Recalling a Witness

Foundations and Applications of Monotonic State

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Monotonicity is really useful!

Its essence can be captured very neatly!

Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
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Monotonicity in program verification

- Consider a program operating on **set-valued state**

$$\texttt{insert v; complex\_procedure(); } \texttt{assert } (\texttt{v} \in \texttt{get()})$$

- To prove the assertion (say, in a Floyd-Hoare style logic), we could prove that the code maintains a **stateful invariant**

$$\{\lambda\, \texttt{s}\, .\, \texttt{v} \in \texttt{s}\}\ \texttt{complex\_procedure()}\ \{\lambda\, \texttt{s}\, .\, \texttt{v} \in \texttt{s}\}$$

  - likely that we have to **carry** $\lambda\, \texttt{s}\, .\, \texttt{v} \in \texttt{s}$ **through** the proof of c_p
    - **does not guarantee** that $\lambda\, \texttt{s}\, .\, \texttt{v} \in \texttt{s}$ holds at every point in c_p
    - **sensitive** to proving that c_p maintains $\lambda\, \texttt{s}\, .\, \texttt{w} \in \texttt{s}$ for some other w

- However, if c_p **never removes**, then $\lambda\, \texttt{s}\, .\, \texttt{v} \in \texttt{s}$ is **stable**, and we would like the program logic to give us $\texttt{v} \in \texttt{get()}$ "**for free**"

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Monotonicity in programming

- **Programming** also relies on **monotonicity**,

  even if you don't realise it!

- Consider ML-style typed **references** `r:ref a`
  - `r` is a **proof of existence** of an `a`-typed value in the heap

- Correctness relies on **monotonicity**!
  1) Allocation **stores** an `a`-typed value in the heap
  2) Writes **don't change type** and there is **no deallocation**
  3) So, given a ref. `r`, it is **guaranteed to point** to an `a`-typed value

- Baked into the memory models of most languages
- We derive them from **global state + general monotonicity**

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- In this talk
  - our **motivating example** and **monotonic counters**
  - **typed references** (`ref t`) and **untyped references** (`uref`)
  - more flexibility with **monotonic references** (`mref t rel`)
- More in the paper
  - temporarily **violating monotonicity** via snapshots
  - two substantial case studies
    - a **secure file-transfer** application
    - Ariadne **state continuity** protocol [Strackx, Piessens 2016]
  - pointers to other works in F* relying on monotonicity for
    - sophisticated **region-based memory models** [`fstar-lang.org`]
    - **crypto** and **TLS verification** [`project-everest.github.io`]

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Key ideas behind our general framework

- We focus on **monotonic programs** and **stable predicates**
  - per verification task, we **choose a preorder** `rel` on states
    - set inclusion, heap inclusion, increasing counter values, ...
  - a stateful program e is **monotonic** (wrt. `rel`) when
    $$\forall\, s\, e'\, s'.\ (e, s) \rightsquigarrow^* (e', s') \implies \mathtt{rel}\ s\ s'$$
  - a stateful predicate `p` is **stable** (wrt. `rel`) when
    $$\forall\, s\, s'.\ \mathtt{p}\ s\ \wedge\ \mathtt{rel}\ s\ s' \implies \mathtt{p}\ s'$$
- **Our solution:** extend Hoare-style program logics (e.g., F*) with
  - a means to **witness** the validity of `p` s in some state s
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Recap: Ordinary global state in F*

- F* is an ML-like dependently typed language, aimed at verification
- F* supports Hoare-style reasoning about state via the **comp. type**

$$\mathtt{ST}_{\mathtt{state}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where

$$\mathtt{pre} : \mathtt{state} \to \mathrm{Type} \qquad \mathtt{post} : \mathtt{state} \to \mathtt{t} \to \mathtt{state} \to \mathrm{Type}$$

- ST is an abstract pre-postcondition refinement of

$$\mathtt{st}\ \mathtt{t} \stackrel{\text{def}}{=} \mathtt{state} \to \mathtt{t} * \mathtt{state}$$

- The global state **actions** have types

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New: Monotonic global state in F*

- We capture monotonic state with a new **computational type**

$$\mathtt{MST}_{\mathtt{state},\mathtt{rel}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where pre and post are typed as in ST

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- To ensure **monotonicity**, the **put** action gets a precondition

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$$\mathtt{mst}\ \mathtt{t} \stackrel{\mathrm{def}}{=} \mathtt{s}_0{:}\mathtt{state} \to \mathtt{t} * \mathtt{s}_1{:}\mathtt{state}\{\mathtt{rel}\ \mathtt{s}_0\ \mathtt{s}_1\}$$

New: Monotonic global state in F*

- We capture monotonic state with a new **computational type**

$$\mathtt{MST}_{\mathtt{state},\mathtt{rel}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where pre and post are typed as in ST

- The **get** action is typed as in ST

$$\mathtt{get} : \mathtt{unit} \to \mathtt{MST}\ \mathtt{state}\ (\mathtt{requires}\ (\lambda\ \_.\top))\ (\mathtt{ensures}\ (\lambda\ \mathtt{s}_0\ \mathtt{s}\ \mathtt{s}_1\ .\ \mathtt{s}_0 = \mathtt{s} = \mathtt{s}_1))$$

- To ensure **monotonicity**, the **put** action gets a precondition

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New: Recalling a Witness

- We introduce a **logical capability** (a **modality** in ongoing work)

$$\mathtt{witnessed} : (\mathtt{state} \to \mathrm{Type}) \to \mathrm{Type}$$

  together with a **weakening principle** (**functoriality**)

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Outline

- Monotonic state by example
- Key ideas behind our general framework
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

The motivating example revisited

- Recall the program operating on the **set-valued state**

  $$\texttt{insert v};\ \texttt{complex\_procedure}();\ \texttt{assert}\ (\texttt{v} \in \texttt{get}())$$

  - We pick **set inclusion** $\subseteq$ as our preorder `rel` on states

  - We **prove the assertion** by inserting a `witness` and `recall`

$$\texttt{insert v};\ \texttt{witness}\ (\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s});\ \texttt{c\_p}();\ \texttt{recall}\ (\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s});\ \texttt{assert}\ (\texttt{v} \in \texttt{get}())$$

  - For **any other** `w`, wrapping

    $$\texttt{insert w};\ [\ \ ];\ \texttt{assert}\ (\texttt{w} \in \texttt{get}())$$

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ML-style typed references (local state)

- First, we define a type of **heaps** as a finite map

$\mathtt{type\ heap} =$

$\quad | \ \mathtt{H} : \mathtt{h}{:}(\mathbb{N} \to \mathtt{cell}) \to \mathtt{ctr}{:}\mathbb{N}\{\forall\, \mathtt{n} \,.\, \mathtt{ctr} \leq \mathtt{n} \implies \mathtt{h\ n} = \mathtt{Unused}\} \to \mathtt{heap}$

where

$\mathtt{type\ cell} =$

$\quad | \ \mathtt{Unused} : \mathtt{cell}$

$\quad | \ \mathtt{Used} : \mathtt{a}{:}\mathrm{Type} \to \mathtt{v}{:}\mathtt{a} \to \mathtt{cell}$

- Next, we define a **preorder** on heaps (**heap inclusion**)

$\mathtt{let\ heap\_inclusion}\ (\mathtt{H}\ \mathtt{h}_0\ \_)\ (\mathtt{H}\ \mathtt{h}_1\ \_) = \forall\, \mathtt{id} \,.\, \mathtt{match}\ \mathtt{h}_0\ \mathtt{id} \,,\, \mathtt{h}_1\ \mathtt{id}\ \mathtt{with}$

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- As a result, we can define new **local state effect**

$$\mathtt{MLST}\ \mathtt{t}\ \mathtt{pre}\ \mathtt{post} \stackrel{\text{def}}{=} \mathtt{MST}_{\mathtt{heap},\mathtt{heap\_inclusion}}\ \mathtt{t}\ \mathtt{pre}\ \mathtt{post}$$

- Next, we define the type of **references** using monotonicity

```
abstract type ref a = id:ℕ{witnessed (λh . contains h id a)}
```

where

```
let contains (H h _) id a =
  match h id with
    | Used b _ → a = b
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- Important: `contains` is **stable** wrt. `heap_inclusion`

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- Finally, we define `MLST`'s **actions** using `MST`'s actions
  - `let alloc (a:Type) (v:a) : MLST (ref a) ... = ...`
    - **get** the current heap
    - **create** a fresh ref., and **add** it to the heap
    - **put** the updated heap back
    - **witness** that the created ref. is in the heap
  - `let read (r:ref a) : MLST t ... = ...`
    - **recall** that the given ref. is in the heap
    - **get** the current heap
    - **select** the given reference from the heap
  - `let write (r:ref a) (v:a) : MLST unit ... = ...`
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Adding untyped and monotonic references

- **Untyped references** (`uref`) with strong updates
  - Used `heap` cells are extended with **tags**

    where

    ```
    | Used : a:Type → v:a → t:tag → cell

    type tag = Typed : tag | Untyped : tag
    ```

  - `urefs` can be extended to also support **deallocation**

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    ```

  - `mrefs` provide **more flexibility** with ref.-wise monotonicity

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    where

    $$\mathtt{type}\ \mathtt{tag}\ \mathtt{a}\ =\ \mathtt{Typed} : \mathtt{rel{:}preorder}\ \mathtt{a} \to \mathtt{tag}\ \mathtt{a}\ \mid\ \mathtt{Untyped} : \mathtt{tag}\ \mathtt{a}$$

  - `mrefs` provide **more flexibility** with ref.-wise monotonicity

Adding untyped and monotonic references

- **Untyped references** (`uref`) with strong updates
  - Used heap cells are extended with **tags**

    $$| \ \texttt{Used} : \mathrm{a{:}Type} \rightarrow \texttt{v}{:}\mathrm{a} \rightarrow \texttt{t:tag} \rightarrow \texttt{cell}$$

    where

    $$\texttt{type tag} \ = \ \texttt{Typed} : \texttt{tag} \ | \ \texttt{Untyped} : \texttt{tag}$$

  - `urefs` can be extended to also support **deallocation**

- **Monotonic references** (`mref a rel`)
  - Used heap cells are extended with **typed tags**

    $$| \ \texttt{Used} : \mathrm{a{:}Type} \rightarrow \texttt{v}{:}\mathrm{a} \rightarrow \texttt{t:tag} \ \mathrm{a} \rightarrow \texttt{cell}$$

    where

    $$\texttt{type tag} \ \mathrm{a} \ = \ \texttt{Typed} : \texttt{rel:preorder} \ \mathrm{a} \rightarrow \texttt{tag} \ \mathrm{a} \ | \ \texttt{Untyped} : \texttt{tag} \ \mathrm{a}$$

  - `mrefs` provide **more flexibility** with ref.-wise monotonicity

Conclusion

- Monotonicity
  - can be distilled into a **simple** and **general** framework
  - is **useful** for **programming** (refs.) and **verification** (Prj. Everest)

- See the paper for
  - further **examples** and **case studies**
  - **meta-theory** and **correctness results** for `MST`
    - based on an instrumented operational semantics

$$(\mathtt{witness}\ x.\varphi\ ,\ s\ ,\ W) \ \rightsquigarrow\ (\mathtt{return}\ ()\ ,\ s\ ,\ W \cup \{x.\varphi\})$$

    - and cut elimination for the `witnessed`-logic
  - first steps towards **monadic reification** for `MST`
    - useful for extrinsic reasoning, e.g., for relational properties
    - but have to be careful when breaking abstraction

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Thank you!

Interested in doing an F* internship?

Get in touch with the F* team!

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Appendix: `witnessed` as a modality

- Part of **ongoing work** into improving **mon. reification** for `MST`

- `state`-indexed **Kripke-semantics**

$$[\![\texttt{witnessed}\ \texttt{p}]\!](\texttt{s}) \stackrel{\text{def}}{=} \forall\, \texttt{s}'.\, \texttt{rel}\ \texttt{s}\ \texttt{s}' \implies [\![\texttt{p}\ \texttt{s}']\!](\texttt{s})$$

- Allows us to validate **additional properties**, such as

$$\texttt{p} \iff \texttt{witnessed}\ (\texttt{fun}\ \_ \to \texttt{p})$$

$$\texttt{witnessed}\ \texttt{p} \iff \texttt{witnessed}\ (\texttt{fun}\ \_ \to \texttt{witnessed}\ \texttt{p})$$

$$\texttt{witnessed}\ \texttt{p} \wedge \texttt{witnessed}\ \texttt{q} \iff \texttt{witnessed}\ (\texttt{fun}\ \texttt{s} \to \texttt{p}\ \texttt{s} \wedge \texttt{q}\ \texttt{s})$$

$$\ldots$$

Appendix: monotonicity and sep. logic

- E.g., in PCM-based sep. logics, one can reason about monotonic counters using **freely duplicable** (stable) **predicates**

$$\mathsf{MC}(c, i)$$

  describing that counter c is at least i [Jensen, Birkedal'12]

- To also reason about **precise counter values**, we need a more sophisticated encoding also involving **exclusively owned preds.**

- Instead, we stayed within (non-sep.) Hoare logics because
  - we wanted to focus on the **essence of monotonicity**
  - it **scales well** due to lending itself to SMT-based automation