Comodels as a gateway for interacting with the external world

Danel Ahman

(joint work with Andrej Bauer)

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Shonan, 26 March 2019



Computational effects in FP

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• Using monads (e.g., as in HASKELL)

```
type St a = String \rightarrow (a, String)

f :: St a \rightarrow St (a,a)

f c = c >>= (\x \rightarrow c >>= (\y \rightarrow return (x,y)))
```

• Using algebraic effects and handlers (e.g., as in Eff)

```
effect Get : int effect Put : int \rightarrow unit  
(*: int \rightarrow a*int!\{\} *)
let g (c:unit \rightarrow a!{Get,Put}) = with st_h handle (perform (Put 42); c ())
```

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effect Get : int effect Put : int \rightarrow unit (*: int \rightarrow a*int!\{\} *) let g (c:unit \rightarrow a!{Get,Put}) = with st_h handle (perform (Put 42); c ())
```

Works well for effects that can be represented as pure data!
 But what about effects that need access to the external world?

• Declare a **signature** of monads or algebraic effects

effect RandomFloat : float \rightarrow float

```
type IO a  \begin{array}{llll} & & & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ &
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• Declare a **signature** of monads or algebraic effects

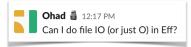
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type IO a  \begin{array}{llll} & & & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ &
```

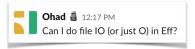
```
effect Raise : string \rightarrow empty

effect RandomInt : int \rightarrow int
effect RandomFloat : float \rightarrow float
```

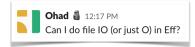
• And then treat them **specially** in the compiler, e.g.,

```
let rec top_handle op =
  match op with
  | ...
```









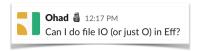


Ohad 🖥 8:35 PM
So here's the hack I added We should do something a bit more principled.

In pervasives.eff:

effect Write : (string*string) -> unit

in eval.ml, under let rec top_handle op = add the case:



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So here's the hack I added We should do something a bit more principled
In pervasives.eff:
 effect Write : (string*string) -> unit
in eval.ml under let rec top handle op = add the case:
     | "Write" ->
        (match v with
         | V.Tuple vs ->
            let (file_name :: str :: _) = List.map V.to_str vs in
            let file_handle = open_out_gen
                                 [Open_wronly
                                 :Open_append
                                 :Open creat
                                 :Open text
                                 ] 0o666 file_name in
            Printf.fprintf file_handle "%s" str:
            close out file handle:
            top handle (k V.unit value)
```

This talk — a principled (co)algebraic approach!

• let f (s:string) =
 let fh = fopen "foo.txt" in
 fwrite fh (s^s);
 fclose fh;
 return fh

let g s =
 let fh = f s in fread fh

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We could resolve this by typing fh linearly (but s non-linearly)

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- We could resolve this by typing fh linearly (but s non-linearly)
- But what if we wrap f in a handler?

```
let h = handler

| effect (FWrite fh s k) \rightarrow return fh

let g s = with h handle f ()
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- But what if we wrap f in a handler?

```
let h = handler

| effect (FWrite fh s k) \rightarrow return fh

let g s = with h handle f () (* dangling fh ! *)
```



So, how could we solve these issues?

- We could try using existing programming mechanisms, e.g.,
 - Modules and abstraction, e.g., System.IO

• Linear (and non-linear) types and effects

```
linear type fhandle  {\bf effect} \ \ {\sf FClose} \ : \ ({\bf linear} \ \ {\sf fhandle}) \to {\sf unit}   {\bf linear} \ \ {\bf effect} \ \ {\sf FClose} \ : \ {\sf fhandle} \to {\sf unit}
```

• Handlers with **finally clauses**

So, how could we solve these issues?

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 - Modules and abstraction, e.g., System.IO

```
type IO a \mathsf{hClose} \ :: \ \mathsf{Handle} \to \mathsf{IO} \ \ ()
```

• Linear (and non-linear) types and effects

```
linear type fhandle  {\bf effect} \ \ {\sf FClose} \ : \ ({\bf linear} \ \ {\sf fhandle}) \to {\sf unit}   {\bf linear} \ \ {\bf effect} \ \ {\sf FClose} \ : \ {\sf fhandle} \to {\sf unit}
```

- Handlers with **finally clauses**
- Problem: They don't really capture the essence of the problem



• Let's look at HASKELL's IO monad again

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- A common explanation is to think of functions

$$a \rightarrow IO b$$

as

$$\mathsf{a} \to (\mathsf{RealWorld} \to (\mathsf{b}, \mathsf{RealWorld}))$$

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- With the System.IO module abstraction ensuring that
 - We can't get our hands on RealWorld it's **not material**
 - The RealWorld is affected linearly
 - We don't ask more from RealWorld than it can provide

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But wait a minute! RealWorld looks a lot like a comodel!

 $\mathsf{hGetLine} : (\mathsf{Handle}, \mathsf{RealWorld}) \to (\mathsf{String}, \mathsf{RealWorld})$

 $\mathsf{hClose} : (\mathsf{Handle}, \mathsf{RealWorld}) \to ((), \mathsf{RealWorld})$

I.e., IO is about the external world rather than internal effects!

• A signature Σ is a set of operation symbols

 $op : A \leadsto B$ (in univ. alg., |A|-many op : |B|)

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$$A \leadsto B$$
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• A model/algebra/handler $\mathcal M$ of Σ is given by

$$\mathcal{M} = \langle M : \mathsf{Set} , \{ \mathsf{op}_{\mathcal{M}} : A \times M^B \longrightarrow M \}_{\mathsf{op} \in \Sigma} \rangle$$

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• A comodel/coalgebra/cohandler W of Σ is given by

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- Intutively, comodels describe a notion of state/world, e.g.,
 - Operational semantics using a tensor of a model and a comodel (Plotkin & Power, Abou-Saleh & Pattinson)
 - Stateful runners of effectful programs

- (Uustalu)
- Default top-level behaviour of alg. effects (Bauer & Pretnar)

Now external world explicit, but dangling fh etc still possible

Now **external world** explicit, but **dangling** fh etc **still possible**

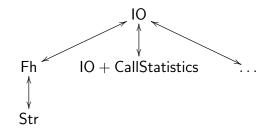
Better, but have to explicitly open and thread through fh

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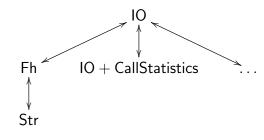
Solution: Modular treatment of external worlds

• Examples of **modularity** we might want from comodels



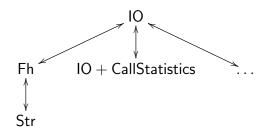
- Fh "world which consists of exactly one fh"
- Fh \longrightarrow IO "call fclose with stored fh"

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- Fh "world which consists of exactly one fh"
- ullet Fh \longrightarrow IO "call fclose with stored fh"
- Str "world that is **blissfully unaware** of **fh**"

• Examples of modularity we might want from comodels



- Fh "world which consists of exactly one fh"
- ullet IO \longrightarrow Fh "call fopen with foo.txt, store returned fh"
- Fh \longrightarrow IO "call fclose with stored fh"
- Str "world that is **blissfully unaware** of fh"
- Observation: IO ←→ Fh and other ←→ look a lot like lenses

• Our **general framework** on the file operations example

```
let f (s:string) =
    using
    Fh @ (fopen_of_io "foo.txt")
    cohandle
    fwrite_of_fh (s^s)
    finally
    x @ fh → fclose_of_io fh
```

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```
let f(s:string) =
                                       (* in IO *)
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    Fh @ (fopen_of_io "foo.txt") (* in IO *)
  cohandle
                                       (* in Fh *)
    fwrite_of_fh (s^s)
  finally
    \times @ fh \rightarrow fclose_of_io fh
                                       (* in IO *)
```

where

```
Fh =
                                   (* W = fhandle *)
 { co\_fread \_ @ fh \rightarrow ...,
   co_fwrite s @ fh → fwrite_of_io s fh;
                          return ((), fh) }
     (* co\_fread : (unit * W) \rightarrow (string * W) *)
     (* co_fwrite : (string * W) \rightarrow (unit * W) *)
```

• The modularity aspect of our general framework

```
let f(s:string) =
  using Fh @ (fopen_of_io "foo.txt")
  cohandle
     using Str @ (fread_of_fh ())
     cohandle
       write_of_str (s^s)
     finally
       0 \text{ s} \rightarrow \text{fwrite of fh s}
  finally
     _{-} @ fh \rightarrow fclose_of_io fh
```

where

```
Str = \{ co\_write s @ s' \rightarrow (* W = string *) \\ return ((),s'^s) \}
```

• Comodels can also **extend** the (intermediate) external world

where

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where

• Can also track **nondet./prob. choice results**, etc

Types

$$A, B, W ::= b \mid 1 \mid A \times B \mid 0 \mid A + B \mid A \xrightarrow{\Sigma} B$$

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```
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```

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• Signatures

$$\Sigma ::= \{ \mathsf{op}_1 : A_1 \leadsto B_1 , \ldots, \mathsf{op}_n : A_n \leadsto B_n \}$$

• Terms

$$c ::= \mathbf{return} \ v \mid \mathbf{let} \ x = c_1 \ \mathbf{in} \ c_2 \mid v_1 v_2 \mid$$
 op $v \mid$ (comodel op.) using $C @ c_i$ cohandle c finally $x @ w \rightarrow c_f$

(simple setting, only comodel ops. and no handlers (wait few slides))

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Terms

```
v ::= x \mid \dots
```

$$c ::=$$
 return $v \mid$ let $x = c_1$ in $c_2 \mid v_1 v_2 \mid$ op $v \mid$ (comodel op.) using $C @ c_i$ cohandle c finally $x @ w \rightarrow c_f$

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• Comodels (cohandlers)

$$C ::= \{ \overline{op}_1 \times @ w \rightarrow c_1, \ldots, \overline{op}_n \times @ w \rightarrow c_n \}$$

• Typing judgements

$$\Gamma \vdash v : A \qquad \Gamma \vdash c : A$$

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 $\Gamma \stackrel{\mathbf{\Sigma}}{\vdash}$ using D @ c_i cohandle c finally x @ $w \rightarrow c_f : B$

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$$\Gamma \stackrel{\mathsf{E}}{\vdash} \mathsf{D}$$
 comodel of Σ' with carrier W_{D} $\Gamma \stackrel{\mathsf{E}}{\vdash} c_i : W_{\mathsf{D}}$

$$\Gamma \stackrel{\mathsf{E}'}{\vdash} c : A \qquad \Gamma, x : A, w : W_{\mathsf{D}} \stackrel{\mathsf{E}}{\vdash} c_f : B$$

$$\Gamma \stackrel{\mathsf{E}}{\vdash} \mathbf{using} \; \mathsf{D} \; @ \; c_i \; \mathbf{cohandle} \; c \; \mathbf{finally} \; x \; @ \; w \to c_f : B$$

and

$$\frac{\mathsf{op}: A \leadsto B \in \Sigma \qquad \Gamma \vdash v: A}{\Gamma \vdash^{\Sigma} \mathsf{op} \ v: B}$$

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- Term interpretation looks very similar to alg. effects:

$$\llbracket \Gamma \vdash v : A \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow \llbracket A \rrbracket \qquad \llbracket \Gamma \overset{\Sigma}{\vdash} c : A \rrbracket : \llbracket \Gamma \rrbracket \longrightarrow T_{\Sigma} \llbracket A \rrbracket$$

• un-cohandled operations wait for a suitable external world!

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- un-cohandled operations wait for a suitable external world!
- The interesting part is the interpretation of

$$\Gamma \stackrel{\vdash}{\vdash}$$
 using D @ c_i cohandle c finally x @ $w \rightarrow c_f : B$

which is based on the linear state-passing translation

$$\llbracket \mathsf{D} \rrbracket \in \mathsf{Comodel}(\Sigma')$$

$$\mathsf{cohandle_with}_{\llbracket \mathsf{D} \rrbracket} : \mathcal{T}_{\Sigma'} \llbracket A \rrbracket \longrightarrow (\llbracket W_\mathsf{D} \rrbracket \to \llbracket A \rrbracket \times \llbracket W_\mathsf{D} \rrbracket)$$

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- \bullet For example, consider the $\mbox{\bf big-step}$ evaluation of $\mbox{\bf using D}$...

```
 ((\overrightarrow{(C, w_0)}, (C', w'_0)), c_i) \downarrow ((\overrightarrow{(C, w_1)}, (C', w'_1)), \text{ return } w''_0) 
((\overrightarrow{(C, w_1)}, (C', w'_1), (D, w''_0)), c) \downarrow (((\overrightarrow{(C, w_2)}, (C', w'_2), (D, w''_1)), \text{ return } v) 
(((\overrightarrow{(C, w_2)}, (C', w'_2)), c_f[v/x, w''_1/w]) \downarrow (((\overrightarrow{(C, w_3)}, (C', w'_3)), \text{ return } v')
```

$$((\overrightarrow{(C,w_0)},(C',w_0')) , \textbf{ using } D @ c_i \textbf{ cohandle } c \textbf{ finally } x @ w \rightarrow c_f)$$

$$\qquad \qquad \qquad \downarrow$$

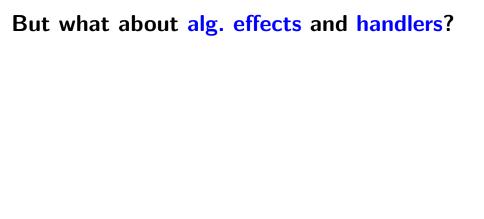
$$((\overrightarrow{(C,w_3)},(C',w_3')) , \textbf{ return } v')$$

- Regarding op. semantics, e.g., consider confs. $(\overrightarrow{(C,w)}, c)$
- \bullet For example, consider the $\mbox{\bf big-step}$ evaluation of $\mbox{\bf using D}$...

```
 ((\overrightarrow{(C, w_0)}, (C', w'_0)), c_i) \downarrow ((\overrightarrow{(C, w_1)}, (C', w'_1)), \text{ return } w''_0) 
((\overrightarrow{(C, w_1)}, (C', w'_1), (D, w''_0)), c) \downarrow ((\overrightarrow{(C, w_2)}, (C', w'_2), (D, w''_1)), \text{ return } v) 
((\overrightarrow{(C, w_2)}, (C', w'_2)), c_f[v/x, w''_1/w]) \downarrow ((\overrightarrow{(C, w_3)}, (C', w'_3)), \text{ return } v')
```

```
\left(\begin{array}{c} ((\overrightarrow{(\mathsf{C},w_0)},(\mathsf{C}',w_0')) \;,\; \mathbf{using}\; \mathsf{D}\; @\; c_i\; \mathbf{cohandle}\; c\; \mathbf{finally}\; x\; @\; w \to c_f\;\right) \\ & \qquad \qquad \qquad \qquad \downarrow \\ & \qquad \qquad \left(\begin{array}{c} ((\overrightarrow{(\mathsf{C},w_3)},(\mathsf{C}',w_3')) \;,\; \mathbf{return}\; v'\;\right) \end{array}\right)
```

The interpretation of operations uses the co-operations of Cs
 In fact, is parametric in the semantics of (outer) co-operations



• At least **two** (orthogonal) research directions (**ongoing work**)

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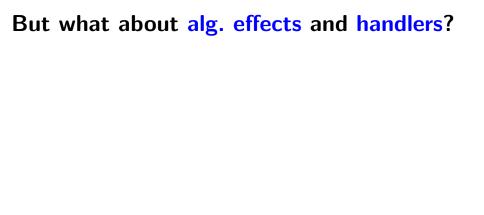
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```

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- Algebraic operations (in the sense of EFF) allowed in c,
 but they must not be allowed to escape cohandle (for linearity)
- To escape, have to use the co-operations of the external world
- The continuations of handlers in c are delimited by cohandle
- How do multi-handlers fit here? Interacting handlers-cohandlers?



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- Important: finally does not (!) jump back into cohandle
- Algebraic operations only allowed to appear in co-operations

Conclusions

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- We're making them into a **modular programming abstraction**
- Linearity by leaving outer worlds implicit (via comodel ops.)
- \bullet System.IO , Koka's $% \left(1\right) =\left(1\right) =\left(1\right) \left(1\right) =\left(1\right) \left(1\right) =\left(1\right) \left(1\right) \left(1\right) \left(1\right) =\left(1\right) \left(1\right$

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- \bullet System.IO , Koka's initially & finally , Python's with , \dots

Ongoing and future work

- Work out all the formal details of what I have shown you today
- Algebraic effects and (multi-)handlers
- More examples and use cases
- Clarify the connection with (effectful) lenses
- Combinatorics of comodels and their lens-like relationships