Comodels as a gateway for interacting with the external world

Danel Ahman

(joint work with Andrej Bauer)

Shonan, 27 March 2019

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Computational effects in FP

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- Using **monads** (e.g., as in HASKELL)

```
type St a = String → (a,String)

f :: St a → St (a,a)
f c = c >>= (\x → c >>= (\y → return (x,y)))
```

- Using **algebraic effects** and **handlers** (e.g., as in EFF)

```
effect Get : int
effect Put : int → unit
                                     (*: int → a*int!{} *)
let g (c:unit → a!{Get,Put}) =
  with st_h handle (perform (Put 42); c ())
```

Computational effects in FP

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  with st_h handle (perform (Put 42); c ())
```

- Both are good for **faking comp. effects** in a pure language!

  But what about effects that need access to the **external world**?

External world in FP

- Declare a **signature** of monads or algebraic effects

```
type IO a

openFile  :: FilePath → IOMode → IO Handle
hGetLine  :: Handle → IO String
hClose    :: Handle → IO ()
```

```
effect Read  : string
effect Raise : string → empty

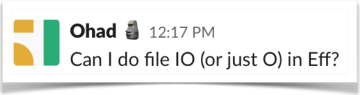
effect RandomInt   : int → int
effect RandomFloat : float → float
```

- And then treat them **specially** in the compiler, e.g.,

```
let rec top_handle op =
  match op with
  | ...
```

External world in FP

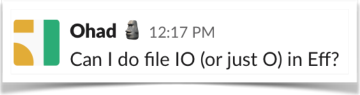
External world in FP



Ohad 🗿 12:17 PM

Can I do file IO (or just O) in Eff?

External world in FP



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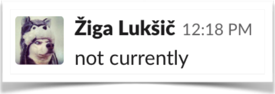
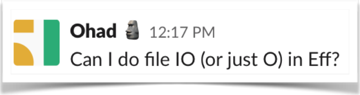


Žiga Lukšič 12:18 PM

not currently

External world in FP



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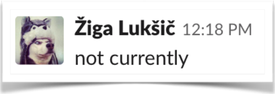

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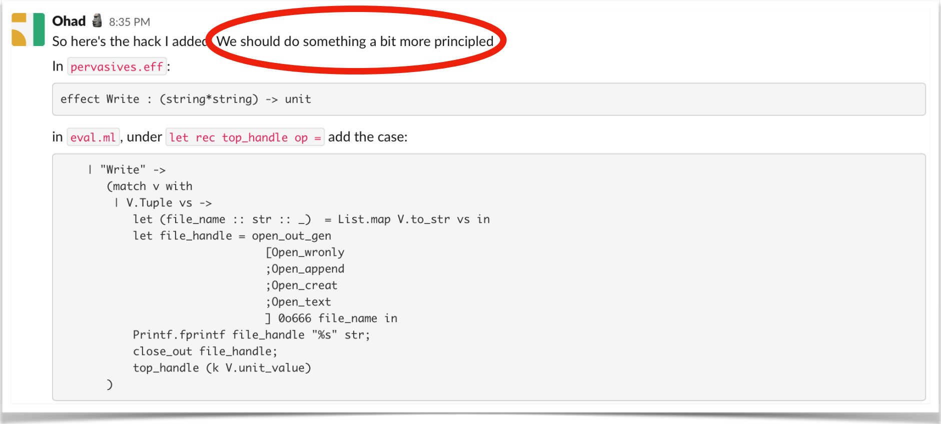

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So here's the hack I added. We should do something a bit more principled

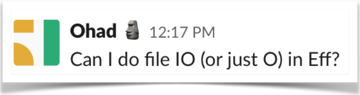
In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
    | "Write" ->
      (match v with
       | V.Tuple vs ->
         let (file_name :: str :: _)  = List.map V.to_str vs in
         let file_handle = open_out_gen
                             [Open_wronly
                             ;Open_append
                             ;Open_creat
                             ;Open_text
                             ] 0o666 file_name in
         Printf.fprintf file_handle "%s" str;
         close_out file_handle;
         top_handle (k V.unit_value)
      )
```

External world in FP



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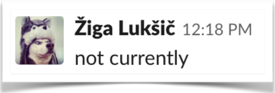

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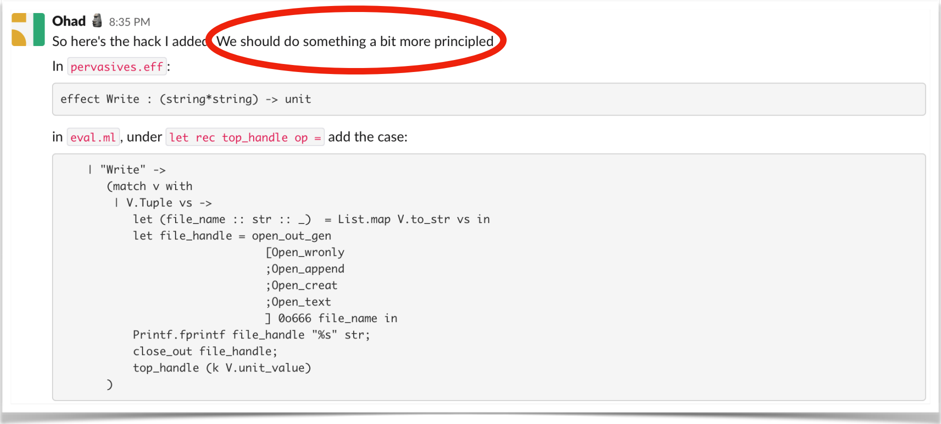

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This talk — a principled (co)algebraic approach!

Another issue — linearity or lack thereof

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-
  ```
  let f (s:string) =
    let fh = fopen ”foo.txt” in
    fwrite fh (s^s);
    fclose fh;
    return fh

  let g s =
    let fh = f s in fread fh
  ```

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- But what if we wrap `f` in a **handler**?

  ```
  let h = handler
          | effect (FWrite fh s k) → return fh

  let g s = with h handle f ()
  ```

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- But what if we wrap `f` in a **handler**?

  ```
  let h = handler
          | effect (FWrite fh s k) → return fh

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So, how could we solve these issues?

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- We could try using **existing PL techniques**, e.g.,
  - **Modules** and **abstraction**, e.g., `System.IO`

    ```
    type IO a

    hClose :: Handle → IO ()
    ```

  - **Linear** (and **non-linear**) **types** and **effects**

    ```
     linear type fhandle

     effect FClose : (linear fhandle) → unit

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  - Handlers with **`finally`** **clauses**

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  - Handlers with `finally` **clauses**
- **Problem:** They don't really capture the **essence of the problem**

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- A common explanation is to think of functions

  $$\mathsf{a} \rightarrow \mathsf{IO\ b}$$

  as

  $$\mathsf{a} \rightarrow (\mathsf{RealWorld} \rightarrow (\mathsf{b}, \mathsf{RealWorld}))$$

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- With the `System.IO` **module abstraction** ensuring that
  - We **cannot get our hands on** RealWorld
  - We have the impression of RealWorld **used linearly**
  - We **don't ask more** from RealWorld than it can provide

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But wait a minute! RealWorld looks a lot like a **comodel**!

$$\mathsf{hGetLine} : (\mathsf{Handle}, \mathsf{RealWorld}) \to (\mathsf{String}, \mathsf{RealWorld})$$

$$\mathsf{hClose} \quad : (\mathsf{Handle}, \mathsf{RealWorld}) \to ((), \mathsf{RealWorld})$$

I.e., **IO** is about the **external world** rather than internal effects!

Important: co-operations (hClose) make a **promise to return**!

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- Intutively, comodels describe **evolution of the world** W
  - Operational semantics using a tensor of a model and a comodel (Plotkin & Power, Abou-Saleh & Pattinson)
  - Stateful runners of effectful programs (Uustalu)
  - Linear state-passing translation (Møgelberg and Staton)
  - Top-level behaviour of alg. effects in Eff v2 (Bauer & Pretnar)

Comodels as a gateway to the external world

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  ```
  let f (s:string) =
    using IO cohandle
      let fh = fopen "foo.txt" in
      fwrite fh (s^s);
      fclose fh                                    (* in IO *)
  ```

  Now **external world** explicit, but **dangling** `fh` etc **still possible**

Comodels as a gateway to the external world

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- **Solution: Modular treatment** of **external worlds**

Comodels as a gateway to the external world

- Examples of **modularity** we might want from comodels

$$\begin{tikzcd}
 & \mathrm{IO} \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] & & \text{(ext. world)} \\
\mathrm{Fh} & & \dots & \text{(inner world)}
\end{tikzcd}$$

- Fh — “**world** which consists of **exactly one** `fh`”
- IO $\longrightarrow$ Fh — “call `fopen` with `foo.txt`, store returned `fh`”
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Comodels as a gateway to the **external world**

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$$\begin{tikzcd}
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Comodels as a gateway to the external world

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- Fh $\longrightarrow$ IO — “call `fclose` with stored `fh`”

- Str — “world that is **blissfully unaware** of `fh`”

- **Observation:** IO $\longleftrightarrow$ Fh and other $\longleftrightarrow$ look a lot like **lenses**

Comodels as a gateway to the external world

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- Our **general framework** on the file operations example

```
let f (s:string) =
  using
    Fh @ (fopen_of_io "foo.txt")
  cohandle
    fwrite_of_fh (s^s)
  finally
    x @ fh → fclose_of_io fh
```

Comodels as a gateway to the external world

- Our **general framework** on the file operations example

```
let f (s:string) =                          (* in IO *)
  using
    Fh @ (fopen_of_io ”foo.txt”)            (* in IO *)
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    fwrite_of_fh (s^s)                        (* in Fh *)
  finally
    x @ fh → fclose_of_io fh                  (* in IO *)
```

where

```
Fh =                                  (* W = fhandle *)
 { co_fread  _ @ fh → ...,
   co_fwrite s @ fh → fwrite_of_io s fh;
                      return ((),fh)      }

     (* co_fread  : (unit * W) → (string * W) *)
    (* co_fwrite : (string * W) → (unit * W) *)
```

Comodels as a gateway to the external world

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- The **modularity aspect** of our general framework

```
let f (s:string) =                          (* in IO *)
  using Fh @ (fopen_of_io "foo.txt")
  cohandle

    using Str @ (fread_of_fh ())            (* in Fh *)
    cohandle
      write_of_str (s^s)                    (* in Str *)
    finally
      _ @ s → fwrite_of_fh s

  finally
    _ @ fh → fclose_of_io fh
```

where

```
Str = { co_write s @ s' →                   (* W = string *)
             return ((),s'^s) }
```

Comodels as a gateway to the external world

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- Comodels can also **extend** the (intermediate) external world

```
let f (s:string) =                          (* in IO *)
  using Stats @ (fopen_of_io "foo.txt")
  cohandle
    fwrite_of_stats (s^s)
  finally
    _ @ (fh,c) →
          let fh' = fopen_of_io "stats.txt" in
          fwrite_of_io fh' c; fclose_of_io fh';
          fclose_of_io fh
```

where

```
Stats =                            (* W = fhandle * nat*)
  { co_fread  _ @ (fh,c) → ...,
    co_fwrite s @ (fh,c) → ...,
    co_reset  _ @ (fh,c) → return ((),(fh,0)) }
```

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 let f (s:string) =                                   (* in IO *)
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   cohandle
     fwrite_of_stats (s^s)
   finally
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where

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  { co_fread  _ @ (fh,c) → ...,
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```

- Can also track **nondet./prob. choice results**, etc

Comodels as a gateway to the external world

Comodels as a gateway to the external world

- The external world could also be **pure**

```
 let f (s:string) =                          (* in Pure *)
   using Str @ (return ”default value”)
   cohandle
     ...
     let s = read_of_str () in
     if (s == ”foo”)
     then (...; write_of_str ”bar”; ...)
     else (...)
   finally
     x @ s → return x
```

where

```
Str =                                   (* W = string *)
  { co_read   _ @ s  → return (s,s)   ,
    co_write s @ s' → return ((),s') }
```

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$$A, B, W \;\; ::= \;\; \mathsf{b} \mid 1 \mid A \times B \mid 0 \mid A + B \mid A \xrightarrow{\Sigma} B$$

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$$A, B, W \ ::= \ \mathsf{b} \mid 1 \mid A \times B \mid 0 \mid A + B \mid A \xrightarrow{\Sigma} B$$

- Signatures of **worlds**

$$\boldsymbol{\omega} \ ::= \ \{ \ \mathsf{op}_1 : A_1 \rightsquigarrow B_1 \ , \ \ldots \ , \ \mathsf{op}_n : A_n \rightsquigarrow B_n \ \}$$

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- **Computation terms** (value terms are unsurprising)

$$\begin{array}{rcll} c & ::= & \mathbf{return}\ v \mid \mathbf{let}\ x = c_1\ \mathbf{in}\ c_2 \mid v_1 v_2 & \\ & \mid & \widehat{\mathsf{op}}\ v & \text{(comodel op.)} \\ & \mid & \mathbf{using}\ C \mathbin{@} c_i\ \mathbf{cohandle}\ c\ \mathbf{finally}\ x \mathbin{@} w \to c_f & \end{array}$$

So what's happening more formally?

- Core calculus for cohandlers (wo/ handlers ⇒ wait few slides)
- **Types**

$$A, B, W \ ::= \ \mathsf{b} \mid 1 \mid A \times B \mid 0 \mid A + B \mid A \xrightarrow{\Sigma} B$$

- Signatures of **worlds**

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- **Comodels** (**cohandlers**)

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and

$$\frac{\mathrm{op} : A \leadsto B \in \omega \qquad \Gamma \vdash v : A}{\Gamma \vDash^{\omega} \widehat{\mathrm{op}}\ v : B}$$

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- The interesting part is the interpretation of **cohandling**

$$\Gamma \overset{\omega}{\vDash} \mathbf{using}\ \mathrm{D} \mathbin{@} c_i\ \mathbf{cohandle}\ c\ \mathbf{finally}\ x \mathbin{@} w \to c_f : B$$

which is based on the **linear state-passing translation**, i.e.,

$$\frac{\llbracket \mathrm{D} \rrbracket \in \mathrm{Comod}_{\omega'}(\mathrm{Kleisli}(T_{\omega}))}{\mathrm{cohandle\_with}_{\llbracket \mathrm{D} \rrbracket} : T_{\omega'}\, \llbracket A \rrbracket \longrightarrow \Big(\llbracket W_{\mathrm{D}} \rrbracket \to T_{\omega}\, (\llbracket A \rrbracket \times \llbracket W_{\mathrm{D}} \rrbracket) \Big)}$$

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- The interpretation of **operations** uses the **co-operations** of Cs

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  it is natural to want that

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- Where do **multi-handlers** fit? Co-operating handlers-cohandlers?

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```
using
  C @ c_i
cohandle
  fwrite_of_d s;          (* co_fwrite throws e *)
  fread ()
finally
  | x @ w → c_f
  | throw e → c_do_some_cleanup
  | op x k → ...
```

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- System.IO, KOKA's **initially** & **finally**, PYTHON's **with**, . . .

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- `System.IO`, Koka's **initially** & **finally**, Python's **with**, ...

Ongoing work

- **Algebraic effects** and **(multi-)handlers**
- More **examples** and **use cases**
- Clarify the connection with **(effectful) lenses**
- **Combinatorics** of comodels and their lens-like relationships