Runners in action

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Today's plan

- **Computational effects** and **external resources** in PL
- **Issues with standard approaches** to **external resources**
- **Runners** – a natural model for **top-level runtime**
- **T-runners** – for also modelling **non-top-level runtimes**
- Turn **T**-runners into a **useful programming construct**
- Demonstrate the use of runners through **programming examples**

Computational effects

and

external resources

Computational effects in PL

Computational effects in PL

- Using **monads** (as in Haskell)

```
type St a = String → (a,String)
instance St Monad where
  ...

f :: St a → St (a,a)
f c = c >>= (\ x → c >>= (\ y → return (x,y)))
```

- Using **alg. effects** and **handlers** (as in Eff, Frank, Koka)

```
effect Get : unit → int
effect Put : int → unit

let g (c:unit → a!{Get,Put}) : int → a ∗ int ! {} =
  with st_handler handle (perform (Put 42); c ())
```

Computational effects in PL

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```

- Good for **simulating comp. effects** in a pure language!

  But what about effects that need access to the **external world**?

External resources in PL

External resources in PL

- Declare a **signature of monads** or **algebraic effects**, e.g.,

```
(* System.IO *)
type IO a
openFile :: FilePath → IOMode → IO Handle
```

```
(* pervasives.eff *)
effect RandomInt : int → int
effect RandomFloat : float → float
```

- And then **treat them specially** in the compiler, e.g., in Eff

```
(* eff/src/backends/runtime/eval.ml *)
let rec top_handle op =
  match op with
  | Value v → v
  | Call (RandomInt, v, k) →
      top_handle (k (Const.of_integer (Random.int (Value.to_int v))))
  | ...
```

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but there are **some issues** with that approach ...

First issue

- Difficult to cover all possible use cases
  - **external resources hard-coded** into the top-level runtime
  - **non-trivial to change** what's available and how it's implemented

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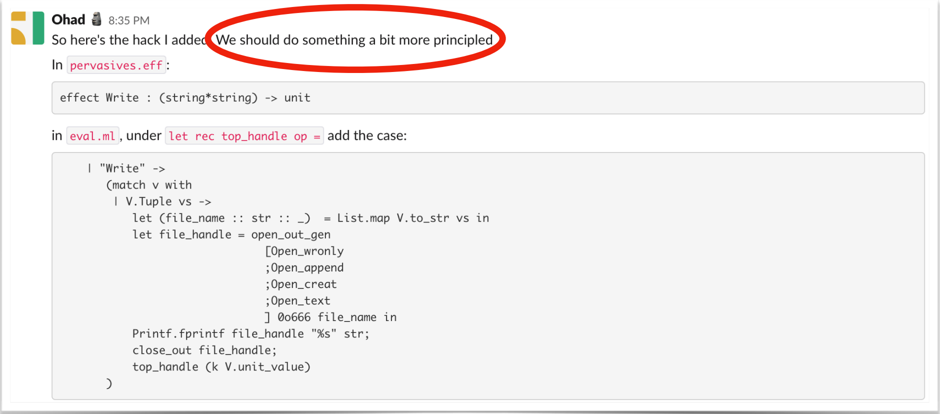

Ohad 8:35 PM

So here's the hack I added. We should do something a bit more principled

In `pervasives.eff`:

```
effect Write : (string*string) -> unit
```

in `eval.ml`, under `let rec top_handle op =` add the case:

```
    | "Write" ->
      (match v with
       | V.Tuple vs ->
          let (file_name :: str :: _)  = List.map V.to_str vs in
          let file_handle = open_out_gen
                              [Open_wronly
                              ;Open_append
                              ;Open_creat
                              ;Open_text
                              ] 0o666 file_name in
          Printf.fprintf file_handle "%s" str;
          close_out file_handle;
          top_handle (k V.unit_value)
      )
```

This talk — a principled modular (co)algebraic approach!

Second issue

- **Lack of linearity** for external resources

```
let f (s:string) =
  let fh = fopen "foo.txt" in
  fwrite (fh,s^s);
  fclose fh;
  return fh

let g s =
  let fh = f s in fread fh                    (* fh not open any more ! *)
```

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  let fh = f s in fread fh                    (* fh not open any more ! *)
```

- We shall address these kinds of issues **indirectly (!)**:
  - by **not** introducing a linear typing discipline
  - instead we make it convenient to **hide external resources**

    (addressing stronger typing disciplines in the future)

Third issue

- **Excessive generality** of effect handlers

```
let f (s:string) =
  let fh = fopen "foo.txt" in
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  fclose fh

let h = handler { fwrite (fh,s) k → return () }
```

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- **Excessive generality** of effect handlers

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  let fh = fopen "foo.txt" in
  fwrite (fh,s^s);
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let h = handler { fwrite (fh,s) k → return () }
```

- But misuse of external resources can also be **purely accidental**

```
let g (s1 s2:string) =
  let fh = fopen "foo.txt" in
  let b = choose () in
  if b then (fwrite (fh,s1^s2)) else (fwrite (fh,s2^s1));
  fclose fh

let nd_handler =
  handler { choose () k → return (k true ++ k false) }
```

Third issue

- **Excessive generality** of effect handlers

```
let f (s:string) =
  let fh = fopen "foo.txt" in
  fwrite (fh,s^s);
  fclose fh

let h = handler { fwrite (fh,s) k → return () }
```

- We shall address these kinds of issues **directly (!!)**,
  - by proposing a **restricted form of handlers** for resources
  - that supports **controlled initialisation** and **finalisation**,
  - (and in the future limit how general handlers can be used)

Runners

A natural model of top-level runtime

A natural model of top-level runtime

- Given a **signature**[1] Σ of operation symbols ($A_{\mathsf{op}}, B_{\mathsf{op}}$ are sets)

$$\mathsf{op} : A_{\mathsf{op}} \rightsquigarrow B_{\mathsf{op}}$$

  a **runner**[2] $\mathcal{R}$ for Σ is given by a carrier $|\mathcal{R}|$ and co-operations

$$\Big(\overline{\mathsf{op}}_{\mathcal{R}} : A_{\mathsf{op}} \times |\mathcal{R}| \longrightarrow B_{\mathsf{op}} \times |\mathcal{R}|\Big)_{\mathsf{op} \in \Sigma}$$

  where think of $|\mathcal{R}|$ as a set of **runtime configurations**

[1] We consider runners for signatures, but the work generalises to alg. theories.
[2] In the literature also known as **comodels** for Σ (or for an algebraic theory).

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  where think of $|\mathcal{R}|$ as a set of **runtime configurations**

- For example, a natural **runner $\mathcal{R}$ for S-valued state** signature

  $$\left\{ \quad \mathsf{get} : \mathbb{1} \rightsquigarrow S \quad , \quad \mathsf{set} : S \rightsquigarrow \mathbb{1} \quad \right\}$$

  is given by

  $$|\mathcal{R}| \stackrel{\text{def}}{=} S \qquad \overline{\mathsf{get}}_{\mathcal{R}}\,(\star, s) \stackrel{\text{def}}{=} (s, s) \qquad \overline{\mathsf{set}}_{\mathcal{R}}\,(s', s) \stackrel{\text{def}}{=} (\star, s')$$

---

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A natural model of top-level runtime ctd.

- Runners/comodels have been used for
  - **operational semantics** using tensors of models and comodels [Plotkin and Power '08]
  - **top-level implementation of algebraic effects** in EFF [Bauer and Pretnar '15]

  and

  - **linear-use state-passing translation** [Møgelberg and Staton '11, '14]
  - **stateful running** of algebraic effects [Uustalu '15]

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- The latter explicitly rely on one-to-one correspondence between
  - **runners** $\mathcal{R}$
  - **monad morphisms**[3] $\mathsf{r} : \mathbf{Free}_{\Sigma}(-) \longrightarrow \mathbf{St}_{|\mathcal{R}|}$

[3] $\mathbf{Free}_{\Sigma}(X)$ is the free monad ind. defined with leaves $\mathsf{val}\, x$ and nodes $\mathsf{op}(a, \kappa)$.

A natural model of **top-level runtime** ctd.

- So, runners $\mathcal{R}$ are a natural model of **top-level runtime**

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- So, runners $\mathcal{R}$ are a natural model of **top-level runtime**
- But what if this runtime is not **the** runtime?
  - hardware vs OSs
  - OSs vs VMs
  - VMs vs sandboxes

  but also

  - browsers vs web pages
  - ...

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- So, runners $\mathcal{R}$ are a natural model of **top-level runtime**
- But what if this runtime is not ****the**** runtime?
  - hardware vs OSs
  - OSs vs VMs
  - VMs vs sandboxes

  but also
  - browsers vs web pages
  - ...
- Unfortunately, runners, as defined above, are **not readily able to**
  - use **external resources**
  - **signal failure** caused by unavoidable circumstances
- But is there a **useful generalisation** that would achieve this?

Effectful runners for modular top-levels

Effectful runners for modular top-levels

- Møgelberg and Staton usefully observed that a **runner** $\mathcal{R}$ is equivalently simply a family of **generic effects** for $\mathbf{St}_{|\mathcal{R}|}$, i.e.,

$$\left(\overline{\mathsf{op}}_{\mathcal{R}} : A_{\mathsf{op}} \longrightarrow \mathbf{St}_{|\mathcal{R}|}\, B_{\mathsf{op}}\right)_{\mathsf{op} \in \Sigma}$$

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- The one-to-one correspondence with **monad morphisms**

$$\mathsf{r} : \mathbf{Free}_{\Sigma}(-) \longrightarrow \mathbf{T}$$

  simply amounts to the **universal property of free models**, i.e.,

$$\mathsf{r}_X\,(\mathsf{val}\, x) = \eta_X\, x \qquad\qquad \mathsf{r}_X\,(\mathsf{op}(a, \kappa)) = \underbrace{(\mathsf{r}_X \circ \kappa)^{\dagger}(\overline{\mathsf{op}}_{\mathcal{R}}\, a)}_{\mathsf{op}_{\mathcal{M}}(a, \mathsf{r}_X \circ \kappa)}$$

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- Observe that κ appears in a **tail call position** on the right!

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- What would be a **useful class of monads** $\mathbf{T}$ to use?

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- We want a runner to be a bit like a **kernel of an OS**, i.e., to
  - **(i)** provide management of **(internal) resources**
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- **Algebraically** (and pragmatically), this amounts to taking
  - **(i)** $\mathsf{getenv} : \mathbb{1} \rightsquigarrow C \quad \& \quad \mathsf{setenv} : C \rightsquigarrow \mathbb{1}$
  - **(ii)** $\mathsf{op} : A_{\mathsf{op}} \rightsquigarrow B_{\mathsf{op}}$ (op $\in \Sigma'$, for some external Σ')
  - **(iii)** $\mathsf{kill} : S \rightsquigarrow \mathbb{0}$

  s.t., **(i)** satisfy state equations; and **(i)** commute with **(ii)** and **(iii)**

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  s.t., **(i)** satisfy state equations; and **(i)** commute with **(ii)** and **(iii)**

- The **induced monad** is then isomorphic to

$$\mathbf{T}\,X \quad \stackrel{\text{def}}{=} \quad C \Rightarrow \mathbf{Free}_{\Sigma'}\big(X \times C + S\big)$$

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- The corresponding **T-runners** $\mathcal{R}$ for Σ are then of the form

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- With this, our **T-runners** $\mathcal{R}$ for Σ are (with "primitive" excs.)

$$\left(\overline{\mathsf{op}}_{\mathcal{R}} : A_{\mathsf{op}} \longrightarrow \mathbf{K}_{\Sigma', E_{\mathsf{op}}, S, C}\, B_{\mathsf{op}}\right)_{\mathsf{op} \in \Sigma}$$

  where we call $\mathbf{K}_{\Sigma,E,S,C}$ a **kernel monad** (the sum of **T** and excs.)

$$\mathbf{K}_{\Sigma', E_{\mathsf{op}}, S, C}\, B_{\mathsf{op}} \quad \stackrel{\text{def}}{=} \quad C \Rightarrow \mathbf{Free}_{\Sigma'}\left(\left(B_{\mathsf{op}} + E_{\mathsf{op}}\right) \times C + S\right)$$

T-runners as a programming construct

(towards a core calculus for runners)

T-runners as a programming construct

- First, we include **T-runners** for Σ

$$\left(\overline{\mathsf{op}}_{\mathcal{R}} : A_{\mathsf{op}} \longrightarrow \mathbf{K}_{\Sigma',E_{\mathsf{op}},S,C}\, B_{\mathsf{op}}\right)_{\mathsf{op}\,\in\,\Sigma}$$

in our language **as values**, and **co-ops. as kernel code**, i.e.,

```
let R = { op₁ x₁ → K₁ , ... , opₙ xₙ → Kₙ }_C
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T-runners as a **programming construct**

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let R = {  op₁ x₁ → K₁  ,  ...  ,  opₙ xₙ → Kₙ  }_C
```

- For instance, we can provide **write-only file access** as

```
let R_FH = {
  write s → if (length s > maxSize)
            then (raise WriteSizeExceeded)
            else  (let fh = getenv () in
                     if (isValid fh) then (fwrite (fh,s)) else (kill IOError))
}_FileHandle
```

  where

$$\Sigma \stackrel{\text{def}}{=} \{\ \mathsf{write} : \mathsf{String} \rightsquigarrow 1\ !\ E \cup \{\mathsf{WriteSizeExceeded}\}\ \}$$

$$\left(\mathsf{fwrite} : \mathsf{FileHandle} \times \mathsf{String} \rightsquigarrow 1\ !\ E\right) \in \Sigma' \qquad S = \{\ \mathsf{IOError}\ \}$$

Controlled initialisation and finalisation

Controlled **initialisation** and **finalisation**

- Recall that the components r_X of the monad morphism

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induced by a $\mathbf{T}$-runner $\mathcal{R}$ are all **tail-recursive**

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induced by a $\mathbf{T}$-runner $\mathcal{R}$ are all **tail-recursive**

- We make use of it to **run user code**:

```
using R @ M_init
run M
finally {return x @ c → M_ret , ... raise e @ c → M_e ... , ... kill s → M_s ...}
```

where

(**user monads**)

- Ms are **user code**, modelled using $\mathbf{U}_{\Sigma,E}\, X \stackrel{\text{def}}{=} \mathbf{Free}_\Sigma(X + E)$

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  - $\mathsf{M}_{\mathsf{init}}$ produces the **initial kernel state**
  - M is the user code being **run using the runner** R
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  - $\mathsf{M}_{\mathsf{ret}}$, M_{e}, M_{s} **finalise** for return values, exceptions, and signals
  - $\mathsf{M}_{\mathsf{ret}}$ and M_{e} **depend on the final state** c, but M_{s} **does not**

Controlled initialisation and finalisation ctd.

- For instance, we can define a Python-esque **with construct**

```
with fileName do M
=
using R_FH @ (fopen fileName)
run M
finally {
  return x @ fh → fclose fh; return x ,
  raise WriteSizeExceeded @ fh → fclose fh; return () ,
  raise e @ fh → fclose fh; raise e , (* other exceptions in E are re-raised *)
  kill IOError → ... }
```

Controlled initialisation and finalisation ctd.

- For instance, we can define a PYTHON-esque **with construct**

```
with fileName do M
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using R_FH @ (fopen fileName)
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  raise WriteSizeExceeded @ fh → fclose fh; return () ,
  raise e @ fh → fclose fh; raise e , (∗ other exceptions in E are re-raised ∗)
  kill IOError → ... }
```

  - the **file handle is hidden** from `M`
  - `M` **can only call** write : String $\rightsquigarrow$ 1 ! $E \cup \{\text{WriteSizeExceeded}\}$ but **not** (the external operations) `fopen`, `fclose`, and `fwrite`
  - `fopen` and `fclose` are **limited to initialisation-finalisation**
  - `M` can itself also catch WriteSizeExceeded to **re-try writing**

A core calculus for programming with runners

Core calculus (types and judgements)

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- **Values**

$$\Gamma \vdash V : X \qquad\qquad \Gamma \vdash V \equiv W : X$$

- **User computations**

$$\Gamma \vdash M : X \,!\, (\Sigma, E) \qquad\qquad \Gamma \vdash M \equiv N : X \,!\, (\Sigma, E)$$

- **Kernel computations**

$$\Gamma \vdash K : X \lightning (\Sigma, E, S, C) \qquad\qquad \Gamma \vdash K \equiv L : X \lightning (\Sigma, E, S, C)$$

Core calculus (types and judgements)

- **Values**

$$\Gamma \vdash V : X \qquad\qquad \Gamma \vdash V \equiv W : X$$

- **User computations**

$$\Gamma \vdash M : X \,!\, (\Sigma, E) \qquad\qquad \Gamma \vdash M \equiv N : X \,!\, (\Sigma, E)$$

- **Kernel computations**

$$\Gamma \vdash K : X \lightning (\Sigma, E, S, C) \qquad\qquad \Gamma \vdash K \equiv L : X \lightning (\Sigma, E, S, C)$$

- **Ground types** (for types of operations and kernel state)

$$A, B, C \quad ::= \quad \mathsf{B} \mid 1 \mid 0 \mid A \times B \mid A + B$$

- **Types**

$$\begin{array}{rcl} X, Y & ::= & \mathsf{B} \mid 1 \mid 0 \mid X \times Y \mid X + Y \\ & \mid & X \to Y \,!\, (\Sigma, E) \\ & \mid & X \to Y \lightning (\Sigma, E, S, C) \\ & \mid & \Sigma \Rightarrow (\Sigma', S, C) \end{array}$$

Core calculus (user computations)

$$
\begin{array}{lll}
M, N & ::= \mathsf{return}\ V & \text{value} \\
& \mid\ \mathsf{try}\ M\ \mathsf{with}\ \{\mathsf{return}\ x \mapsto N, (\mathsf{raise}\ e \mapsto N_e)_{e \in E}\} & \text{exception handler} \\
& \mid\ V\ W & \text{application} \\
& \mid\ \mathsf{match}\ V\ \mathsf{with}\ \{\langle x, y\rangle \mapsto M\} & \text{product elimination} \\
& \mid\ \mathsf{match}\ V\ \mathsf{with}\ \{\}_X & \text{empty elimination} \\
& \mid\ \mathsf{match}\ V\ \mathsf{with}\ \{\mathsf{inl}\ x \mapsto M, \mathsf{inr}\ y \mapsto N\} & \text{sum elimination} \\
& \mid\ \mathsf{op}_X(V, (x \,.\, M), (N_e)_{e \in E_{\mathsf{op}}}) & \text{operation call} \\
& \mid\ \mathsf{raise}_X\ e & \text{raise exception} \\
& \mid\ \mathsf{using}\ V \mathbin{@} W\ \mathsf{run}\ M\ \mathsf{finally}\ \{ & \text{run} \\
& \qquad \mathsf{return}\ x \mathbin{@} c \mapsto N, & \\
& \qquad (\mathsf{raise}\ e \mathbin{@} c \mapsto N_e)_{e \in E}, & \\
& \qquad (\mathsf{kill}\ s \mapsto N_s)_{s \in S}\} & \\
& \mid\ \mathsf{kernel}\ K \mathbin{@} V\ \mathsf{finally}\ \{ & \text{switch to kernel mode} \\
& \qquad \mathsf{return}\ x \mathbin{@} c \mapsto N, & \\
& \qquad (\mathsf{raise}\ e \mathbin{@} c \mapsto N_e)_{e \in E}, & \\
& \qquad (\mathsf{kill}\ s \mapsto N_s)_{s \in S}\} &
\end{array}
$$

Core calculus (kernel computations)

$$
\begin{array}{rcll}
K, L & ::= & \mathsf{return}_C\, V & \text{value} \\
& \mid & \mathsf{try}\ K\ \mathsf{with}\ \{\mathsf{return}\ x \mapsto L, (\mathsf{raise}\ e \mapsto L_e)_{e \in E}\} & \text{exception handler} \\
& \mid & V\, W & \text{application} \\
& \mid & \mathsf{match}\ V\ \mathsf{with}\ \{\langle x, y \rangle \mapsto K\} & \text{product elimination} \\
& \mid & \mathsf{match}\ V\ \mathsf{with}\ \{\}_{X @ C} & \text{empty elimination} \\
& \mid & \mathsf{match}\ V\ \mathsf{with}\ \{\mathsf{inl}\ x \mapsto K, \mathsf{inr}\ y \mapsto L\} & \text{sum elimination} \\
& \mid & \mathsf{op}_{X @ C}(V, (x \,.\, K), (L_e)_{e \in E_{\mathsf{op}}}) & \text{operation call} \\
& \mid & \mathsf{raise}_{X @ C}\, e & \text{raise exception} \\
& \mid & \mathsf{kill}_{X @ C}\, s & \text{send signal} \\
& \mid & \mathsf{getenv}_C(c \,.\, K) & \text{get state} \\
& \mid & \mathsf{setenv}(V, K) & \text{set state} \\
& \mid & \mathsf{user}\ M\ \mathsf{with}\ \{\mathsf{return}\ x \mapsto K, (\mathsf{raise}\ e \mapsto L_e)_{e \in E}\} & \text{switch to user mode}
\end{array}
$$

Core calculus (type system)

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- For example, the **typing rule for runners** is

$$\frac{\Sigma = \{\, \mathsf{op}_1 \,, \, \ldots \,, \, \mathsf{op}_n \,\} \qquad \big(\; \Gamma, x_i \!:\! A_{\mathsf{op}_i} \vdash K_i : B_{\mathsf{op}_i} \lightning (\Sigma', E_{\mathsf{op}_i}, S, C) \;\big)_{1 \leqslant i \leqslant n}}{\Gamma \vdash \{\, \mathsf{op}_1\, x_1 \mapsto K_1 \,, \, \ldots \,, \, \mathsf{op}_n\, x_n \mapsto K_n \,\}_C : \Sigma \Rrightarrow (\Sigma', S, C)}$$

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- and the **typing rule for running user comps.** is

$$\frac{\begin{array}{c} \Gamma \vdash V : \Sigma \Rightarrow (\Sigma', S, C) \qquad \Gamma \vdash W : C \qquad \Gamma \vdash M : X \,!\, (\Sigma, E) \\ \Gamma, x \!:\! X, c \!:\! C \vdash N_{ret} : Y \,!\, (\Sigma', E') \qquad \big(\, \Gamma, c \!:\! C \vdash N_e : Y \,!\, (\Sigma', E') \,\big)_{e \in E} \\ \big(\, \Gamma \vdash N_s : Y \,!\, (\Sigma', E') \,\big)_{s \in S} \end{array}}{\begin{array}{l} \Gamma \vdash \mathbf{using}\ V \ @\ W\ \mathbf{run}\ M\ \mathbf{finally}\ \{\, \mathbf{return}\ x \ @\ c \mapsto N_{ret} \,, \\ \qquad\qquad\qquad\qquad\qquad\qquad \big(\mathbf{raise}\ e \ @\ c \mapsto N_e\big)_{e \in E} \,, \\ \qquad\qquad\qquad\qquad\qquad\qquad \big(\mathbf{kill}\ s \mapsto N_s\big)_{s \in S} \,\} : Y \,!\, (\Sigma', E') \end{array}}$$

Core calculus (equational theory)

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- For example, the β-**equations for running user comps.** are

$$\Gamma \vdash \textbf{using } V \mathbin{@} W \textbf{ run } (\textbf{return } V') \textbf{ finally } F \equiv N_{ret}[V'/x, W/c] : Y \,!\, (\Sigma', E')$$

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$$\begin{aligned}
&\Gamma \vdash \textbf{using } R \ @ \ W \textbf{ run } \big(\mathsf{op}_X \,(V, (y.M), (M_e)_{e \in E_{\mathsf{op}}})\big) \textbf{ finally } F \\
&\quad \equiv \textbf{kernel } K_{op}[V/x_{op}] \ @ \ W \textbf{ finally } \{ \\
&\qquad\qquad \textbf{return } y \ @ \ c' \mapsto \textbf{using } R \ @ \ c' \textbf{ run } M \textbf{ finally } F \ , \\
&\qquad\qquad \big(\textbf{raise } e \ @ \ c' \mapsto \textbf{using } R \ @ \ c' \textbf{ run } M_e \textbf{ finally } F\big)_{e \in E_{\mathsf{op}}} \ , \\
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$$\begin{aligned}
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\equiv\ & \mathbf{kernel}\ K_{op}[V/x_{op}] \ @\ W\ \mathbf{finally}\ \{ \\
& \qquad \mathbf{return}\ y \ @\ c' \mapsto \mathbf{using}\ R \ @\ c'\ \mathbf{run}\ M\ \mathbf{finally}\ F\ , \\
& \qquad \big(\mathbf{raise}\ e \ @\ c' \mapsto \mathbf{using}\ R \ @\ c'\ \mathbf{run}\ M_e\ \mathbf{finally}\ F\big)_{e \,\in\, E_{\mathrm{op}}}\ , \\
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\end{aligned}$$

and the β-**equation for signal handling** is

$$\Gamma \vdash \mathbf{kernel}\ (\mathbf{kill}_{X @ C}\ s) \ @\ W\ \mathbf{finally}\ F \equiv N_s : Y \,!\, (\Sigma', E')$$

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&\quad \equiv \textbf{kernel } K_{op}[V/x_{op}] \mathbin{@} W \textbf{ finally } \{ \\
&\qquad\quad \textbf{return } y \mathbin{@} c' \mapsto \textbf{using } R \mathbin{@} c' \textbf{ run } M \textbf{ finally } F\ , \\
&\qquad\quad \big(\textbf{raise } e \mathbin{@} c' \mapsto \textbf{using } R \mathbin{@} c' \textbf{ run } M_e \textbf{ finally } F\big)_{e \in E_{\mathsf{op}}}\ , \\
&\qquad\quad \big(\textbf{kill } s \mapsto N_s\big)_{s \in S}\ \} : Y \,!\, (\Sigma', E')
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and the β**-equation for signal handling** is

$$\Gamma \vdash \textbf{kernel } (\textbf{kill}_{X@C}\ s) \mathbin{@} W \textbf{ finally } F \equiv N_s : Y \,!\, (\Sigma', E')$$

and kernel comp. equations include **kernel theory equations**

Core calculus (subtyping)

- The calculus also includes **subtyping**, and **subsumption rules**

$$\frac{\Gamma \vdash V : A \qquad A <: B}{\Gamma \vdash V : B}$$

$$\frac{\Gamma \vdash M : A \,!\, (\Sigma, E) \qquad \Sigma \subseteq \Sigma' \qquad A <: B \qquad E \subseteq E'}{\Gamma \vdash M : B \,!\, (\Sigma', E')}$$

$$\frac{\begin{array}{c}\Gamma \vdash K : A \,\lightning\, (\Sigma, E, S, C) \qquad \Sigma \subseteq \Sigma' \\ A <: B \qquad E \subseteq E' \qquad S \subseteq S' \qquad C \equiv C'\end{array}}{\Gamma \vdash K : B \,\lightning\, (\Sigma', E', S', C')}$$

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- We use $C \equiv C'$ to have (standard) **proof-irrelevant subtyping**
- Otherwise, instead of just $C <: C'$, we would need a **lens** $C' \leftrightarrow C$

Core calculus (semantics)

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- **Monadic semantics**, for concreteness in **Set**, using
  - **user monads** $\mathbf{U}_{\Sigma,E}\,X \;\stackrel{\text{def}}{=}\; \mathbf{Free}_{\Sigma}(X + E)$
  - **kernel monads** $\mathbf{K}_{\Sigma,E,S,C}\,X \;\stackrel{\text{def}}{=}\; C \Rightarrow \mathbf{Free}_{\Sigma}\big((X + E) \times C + S\big)$

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- (At a high level) the **judgements are interpreted** as maps

$$[\![\Gamma \vdash V : X]\!] : [\![\Gamma]\!] \longrightarrow [\![X]\!]$$

$$[\![\Gamma \vdash M : X \,!\, (\Sigma, E)]\!] : [\![\Gamma]\!] \longrightarrow \mathbf{U}_{\Sigma,E}[\![X]\!]$$

$$[\![\Gamma \vdash K : X \,\lightning\, (\Sigma, E, S, C)]\!] : [\![\Gamma]\!] \longrightarrow \mathbf{K}_{\Sigma,E,S,[\![C]\!]}[\![X]\!]$$

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- **Theorem:** The semantics is coherent (**subtyping!**) and sound.

Core calculus (semantics ctd.)

- In order to prove **coherence** of the semantics, we actually work in the (total category $\mathbf{Sub}(\mathbf{Set})$ of the) **subset fibration**

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- For instance, **kernel computations** are interpreted as

$$
\begin{tikzcd}
\llbracket \Gamma \rrbracket \arrow[rr, "\llbracket \Gamma \vdash K \,:\, X \,\lightning\, (\Sigma, E, S, C) \rrbracket"] \arrow[d, "\subseteq"'] & & \mathbf{K}_{\Sigma, E, S, \llbracket C \rrbracket} \llbracket X \rrbracket \arrow[d, "\subseteq"] \\
\llbracket\!\llbracket \Gamma^s \rrbracket\!\rrbracket \arrow[rr, "\llbracket\!\llbracket \Gamma^s \vdash K \,:\, X^s \,\lightning\, C \rrbracket\!\rrbracket"'] & & \mathbf{K}_{\mathcal{O} + \{\lightning\}, \mathcal{E}, \mathcal{S}, \llbracket\!\llbracket C \rrbracket\!\rrbracket} \llbracket\!\llbracket X^s \rrbracket\!\rrbracket
\end{tikzcd}
$$

where $\Gamma^s \vdash K : X^s \lightning C$ is a **skeletal kernel typing judgement** and use the extra op. $\lightning : 1 \rightsquigarrow 0 \,!\, \{\}$ to model **runtime errors**

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- No essential obstacles to extending to **Sub**(**Cpo**) and beyond

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$$\begin{tikzcd}
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[\![\![\Gamma^s]\!]\!] \arrow[rr, "{[\![\![\Gamma^s \vdash K : X^s \lightning C]\!]\!]}"'] & & \mathbf{K}_{\mathcal{O}+\{\lightning\},\mathcal{E},\mathcal{S},[\![\![C]\!]\!]}[\![\![X^s]\!]\!]
\end{tikzcd}$$

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- No essential obstacles to extending to **Sub**(**Cpo**) and beyond

- **Ground type restriction** on C simplifies the sem. ($[\![\![C]\!]\!] = [\![C]\!]$)

Core calculus (semantics ctd.)

$$
\begin{aligned}
\llbracket \Gamma \vdash \textbf{using}\ V\ \textbf{@}\ W\ \textbf{run}\ M\ \textbf{finally}\ \{\ & \textbf{return}\ x\ \textbf{@}\ c \mapsto N_{ret}\ , \\
& \left(\textbf{raise}\ e\ \textbf{@}\ c \mapsto N_e\right)_{e \in E}\ , \\
& \left(\textbf{kill}\ s \mapsto N_s\right)_{s \in S}\ \} : Y\,!\,(\Sigma', E')\rrbracket_\gamma \ \stackrel{\text{def}}{=}\ \ldots
\end{aligned}
$$

Core calculus (semantics ctd.)

$$\begin{aligned}\llbracket \Gamma \vdash \textbf{using } V \mathbin{@} W \textbf{ run } M \textbf{ finally } \{ \; & \textbf{return } x \mathbin{@} c \mapsto N_{ret} \; , \\ & \big(\textbf{raise } e \mathbin{@} c \mapsto N_e\big)_{e \in E} \; , \\ & \big(\textbf{kill } s \mapsto N_s\big)_{s \in S} \; \} : Y \,!\, (\Sigma', E') \rrbracket_\gamma \stackrel{\text{def}}{=} \; \ldots\end{aligned}$$

- $\llbracket V \rrbracket_\gamma \;=\; \mathcal{R} \;=\; \Big(\overline{\mathsf{op}}_{\mathcal{R}} : \llbracket A_{\mathsf{op}} \rrbracket \longrightarrow \mathbf{K}_{\Sigma', E_{\mathsf{op}}, S, \llbracket C \rrbracket} \, \llbracket B_{\mathsf{op}} \rrbracket\Big)_{\mathsf{op} \in \Sigma}$
- $\llbracket \textbf{return} \; \mathrm{x} \mathbin{@} \mathrm{c} \to N_{ret} \rrbracket_\gamma \;\in\; \llbracket X \rrbracket \times \llbracket C \rrbracket \longrightarrow \mathbf{U}_{\Sigma', E'} \, \llbracket Y \rrbracket$
- $\llbracket (\textbf{raise} \; \mathrm{e} \mathbin{@} \mathrm{c} \to N_e)_{e \in E} \rrbracket_\gamma \;\in\; E \times \llbracket C \rrbracket \longrightarrow \mathbf{U}_{\Sigma', E'} \, \llbracket Y \rrbracket$
- $\llbracket (\textbf{kill} \; \mathrm{s} \to N_s)_{s \in S} \rrbracket_\gamma \;\in\; S \longrightarrow \mathbf{U}_{\Sigma', E'} \, \llbracket Y \rrbracket$

Core calculus (semantics ctd.)

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\begin{aligned}
[\![\Gamma \vdash \textbf{using } V \;@\; W \textbf{ run } M \textbf{ finally } \{ & \textbf{ return } x \;@\; c \mapsto N_{ret} \;, \\
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\end{aligned}
$$

- $[\![V]\!]_\gamma \;=\; \mathcal{R} \;=\; \left(\overline{\mathsf{op}}_{\mathcal{R}} : [\![A_{\mathsf{op}}]\!] \longrightarrow \mathbf{K}_{\Sigma', E_{\mathsf{op}}, S, [\![C]\!]}\, [\![B_{\mathsf{op}}]\!]\right)_{\mathsf{op} \in \Sigma}$
- $[\![\textbf{return } \mathsf{x} \;@\; \mathsf{c} \to N_{ret}]\!]_\gamma \;\in\; [\![X]\!] \times [\![C]\!] \longrightarrow \mathbf{U}_{\Sigma', E'}\, [\![Y]\!]$
- $[\![(\textbf{raise } \mathsf{e} \;@\; \mathsf{c} \to N_e)_{e \in E}]\!]_\gamma \;\in\; E \times [\![C]\!] \longrightarrow \mathbf{U}_{\Sigma', E'}\, [\![Y]\!]$
- $[\![(\textbf{kill } \mathsf{s} \to N_s)_{s \in S}]\!]_\gamma \;\in\; S \longrightarrow \mathbf{U}_{\Sigma', E'}\, [\![Y]\!]$
- allowing us to use the **free model property** to construct

$$
\mathbf{U}_{\Sigma, E}\, [\![X]\!] \xrightarrow{\;\mathsf{r}_{[\![X]\!] + E}\;} \mathbf{K}_{\Sigma', E, S, [\![C]\!]}\, [\![X]\!] \xrightarrow{\;(\lambda [\![N_{ret}]\!]_\gamma)^\ddagger\;} \underbrace{[\![C]\!] \Rightarrow \mathbf{U}_{\Sigma', E'}\, [\![Y]\!]}_{\text{carrier of ker. th. model}}
$$

Core calculus (semantics ctd.)

$$\boxed{\begin{aligned}\llbracket \Gamma \vdash \textbf{using } V \mathbin{@} W \textbf{ run } M \textbf{ finally } \{ & \textbf{return } x \mathbin{@} c \mapsto N_{ret} \,, \\ & \left(\textbf{raise } e \mathbin{@} c \mapsto N_e\right)_{e \in E} \,, \\ & \left(\textbf{kill } s \mapsto N_s\right)_{s \in S} \,\} : Y \,!\, (\Sigma', E') \rrbracket_\gamma \stackrel{\text{def}}{=} \ \dots\end{aligned}}$$

- $\llbracket V \rrbracket_\gamma \; = \; \mathcal{R} \; = \; \left(\overline{\mathsf{op}}_{\mathcal{R}} : \llbracket A_{\mathsf{op}} \rrbracket \longrightarrow \mathbf{K}_{\Sigma', E_{\mathsf{op}}, S, \llbracket C \rrbracket} \, \llbracket B_{\mathsf{op}} \rrbracket \right)_{\mathsf{op} \in \Sigma}$
- $\llbracket \textbf{return } \mathsf{x} \mathbin{@} \mathsf{c} \to N_{ret} \rrbracket_\gamma \; \in \; \llbracket X \rrbracket \times \llbracket C \rrbracket \longrightarrow \mathbf{U}_{\Sigma', E'} \, \llbracket Y \rrbracket$
- $\llbracket (\textbf{raise } \mathsf{e} \mathbin{@} \mathsf{c} \to N_e)_{e \in E} \rrbracket_\gamma \; \in \; E \times \llbracket C \rrbracket \longrightarrow \mathbf{U}_{\Sigma', E'} \, \llbracket Y \rrbracket$
- $\llbracket (\textbf{kill } \mathsf{s} \to N_s)_{s \in S} \rrbracket_\gamma \; \in \; S \longrightarrow \mathbf{U}_{\Sigma', E'} \, \llbracket Y \rrbracket$
- allowing us to use the **free model property** to construct

$$\mathbf{U}_{\Sigma, E} \, \llbracket X \rrbracket \xrightarrow{\mathsf{r}_{\llbracket X \rrbracket + E}} \mathbf{K}_{\Sigma', E, S, \llbracket C \rrbracket} \, \llbracket X \rrbracket \xrightarrow{(\lambda \llbracket N_{ret} \rrbracket_\gamma)^\ddagger} \underbrace{\llbracket C \rrbracket \Rightarrow \mathbf{U}_{\Sigma', E'} \, \llbracket Y \rrbracket}_{\text{carrier of ker. th. model}}$$

- and then apply the resulting composite map to

$$\llbracket M \rrbracket_\gamma \; \in \; \mathbf{U}_{\Sigma, E} \, \llbracket X \rrbracket \qquad \text{and} \qquad \llbracket W \rrbracket_\gamma \; \in \; \llbracket C \rrbracket$$

Core calculus (finalisation)

$$\begin{aligned}\Gamma \vdash \textbf{using}\ V\ @\ W\ \textbf{run}\ M\ \textbf{finally}\ \{\ & \textbf{return}\ x\ @\ c \mapsto N_{ret}\ , \\ & \big(\textbf{raise}\ e\ @\ c \mapsto N_e\big)_{e \in E}\ , \\ & \big(\textbf{kill}\ s \mapsto N_s\big)_{s \in S}\ \} : Y \,!\, (\Sigma', E')\end{aligned}$$

Core calculus (finalisation)

$$\boxed{\begin{array}{l}\Gamma \vdash \textbf{using}\ V\ @\ W\ \textbf{run}\ M\ \textbf{finally}\ \{\ \textbf{return}\ x\ @\ c \mapsto N_{ret}\ , \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \big(\textbf{raise}\ e\ @\ c \mapsto N_e\big)_{e \in E}\ , \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \big(\textbf{kill}\ s \mapsto N_s\big)_{s \in S}\ \} : Y\,!\,(\Sigma', E')\end{array}}$$

- The **finally** block $(N_{ret}, N_e, \ldots, N_s, \ldots)$ determines **fin. maps**

$$\phi_\gamma : (\llbracket X \rrbracket + E) \times \llbracket C \rrbracket + S \longrightarrow \mathbf{U}_{\Sigma', E'}\, \llbracket Y \rrbracket \qquad\qquad (\gamma \in \llbracket \Gamma \rrbracket)$$

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- **Theorem (finalisation):**
  - for every environment $\gamma \in \llbracket \Gamma \rrbracket$,
  - there exists a comp. tree $t \in \mathbf{Free}_{\Sigma'}\big((\llbracket X \rrbracket + E) \times \llbracket C \rrbracket + S\big)$,
  - such that **running factors through finalisation**, i.e.,

$$\llbracket \Gamma \vdash \textbf{using}\ V\ @\ W\ \textbf{run}\ M\ \textbf{finally}\ F : Y\,!\,(\Sigma', E') \rrbracket_\gamma = \phi_\gamma^\dagger\ t$$

Implementing runners

Experimenting with the theory in practice

Experimenting with the **theory in practice**

- A **small experimental language** COOP[4]
  - Implements the core calculus with few extras
  - The interpreter is directly based on the equational theory
  - Top-level containers for running external (OCaml) code
  - `https://github.com/andrejbauer/coop`

[4] coop [/ku:p/] – a cage where small animals are kept, especially chickens

Experimenting with the theory in practice

- A **small experimental language** COOP[4]
  - Implements the core calculus with few extras
  - The interpreter is directly based on the equational theory
  - Top-level containers for running external (OCaml) code
  - `https://github.com/andrejbauer/coop`
- A HASKELL **library** HASKELL-COOP
  - A shallow-embedding of the core calculus in HASKELL
  - Uses one of the Freer monad implementations underneath
  - Operational aspects implement the denotational semantics
  - Top-level containers for arbitrary HASKELL monads
  - Examples make use of HASKELL's features (GADTs, ...)
  - `https://github.com/danelahman/haskell-coop`

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Runners in action

Runners can be vertically nested

Runners can be **vertically nested**

- ```
  using R_FH @ (fopen fileName)
  run (
    using R_FC @ (return "")
    run M
    finally {
      return x @ str → write str; return x ,
      raise WriteSizeExceeded @ str → write str; raise WriteSizeExceeded }
  )
  finally {
    return x @ fh → ... , raise e @ fh → ... , kill IOError → ... }
  ```

  where the **file contents runner** (with $\Sigma' = \{\}$) is defined as

  ```
  let R_FC = {
    write str' → let str = getenv () in
                 if (length (str^str') > max) then (raise WriteSizeExceeded)
                                              else (setenv (str^str'))
  }_String
  ```
  ```

Vertical nesting for instrumentation

Vertical nesting for **instrumentation**

- ```
  using R_Cost @ (return 0)
  run M
  finally {
    return x @ c → report_cost c; return x ,
    raise e @ c → report_cost c; raise e ,
    kill s → raise SignalHappenedException }
  ```

  where the **cost model runner** is defined as

  ```
  let R_Cost = {
    ... ,
    op a → let c = getenv () in
           setenv (c + 1);
           op a ,                    (* forwards op outwards *)
    ...
  }_Nat
  ```

- The runner $\mathsf{R}_{\mathsf{Cost}}$ implements the same sig. Σ that `M` is using
- As a result, the runner $\mathsf{R}_{\mathsf{Cost}}$ is **invisible** from `M`'s viewpoint
  ```

Vertical nesting for active monitoring

Vertical nesting for **active monitoring**

- First, we define a runner for **integer-valued ML-style state** as

```
type IntHeap = (Nat → (Int + 1)) × Nat                                type Ref = Nat
```

```
let R_IntState = {
  alloc x → let h = getenv () in                    (* alloc : Int ⇝ Ref ! {} *)
            let (r,h') = heapAlloc h x in
            setenv h';
            return r ,

  deref r → let h = getenv () in                    (* deref : Ref ⇝ Int ! {} *)
            match (heapSel h r) with
            | inl x → return x
            | inr () → kill ReferenceDoesNotExist ,

  assign r y → let h = getenv () in                 (* assign : Ref × Int ⇝ 1 ! {} *)
               match (heapUpd h r y) with
               | inl h' → setenv h'
               | inr () → kill ReferenceDoesNotExist
}_IntHeap
```

Vertical nesting for active monitoring ctd.

- Next, we define $F^{\star}$-style **monotonic state** on top of R_{IntState}

Vertical nesting for **active monitoring** ctd.

- Next, we define F*-style **monotonic state** on top of R_{IntState}

```
type MonMemory = Ref → (Ord + 1)          type Ord = Int → Int → Bool
```

```
let R_MonState = {
  mAlloc x rel → let r = alloc x in                  (* : Int × Ord ⇝ Ref ! {} *)
                 let m = getenv () in
                 setenv (memAdd m r rel);
                 return r,

  mDeref r → deref r ,                                     (* : Ref ⇝ Int ! {} *)

  mAssign r y → let x = deref r in                  (* : Ref × Int ⇝ 1 ! {MV} *)
                let m = getenv () in
                match (memSel m r) with
                | inl rel → if (rel x y)
                            then (assign r y)
                            else (raise MonotonicityViolation)
                | inr → kill PreorderDoesNotExist
}_MonMemory
```

Vertical nesting for **active monitoring** ctd.

- We can then perform **runtime monotonicity verification** as

Vertical nesting for **active monitoring** ctd.

- We can then perform **runtime monotonicity verification** as

```
using R_IntState @ ((fun _ → inr ()) , 0)        (∗ init. empty ML−style heap ∗)
run (

  using R_MonState @ (fun _ → inr ())        (∗ init. empty preorders memory ∗)
  run (

    let r = mAlloc 0 (⩽) in
    mAssign r 1;
    mAssign r 0;        (∗ R_MonState raises MonotonicityViolation exception ∗)
    mAssign r 2

  )
  finally { ... , raise MonotonicityViolation @ m → ... , ... }

)
finally { ... }
```

Runners can also be horizontally paired

Runners can also be horizontally paired

- Given runners for

```
let R₁ = { ... , op₁ᵢ x₁ᵢ → K₁ᵢ , ... }_C₁          (* : Σ₁ ⇒ (Σ'₁, S₁, C₁) *)
let R₂ = { ... , op₂ⱼ x₂ⱼ → K₂ⱼ , ... }_C₂          (* : Σ₂ ⇒ (Σ'₂, S₂, C₂) *)
```

we can **pair them** to get the runner

```
let R = { ... ,                 (* : Σ₁ + Σ₂ ⇒ (Σ'₁ + Σ'₂, S₁ + S₂, C₁ × C₂) *)
  op₁ᵢ x₁ᵢ  → let (c,c') = getenv () in
              user (kernel (K₁ᵢ x₁ᵢ) @ c finally {
                        return y @ c'' → return (inl (inl y,c'')),
                        raise e @ c'' → return (inl (inr e,c'')),  (* e ∈ E_op₁ᵢ *)
                        kill s → return (inr s) }                    (* s ∈ S₁ *)
              finally {
                return (inl (inl y,c'')) → setenv (c'',c'); return y,
                return (inl (inr e,c'')) → setenv (c'',c'); raise e,
                return (inr s) → kill s },
  ... ,
  op₂ⱼ x₂ⱼ → ... , ... }_C₁×C₂
```

Runners can also be **horizontally paired**

- Given runners for

```
let R₁ = { … ,  op₁ᵢ x₁ᵢ → K₁ᵢ  , … }_C₁              (* : Σ₁ ⇒ (Σ'₁, S₁, C₁) *)
let R₂ = { … ,  op₂ⱼ x₂ⱼ → K₂ⱼ  , … }_C₂              (* : Σ₂ ⇒ (Σ'₂, S₂, C₂) *)
```

  we can **pair them** to get the runner

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let R = { … ,                     (* : Σ₁ + Σ₂ ⇒ (Σ'₁ + Σ'₂, S₁ + S₂, C₁ × C₂) *)
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                return (inr s) → kill s },
  … ,
  op₂ⱼ x₂ⱼ → … , … }_C₁×C₂
```

- For instance, this way we can build a runner for **IO and state**

Other examples

Other examples

- More general forms of **(ML-style) state** (for general `Ref A`)
  - if the host language allows it, we use GADTs, etc for safety
  - some examples extract a footprint from a larger memory

- **Combinations** of different effects and runners
  - in particular the combination of IO and state
  - good use case for both vertical and horizontal composition

- KOKA-style **ambient values** and **ambient functions**
  - **ambient values** are essentially **mutable variables/parameters**
  - **ambient functions** are **applied in their lexical context**
  - a runner that treats **amb. fun. application as a co-operation**
  - amb. funs. are stored in a context-depth-sensitive heap
  - the appl. co-operation restores the heap to the lexical context

Other examples (ambient vals. and funs.)

- **Ambient values**

```
ambient val f : int → int
ambient val x : int

with val x = 4
with val f = fun y → x + y
with val x = 2

f 1                                        (* Returns 3 *)
```

Other examples (ambient vals. and funs.)

- **Ambient values**

```
ambient val f : int → int
ambient val x : int

with val x = 4
with val f = fun y → x + y
with val x = 2

f 1                                    (* Returns 3 *)
```

- **Ambient functions**

```
ambient fun f : int → int
ambient val x : int

with val x = 4
with fun f = fun y → x + y
with val x = 2

f 1                                    (* Returns 5 *)
```

Other examples (ambient vals. and funs.)

```haskell
module AmbientsTests where

import Control.Runner
import Control.Runner.Ambients

ambFun :: AmbVal Int -> Int -> AmbEff Int
ambFun x y =
  do x <- getVal x;
     return (x + y)

test1 :: AmbEff Int
test1 =
  withAmbVal
    (4 :: Int)
    (\ x ->
      withAmbFun
        (ambFun x)
        (\ f ->
          do rebindVal x 2;
             applyFun f 1))

test2 = ambTopLevel test1
```

Other examples (ambient vals. and funs.)

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        (\ f ->
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test2 = ambTopLevel test1
```

```haskell
module Control.Runner.Ambients

...

ambCoOps :: Amb a -> Kernel sig AmbHeap a
ambCoOps (Bind f) =
  do h <- getEnv;
     (f,h') <- return (ambHeapAlloc h f);
     setEnv h';
     return f
ambCoOps (Apply f x) =
  do h <- getEnv;
     (f,d) <- return (ambHeapSel h f (depth h));
     user
       (run
          ambRunner
          (return (h {depth = d}))
          (f x)
          ambFinaliser)
       return
ambCoOps (Rebind f g) =
  do h <- getEnv;
     setEnv (ambHeapUpd h f g)

ambRunner :: Runner '[Amb] sig AmbHeap
ambRunner = mkRunner ambCoOps
```

Wrapping up

- **Runners** are a natural model of **top-level runtime**
- We propose **T-runners** to also model **non-top-level runtimes**
- We have turned **T**-runners into a **(useful ?) programming construct** with controlled **initialisation** and **finalisation**
- Despite being **affine**, **T**-runners still **cover a lot of ground**
- Two **implementations**: COOP & HASKELL-COOP
- **Ongoing** and **future:** handlers, lenses in subtyping and semantics, cat. of runners, concurrency, refinement typing, compilation, ...

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