Comodels as a gateway for interacting with the external world

Danel Ahman

(joint work with Andrej Bauer)

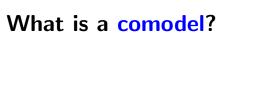
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Ljubljana, 21 March 2019



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$$\mathcal{M} = \langle M : \mathsf{Set} , \{ \mathsf{op}_{\mathcal{M}} : A \times M^B \longrightarrow M \}_{\mathsf{op} \in \Sigma} \rangle$$

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- Intutively, comodels describe a notion of state/world, e.g.,
 - Operational semantics using a tensor of a model and a comodel (Plotkin & Power, Abou-Saleh & Pattinson)
 - Stateful runners of effectful programs (Uustalu)
 - Default top-level behaviour of alg. effects (Bauer & Pretnar)



• Using monads (e.g., as in HASKELL)

```
type St a = String \rightarrow (a, String)

f :: St a \rightarrow St (a,a)

f c = c >>= (\x \rightarrow c >>= (\y \rightarrow return (x,y)))
```

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• Using algebraic effects and handlers (e.g., as in EFF)

```
effect Get : int effect Put : int \rightarrow unit (*: int \rightarrow a*int!\{\} *) let g (c:unit \rightarrow a!{Get,Put}) = with st_h handle (perform (Put 42); c ())
```

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```

Works well for effects that can be represented as pure data!
 But what about effects that need access to the external world?

• Declare a **signature** of monads/effects

```
type IO a  \begin{tabular}{lll} \textbf{openFile} & :: & FilePath $\rightarrow$ IOMode $\rightarrow$ IO Handle \\ \textbf{hGetLine} & :: & Handle $\rightarrow$ IO String \\ \textbf{hClose} & :: & Handle $\rightarrow$ IO () \\ \end{tabular}
```

• Declare a **signature** of monads/effects

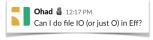
effect RandomFloat : float \rightarrow float

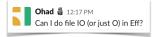
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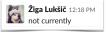
```
effect Raise : string \rightarrow empty

effect RandomInt : int \rightarrow int
effect RandomFloat : float \rightarrow float
```

• And then treat it **specially** in the compiler, e.g.,







```
Ohad 3 12:17 PM
Can I do file IO (or just O) in Eff?
```



```
Ohad 🏭 8:35 PM
So here's the hack I added We should do something a bit more principled
 In pervasives.eff:
  effect Write : (string*string) -> unit
 in eval.ml, under let rec top_handle op = add the case:
      | "Write" ->
         (match v with
           | V.Tuple vs ->
             let (file_name :: str :: _) = List.map V.to_str vs in
             let file_handle = open_out_gen
                                  [Open_wronly
                                  :Open append
                                  :Open_creat
                                  :Open text
                                 ] 0o666 file_name in
             Printf.fprintf file handle "%s" str:
             close_out file_handle;
             top handle (k V.unit value)
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This talk — a principled (co)algebraic approach!

```
let f(s:string) =
  let fh = fopen "foo.txt" in
  fwrite fh (s^s);
  fclose fh:
```

return fh

let fh = f s in fread fh

let g s =

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let fh = f s in fread fh (* fh not open ! *)

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- We could resolve this by typing file handles linearly
- But we want other values (e.g., strings) to be used non-linearly!

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```
let h = handler
| effect (FWrite fh s k) → return s

let g s = with h handle f s (* dangling fh ! *)
```



So, how could we solve these issues?

- Using existing programming mechanisms, e.g.,
 - Modules and abstraction, e.g., System.IO

• Linear (and non-linear) types

```
linear type fhandle  {\bf effect} \ \ {\sf FClose} \ : \ ({\bf linear} \ \ {\sf fhandle}) \to {\sf unit}   {\bf linear} \ \ {\bf effect} \ \ {\sf FClose} \ : \ {\sf fhandle} \to {\sf unit}
```

• Handlers with **finally clauses**

So, how could we solve these issues?

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```
type IO a \mathsf{hClose} \ :: \ \mathsf{Handle} \to \mathsf{IO} \ \ ()
```

• Linear (and non-linear) types

```
linear type fhandle  {\bf effect} \  \, {\sf FClose} \, : \, ({\bf linear} \  \, {\sf fhandle}) \rightarrow {\sf unit}   {\bf linear} \  \, {\bf effect} \  \, {\sf FClose} \, : \, {\sf fhandle} \rightarrow {\sf unit}
```

- Handlers with **finally clauses**
- Problem: They don't really explain the essence of the solution



• Let's look at HASKELL's IO monad again

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- A common explanation is to think of functions

$$a \rightarrow IO b$$

as

$$\mathsf{a} \to (\mathsf{RealWorld} \to (\mathsf{b}, \mathsf{RealWorld}))$$

which is the same as

$$(\mathsf{a},\mathsf{RealWorld}) \to (\mathsf{b},\mathsf{RealWorld})$$

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which is the same as

$$(a, RealWorld) \rightarrow (b, RealWorld)$$

- With the System.IO module abstraction ensuring that
 - We can't get our hands on RealWorld it's an idea of the world
 - The real world is affected linearly
 - We don't ask more from the real world than it can provide

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which is the same as

But wait a minute, RealWorld looks a lot like a comodel!

 $\mathsf{hGetLine}: (\mathsf{Handle}, \mathsf{RealWorld}) \to (\mathsf{String}, \mathsf{RealWorld})$

 $\mathsf{hClose} = \mathsf{:} (\mathsf{Handle}, \mathsf{RealWorld}) o ((), \mathsf{RealWorld})$

So, IO is more about in which external world our program is in!

Comodels as a gateway to the external world

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• let f (s:string) =
 using IO run
 let fh = fopen "foo.txt" in
 fwrite fh (s^s);
 fclose fh

Comodels as a gateway to the external world

• Now external world explicit, but dangling fh etc still possible

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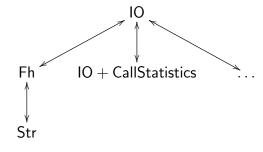
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• Better, but have to explicitly open and thread through fh

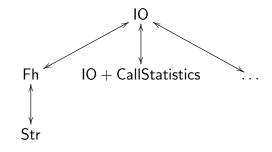
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- Better, but have to explicitly open and thread through fh
- Solution: Modular treatment of external worlds

• Examples of **modularity** we might want from comodels



Examples of modularity we might want from comodels



- Fh "world which consists of exactly one fh"
- Fh \longrightarrow IO "call fclose with stored fh"
- Observation: IO ←→ Fh and other ←→ look a lot like lenses

• Our general framework on the file operations example

• Our **general framework** on the file operations example

```
let f(s:string) =
                                (* @ IO : unit *)
    using Fh
    starting_with (fopen_of_io "foo.txt")
    run
      fwrite_of_fh (s^s) (* @ Fh : unit *)
    ending_with (fun _ fh → fclose_of_io fh)
where
```

```
Fh =
   \langle W = fhandle,
    co_fread((),fh) = \dots,
    co_fwrite (s,fh) = fwrite_of_io s fh;
                           return ((),fh)>
(* co\_fread : (unit * W) \rightarrow (string * W) @ IO *)
(* co_fwrite : (string * W) \rightarrow (unit * W) @ IO *)
```

• The modularity aspect of our general framework

```
let f(s:string) =
                              (* @ IO : unit *)
 using Fh
 starting_with (fopen_of_io "foo.txt")
 run
    using Str
    starting_with (fread_of_fh ())
    run
      fwrite_of_str(s^s) (* @ Str: unit *)
   ending_with (fun _ s → fwrite_of_fh s)
 ending_with (fun _ fh → fclose_of_io fh)
```

where

```
\mathsf{Str} = \langle \mathsf{W} = \mathsf{string} \; , \; \ldots \; 
angle
```

• Comodels can also **extend** the (intermediate) external world(s)

```
(* @ IO : unit *)
let f(s:string) =
  using Stats
  starting_with (fopen_of_io "foo.txt")
  run
    fwrite_of_stats (s^s) (* @ Stats : unit *)
  ending_with
    (fun _{-} (fh,c) \rightarrow
      let fh' = fopen_of_io "stats.txt" in
      fwrite_of_io fh'c;
      fclose_of_io fh'; fclose_of_io fh)
```

where

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\mathsf{Stats} = \langle \mathsf{W} = (\mathsf{fhandle} * \mathsf{nat}), \ldots \rangle
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where

```
\mathsf{Stats} = \langle \mathsf{W} = (\mathsf{fhandle} * \mathsf{nat}), \ldots \rangle
```

- We can also track of nondet./prob. choice results, and alike
- Could we also use comodels for dynamic (NI) monitoring?

• **Typing judgement** for computations $\Gamma \vdash c \ @ \ \vec{C} : A$

- **Typing judgement** for computations $\Gamma \vdash c \bigcirc \vec{C} : A$
- The two central typing rules are (U is the "universe", aka IO)

```
\Gamma \vdash D \text{ comodel } @ \vec{C} \qquad D \neq U
\Gamma \vdash c_s @ \vec{C} : D.W \qquad \Gamma \vdash c @ \vec{C}, D : A \qquad \Gamma, x : A, w : D.W \vdash c_e @ \vec{C} : A
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• **Typing judgement** for computations $\Gamma \vdash c \bigcirc \vec{C} : A$

run c

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\Gamma \vdash D \text{ comodel } @ \overrightarrow{C} \qquad D \neq U
\Gamma \vdash c_s @ \overrightarrow{C} : D.W \qquad \Gamma \vdash c @ \overrightarrow{C}, D : A \qquad \Gamma, x : A, w : D.W \vdash c_e @ \overrightarrow{C} : A
\Gamma \vdash \textbf{using } D
\textbf{starting\_with } c_s
```

and

```
\frac{\Gamma \vdash \mathsf{D} \; \mathsf{comodel} \; \mathbf{0} \; \vec{\mathsf{C}} \quad \mathsf{op} : A \leadsto B \in \mathsf{D}.\Sigma \quad \Gamma \vdash \nu : A}{\Gamma \vdash \mathsf{op} \; \nu \; \mathbf{0} \; \vec{\mathsf{C}}, \mathsf{D} : B}
```

ending_with $(x.w.c_e)$ \bigcirc \overrightarrow{C} : A

Regarding the denotational semantics, the idea is to interpret

$$\Gamma \vdash c \bigcirc \overrightarrow{C} : A$$

as

$$\llbracket \Gamma \vdash c @ \vec{\mathsf{C}} : A \rrbracket : \llbracket \vec{\mathsf{C}} \rrbracket \longrightarrow \llbracket A \rrbracket \times \llbracket \vec{\mathsf{C}} \rrbracket$$

which in its essence is very similar to Møgelberg and Staton's comodels-based **linear state-passing transformation**

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• Regarding **operational semantics**, the idea is to consider confs.

$$(\overrightarrow{(C,w)}, c)$$

either in a big- or small-step style

• where $\overrightarrow{(C, w)}$ is a stack of worlds

• For example, consider the **big-step evaluation** of **using** D ...

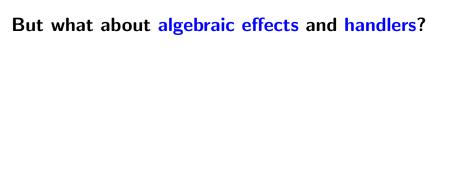
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```
 ((\overrightarrow{(C, w_0)}, (C', w'_0)), c_s) \downarrow ((\overrightarrow{(C, w_1)}, (C', w'_1)), return w''_0) 
((\overrightarrow{(C, w_1)}, (C', w'_1), (D, w''_0)), c) \downarrow ((\overrightarrow{(C, w_2)}, (C', w'_2), (D, w''_1)), return v) 
((\overrightarrow{(C, w_2)}, (C', w'_2)), c_e[v/x, w''_1/w]) \downarrow ((\overrightarrow{(C, w_3)}, (C', w'_3)), return v')
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((\overrightarrow{(C, w_1)}, (C', w'_1), (D, w''_0)), c) \Downarrow ((\overrightarrow{(C, w_2)}, (C', w'_2), (D, w''_1)), return v) 
((\overrightarrow{(C, w_2)}, (C', w'_2)), c_e[v/x, w''_1/w]) \Downarrow ((\overrightarrow{(C, w_3)}, (C', w'_3)), return v')
```

 The interpretation of operations uses the co-operations of Cs, naturally traversing the stack of (intermediate) external worlds



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- The continuations of handlers in c are delimited by run
 Again, so as to ensure linearity and reaching ending_with
- How do multi-handlers fit in here? Tensor products of some sort?

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- Comodels as a gateway for interacting with the external world
- We're making them into a **modular programming abstraction**
- Linearity by leaving outer worlds implicit (via comod. alg. ops.)
- \bullet System.IO , Koka's $% \left(1\right) =\left(1\right) =\left(1\right) \left(1\right) =\left(1\right) \left(1\right)$

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- We're making them into a **modular programming abstraction**
- Linearity by leaving outer worlds implicit (via comod. alg. ops.)
- \bullet System.IO , Koka's initially & finally , Python's with , \dots

Ongoing and future work

- Work out all the **formal details** of what I have shown you today
- Algebraic effects and (multi-)handlers
- More examples and use cases (Matija, the Eff Architecture?)
- Clarify the connection with (effectful) lenses
- Combinatorics of comodels and their lens-like relationships