2, S_2, C_2) *)$

we can **pair them** to get the runner

**let**  $R = \{ \dots, \quad (* : \Sigma_1 + \Sigma_2 \Rightarrow (\Sigma'_1 + \Sigma'_2, S_1 + S_2, C_1 \times C_2) *)$   
     $\text{op}_{1i} \ x_{1i} \rightarrow \text{let } (c, c') = \text{getenv } () \text{ in}$   
        **user** (**kernel** ( $K_{1i} \ x_{1i}$ ) **@**  $c$  **finally** {

**return**  $y \text{ @ } c'' \rightarrow \text{return } (\text{inl } (\text{inl } y, c''))$ ,

**raise**  $e \text{ @ } c'' \rightarrow \text{return } (\text{inl } (\text{inr } e, c''))$ ,

**kill**  $s \rightarrow \text{return } (\text{inr } s)$  }

$(* \ e \in E_{\text{op}_{1i}} \ *)$   
 $(* \ s \in S_1 \ *)$

**finally** {

**return**  $(\text{inl } (\text{inl } y, c'')) \rightarrow \text{setenv } (c'', c'); \text{return } y$ ,

**return**  $(\text{inl } (\text{inr } e, c'')) \rightarrow \text{setenv } (c'', c'); \text{raise } e$ ,

**return**  $(\text{inr } s) \rightarrow \text{kill } s$  }

$\dots,$   
     $\text{op}_{2j} \ x_{2j} \rightarrow \dots, \dots \}_{C_1 \times C_2}$

# Runners can also be horizontally paired

- Given runners for

```
let R1 = { ... , op1i x1i → K1i , ... }C1 (* : Σ1 ⇒ (Σ'1, S1, C1) *)  
let R2 = { ... , op2j x2j → K2j , ... }C2 (* : Σ2 ⇒ (Σ'2, S2, C2) *)
```

we can **pair them** to get the runner

```
let R = { ... , (* : Σ1 + Σ2 ⇒ (Σ'1 + Σ'2, S1 + S2, C1 × C2) *)  
  op1i x1i → let (c, c'') = getenv () in  
    user (kernel (K1i x1i) @ c finally {  
      return y @ c'' → return (inl (inl y, c'')),  
      raise e @ c'' → return (inl (inr e, c'')), (* e ∈ Eop1i *)  
      kill s → return (inr s) } (* s ∈ S1 *)  
    finally {  
      return (inl (inl y, c'')) → setenv (c'', c'); return y,  
      return (inl (inr e, c'')) → setenv (c'', c'); raise e,  
      return (inr s) → kill s },  
  ... ,  
  op2j x2j → ..., ... }C1 × C2
```

- For instance, this way we can build a runner for IO **and** state



## Other examples

# Other examples

- More general forms of **(ML-style) state** (for general Ref A)
  - if the host language allows it, we use GADTs, etc for safety
  - some examples extract a footprint from a larger memory
- **Combinations** of different effects and runners
  - in particular the combination of IO and state
  - good use case for both vertical and horizontal composition
- KOKA-style **ambient values** and **ambient functions**
  - **ambient values** are essentially **mutable variables/parameters**
  - **ambient functions** are **applied in their lexical context**
  - a runner that treats **amb. fun. application as a co-operation**
  - amb. funs. are stored in a context-depth-sensitive heap
  - the appl. co-operation restores the heap to the lexical context

# Other examples (ambient vals. and funs.)

- In KOKA one can use **ambient values**

```
ambient val f : int → int
```

```
ambient val x : int
```

```
with val x = 4
```

```
with val f = fun y → x + y
```

```
with val x = 2
```

```
f 1
```

(\* What's the result here? \*)

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```
with val x = 2
```

```
f 1
```

(\* What's the result here? \*)

- But one can also use **ambient functions**

```
ambient fun f : int → int
```

```
ambient val x : int
```

```
with val x = 4
```

```
with fun f = fun y → x + y
```

```
with val x = 2
```

```
f 1
```

(\* What's the result here? \*)

# Other examples (ambient vals. and funs.)

```
module AmbientsTests where

import Control.Runner
import Control.Runner.Ambients

ambFun :: AmbVal Int -> Int -> AmbEff Int
ambFun x y =
  do x <- getVal x;
  return (x + y)

test1 :: AmbEff Int
test1 =
  withAmbVal
    (4 :: Int)
    (\ x ->
      withAmbFun
        (ambFun x)
        (\ f ->
          do rebindVal x 2;
          applyFun f 1))

test2 = ambTopLevel test1
```

# Other examples (ambient vals. and funs.)

```
module AmbientsTests where

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          do rebindVal x 2;
          applyFun f 1))

test2 = ambTopLevel test1
```

```
module Control.Runner.Ambients

...

ambCoOps :: Amb a -> Kernel sig AmbHeap a
ambCoOps (Bind f) =
  do h <- getEnv;
  (f,h') <- return (ambHeapAlloc h f);
  setEnv h';
  return f
ambCoOps (Apply f x) =
  do h <- getEnv;
  (f,d) <- return (ambHeapSel h f (depth h));
  user
    (run
      ambRunner
        (return (h {depth = d}))
        (f x)
        ambFinaliser)
  return
ambCoOps (Rebind f g) =
  do h <- getEnv;
  setEnv (ambHeapUpd h f g)

ambRunner :: Runner '[Amb] sig AmbHeap
ambRunner = mkRunner ambCoOps
```

# Wrapping up

- **Runners** are a natural model of **top-level runtime**
- We propose **T-runners** to also model **non-top-level runtimes**
- We have turned **T-runners** into a **(practical ?) programming construct**, that supports controlled initialisation and finalisation
- I showed you some **combinators** and **programming examples**
- Two **implementations**: COOP & HASKELL-COOP
- **Future work**: lenses in subtyping and semantics, cat. of runners, adding handlers, case studies, refinement typing, compilation, ...

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