Comodels as a gateway for interacting with the external world

Danel Ahman

(joint work with Andrej Bauer)

Shonan, 28 March 2019

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Computational effects in FP

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- Using **monads** (as in Haskell)

```
type St a = String → (a,String)

f :: St a → St (a,a)
f c = c >>= (\x → c >>= (\y → return (x,y)))
```

- Using **alg. effects** and **handlers** (as in Eff, Frank, Koka)

```
effect Get : int
effect Put : int → unit
                                          (*: int → a*int!{} *)
let g (c:unit → a!{Get,Put}) =
  with st_h handle (perform (Put 42); c ())
```

Computational effects in FP

- Using **monads** (as in Haskell)

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type St a = String → (a, String)

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```

- Both are good for **faking comp. effects** in a pure language!

  But what about effects that need access to the **external world**?

External world in FP

- Declare a **signature** of monads or algebraic effects

```
type IO a

openFile  :: FilePath → IOMode → IO Handle
hGetLine  :: Handle → IO String
hClose    :: Handle → IO ()
```

```
effect Read  : string
effect Raise : string → empty

effect RandomInt   : int → int
effect RandomFloat : float → float
```

- And then treat them **specially** in the compiler, e.g.,

```
let rec top_handle op =
  match op with
  | ...
```

External world in FP

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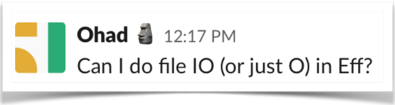

External world in FP

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So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
    | "Write" ->
      (match v with
       | V.Tuple vs ->
         let (file_name :: str :: _)  = List.map V.to_str vs in
         let file_handle = open_out_gen
                             [Open_wronly
                             ;Open_append
                             ;Open_creat
                             ;Open_text
                             ] 0o666 file_name in
         Printf.fprintf file_handle "%s" str;
         close_out file_handle;
         top_handle (k V.unit_value)
      )
```

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This talk — a principled (co)algebraic approach!

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- ```
  let f (s:string) =
    let fh = fopen "foo.txt" in
    fwrite fh (s^s);
    fclose fh;
    return fh

  let g s =
    let fh = f s in fread fh
  ```
  ```

Another issue — linearity or lack thereof

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- Even worse when we wrap `f` in a **handler**?

  ```
  let h = handler
            | effect (FWrite fh s k) → return ()

  let g' s =
    with h handle f ()
  ```
  ```

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- Even worse when we wrap `f` in a **handler**?

  ```
  let h = handler
          | effect (FWrite fh s k) → return ()

  let g' s =
    with h handle f ()          (* dangling fh ! *)
  ```
  ```

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- We could try using **existing PL techniques**, e.g.,
  - **Modules** and **abstraction**, e.g., `System.IO`

    ```
    type IO a

    hClose :: Handle → IO ()
    ```

  - **Linear** (and **non-linear**) **types** and **effects**

    ```
    linear type fhandle

    effect FClose : (linear fhandle) → unit

    linear effect FClose : fhandle → unit
    ```

  - Handlers with `finally` **clauses**

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```
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  - Handlers with `finally` **clauses**
- **Problem:** They don't really capture the **essence of the problem**

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- A common explanation is to think of functions

$$\mathsf{a} \to \mathsf{IO\ b}$$

as

$$\mathsf{a} \to (\mathsf{RealWorld} \to (\mathsf{b}, \mathsf{RealWorld}))$$

which is the same as

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- With the `System.IO` **module abstraction** ensuring that
  - We **cannot get our hands on** RealWorld
  - We have the impression of RealWorld **used linearly**
  - We **don't ask more** from RealWorld than it can provide

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But wait a minute! RealWorld looks a lot like a **comodel**!

$$\mathsf{hGetLine} : (\mathsf{Handle}, \mathsf{RealWorld}) \to (\mathsf{String}, \mathsf{RealWorld})$$

$$\mathsf{hClose} \quad : (\mathsf{Handle}, \mathsf{RealWorld}) \to ((), \mathsf{RealWorld})$$

Important: co-operations (hClose) make a **promise to return**!

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$$\mathcal{M} = \langle\ M : \mathsf{Set}\ ,\ \{\mathsf{op}_{\mathcal{M}} : A \times M^B \longrightarrow M\}_{\mathsf{op} \in \Sigma}\ \rangle$$

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- Intutively, comodels describe **evolution of the world** W
  - Operational semantics using a tensor of a model and a comodel (Plotkin & Power, Abou-Saleh & Pattinson)
  - Stateful runners of effectful programs (Uustalu)
  - Linear state-passing translation (Møgelberg and Staton)
  - Top-level behaviour of alg. effects in Eff v2 (Bauer & Pretnar)

Comodels as a gateway to the external world

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- ```
  let f (s:string) =
    using IO cohandle
      let fh = fopen "foo.txt" in
      fwrite fh (s^s);
      fclose fh                              (* in IO *)
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  Now **external world** explicit, but **dangling** `fh` etc **still possible**
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  Better, but **have to explicitly open** and **thread through** `fh`

Comodels as a gateway to the **external world**

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- **Solution: Modular treatment** of **external worlds**

Comodels as a gateway to the **external world**

- Examples of **modularity** we might want from comodels

$$
\begin{tikzcd}
 & \mathsf{IO} & & \text{(ext. world)} \\
\mathsf{Fh} \arrow[ur, <->] & & \ldots \arrow[ul, <->] & \text{(inner world)}
\end{tikzcd}
$$

- Fh — “**world** which consists of **exactly one** `fh`”
- IO $\longrightarrow$ Fh — “call `fopen` with `foo.txt`, store returned `fh`”
- Fh $\longrightarrow$ IO — “call `fclose` with stored `fh`”

Comodels as a gateway to the external world

- Examples of **modularity** we might want from comodels

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\begin{tikzcd}
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- $\mathsf{IO} \longrightarrow \mathsf{Fh}$ — "call `fopen` with `foo.txt`, store returned `fh`"
- $\mathsf{Fh} \longrightarrow \mathsf{IO}$ — "call `fclose` with stored `fh`"
- Str — "world that is **blissfully unaware** of `fh`"
- **Observation:** $\mathsf{IO} \longleftrightarrow \mathsf{Fh}$ and other $\longleftrightarrow$ look a lot like **lenses**

Comodels as a gateway to the external world

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- Our **general framework** on the file operations example

```
let f (s:string) =
  using
    Fh @ (fopen_of_io "foo.txt")
  cohandle
    fwrite_of_fh (s^s)
  finally
    x @ fh → fclose_of_io fh
```

Comodels as a gateway to the external world

- Our **general framework** on the file operations example

```
let f (s:string) =                                   (* in IO *)
  using
    Fh @ (fopen_of_io "foo.txt")                     (* in IO *)
  cohandle
    fwrite_of_fh (s^s)                               (* in Fh *)
  finally
    x @ fh → fclose_of_io fh                         (* in IO *)
```

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```
let f (s:string) =                         (* in IO *)
  using
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    fwrite_of_fh (s^s)                     (* in Fh *)
  finally
    x @ fh → fclose_of_io fh               (* in IO *)
```

where

```
Fh =                                 (* W = fhandle *)
 { co_fread   _ @ fh → ... ,
   co_fwrite s @ fh → fwrite_of_io s fh;
                      return ((),fh)      }

    (* co_fread  : (unit * W) → (string * W) *)
   (* co_fwrite : (string * W) → (unit * W) *)
```

Comodels as a gateway to the external world

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- The **modularity aspect** of our general framework

```
let f (s:string) =                                    (* in IO *)
  using Fh @ (fopen_of_io "foo.txt")
  cohandle

    using Str @ (fread_of_fh ())                      (* in Fh *)
    cohandle
      write_of_str (s^s)                              (* in Str *)
    finally
      _ @ s → fwrite_of_fh s

  finally
    _ @ fh → fclose_of_io fh
```

where

```
Str = { co_write s @ s' →                        (* W = string *)
              return ((),s'^s) }
```

Comodels as a gateway to the external world

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- Comodels can also **extend** the (intermediate) external world

```
let f (s:string) =                                   (* in IO *)
  using Stats @ (fopen_of_io "foo.txt")
  cohandle
    fwrite_of_stats (s^s)
  finally
    _ @ (fh,c) →
          let fh' = fopen_of_io "stats.txt" in
          fwrite_of_io fh' c; fclose_of_io fh';
          fclose_of_io fh
```

where

```
Stats =                                    (* W = fhandle * nat*)
  { co_fread   _ @ (fh,c) → ...,
    co_fwrite  s @ (fh,c) → ...,
    co_reset   _ @ (fh,c) → return ((),(fh,0)) }
```

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where

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Stats =                          (* W = fhandle * nat*)
  { co_fread  _ @ (fh,c) → ...,
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```

- Can also track **nondet./prob. choice results**, etc

Comodels as a gateway to the external world

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- The external world could also be **pure**

```
let f (s:string) =                    (* in Pure *)
  using Str @ (return "default value")
  cohandle
    ...
    let s = read_of_str () in
    if (s == "foo")
    then (...; write_of_str "bar"; ...)
    else (...)
  finally
    x @ s → return x
```

where

```
Str =                                 (* W = string *)
  { co_read  _ @ s  → return (s,s)    ,
    co_write s @ s' → return ((),s') }
```

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$$A, B, W \;::=\; \mathsf{b} \mid 1 \mid A \times B \mid 0 \mid A + B \mid A \xrightarrow{\Sigma} B$$

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- Signatures of **worlds**

$$\boldsymbol{\omega} \ ::= \ \{\, \mathsf{op}_1 : A_1 \rightsquigarrow B_1 \ , \ \ldots \ , \ \mathsf{op}_n : A_n \rightsquigarrow B_n \,\}$$

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- **Computation terms** (value terms are unsurprising)

$$\begin{array}{lll} c & ::= & \mathbf{return}\ v \mid \mathbf{let}\ x = c_1\ \mathbf{in}\ c_2 \mid v_1 v_2 \\ & \mid & \widehat{\mathsf{op}}\ v \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(comodel op.)} \\ & \mid & \mathbf{using}\ C\ @\ c_i\ \mathbf{cohandle}\ c\ \mathbf{finally}\ x\ @\ w \to c_f \end{array}$$

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- **Comodels (cohandlers)**

$$C \;::=\; \{\, \overline{\mathsf{op}}_1\ x\ @\ w \to c_1 \,,\; \ldots \,,\; \overline{\mathsf{op}}_n\ x\ @\ w \to c_n \,\}$$

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- **Typing judgements**

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$$\Gamma \vdash v : A \qquad\qquad \Gamma \vDash^{\omega} c : A$$

- The two central **typing rules** are

$$\frac{\Gamma \vDash^{\omega} \mathrm{D} \text{ comodel of } \omega' \text{ with carrier } W_{\mathrm{D}} \qquad \Gamma \vDash^{\omega} c_i : W_{\mathrm{D}} \qquad \Gamma \vDash^{\omega'} c : A \qquad \Gamma, x : A, w : W_{\mathrm{D}} \vDash^{\omega} c_f : B}{\Gamma \vDash^{\omega} \textbf{using } \mathrm{D} \mathbin{@} c_i \textbf{ cohandle } c \textbf{ finally } x \mathbin{@} w \to c_f : B}$$

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and

$$\frac{\mathrm{op} : A \rightsquigarrow B \in \omega \qquad \Gamma \vdash v : A}{\Gamma \vDash^{\omega} \widehat{\mathrm{op}}\ v : B}$$

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- **Denotational semantics** is heavily inspired by Møgelberg and Staton's **linear state-passing translation**

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  - **un-cohandled operations wait for a suitable external world**!

- The interesting part is the interpretation of **cohandling**

$$\Gamma \vDash^{\omega} \textbf{using}\ \mathrm{D} \mathbin{@} c_i\ \textbf{cohandle}\ c\ \textbf{finally}\ x \mathbin{@} w \to c_f : B$$

  which is based on the **linear state-passing translation**, i.e.,

$$\frac{[\![\mathrm{D}]\!] \in \mathrm{Comod}_{\omega'}(\mathrm{Kleisli}(T_{\omega}))}{\mathrm{cohandle\_with}_{[\![\mathrm{D}]\!]} : T_{\omega'}\,[\![A]\!] \longrightarrow \Big([\![W_{\mathrm{D}}]\!] \to T_{\omega}\,([\![A]\!] \times [\![W_{\mathrm{D}}]\!])\Big)}$$

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- The interpretation of **operations** uses the **co-operations** of Cs

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- Where do **multi-handlers** fit? Co-operating handlers-cohandlers?

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```
using
  D @ c_i
cohandle
  fwrite_of_d s;          (* co_fwrite throws e *)
  fread ()
finally
  | x @ w → c_f
  | throw e → c_do_some_cleanup
  | op x k → ...
```

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- `System.IO`, Koka's **initially** & **finally**, Python's **with**, ...
- Convenient for **general FFI**

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Some ongoing work

- Interaction with **algebraic effects** and **(multi-)handlers**
- Clarify the connection with **(effectful) lenses**
- **Combinatorics** of comodels and their lens-like relationships