Recalling a Witness

Foundations and Applications of Monotonic State

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Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

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Monotonicity in program verification

- Consider a program operating on **set-valued state**

$$\texttt{insert v; complex\_procedure(); } \texttt{assert } (\texttt{v} \in \texttt{get()})$$

- To prove the assertion (say, in a Floyd-Hoare style logic), we could prove that the code maintains a **stateful invariant**

$$\{\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}\}\ \texttt{complex\_procedure()}\ \{\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}\}$$

  - likely that we have to **carry** $\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}$ **through** the proof of c_p
    - **does not guarantee** that $\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}$ holds at every point in c_p
    - **sensitive** to proving that c_p maintains $\lambda\,\texttt{s}\,.\,\texttt{w} \in \texttt{s}$ for some other $\texttt{w}$

- However, if c_p **does not remove**, then $\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s}$ is **stable**, and we would like the program logic to give us $\texttt{v} \in \texttt{get()}$ "**for free**"

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Monotonicity in programming

- **Programs** also rely on **monotonicity**, even if you don't realise it!

- Consider ML-style typed **references** `r:ref a`
  - `r` is a **proof of existence** of an `a`-typed value in the heap

- Correctness relies on **monotonicity**!
  1) Allocation **stores** an `a`-typed value in the heap
  2) Writes **don't change type** and there is **no deallocation**
  3) So, given a ref. `r`, it is **guaranteed to point** to an `a`-typed value

- Baked into the memory models of most languages

- We derive them from **global state** + **general monotonicity**

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- In this talk
  - our **motivating example** and **monotonic counters**
  - **typed references** (`ref t`) and **untyped references** (`uref`)
  - more flexibility with **monotonic references** (`mref t rel`)
- More in the paper
  - temporarily **violating monotonicity** via snapshots
  - two substantial case studies
    - a **secure file-transfer** application
    - Ariadne **state continuity** protocol [Strackx, Piessens 2016]
  - pointers to other works in F* relying on monotonicity for
    - sophisticated **region-based memory models** [`fstar-lang.org`]
    - **crypto** and **TLS verification** [`project-everest.github.io`]

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Key ideas behind our general framework

- We focus on **monotonic programs** and **stable predicates**
  - per verification task, we **choose a preorder** `rel` on states
    - set inclusion, heap inclusion, increasing counter values, ...
  - a stateful program e is **monotonic** (wrt. `rel`) when
    $$\forall\, \mathrm{s}\, \mathrm{e}'\, \mathrm{s}'.\ (\mathrm{e}, \mathrm{s}) \rightsquigarrow^* (\mathrm{e}', \mathrm{s}') \implies \texttt{rel}\ \mathrm{s}\ \mathrm{s}'$$
  - a stateful predicate `p` is **stable** (wrt. `rel`) when
    $$\forall\, \mathrm{s}\, \mathrm{s}'.\ \texttt{p}\ \mathrm{s}\ \wedge\ \texttt{rel}\ \mathrm{s}\ \mathrm{s}' \implies \texttt{p}\ \mathrm{s}'$$
- **Our solution:** extend Hoare-style program logics (e.g., F*) with
  - a means for turning a `p` into a **state-independent proposition**
  - a means to **witness** the validity of `p` s in some state s
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Recap: Ordinary global state in F*

- F* is an ML-like dependently typed language, aimed at verification
- F* supports Hoare-style reasoning about state via the **comp. type**

$$\mathtt{ST}_{\mathtt{state}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where

$$\mathtt{pre} : \mathtt{state} \to \mathrm{Type} \qquad \mathtt{post} : \mathtt{state} \to \mathtt{t} \to \mathtt{state} \to \mathrm{Type}$$

- $\mathtt{ST}$ is an **abstract** pre-postcondition refinement of

$$\mathtt{st}\ \mathtt{t} \stackrel{\mathsf{def}}{=} \mathtt{state} \to \mathtt{t} * \mathtt{state}$$

- The global state **actions** have types

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New: Monotonic global state in F*

- We capture monotonic state with a new **computation type**

$$\mathtt{MST}_{\mathtt{state},\mathtt{rel}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where pre and post are typed as in ST

- The **get** action is typed as in ST

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- To ensure **monotonicity**, the **put** action gets a precondition

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$$\mathtt{mst}\ \mathtt{t} \stackrel{\mathrm{def}}{=} \mathtt{s_0}{:}\mathtt{state} \rightarrow \mathtt{t} * \mathtt{s_1}{:}\mathtt{state}\{\mathtt{rel}\ \mathtt{s_0}\ \mathtt{s_1}\}$$

New: Monotonic global state in F*

- We capture monotonic state with a new **computation type**

$$\mathtt{MST}_{\mathtt{state},\mathtt{rel}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where pre and post are typed as in ST

- The **get** action is typed as in ST

$$\begin{aligned}\mathtt{get} : \mathtt{unit} \to \mathtt{MST}\ \mathtt{state}\ &(\mathtt{requires}\ (\lambda\, \_ . \top))\\ &(\mathtt{ensures}\ (\lambda\, \mathtt{s}_0\, \mathtt{s}\, \mathtt{s}_1 \,.\, \mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))\end{aligned}$$

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New: Recalling a Witness

- We introduce a **logical capability** (a **modality** in ongoing work)

$$\texttt{witnessed} : (\texttt{state} \to \texttt{Type}) \to \texttt{Type}$$

  together with a **weakening principle** (**functoriality**)

$$\texttt{wk} : \texttt{p},\texttt{q}{:}(\texttt{state} \to \texttt{Type}) \to \texttt{Lemma}\ (\texttt{requires}\ (\forall\, \texttt{s}\,.\, \texttt{p s} \implies \texttt{q s}))$$
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- We add a **stateful introduction rule** for `witnessed`

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Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

The motivating example revisited

- Recall the program operating on the **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert } (\mathrm{v} \in \mathtt{get}())$$

  - We pick **set inclusion** $\subseteq$ as our preorder `rel` on states
  - We **prove the assertion** by inserting a `witness` and `recall`

$$\texttt{insert v; witness } (\lambda\, \mathrm{s} \,.\, \mathrm{v} \in \mathrm{s})\texttt{; c\_p(); recall } (\lambda\, \mathrm{s} \,.\, \mathrm{v} \in \mathrm{s})\texttt{; assert } (\mathrm{v} \in \mathtt{get}())$$

  - For **any other** w, wrapping

$$\texttt{insert w; [ ]; assert } (\mathrm{w} \in \mathtt{get}())$$

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ML-style typed references (local state)

- First, we define a type of **heaps**

  $\texttt{type heap} =$

  $\quad | \ \texttt{H} : \texttt{h}{:}(\mathbb{N} \to \texttt{cell}) \to \texttt{ctr}{:}\mathbb{N}\{\forall\,\texttt{n}\,.\,\texttt{ctr} \leq \texttt{n} \implies \texttt{h}\ \texttt{n} = \texttt{Unused}\} \to \texttt{heap}$

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- As a result, we can define new **local state effect**

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$$\mathtt{abstract\ type\ ref\ a = id{:}\mathbb{N}\{witnessed\ (\lambda\, h\, .\, contains\ h\ id\ a)\}}$$

where

```
let contains (H h _) id a =
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- Important: `contains` is **stable** wrt. `heap_inclusion`

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- Finally, we define `LST`'s **actions** using `MST`'s actions
  - `let alloc` (a:Type) (v:a) : `LST` (`ref a`) `...` = `...`
    - **get** the current heap
    - **create** a fresh ref., and **add** it to the heap
    - **put** the updated heap back
    - **witness** that the created ref. is in the heap
  - `let read` (r:`ref a`) : `LST t ...` = `...`
    - **recall** that the given ref. is in the heap
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    - **select** the given reference from the heap
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Adding untyped and monotonic references

- **Untyped references** (`uref`) with strong updates
  - Used `heap` cells are extended with **tags**

    where
    $$| \ \texttt{Used} : \mathrm{a{:}Type} \rightarrow \mathrm{v{:}a} \rightarrow \mathrm{t{:}tag} \rightarrow \texttt{cell}$$
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    $$\texttt{type tag} = \texttt{Typed} : \texttt{tag} \ | \ \texttt{Untyped} : \texttt{tag}$$

  - `urefs` can be extended to also support **deallocation**

- **Monotonic references** (`mref a rel`)
  - Used heap cells are extended with **typed tags**

    $$| \ \texttt{Used} : \texttt{a:Type} \to \texttt{v:a} \to \texttt{t:tag a} \to \texttt{cell}$$

    where

    $$\texttt{type tag a} = \texttt{Typed} : \texttt{rel:preorder a} \to \texttt{tag a} \ | \ \texttt{Untyped} : \texttt{tag a}$$

  - `mrefs` provide **more flexibility** with ref.-wise monotonicity

Conclusion

- In conclusion
  - making use of monotonicity is very **useful** in verification
  - using monotonicity can be distilled into a **simple** interface
  - useful for **programming** (refs.) and **verification** (Prj. Everest)

- See the paper for
  - further **examples** and **case studies**
  - **meta-theory** and **correctness results** for `MST`
    - based on an instrumented operational semantics

$$(\texttt{witness}\ x.\varphi\ ,\ s\ ,\ W)\ \rightsquigarrow\ (\texttt{return}\ ()\ ,\ s\ ,\ W \cup \{x.\varphi\})$$

    - and cut elimination for the `witnessed`-logic
  - first steps towards **monadic reification** for `MST`
    - useful for extrinsic reasoning, e.g., for relational properties
    - but have to be careful when breaking abstraction

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Thank you!

Questions?

Appendix: `witnessed` as a modality

- **Ongoing work**

- Instead of taking `witnessed` as primitive,
  could extend F*'s logic with **hybrid modal operators**

$$\downarrow : (\mathtt{state} \to \mathtt{Type}) \to \mathtt{Type} \qquad \mathtt{@} : \mathtt{Type} \to \mathtt{state} \to \mathtt{Type}$$

- And then define

$$\mathtt{witnessed}\ \mathtt{p} \stackrel{\text{def}}{=} \downarrow (\mathtt{fun}\ \mathtt{s} \to \forall\, \mathtt{s}'.\mathtt{rel}\ \mathtt{s}\ \mathtt{s}' \implies \mathtt{p}\ \mathtt{s}')$$

  which internalises the `state`-indexed **Kripke-semantics**

$$[\![\mathtt{witnessed}\ \mathtt{p}]\!](\mathtt{s}) \stackrel{\text{def}}{=} \forall\, \mathtt{s}'.\mathtt{rel}\ \mathtt{s}\ \mathtt{s}' \implies [\![\mathtt{p}\ \mathtt{s}']\!](\mathtt{s})$$

- Validates **additional properties** of `witnessed`, e.g.,

$$\mathtt{witnessed}\ \mathtt{p} \land \mathtt{witnessed}\ \mathtt{q} \implies \mathtt{witnessed}\ (\mathtt{fun}\ \mathtt{s} \to \mathtt{p}\ \mathtt{s} \land \mathtt{q}\ \mathtt{s})$$