

Comodels as a gateway for interacting with the **external world**

Danel Ahman

(joint work with Andrej Bauer)

Ljubljana, 21 March 2019

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What is a **comodel**?

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- A **signature** Σ is a set of operation symbols

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- Intuitively, comodels describe a notion of **state/world**, e.g.,
 - **Operational semantics** as a tensor of a model and comodel
(Plotkin & Power, Abou-Saleh & Pattinson)
 - **Stateful runners** of effectful programs (Uustalu)
 - Default **top-level behaviour** of alg. effects (Bauer & Pretnar)

Computational effects in FP

Computational effects in FP

- Using **monads** (e.g., as in HASKELL)

```
type St a = String → (a, String)
```

```
f :: St a → St (a, a)
```

```
f c = c >>= (\x → c >>= (\y → return (x, y)))
```

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- Using **algebraic effects** and **handlers** (e.g., as in EFF)

```
effect Get : int
```

```
effect Put : int → unit
```

```
(*: int → a*int!{} *)
```

```
let g (c: unit → a!{Get, Put}) =
```

```
  with st_h handle (perform (Put 42); c ())
```

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(*: int → a*int!{ } *)
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let g (c: unit → a!{ Get, Put }) =
```

```
  with st_h handle (perform (Put 42); c ())
```

- Works well for effects that can be **represented** as pure data!

But what about effects that need access to the **external world**?

External world in FP

- Declare a **signature** of monads/effects

```
type IO a
```

```
openFile  :: FilePath → IOMode → IO Handle
```

```
hGetLine  :: Handle → IO String
```

```
hClose    :: Handle → IO ()
```

External world in FP

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```
effect Read    : string
```

```
effect Raise   : string → empty
```

```
effect RandomInt    : int → int
```

```
effect RandomFloat  : float → float
```

External world in FP

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effect Read    : string
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effect RandomInt    : int → int
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effect RandomFloat : float → float
```

- And then treat it **specially** in the compiler, e.g.,

```
let rec top_handle op =
```

```
    match op with
```

```
    | ...
```

External world in FP

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Ohad 🤖 12:17 PM

Can I do file IO (or just O) in Eff?

External world in FP



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Žiga Lukšič 12:18 PM

not currently

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So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```

| "Write" ->
  (match v with
  | V.Tuple vs ->
    let (file_name :: str :: _) = List.map V.to_str vs in
    let file_handle = open_out_gen
      [Open_wronly
       ;Open_append
       ;Open_creat
       ;Open_text
       ] 0o666 file_name in
    Printf.fprintf file_handle "%s" str;
    close_out file_handle;
    top_handle (k V.unit_value)
  )
```

External world in FP



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```

This talk — a principled (co)algebraic approach!

Another issue — **linearity** or lack thereof

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- ```
let f (s:string) =
 let fh = fopen "foo.txt" in
 fwrite fh (s^s);
 fclose fh;
 return fh

let g s =
 let fh = f s in fread fh
```

## Another issue — **linearity** or lack thereof

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- We could resolve this by typing file handles **linearly**
  - But we want other values (e.g., strings) to be used **non-linearly**!

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- We could resolve this by typing file handles **linearly**
 - But we want other values (e.g., strings) to be used **non-linearly**!
- But what if we wrap `f` in a **handler**?

```
let h = handler  
  | effect (FWrite fh s k) → return s  
  
let g s = with h handle f s
```


Another issue — **linearity** or lack thereof

- ```
let f (s:string) =
 let fh = fopen "foo.txt" in
 fwrite fh (s^s);
 fclose fh;
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let g s =
 let fh = f s in fread fh (* fh not open ! *)
```
- We could resolve this by typing file handles **linearly**
  - But we want other vars. (e.g., strings) to be used **non-linearly**
- But what if we wrap `f` in a **handler**?

```
let h = handler
 | effect (FWrite fh s k) → return s

let g s = with h handle f s (* dangling fh ! *)
```

**So, how could we solve these issues?**

# So, how could we solve these issues?

- Using **existing programming mechanisms**, e.g.,
  - Modules and abstraction

```
module System.IO where

type IO a

hClose :: Handle → IO ()
```

- Linear (and non-linear) types

```
linear type fhandle

effect FClose : (linear fhandle) → unit

linear effect FClose : fhandle → unit
```

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  - Modules and abstraction

```
module System.IO where

type IO a

hClose :: Handle → IO ()
```

- Linear (and non-linear) types

```
linear type fhandle

effect FClose : (linear fhandle) → unit

linear effect FClose : fhandle → unit
```

- **Problem:** They don't really explain the **essence of the solution**

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- Let's look at HASKELL's **IO monad** again
- A common explanation is to think of functions

$$a \rightarrow \text{IO } b$$

as

$$a \rightarrow (\text{RealWorld} \rightarrow (b, \text{RealWorld}))$$

which is the same as

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- With the `System.IO` **module abstraction** ensuring that
  - We can't get our hands on **RealWorld** — it's an idea of the world
  - The **real world** is affected linearly
  - We don't ask more from the **real world** than it can provide



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But wait a minute, **RealWorld** looks a lot like a **comodel**!

`hGetLine` : `(Handle, RealWorld) → (String, RealWorld)`

`hClose` : `(Handle, RealWorld) → ((), RealWorld)`

So, **IO** is more about in which **external world** our program is in!

**Comodels** as a gateway to the **external world**

## Comodels as a gateway to the external world

- ```
let f (s:string) =  
    using IO run  
        let fh = fopen "foo.txt" in  
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        fclose fh
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Comodels as a gateway to the external world

- ```
let f (s:string) =
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 (** @ IO : unit **)
- Now **external world** explicit, but **dangling** `fh` etc **still possible**

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- ```
let f (s:string) =  
    using IO run  
        let fh = fopen "foo.txt" in  
        fwrite fh (s^s) (* @ IO : unit *)  
    ending_with (fclose fh)
```

Comodels as a gateway to the external world

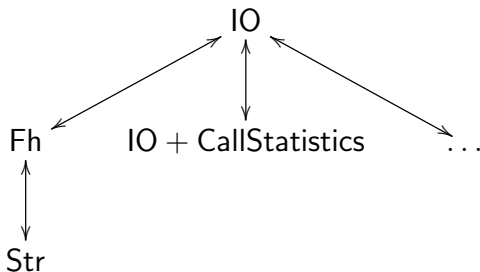
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- Better, but **have to explicitly open** and **thread through** `fh`

Comodels as a gateway to the external world

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- Better, but **have to explicitly open** and **thread through** `fh`
- **Solution:** Modular treatment of **external worlds**

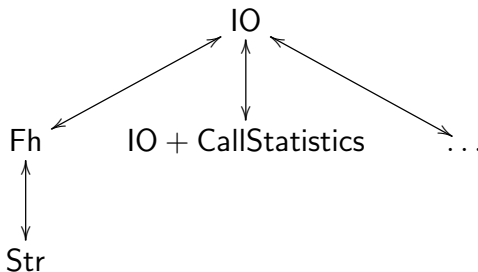
Comodels as a gateway to the external world

- Examples of **modularity** we might want from comodels



Comodels as a gateway to the external world

- Examples of **modularity** we might want from comodels



- Fh — “world which consists of exactly one fh”
- IO \longrightarrow Fh — “call `fopen` with `foo.txt`, store returned fh”
- Fh \longrightarrow IO — “call `fclose` with stored fh”
- Observation:** IO \longleftrightarrow Fh and other \longleftrightarrow look a lot like **lenses**

Comodels as a gateway to the **external world**

Comodels as a gateway to the external world

- Our **general framework** on the file operations example

```
let f (s:string) = (* @ IO : unit *)  
  using Fh  
  starting_with (fopen_of_io "foo.txt")  
  run  
    fwrite_of_fh (s^s) (* @ Fh : unit *)  
  ending_with (fun _ fh → fclose_of_io fh)
```

Comodels as a gateway to the external world

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  run
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  ending_with (fun _ fh → fclose_of_io fh)
```

where

```
Fh =
  ⟨W = fhandle ,
    co_fread ((),fh) = ... ,
    co_fwrite (s,fh) = fwrite_of_io s fh;
    return ((),fh)⟩
```

```
(* co_fread : (unit * W) → (string * W) @ IO *)
(* co_fwrite : (string * W) → (unit * W) @ IO *)
```

Comodels as a gateway to the **external world**

Comodels as a gateway to the external world

- The **modularity aspect** of our general framework

```
let f (s:string) = (* @ IO : unit *)
  using Fh
  starting_with (fopen_of_io "foo.txt")
  run

  using Str
  starting_with (fread_of_fh ())
  run
  fwrite_of_str (s^s) (* @ Str : unit *)
  ending_with (fun _ s → fwrite_of_fh s)

  ending_with (fun _ fh → fclose_of_io fh)
```

where

```
Str = ⟨ W = string , ... ⟩
```

Comodels as a gateway to the **external world**

Comodels as a gateway to the external world

- Comodels can also **extend** the (intermediate) external world(s)

```
let f (s:string) = (* @ IO : unit *)
  using Stats
  starting_with (fopen_of_io "foo.txt")
  run
    fwrite_of_stats (s^s) (* @ Stats : unit *)
  ending_with
    (fun _ (fh,c) →
      let fh' = fopen_of_io "stats.txt" in
      fwrite_of_io fh' c;
      fclose_of_io fh'; fclose_of_io fh)
```

where

```
Stats = ⟨ W = (fhandle * nat), ... ⟩
```

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```

where

```
Stats = ⟨ W = (fhandle * nat), ... ⟩
```

- We can also track of **nondet./prob. choice results**, and alike
- Could we also use comodels for **dynamic (NI) monitoring**?

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- **Typing judgement** for computations $\Gamma \vdash c @ \vec{C} : A$

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- The two central **typing rules** are (U is the “universe”, aka IO)

$\Gamma \vdash D \text{ comodel } @ \vec{C} \quad D \neq U$

$\Gamma \vdash c_s @ \vec{C} : D.W \quad \Gamma \vdash c @ \vec{C}, D : A \quad \Gamma, x:A, w:D.W \vdash c_e @ \vec{C} : A$

$\Gamma \vdash$ **using** D
 starting_with c_s
 run c
 ending_with $(x.w.c_e) @ \vec{C} : A$

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and

$\Gamma \vdash D \text{ comodel } @ \vec{C} \quad \text{op} : A \rightsquigarrow B \in D.\Sigma \quad \Gamma \vdash v : A$

 $\Gamma \vdash \text{op } v @ \vec{C}, D : B$

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- Regarding the **denotational semantics**, the idea is to interpret

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as

$$\llbracket \Gamma \vdash c @ \vec{C} : A \rrbracket : \llbracket \vec{C} \rrbracket \multimap \llbracket A \rrbracket \times \llbracket \vec{C} \rrbracket$$

which in its essence is very similar to Møgelberg and Staton's comodels-based **linear state-passing transformation**

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which in its essence is very similar to Møgelberg and Staton's comodels-based **linear state-passing transformation**

- Regarding **operational semantics**, the idea is to consider confs.

$$(\overrightarrow{(C, w)}, c)$$

either in a **big-** or **small-step** style

- where $\overrightarrow{(C, w)}$ is a stack of worlds

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$$((\overrightarrow{(C, w_0)}), (C', w'_0)) , c_s) \Downarrow ((\overrightarrow{(C, w_1)}), (C', w'_1)) , \text{return } w''_0)$$

$$((\overrightarrow{(C, w_1)}), (C', w'_1), (D, w''_0)) , c) \Downarrow ((\overrightarrow{(C, w_2)}), (C', w'_2), (D, w''_1)) , \text{return } v)$$

$$((\overrightarrow{(C, w_2)}), (C', w'_2)) , c_e[v/x, w''_1/w]) \Downarrow ((\overrightarrow{(C, w_3)}), (C', w'_3)) , \text{return } v')$$

$$((\overrightarrow{(C, w_0)}), (C', w'_0)) , \text{using D s_w } c_s \text{ run c e_w } (x.w.c_e))$$

$$\Downarrow$$

$$((\overrightarrow{(C, w_3)}), (C', w'_3)) , \text{return } v')$$

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- The interpretation of **operations** uses the **co-operations** of C_s , naturally traversing the stack of (intermediate) external worlds

But what about **algebraic effects** and **handlers**?

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- An interesting question for **future work**, but feels natural that in

```
using C
starting_with c_s
run c
ending_with (fun x w → c_e)
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- One can use **algebraic operations** (in the sense of EFF) in `c`, but they must not be allowed to escape `run` (for linearity)

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It might make sense to allow alg. ops. to escape `c_s` and `c_e`
- The continuations of **handlers** in `c` are delimited by `run`
Again, so as to ensure linearity and reaching `ending_with`

But what about **algebraic effects** and **handlers**?

- An interesting question for **future work**, but feels natural that in

```
using C
starting_with c_s
run c
ending_with (fun x w → c_e)
```

- One can use **algebraic operations** (in the sense of EFF) in `c`, but they must not be allowed to escape `run` (for linearity)
- To escape, have to use the **co-operations** of the **external world**
It might make sense to allow alg. ops. to escape `c_s` and `c_e`
- The continuations of **handlers** in `c` are delimited by `run`
Again, so as to ensure linearity and reaching `ending_with`
- How do **multi-handlers** fit in the picture? Tensors of some sort?

Conclusions

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- We're making them into a **modular programming abstraction**
- **Linearity** by leaving **outer worlds** implicit (via comod. alg. ops.)
- System.IO , KOKA's **initially** & **finally** , PYTHON's **with** , ...

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- **Comodels** as a gateway for interacting with the **external world**
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- `System.IO` , KOKA's **initially** & **finally** , PYTHON's **with** , ...

Ongoing and future work

- Work out all the **formal details** of what I have shown you today
- **Algebraic effects** and **(multi-)handlers**
- More **examples** and **use cases** (Matija, the Eff Architecture?)
- Clarify the connection with **(effectful) lenses**
- **Combinatorics** of comodels and their lens-like relationships