

Interacting with the **external world** using **comodels** (aka **runners**)

Danel Ahman

(joint work with Andrej Bauer)

University of Ljubljana, Slovenia

Gallinette seminar, Nantes, 14.10.2019

The plan

- **Computational effects** and **external resources** in PL
- **Issues** with the standard approaches to **external resources**
- **Runners** – a natural model for **top-level runtime**
- **T-runners** – for also modelling **non-top-level runtimes**
- Turning **T**-runners into a **useful programming construct**
- Some **programming examples**

Computational effects
and
external resources

Computational effects in PL

Computational effects in PL

- Using **monads** (as in HASKELL)

```
type St a = String → (a,String)
```

```
f :: St a → St (a,a)
```

```
f c = c >>= (\ x → c >>= (\ y → return (x,y)))
```

- Using **alg. effects** and **handlers** (as in EFF, FRANK, KOKA)

```
effect Get : int
```

```
effect Put : int → unit
```

```
let g (c:Unit → a!{Get,Put}) =
```

```
  with st_handler handle (perform (Put 42); c ()) (* : int → a * int *)
```

Computational effects in PL

- Using **monads** (as in HASKELL)

```
type St a = String → (a,String)
```

```
f :: St a → St (a,a)
```

```
f c = c >>= (\ x → c >>= (\ y → return (x,y)))
```

- Using **alg. effects** and **handlers** (as in EFF, FRANK, KOKA)

```
effect Get : int
```

```
effect Put : int → unit
```

```
let g (c:Unit → a!{Get,Put}) =
```

```
  with st_handler handle (perform (Put 42); c ()) (* : int → a * int *)
```

- Both are good for **faking comp. effects** in a pure language!
But what about effects that need access to the **external world**?

External resources in PL

External resources in PL

- Declare a **signature of monads** or **algebraic effects**, e.g.,

```
(* System.IO *)  
type IO a  
openFile :: FilePath → IOMode → IO Handle
```

```
(* pervasives.eff *)  
effect RandomInt : int → int  
effect RandomFloat : float → float
```

- And then **treat them specially** in the compiler, e.g.,

```
(* eff/src/backends/eval.ml *)  
let rec top_handle op =  
  match op with  
  | ...
```


External resources in PL

- Declare a **signature of monads** or **algebraic effects**, e.g.,

```
(* System.IO *)  
type IO a  
openFile :: FilePath → IOMode → IO Handle
```

```
(* pervasives.eff *)  
effect RandomInt : int → int  
effect RandomFloat : float → float
```

- And then **treat them specially** in the compiler, e.g.,

```
(* eff/src/backends/eval.ml *)  
let rec top_handle op =  
  match op with  
  | ...
```

but there are some issues with that approach ...

First issue

First issue

- Difficult to cover all possible use cases
 - **external resources hard-coded** into the top-level runtime
 - **non-trivial to change** what's available and how it's implemented

First issue

- Difficult to cover all possible use cases
 - **external resources hard-coded** into the top-level runtime
 - **non-trivial to change** what's available and how it's implemented



Ohad 8:35 PM

So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
| "Write" ->
  (match v with
  | V.Tuple vs ->
    let (file_name :: str :: _) = List.map V.to_str vs in
    let file_handle = open_out_gen
      [Open_wronly
       ;Open_append
       ;Open_creat
       ;Open_text
       ] 0o666 file_name in
    Printf.fprintf file_handle "%s" str;
    close_out file_handle;
    top_handle (k V.unit_value)
  )
```

First issue

- Difficult to cover all possible use cases
 - **external resources hard-coded** into the top-level runtime
 - **non-trivial to change** what's available and how it's implemented

 **Ohad** 8:35 PM
So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
| "Write" ->  
  (match v with  
  | V.Tuple vs ->  
    let (file_name :: str :: _) = List.map V.to_str vs in  
    let file_handle = open_out_gen  
                        [Open_wronly  
                        ;Open_append  
                        ;Open_creat  
                        ;Open_text  
                        ] 0o666 file_name in  
    Printf.fprintf file_handle "%s" str;  
    close_out file_handle;  
    top_handle (k V.unit_value)  
  )
```

This talk — a principled modular (co)algebraic approach!

Second issue

Second issue

- **Lack of linearity** for external resources

```
let f (s:string) =  
  let fh = fopen "foo.txt" in  
  fwrite (fh,s^s);  
  fclose fh;  
  return fh
```

```
let g s =  
  let fh = f s in fread fh
```

Second issue

- **Lack of linearity** for external resources

```
let f (s:string) =  
  let fh = fopen "foo.txt" in  
  fwrite (fh,s^s);  
  fclose fh;  
  return fh
```

```
let g s =  
  let fh = f s in fread fh           (* fh not open ! *)
```


Second issue

- **Lack of linearity** for external resources

```
let f (s:string) =  
  let fh = fopen "foo.txt" in  
  fwrite (fh,s^s);  
  fclose fh;  
  return fh  
  
let g s =  
  let fh = f s in fread fh          (* fh not open ! *)
```

- We shall address these kinds of issues **indirectly**,
 - by **not** introducing a linear typing discipline
 - but instead make it convenient to **hide external resources**
(and address stronger typing disciplines in the future)

Third issue

Third issue

- **Excessive generality** of effect handlers

```
let f (s:string) =  
  let fh = fopen "foo.txt" in  
  fwrite (fh,s^s);  
  fclose fh  
  
let h = handler { fwrite (fh,s) k → return () }  
  
let f' s = handle (f "bar") with h
```

Third issue

- **Excessive generality** of effect handlers

```
let f (s:string) =  
  let fh = fopen "foo.txt" in  
  fwrite (fh,s^s);  
  fclose fh  
  
let h = handler { fwrite (fh,s) k → return () }  
  
let f' s = handle (f "bar") with h
```

where misuse of external resources can also be **purely accidental**

```
let g (s:string) =  
  let fh = fopen "foo.txt" in  
  let b = choose () in  
  if b then (fwrite (fh,s)) else (fwrite (fh,s^s));  
  fclose fh  
  
let nd_handler =  
  handler { choose () k → return (k true ++ k false) }
```

Third issue

- **Excessive generality** of effect handlers

```
let f (s:string) =  
  let fh = fopen "foo.txt" in  
  fwrite (fh,s^s);  
  fclose fh  
  
let h = handler { fwrite (fh,s) k → return () }  
  
let f' s = handle (f "bar") with h
```

- We shall address these kinds of issues **directly**,
 - by proposing a **restricted form of handlers** for resources
 - that support **controlled initialisation** and **finalisation**,
 - and **limit** how general handlers can be used

Runners enter the spotlight

A natural model of **top-level runtime**

A natural model of **top-level runtime**

- Given a **signature**¹ Σ of operation symbols ($A_{\text{op}}, B_{\text{op}}$ are sets)

$$\text{op} : A_{\text{op}} \rightsquigarrow B_{\text{op}}$$

a **runner**² \mathcal{R} for Σ is given by a carrier $|\mathcal{R}|$ and co-operations

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \times |\mathcal{R}| \longrightarrow B_{\text{op}} \times |\mathcal{R}| \right)_{\text{op} \in \Sigma}$$

where we think of $|\mathcal{R}|$ as a set of **runtime configurations**

¹We consider runners for signatures, but the work generalises to alg. theories.

²In the literature also known as **comodels** for Σ (or for an algebraic theory).

A natural model of **top-level runtime**

- Given a **signature**¹ Σ of operation symbols ($A_{\text{op}}, B_{\text{op}}$ are sets)

$$\text{op} : A_{\text{op}} \rightsquigarrow B_{\text{op}}$$

a **runner**² \mathcal{R} for Σ is given by a carrier $|\mathcal{R}|$ and co-operations

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \times |\mathcal{R}| \longrightarrow B_{\text{op}} \times |\mathcal{R}| \right)_{\text{op} \in \Sigma}$$

where we think of $|\mathcal{R}|$ as a set of **runtime configurations**

- For example, a natural runner \mathcal{R} for **S-valued state** signature

$$\left\{ \text{get} : \mathbb{1} \rightsquigarrow S \quad , \quad \text{set} : S \rightsquigarrow \mathbb{1} \right\}$$

is given by

$$|\mathcal{R}| \stackrel{\text{def}}{=} S \qquad \overline{\text{get}}_{\mathcal{R}}(\star, s) \stackrel{\text{def}}{=} (s, s) \qquad \overline{\text{set}}_{\mathcal{R}}(s', s) \stackrel{\text{def}}{=} (\star, s')$$

¹We consider runners for signatures, but the work generalises to alg. theories.

²In the literature also known as **comodels** for Σ (or for an algebraic theory).

A natural model of **top-level runtime** ctd.

- Runners/comodels have been used for
 - **operational semantics** using tensors of models and comodels
[Plotkin and Power '08]
 - **top-level implementation** of algebraic effects in EFF
[Bauer and Pretnar '15]and
- **stateful running** of algebraic effects [Uustalu '15]
- **linear-use state-passing translation**
[Møgelberg and Staton '11, '14]

A natural model of **top-level runtime** ctd.

- Runners/comodels have been used for
 - **operational semantics** using tensors of models and comodels [Plotkin and Power '08]
 - **top-level implementation** of algebraic effects in EFF [Bauer and Pretnar '15]and
 - **stateful running** of algebraic effects [Uustalu '15]
 - **linear-use state-passing translation** [Møgelberg and Staton '11, '14]
- The latter explicitly rely on one-to-one correspondence between
 - **runners** \mathcal{R}
 - **monad morphisms**³ $r : \mathbf{Free}_\Sigma(-) \longrightarrow \mathbf{St}_{|\mathcal{R}|}$

³ $\mathbf{Free}_\Sigma(X)$ is the free monad ind. defined with leaves $\text{val } x$ and nodes $\text{op}(a, \kappa)$.

A natural model of **top-level runtime** ctd.

- Recall that for our purposes, we see runners

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \times |\mathcal{R}| \longrightarrow B_{\text{op}} \times |\mathcal{R}| \right)_{\text{op} \in \Sigma}$$

as describing how operations affect **runtime configurations** $|\mathcal{R}|$

A natural model of **top-level runtime** ctd.

- Recall that for our purposes, we see runners

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \times |\mathcal{R}| \longrightarrow B_{\text{op}} \times |\mathcal{R}| \right)_{\text{op} \in \Sigma}$$

as describing how operations affect **runtime configurations** $|\mathcal{R}|$

- But what if this runtime is not ****the**** runtime?
 - hardware vs OS
 - OS vs VMs
 - VMs vs sandboxes

A natural model of **top-level runtime** ctd.

- Recall that for our purposes, we see runners

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \times |\mathcal{R}| \longrightarrow B_{\text{op}} \times |\mathcal{R}| \right)_{\text{op} \in \Sigma}$$

as describing how operations affect **runtime configurations** $|\mathcal{R}|$

- But what if this runtime is not ****the**** runtime?
 - hardware vs OS
 - OS vs VMs
 - VMs vs sandboxes
- Unfortunately, runners, as defined above, are **not readily able to**
 - use **external resources**
 - signal failure** caused by unavoidable circumstances
- But is there a **useful generalisation** that would achieve this?

Effectful runners for modular top-levels

Effectful runners for modular top-levels

- Møgelberg and Staton usefully observed that a **runner** \mathcal{R} is equivalently simply a family of **generic effects** for $\mathbf{St}_{|\mathcal{R}|}$, i.e.,

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow \mathbf{St}_{|\mathcal{R}|} B_{\text{op}} \right)_{\text{op} \in \Sigma}$$

Effectful runners for modular top-levels

- Møgelberg and Staton usefully observed that a **runner** \mathcal{R} is equivalently simply a family of **generic effects** for $\mathbf{St}_{|\mathcal{R}|}$, i.e.,

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow \mathbf{St}_{|\mathcal{R}|} B_{\text{op}} \right)_{\text{op} \in \Sigma}$$

- Building on this, we define a **T-runner** \mathcal{R} for Σ to be given by

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow \mathbf{T} B_{\text{op}} \right)_{\text{op} \in \Sigma}$$

Effectful runners for modular top-levels

- Møgelberg and Staton usefully observed that a **runner** \mathcal{R} is equivalently simply a family of **generic effects** for $\mathbf{St}_{|\mathcal{R}|}$, i.e.,

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow \mathbf{St}_{|\mathcal{R}|} B_{\text{op}} \right)_{\text{op} \in \Sigma}$$

- Building on this, we define a **T-runner** \mathcal{R} for Σ to be given by

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow \mathbf{T} B_{\text{op}} \right)_{\text{op} \in \Sigma}$$

- The one-to-one correspondence with **monad morphisms**

$$r : \mathbf{Free}_{\Sigma}(-) \longrightarrow \mathbf{T}$$

simply amounts to the **universal property of free models**, i.e.,

$$r_X(\text{val } x) = \eta_X x \qquad r_X(\text{op}(a, \kappa)) = \underbrace{(r_X \circ \kappa)^{\dagger}(\overline{\text{op}}_{\mathcal{R}} a)}_{\text{op}_{\mathcal{M}}(a, r_X \circ \kappa)}$$

Effectful runners for modular top-levels

- Møgelberg and Staton usefully observed that a **runner** \mathcal{R} is equivalently simply a family of **generic effects** for $\mathbf{St}_{|\mathcal{R}|}$, i.e.,

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow \mathbf{St}_{|\mathcal{R}|} B_{\text{op}} \right)_{\text{op} \in \Sigma}$$

- Building on this, we define a **T-runner** \mathcal{R} for Σ to be given by

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow \mathbf{T} B_{\text{op}} \right)_{\text{op} \in \Sigma}$$

- The one-to-one correspondence with **monad morphisms**

$$r : \mathbf{Free}_{\Sigma}(-) \longrightarrow \mathbf{T}$$

simply amounts to the **universal property of free models**, i.e.,

$$r_X(\text{val } x) = \eta_X x \qquad r_X(\text{op}(a, \kappa)) = \underbrace{(r_X \circ \kappa)^{\dagger}(\overline{\text{op}}_{\mathcal{R}} a)}_{\text{op}_{\mathcal{M}}(a, r_X \circ \kappa)}$$

- Observe that κ appears in a **tail call position** on the right!

Effectful runners for modular top-levels ctd.

- What would be a **useful class of monads** \mathbf{T} to use?

Effectful runners for modular top-levels ctd.

- What would be a **useful class of monads \mathbf{T}** to use?
- We want a runner to be a bit like a **kernel** of an OS, i.e., to
 - (i) provide management of **(internal) resources**
 - (ii) use further **external resources**
 - (iii) **signal failure** caused by unavoidable circumstances

Effective runners for modular top-levels ctd.

- What would be a **useful class of monads** \mathbf{T} to use?
 - We want a runner to be a bit like a **kernel** of an OS, i.e., to
 - (i) provide management of **(internal) resources**
 - (ii) use further **external resources**
 - (iii) **signal failure** caused by unavoidable circumstances
 - **Algebraically** (and pragmatically), this amounts to taking
 - (i) $\text{getenv} : \mathbb{1} \rightsquigarrow C$, $\text{setenv} : C \rightsquigarrow \mathbb{1}$
 - (ii) $\text{op} : A_{\text{op}} \rightsquigarrow B_{\text{op}}$ ($\text{op} \in \Sigma'$, for some external Σ')
 - (iii) $\text{kill} : S \rightsquigarrow \mathbb{0}$
- s.t., (i) satisfy state equations; and (i) commute with (ii) and (iii)

Effectful runners for modular top-levels ctd.

- What would be a **useful class of monads** \mathbf{T} to use?
- We want a runner to be a bit like a **kernel** of an OS, i.e., to
 - (i) provide management of **(internal) resources**
 - (ii) use further **external resources**
 - (iii) **signal failure** caused by unavoidable circumstances
- **Algebraically** (and pragmatically), this amounts to taking
 - (i) $\text{getenv} : \mathbb{1} \rightsquigarrow C$, $\text{setenv} : C \rightsquigarrow \mathbb{1}$
 - (ii) $\text{op} : A_{\text{op}} \rightsquigarrow B_{\text{op}}$ ($\text{op} \in \Sigma'$, for some external Σ')
 - (iii) $\text{kill} : S \rightsquigarrow \mathbb{0}$s.t., (i) satisfy state equations; and (i) commute with (ii) and (iii)
- The **induced monad** is then isomorphic to

$$\mathbf{T} X \stackrel{\text{def}}{=} C \Rightarrow \mathbf{Free}_{\Sigma'}((X \times C) + S)$$

Effectful runners for modular top-levels ctd.

- The corresponding **T-runners** \mathcal{R} for Σ are then of the form

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow C \Rightarrow \mathbf{Free}_{\Sigma'}((B_{\text{op}} \times C) + S) \right)_{\text{op} \in \Sigma}$$

Effectful runners for modular top-levels ctd.

- The corresponding **T-runners** \mathcal{R} for Σ are then of the form

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow C \Rightarrow \mathbf{Free}_{\Sigma'}((B_{\text{op}} \times C) + S) \right)_{\text{op} \in \Sigma}$$

- Observe that raising signals in S **discards the state**,
but **not all problems are terminal**—they can be recovered from

Effectful runners for modular top-levels ctd.

- The corresponding **T-runners** \mathcal{R} for Σ are then of the form

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow C \Rightarrow \mathbf{Free}_{\Sigma'}((B_{\text{op}} \times C) + S) \right)_{\text{op} \in \Sigma}$$

- Observe that raising signals in S **discards the state**,
but **not all problems are terminal**—they can be recovered from
- Our solution:** consider signatures Σ, Σ' with operation symbols

$$\text{op} : A_{\text{op}} \rightsquigarrow B_{\text{op}} + E_{\text{op}}$$

Effectful runners for modular top-levels ctd.

- The corresponding **T-runners** \mathcal{R} for Σ are then of the form

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow C \Rightarrow \mathbf{Free}_{\Sigma'}((B_{\text{op}} \times C) + S) \right)_{\text{op} \in \Sigma}$$

- Observe that raising signals in S **discards the state**,
but **not all problems are terminal**—they can be recovered from
- Our solution:** consider signatures Σ, Σ' with operation symbols
 $\text{op} : A_{\text{op}} \rightsquigarrow B_{\text{op}} + E_{\text{op}}$ (which we write as $\text{op} : A_{\text{op}} \rightsquigarrow B_{\text{op}} ! E_{\text{op}}$)

Effectful runners for modular top-levels ctd.

- The corresponding **T-runners** \mathcal{R} for Σ are then of the form

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow C \Rightarrow \mathbf{Free}_{\Sigma'}((B_{\text{op}} \times C) + S) \right)_{\text{op} \in \Sigma}$$

- Observe that raising signals in S **discards the state**,
but **not all problems are terminal**—they can be recovered from
- Our solution:** consider signatures Σ, Σ' with operation symbols

$$\text{op} : A_{\text{op}} \rightsquigarrow B_{\text{op}} + E_{\text{op}} \quad (\text{which we write as } \text{op} : A_{\text{op}} \rightsquigarrow B_{\text{op}} ! E_{\text{op}})$$

- With this, our **T-runners** \mathcal{R} for Σ are (with “primitive” excs.)

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow \mathbf{K}_C^{\Sigma' ! E_{\text{op}} \not\downarrow S} B_{\text{op}} \right)_{\text{op} \in \Sigma}$$

where we call $\mathbf{K}_C^{\Sigma' ! E_{\text{op}} \not\downarrow S}$ a **kernel monad**, given by

$$\mathbf{K}_C^{\Sigma' ! E_{\text{op}} \not\downarrow S} X \stackrel{\text{def}}{=} C \Rightarrow \mathbf{Free}_{\Sigma}(((X + E) \times C) + S)$$

T-runners as a programming construct

T-runners as a programming construct

- As our **T-runners** for Σ are of the form

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow \mathbf{K}_C^{\Sigma'!E_{\text{op}} \not\leq S} B_{\text{op}} \right)_{\text{op} \in \Sigma}$$

we accommodate **runners as values** and **co-ops. as kernel code**

```
let R = runner { op1 x1 → K1 , ... , opn xn → Kn } @ C
```

T-runners as a programming construct

- As our **T-runners** for Σ are of the form

$$\left(\overline{\text{op}}_{\mathcal{R}} : A_{\text{op}} \longrightarrow \mathbf{K}_C^{\Sigma' ! E_{\text{op}} \not\prec S} B_{\text{op}} \right)_{\text{op} \in \Sigma}$$

we accommodate **runners as values** and **co-ops. as kernel code**

```
let R = runner { op1 x1 → K1 , ... , opn xn → Kn } @ C
```

- For instance, we can implement a **write-only file handle** as

```
let RFH = runner {  
  write s → if (length s > max)  
    then (raise WriteSizeExceeded)  
    else (let fh = getenv () in  
          if (isValid fh) then (fwrite (fh,s)) else (kill IOError))  
} @ FileHandle
```

where

$$(\text{fwrite} : \text{FileHandle} \times \text{String} \rightsquigarrow 1 ! E) \in \Sigma'$$

$$\Sigma \stackrel{\text{def}}{=} \{ \text{write} : \text{String} \rightsquigarrow 1 ! E \cup \{\text{WriteSizeExceeded}\} \} \quad \text{IOError} \in S$$

Controlled **initialisation** and **finalisation**

Controlled **initialisation** and **finalisation**

- Recall that the components r_X of the monad morphism

$$r : \mathbf{Free}_\Sigma(-) \longrightarrow \mathbf{T}$$

induced by a \mathbf{T} -runner \mathcal{R} are all **tail-recursive**

Controlled **initialisation** and **finalisation**

- Recall that the components r_X of the monad morphism

$$r : \mathbf{Free}_\Sigma(-) \longrightarrow \mathbf{T}$$

induced by a \mathbf{T} -runner \mathcal{R} are all **tail-recursive**

- We make use of it to enable one to **run user code**:

```
using R @ Minit  
run M  
finally {return x @ c → Mret , ... raise e @ c → Me ... , ... kill s → Ms ...}
```

Controlled **initialisation** and **finalisation**

- Recall that the components r_X of the monad morphism

$$r : \mathbf{Free}_\Sigma(-) \longrightarrow \mathbf{T}$$

induced by a \mathbf{T} -runner \mathcal{R} are all **tail-recursive**

- We make use of it to enable one to **run user code**:

```
using R @ Minit  
run M  
finally {return x @ c → Mret , ... raise e @ c → Me ... , ... kill s → Ms ...}
```

where

(a **user monad**)

- M_s are **user code**, modelled using $\mathbf{U}^{\Sigma!E} X \stackrel{\text{def}}{=} \mathbf{Free}_\Sigma(X + E)$

Controlled **initialisation** and **finalisation**

- Recall that the components r_X of the monad morphism

$$r : \mathbf{Free}_\Sigma(-) \longrightarrow \mathbf{T}$$

induced by a \mathbf{T} -runner \mathcal{R} are all **tail-recursive**

- We make use of it to enable one to **run user code**:

```
using R @ Minit  
run M  
finally {return x @ c → Mret , ... raise e @ c → Me ... , ... kill s → Ms ...}
```

where

(a **user monad**)

- M_s are **user code**, modelled using $\mathbf{U}^{\Sigma!E} X \stackrel{\text{def}}{=} \mathbf{Free}_\Sigma(X + E)$
- M_{init} produces the **initial kernel state**
- M is the user code being **run using the runner** R
- M_{ret} , M_e , M_s **finalise** for return values, exceptions, and signals

Controlled **initialisation** and **finalisation**

- Recall that the components r_X of the monad morphism

$$r : \mathbf{Free}_\Sigma(-) \longrightarrow \mathbf{T}$$

induced by a \mathbf{T} -runner \mathcal{R} are all **tail-recursive**

- We make use of it to enable one to **run user code**:

```
using R @ Minit  
run M  
finally {return x @ c → Mret , ... raise e @ c → Me ... , ... kill s → Ms ...}
```

where

(a **user monad**)

- M_s are **user code**, modelled using $\mathbf{U}^{\Sigma!E} X \stackrel{\text{def}}{=} \mathbf{Free}_\Sigma(X + E)$
- M_{init} produces the **initial kernel state**
- M is the user code being **run using the runner** R
- M_{ret} , M_e , M_s **finalise** for return values, exceptions, and signals
- M_{ret} and M_e **depend on the final state** c , but M_s **does not**

Controlled **initialisation** and **finalisation** ctd.

- For instance, we can define a PYTHON-esque **with construct**

```
with fileName do M
=
using RFH @ (fopen fileName)
run M
finally {
  return x @ fh → fclose fh; return x ,
  raise e @ fh → fclose fh; raise e ,
  kill s → ... }
```

- Importantly, here
 - the **file handle is hidden** from M
 - M can only use **write** but not **fopen** and **fclose**
 - write** : $\text{String} \rightsquigarrow 1 ! E \cup \{\text{WriteSizeExceeded}\}$
 - fopen** and **fclose** are limited to **initialisation-finalisation**

**A core calculus for
programming with runners**

Core calculus (syntax)

Core calculus (syntax)

- **Ground types** (types of ops. and kernel state)

$$A, B, C ::= B \mid 1 \mid 0 \mid A \times B \mid A + B$$

- **Types**

$$\begin{aligned} X, Y &::= B \mid 1 \mid 0 \mid X \times Y \mid X + Y \\ &\mid X \xrightarrow{\Sigma} Y ! E \\ &\mid X \xrightarrow{\Sigma} Y ! E \Downarrow S @ C \\ &\mid \Sigma \Rightarrow \Sigma' \Downarrow S @ C \end{aligned}$$

- **Values**

$$\Gamma \vdash V : X$$

- **User computations**

$$\Gamma \vDash M : X ! E$$

- **Kernel computations**

$$\Gamma \vDash K : X ! E \Downarrow S @ C$$

Core calculus (user computations)

$M, N ::=$	<code>return</code>	V	value
	<code>try</code>	M <code>with</code> $\{\text{return } x \mapsto N, (\text{raise } e \mapsto N_e)_{e \in E}\}$	exception handler
	V	W	application
	<code>match</code>	V <code>with</code> $\{\langle x, y \rangle \mapsto M\}$	product elimination
	<code>match</code>	V <code>with</code> $\{\}_X$	empty elimination
	<code>match</code>	V <code>with</code> $\{\text{inl } x \mapsto M, \text{inr } y \mapsto N\}$	sum elimination
	<code>op</code>	$_X(V, (x . M), (N_e)_{e \in E_{\text{op}}})$	operation call
	<code>raise</code>	$_X e$	raise exception
	<code>using</code>	$V @ W$ <code>run</code> M <code>finally</code> $\{$	run
		<code>return</code> $x @ c \mapsto N,$	
		$(\text{raise } e @ c \mapsto N_e)_{e \in E},$	
		$(\text{kill } s \mapsto N_s)_{s \in S}\}$	
	<code>kernel</code>	$K @ V$ <code>finally</code> $\{$	switch to kernel mode
		<code>return</code> $x @ c \mapsto N,$	
		$(\text{raise } e @ c \mapsto N_e)_{e \in E},$	
		$(\text{kill } s \mapsto N_s)_{s \in S}\}$	

Core calculus (kernel computations)

$K, L ::=$	<code>return_C V</code>	value
	<code>try K with {return $x \mapsto L$, (raise $e \mapsto L_e$)_{$e \in E$}}</code>	exception handler
	<code>V W</code>	application
	<code>match V with {$\langle x, y \rangle \mapsto K$}</code>	product elimination
	<code>match V with {}_{$X @ C$}</code>	empty elimination
	<code>match V with {inl $x \mapsto K$, inr $y \mapsto L$}</code>	sum elimination
	<code>op_{$X @ C$}(V, ($x . K$), (L_e)_{$e \in E_{\text{op}}$})</code>	operation call
	<code>raise_{$X @ C$} e</code>	raise exception
	<code>kill_{$X @ C$} s</code>	send signal
	<code>getenv_C($c . K$)</code>	get state
	<code>setenv(V, K)</code>	set state
	<code>user M with {return $x \mapsto K$, (raise $e \mapsto L_e$)_{$e \in E$}}</code>	switch to user mode

Core calculus (type system and eq. theory)

Core calculus (type system and eq. theory)

- For example, the **typing rule for running user comps.** is

$$\begin{array}{c}
 \Gamma \vdash V : \Sigma \Rightarrow \Sigma' \not\downarrow S @ C \quad \Gamma \vdash W : C \\
 \Gamma \Vdash M : X ! E \quad \Gamma, x : X, c : C \Vdash' N_{ret} : Y ! E' \\
 \frac{(\Gamma, c : C \Vdash' N_e : Y ! E')_{e \in E} \quad (\Gamma \Vdash' N_s : Y ! E')_{s \in S}}{\Gamma \Vdash' \text{using } V @ W \text{ run } M \text{ finally } \{ \text{return } x @ c \mapsto N_{ret} , \\
 \text{(raise } e @ c \mapsto N_e)_{e \in E} , \\
 \text{(kill } s \mapsto N_s)_{s \in S} \} : Y ! E'}
 \end{array}$$

Core calculus (type system and eq. theory)

- For example, the **typing rule for running user comps.** is

$$\begin{array}{c}
 \Gamma \vdash V : \Sigma \Rightarrow \Sigma' \not\vdash S @ C \quad \Gamma \vdash W : C \\
 \Gamma \Vdash M : X ! E \quad \Gamma, x : X, c : C \Vdash' N_{ret} : Y ! E' \\
 \frac{(\Gamma, c : C \Vdash' N_e : Y ! E')_{e \in E} \quad (\Gamma \Vdash' N_s : Y ! E')_{s \in S}}{\Gamma \Vdash' \text{using } V @ W \text{ run } M \text{ finally } \{ \text{return } x @ c \mapsto N_{ret} , \\
 \text{(raise } e @ c \mapsto N_e)_{e \in E} , \\
 \text{(kill } s \mapsto N_s)_{s \in S} \} : Y ! E'}
 \end{array}$$

- and the **main β -equation for running user comps.** is

$$\begin{aligned}
 &\Gamma \Vdash' \text{using } R_C @ W \text{ run } (\text{op}_X(V, (x.M), (M_e)_{e \in E_{\text{op}}})) \text{ finally } F \\
 &\equiv \text{kernel } R_{\text{op}}[V] @ W \text{ finally } \{ \\
 &\quad \text{return } x @ c' \mapsto \text{using } R_C @ c' \text{ run } M \text{ finally } F , \\
 &\quad (\text{raise } e @ c' \mapsto \text{using } R_C @ c' \text{ run } M_e \text{ finally } F)_{e \in E_{\text{op}}} , \\
 &\quad (\text{kill } s \mapsto N_s)_{s \in S} \} : Y ! E'
 \end{aligned}$$

Core calculus (type system and eq. theory)

- The calculus also includes **subtyping**, and **subsumption rules**

$$\frac{\Gamma \vdash V : A \quad A <: B}{\Gamma \vdash V : B}$$

$$\frac{\Gamma \Vdash M : A ! E \quad \Sigma \subseteq \Sigma' \quad A <: B \quad E \subseteq E'}{\Gamma \Vdash' M : B ! E'}$$

$$\frac{\begin{array}{cccc} \Gamma \Vdash K : A ! E \downarrow S @ C & \Sigma \subseteq \Sigma' & & \\ A <: B & E \subseteq E' & S \subseteq S' & C = C' \end{array}}{\Gamma \Vdash' K : B ! E' \downarrow S' @ C'}$$

Core calculus (type system and eq. theory)

- The calculus also includes **subtyping**, and **subsumption rules**

$$\frac{\Gamma \vdash V : A \quad A <: B}{\Gamma \vdash V : B}$$

$$\frac{\Gamma \Vdash M : A ! E \quad \Sigma \subseteq \Sigma' \quad A <: B \quad E \subseteq E'}{\Gamma \Vdash' M : B ! E'}$$

$$\frac{\Gamma \Vdash K : A ! E \Downarrow S @ C \quad \Sigma \subseteq \Sigma' \quad A <: B \quad E \subseteq E' \quad S \subseteq S' \quad C = C'}{\Gamma \Vdash' K : B ! E' \Downarrow S' @ C'}$$

- We use $C = C'$ to have (standard) **proof-irrelevant subtyping**
- Otherwise, instead of just $C <: C'$, we would need a **lens** $C' \leftrightarrow C$

Core calculus (semantics)

Core calculus (semantics)

- **Monadic semantics**, for concreteness in **Set**, using
 - **user monads** $\mathbf{U}^{\Sigma!E} X \stackrel{\text{def}}{=} \mathbf{Free}_{\Sigma}(X + E)$
 - **kernel monads** $\mathbf{K}_C^{\Sigma!E \not\vdash S} X \stackrel{\text{def}}{=} C \Rightarrow \mathbf{Free}_{\Sigma}(((X + E) \times C) + S)$

Core calculus (semantics)

- **Monadic semantics**, for concreteness in **Set**, using
 - **user monads** $\mathbf{U}^{\Sigma!E} X \stackrel{\text{def}}{=} \mathbf{Free}_{\Sigma}(X + E)$
 - **kernel monads** $\mathbf{K}_C^{\Sigma!E \not\downarrow S} X \stackrel{\text{def}}{=} C \Rightarrow \mathbf{Free}_{\Sigma}(((X + E) \times C) + S)$
- (At a high level) the **judgements are interpreted** as

$$\llbracket \Gamma \vdash V : X \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow \llbracket X \rrbracket$$

$$\llbracket \Gamma \models M : X ! E \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow \mathbf{U}^{\Sigma!E} \llbracket X \rrbracket$$

$$\llbracket \Gamma \models K : X ! E \not\downarrow S @ C \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow \mathbf{K}_{\llbracket C \rrbracket}^{\Sigma!E \not\downarrow S} \llbracket X \rrbracket$$

Core calculus (semantics ctd.)

- However, to prove **coherence** of the semantics (**subtyping!**), we actually give the semantics in the **subset fibration**

Core calculus (semantics ctd.)

- However, to prove **coherence** of the semantics (**subtyping!**), we actually give the semantics in the **subset fibration**
- For instance, **kernel computations** are interpreted as

$$\begin{array}{ccc}
 \llbracket \Gamma \rrbracket & \xrightarrow{\llbracket \Gamma \models K : X ! E \not\leq S @ C \rrbracket} & \mathbf{K}_{\llbracket C \rrbracket}^{\Sigma ! E \not\leq S} \llbracket X \rrbracket \\
 \downarrow \subseteq & & \downarrow \subseteq \\
 \llbracket \Gamma^s \rrbracket & \xrightarrow{\llbracket \Gamma^s \vdash K : X^s @ C \rrbracket} & \mathbf{K}_{\llbracket C \rrbracket}^{\mathcal{O} ! E \not\leq S + \{\perp\}} \llbracket X^s \rrbracket
 \end{array}$$

where $\Gamma^s \vdash K : X^s @ C$ is a **skeletal kernel typing judgement**

Core calculus (semantics ctd.)

- However, to prove **coherence** of the semantics (**subtyping!**), we actually give the semantics in the **subset fibration**
- For instance, **kernel computations** are interpreted as

$$\begin{array}{ccc}
 \llbracket \Gamma \rrbracket & \xrightarrow{\llbracket \Gamma \models K : X ! E \downarrow S @ C \rrbracket} & \mathbf{K}_{\llbracket C \rrbracket}^{\Sigma ! E \downarrow S} \llbracket X \rrbracket \\
 \downarrow \subseteq & & \downarrow \subseteq \\
 \llbracket \Gamma^s \rrbracket & \xrightarrow{\llbracket \Gamma^s \vdash K : X^s @ C \rrbracket} & \mathbf{K}_{\llbracket C \rrbracket}^{\mathcal{O} ! E \downarrow S + \{\perp\}} \llbracket X^s \rrbracket
 \end{array}$$

where $\Gamma^s \vdash K : X^s @ C$ is a **skeletal kernel typing judgement**

- No essential obstacles to extending to **Sub(Cpo)** and beyond

Core calculus (semantics ctd.)

- However, to prove **coherence** of the semantics (**subtyping!**), we actually give the semantics in the **subset fibration**
- For instance, **kernel computations** are interpreted as

$$\begin{array}{ccc}
 \llbracket \Gamma \rrbracket & \xrightarrow{\llbracket \Gamma \models K : X ! E \not\leq S @ C \rrbracket} & \mathbf{K}_{\llbracket C \rrbracket}^{\Sigma ! E \not\leq S} \llbracket X \rrbracket \\
 \downarrow \subseteq & & \downarrow \subseteq \\
 \llbracket \Gamma^s \rrbracket & \xrightarrow{\llbracket \Gamma^s \vdash K : X^s @ C \rrbracket} & \mathbf{K}_{\llbracket C \rrbracket}^{\mathcal{O} ! \mathcal{E} \not\leq S + \{\perp\}} \llbracket X^s \rrbracket
 \end{array}$$

where $\Gamma^s \vdash K : X^s @ C$ is a **skeletal kernel typing judgement**

- No essential obstacles to extending to **Sub(Cpo)** and beyond
- **Ground type restriction** on C needed to stay within **Sub(...)**
 - Otherwise, analogously to subtyping, we'd need **lenses** instead

Implementing runners

Experimenting with the **theory in practice**

Experimenting with the **theory in practice**

- A **small experimental language** COOP⁴
 - Implements the core calculus with few extras
 - The interpreter is directly based on the denotational semantics
 - Top-level containers for running external (OCaml) code

⁴coop [/ku:p/] – a cage where small animals are kept, especially chickens

Experimenting with the **theory in practice**

- A **small experimental language** COOP⁴
 - Implements the core calculus with few extras
 - The interpreter is directly based on the denotational semantics
 - Top-level containers for running external (OCaml) code
- A **HASKELL library** HASKELL-COOP
 - A shallow-embedding of the core calculus in HASKELL
 - Uses one of the Freer monad implementations underneath
 - Again, the operational aspects implement the denot. semantics
 - Top-level containers for arbitrary HASKELL monads
 - Examples make use of HASKELL's features (GADTs, ...)

⁴coop [/ku:p/] – a cage where small animals are kept, especially chickens

Experimenting with the **theory in practice**

- A **small experimental language** COOP⁴
 - Implements the core calculus with few extras
 - The interpreter is directly based on the denotational semantics
 - Top-level containers for running external (OCaml) code
- A **HASKELL library** HASKELL-COOP
 - A shallow-embedding of the core calculus in HASKELL
 - Uses one of the Freer monad implementations underneath
 - Again, the operational aspects implement the denot. semantics
 - Top-level containers for arbitrary HASKELL monads
 - Examples make use of HASKELL's features (GADTs, ...)
- Both still need some finishing touches, but will be public soon

⁴coop [/ku:p/] – a cage where small animals are kept, especially chickens

Runners in action

Runners can be **vertically nested**

Runners can be **vertically nested**

```
• using RFH @ (fopen fileName)
  run (
    using RFC @ (return "")
    run M
    finally {
      return x @ str → write str; return x ,
      raise e @ str → raise e }
  )
  finally {
    return x @ fh → fclose fh; return x ,
    raise e @ fh → fclose fh; raise e , kill IOError → ... }
```

where the **file contents runner** (with $\Sigma' = \emptyset$) is defined as

```
let RFC = runner {
  write strl → let str = getenv () in
    if (length (str^strl) > max) then (raise WriteSizeExceeded)
    else (setenv (str^strl))
} @ String
```

Vertical nesting for instrumentation

Vertical nesting for instrumentation

- ```
using RSniffer @ (return 0)
run M
finally {
 return x @ c →
 let fh = fopen "nsa.txt" in fwrite (fh,toStr c); fclose fh; return x }
```

where the **instrumenting runner** is defined as

```
let RSniffer = runner {
 ... ,
 op a → let c = getenv () in
 setenv (c + 1);
 op a ,
 ...
} @ Nat
```

(\* forwards op outwards \*)

- The runner  $R_{\text{Sniffer}}$  implements the same sig.  $\Sigma$  that  $M$  is using
- As a result, the runner  $R_{\text{Sniffer}}$  is **invisible** from  $M$ 's viewpoint

# Vertical nesting for **active monitoring**

# Vertical nesting for active monitoring

- First, we define a runner for integer-valued ML-style state as

**type** IntHeap = (Nat  $\rightarrow$  (Int + 1))  $\times$  Nat

**type** Ref = Nat

```
let RIntState = runner {
 alloc x \rightarrow let h = getenv () in (* alloc : Int \rightsquigarrow Ref ! 0 *)
 let (r,h') = heapAlloc h x in
 setenv h';
 return r ,

 deref r \rightarrow let h = getenv () in (* deref : Ref \rightsquigarrow Int ! 0 *)
 match (heapSel h r) with
 | inl x \rightarrow return x
 | inr () \rightarrow kill ReferenceDoesNotExist ,

 assign r y \rightarrow let h = getenv () in (* assign : Ref \times Int \rightsquigarrow 1 ! 0 *)
 match (heapUpd h r y) with
 | inl h' \rightarrow setenv h'
 | inr () \rightarrow kill ReferenceDoesNotExist
}
```

@ IntHeap

## Vertical nesting for **active monitoring** ctd.

- Next we define a runner for **monotonicity layer** on top of  $R_{\text{IntState}}$

# Vertical nesting for **active monitoring** ctd.

- Next we define a runner for **monotonicity layer** on top of  $R_{\text{IntState}}$

**type** MonMemory = Ref  $\rightarrow ((\text{Int} \rightarrow \text{Int} \rightarrow \text{Bool}) + 1)$

```
let RMonState = runner {
 mAlloc x rel \rightarrow let r = alloc x in (* : Int \times Ord \rightsquigarrow Ref ! 0 *)
 let m = getenv () in
 setenv (memAdd m r rel);
 return r,

 mDeref r \rightarrow deref r , (* monDeref : Ref \rightsquigarrow Int ! 0 *)

 mAssign r y \rightarrow let x = deref r in (* : Ref \times Int \rightsquigarrow 1 ! {MV} *)
 let m = getenv () in
 match (memSel m r) with
 | inl rel \rightarrow if (rel x y)
 then (assign r y)
 else (raise MonotonicityViolation)
 | inr \rightarrow kill PreorderDoesNotExist
}
@ MonMemory
```

## Vertical nesting for **active monitoring** ctd.

- We can then perform **runtime monotonicity verification** as

# Vertical nesting for **active monitoring** ctd.

- We can then perform **runtime monotonicity verification** as

```
using RIntState @ ((fun _ → inr ()), 0) (* init empty ML-style heap *)
run (

 using RMonState @ (fun _ → inr ()) (* init empty preorders memory *)
 run (

 let r = mAlloc 0 (≤) in
 mAssign r 1;
 try (mAssign r 0) with {
 return _ → mAssign r 3 ,
 raise MonotonicityViolation → return ();
 }
 mAssign r 2

)
 finally { ... }

)
finally { ... }
```

Runners can also be **horizontally paired**



# Runners can also be horizontally paired

- Given runners for  $\Sigma$  and  $\Sigma'$

```
let R1 = runner { ... , op1i x → K1i , ... } @ C1
let R2 = runner { ... , op2j x → K2j , ... } @ C2
```

we can **pair them** to get a runner for  $\Sigma + \Sigma'$

```
let R = runner { ... ,
 op1i x → let (c,c') = getenv () in
 user (kernel (K1i x) @ c finally {
 return y @ c'' → return (inl (inl y,c'')),
 (raise e @ c'' → return (inl (inr e,c'')))
 })e ∈ Eop,
 (kill s → return (inr s))s ∈ S1 }
 finally {
 return (inl (inl y,c'')) → setenv (c'',c'); return y,
 return (inl (inr e,c'')) → setenv (c'',c'); raise e,
 return (inr s) → kill s },
 ... ,
 op2j x → ...,
 ... } @ C1 × C2
```

(\* analogously to above, just on 2nd comp. of state \*)

# Runners can also be horizontally paired

- Given runners for  $\Sigma$  and  $\Sigma'$

```
let R1 = runner { ... , op1i x → K1i , ... } @ C1
let R2 = runner { ... , op2j x → K2j , ... } @ C2
```

we can **pair them** to get a runner for  $\Sigma + \Sigma'$

```
let R = runner { ... ,
 op1i x → let (c,c') = getenv () in
 user (kernel (K1i x) @ c finally {
 return y @ c'' → return (inl (inl y,c'')),
 (raise e @ c'' → return (inl (inr e,c'')))
 })e ∈ Eop,
 (kill s → return (inr s))s ∈ S1 }
 finally {
 return (inl (inl y,c'')) → setenv (c'',c'); return y,
 return (inl (inr e,c'')) → setenv (c'',c'); raise e,
 return (inr s) → kill s },
 ... ,
 op2j x → ..., (* analogously to above, just on 2nd comp. of state *)
 ... } @ C1 × C2
```

- For instance, this way we can build a runner for **IO and state**

## Other examples (in HASKELL)

## Other examples (in HASKELL)

- More general forms of **(ML-style) state** (for general Ref A)
  - if the host language allows it, we use GADTs, etc for safety
  - some examples extract a footprint from a larger memory
- **Combinations** of different effects and runners
  - in particular the combination of IO and state
  - good use case for both vertical and horizontal composition
- KOKA-style **ambient values** and **ambient functions**
  - **ambient values** are essentially **mutable variables/parameters**
  - **ambient functions** are **applied in their lexical context**
  - a runner that treats **amb. fun. application as a co-operation**
  - amb. funs. are stored in a context-depth-sensitive heap
  - the appl. co-operation restores the heap to the lexical context

# Other examples (ambient functions)

```
module Control.Runner.Ambients

...

ambCoOps :: Amb a -> Kernel sig AmbHeap a
ambCoOps (Bind f) =
 do h <- getEnv;
 (f,h') <- return (ambHeapAlloc h f);
 setEnv h';
 return f
ambCoOps (Apply f x) =
 do h <- getEnv;
 (f,d) <- return (ambHeapSel h f (depth h));
 user
 (run
 ambRunner
 (return (h {depth = d}))
 (f x)
 ambFinaliser)
 return
ambCoOps (Rebind f g) =
 do h <- getEnv;
 setEnv (ambHeapUpd h f g)

ambRunner :: Runner '[Amb] sig AmbHeap
ambRunner = mkRunner ambCoOps
```

```
module AmbientsTests where

import Control.Runner
import Control.Runner.Ambients

ambFun :: AmbVal Int -> Int -> AmbEff Int
ambFun x y =
 do x <- getVal x;
 return (x + y)

test1 :: AmbEff Int
test1 =
 withAmbVal
 (4 :: Int)
 (\ x ->
 withAmbFun
 (ambFun x)
 (\ f ->
 do rebindVal x 2;
 applyFun f 1))

test2 = ambTopLevel test1
```

# Wrapping up

- **Runners** are a natural model of **top-level runtime**
- We propose **T-runners** to also model **non-top-level runtimes**
- We have turned **T-runners** into a **(practical ?) programming construct**, that supports controlled initialisation and finalisation
- I showed you some **combinators** and **programming examples**
- Two **implementations** in the works, COOP & HASKELL-COOP
- **Future:** lenses in subtyping and semantics, category of runners, handlers, larger case studies, refinement typing, compilation, ...

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 834146.



This material is based upon work supported by the Air Force Office of Scientific Research under award number FA9550-17-1-0326.

**Thank you!**







# Core calculus (semantics ctd.)

$$\begin{aligned} \llbracket \Gamma \models^{\Sigma'} \text{ using } V @ W \text{ run } M \text{ finally } \{ \text{return } x @ c \mapsto N_{ret} , \\ (\text{raise } e @ c \mapsto N_e)_{e \in E} , \\ (\text{kill } s \mapsto N_s)_{s \in S} \} : Y ! E' \rrbracket_{\gamma} \stackrel{\text{def}}{=} \dots \end{aligned}$$

- $\llbracket V \rrbracket_{\gamma} = \mathcal{R} = \left( \overline{\text{op}}_{\mathcal{R}} : \llbracket A_{\text{op}} \rrbracket \longrightarrow \mathbf{K}_{\llbracket C \rrbracket}^{\Sigma' ! E_{\text{op}} \not\leq S} \llbracket B_{\text{op}} \rrbracket \right)_{\text{op} \in \Sigma}$
- $\llbracket W \rrbracket_{\gamma} \in \llbracket C \rrbracket$
- $\llbracket M \rrbracket_{\gamma} \in \mathbf{U}^{\Sigma ! E} \llbracket A \rrbracket$
- $\llbracket \text{return } x @ c \mapsto N_{ret} \rrbracket_{\gamma} \in \llbracket A \rrbracket \times \llbracket C \rrbracket \longrightarrow \mathbf{U}^{\Sigma' ! E'} \llbracket B \rrbracket$
- $\llbracket (\text{raise } e @ c \mapsto N_e)_{e \in E} \rrbracket_{\gamma} \in E \times \llbracket C \rrbracket \longrightarrow \mathbf{U}^{\Sigma' ! E'} \llbracket B \rrbracket$
- $\llbracket (\text{kill } s \mapsto N_s)_{s \in S} \rrbracket_{\gamma} \in S \longrightarrow \mathbf{U}^{\Sigma' ! E'} \llbracket B \rrbracket$
- allowing us to use the **free model property** to get

$$\mathbf{U}^{\Sigma ! E} \llbracket A \rrbracket \xrightarrow{r_{\llbracket A \rrbracket} + E} \mathbf{K}_{\llbracket C \rrbracket}^{\Sigma' ! E \not\leq S} \llbracket A \rrbracket \xrightarrow{(\lambda \llbracket N_{ret} \rrbracket_{\gamma})^{\dagger}} \llbracket C \rrbracket \Rightarrow \mathbf{U}^{\Sigma' ! E'} \llbracket B \rrbracket$$

and then apply the resulting composite to  $\llbracket M \rrbracket_{\gamma}$  and  $\llbracket W \rrbracket_{\gamma}$