Recalling a Witness

Foundations and Applications of Monotonic State

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Outline

- Monotonic state by example
- Key ideas behind our approach
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

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Monotonic state and program verification

- Consider a program operating on **set-valued state**

$$\texttt{insert v; complex\_procedure(); assert } (\texttt{v} \in \texttt{get()})$$

- To prove the assertion (say, in a Floyd-Hoare style logic), we could prove that the code maintains a **stateful invariant**

$$\{\lambda\, \texttt{s} \,.\, \texttt{v} \in \texttt{s}\}\ \texttt{complex\_procedure()}\ \{\lambda\, \texttt{s} \,.\, \texttt{v} \in \texttt{s}\}$$

  - likely that we have to **carry** $\lambda\, \texttt{s} \,.\, \texttt{v} \in \texttt{s}$ **through** the proof of c_p
    - **does not guarantee** that $\lambda\, \texttt{s} \,.\, \texttt{v} \in \texttt{s}$ holds at every point in c_p
    - **sensitive** to proving that c_p maintains $\lambda\, \texttt{s} \,.\, \texttt{w} \in \texttt{s}$ for some other w

- However, if c_p **does not remove**, then $\lambda\, \texttt{s} \,.\, \texttt{v} \in \texttt{s}$ is **stable**, and we would like the program logic to give us $\texttt{v} \in \texttt{get()}$ "**for free**"

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Monotonicity is really useful!

- In this talk
  - **motivating example** and **monotonic counters**
  - both **typed** (`ref t`) and **untyped references** (`uref`)
  - more flexibility with **monotonic references** (`mref t rel`)
- More in the paper
  - temporarily performing **non-monotonic updates** via snapshots
  - two substantial case studies
    - a **secure file-transfer** application
    - Ariadne **state continuity** protocol [Strackx, Piessens 2016]
  - pointers to other works in F* relying on monotonicity for
    - sophisticated **region-based memory models** [`fstar-lang.org`]
    - **crypto** and **TLS verification** [`project-everest.github.io`]

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Key ideas behind our approach

- We focus on **monotonic programs** and **stable predicates**
  - per verification task, we **choose a preorder** rel on states
    - set inclusion, heap inclusion, increasing counter values, ...
  - a stateful program e is **monotonic** (wrt. rel) when

$$\forall\, \mathtt{s}\ \mathtt{e}'\ \mathtt{s}'.\ (\mathtt{e}, \mathtt{s}) \rightsquigarrow^* (\mathtt{e}', \mathtt{s}') \implies \mathtt{rel}\ \mathtt{s}\ \mathtt{s}'$$

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- **Our solution:** extend Hoare-style program logics (e.g., F*) with
  - a means for turning a p into a **state-independent proposition**
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Recap: Ordinary global state in F*

- F* is an ML-like dependently typed language, aimed at verification
- F* supports Hoare-style reasoning about state via the **comp. type**

  $$\mathtt{ST_{state}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where

  $$\mathtt{pre} : \mathtt{state} \to \mathtt{Type} \qquad \mathtt{post} : \mathtt{state} \to \mathtt{t} \to \mathtt{state} \to \mathtt{Type}$$

- ST is an **abstract** pre-postcondition refinement of

  $$\mathtt{st}\ \mathtt{t} \stackrel{\text{def}}{=} \mathtt{state} \to \mathtt{t} * \mathtt{state}$$

- The global state **actions** have types

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New: Monotonic global state in F*

- We capture monotonic state with a new **computation type**

$$\mathtt{MST}_{\mathtt{state},\mathtt{rel}}\ \mathtt{t}\ (\mathtt{requires}\ \mathtt{pre})\ (\mathtt{ensures}\ \mathtt{post})$$

  where pre and post are typed as in ST

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$$\begin{aligned}\mathtt{put} : \mathtt{s}{:}\mathtt{state} \to \mathtt{MST}\ \mathtt{unit}\ &(\mathtt{requires}\ (\lambda\, \mathtt{s}_0\, .\, \mathtt{rel}\ \mathtt{s}_0\ \mathtt{s}))\\ &(\mathtt{ensures}\ (\lambda\, \_\, \_\, \mathtt{s}_1\, .\, \mathtt{s}_1 = \mathtt{s}))\end{aligned}$$

- So intuitively, MST is an **abstract** pre-postcondition refinement of

$$\mathtt{mst}\ \mathtt{t} \stackrel{\text{def}}{=} \mathtt{s}_0{:}\mathtt{state} \to \mathtt{t} * \mathtt{s}_1{:}\mathtt{state}\{\mathtt{rel}\ \mathtt{s}_0\ \mathtt{s}_1\}$$

New: Recalling a Witness

- We introduce a **logical capability** (**modality**)

$$\texttt{witnessed} : (\texttt{state} \to \texttt{Type}) \to \texttt{Type}$$

  together with a **weakening principle** (**functoriality**)

$$\texttt{wk} : \mathrm{p,q}{:}(\texttt{state} \to \texttt{Type}) \to \texttt{Lemma}\ (\texttt{requires}\ (\forall\, \mathrm{s}\,.\, \mathrm{p\ s} \implies \mathrm{q\ s}))$$
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- We add a **stateful introduction rule** for `witnessed`

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Outline

- Monotonic state by example
- Key ideas behind our approach
- Accommodating monotonic state in F*
- Some examples of monotonic state at work
- More examples of monotonic state at work (see the paper)
- Monadic reification and reflection (see the paper)
- Meta-theory and correctness results (see the paper)

The motivating example revisited

- Recall the program operating on the **set-valued state**

  $$\texttt{insert v};\ \texttt{complex\_procedure}();\ \texttt{assert}\ (\texttt{v} \in \texttt{get}())$$

  - We pick **set inclusion** $\subseteq$ as our preorder `rel` on states
  - We **prove the assertion** by inserting a `witness` and `recall`

  $$\texttt{insert v};\ \texttt{witness}\ (\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s});\ \texttt{c\_p}();\ \texttt{recall}\ (\lambda\,\texttt{s}\,.\,\texttt{v} \in \texttt{s});\ \texttt{assert}\ (\texttt{v} \in \texttt{get}())$$

  - For **any other** `w`, wrapping

  $$\texttt{insert w};\ [\ \ ];\ \texttt{assert}\ (\texttt{w} \in \texttt{get}())$$

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- **Monotonic counters** are analogous, by using $\mathbb{N}$ and $\leq$, e.g.,

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ML-style typed references (local state)

- First, we define a type of **heaps**

  $\texttt{type heap} =$

  $\quad | \; \texttt{H} : \texttt{h}{:}(\mathbb{N} \to \texttt{cell}) \to \texttt{ctr}{:}\mathbb{N}\{\forall\, \texttt{n} \,.\, \texttt{ctr} \le \texttt{n} \implies \texttt{h n} = \texttt{Unused}\} \to \texttt{heap}$

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  $\texttt{type cell} =$

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- As a result, we can define new **local state effect**

$$\mathtt{LST}\ \mathtt{t}\ \mathtt{pre}\ \mathtt{post}\ \stackrel{\mathsf{def}}{=}\ \mathtt{MST}_{\mathtt{heap},\mathtt{heap\_inclusion}}\ \mathtt{t}\ \mathtt{pre}\ \mathtt{post}$$

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$$\mathtt{abstract\ type\ ref\ a} = \mathtt{id}{:}\mathbb{N}\{\mathtt{witnessed}\ (\lambda\,\mathtt{h}\,.\,\mathtt{contains\ h\ id\ a})\}$$

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let contains (H h _) id a =
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- Observe that `contains` is **stable** wrt. `heap_inclusion`

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- Finally, we define `LST`'s **actions** using `MST`'s actions
  - `let alloc` (a:Type) (v:a) : `LST` (`ref a`) ... = ...
    - **get** the current heap
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    - **witness** that the created ref. is in the heap
  - `let read` (r:`ref a`) : `LST t` ... = ...
    - **recall** that the given ref. is in the heap
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    - **select** the given reference from the heap
  - `let write` (r:`ref a`) (v:a) : `LST unit` ... = ...
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Adding untyped and monotonic references

- **Untyped references** (`uref`) with strong updates
  - Used `heap` cells are extended with **tags**

    where

    $$\mid \texttt{Used} : \mathsf{a{:}Type} \rightarrow \mathsf{v{:}a} \rightarrow \mathsf{t{:}tag} \rightarrow \texttt{cell}$$

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  - useful for **programming** (refs.) and **verification** (Prj. Everest)

- See the paper for
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$$(\texttt{witness}\ x.\varphi\,,\ s\,,\ W) \rightsquigarrow (\texttt{return}\ ()\,,\ s\,,\ W \cup \{x.\varphi\})$$

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Thank you!

Questions?