Applied Stochastic Processes, 18-751, TX Brown, Fall 2017
Homework #1

Due 5pm Monday Sep. 4.

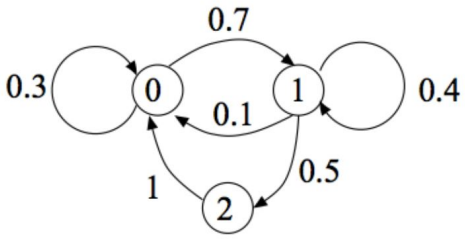
Read Chapter 1, A.1, A.2, and C.3.

Homework Policy: You are encouraged to work with others on the homeworks with the following caveats.

- Caveat 1: You write your own solutions.
- Caveat 2: The top of your homework lists all the other students with whom you collaborated on the assignment.

1. Given two non-empty sets A and B, prove with Venn diagrams each of the following relations:

   (a) $A \cap B^c = A - B$
   (b) $A \cup B^c = (A^c \cap B)^c$
   (c) $B - A \neq A - B$



2. For the above Markov chain, answer the following:

   (a) Is the graph *irreducible*? Is it *periodic*? Justify your answer.
   (b) Compute the invariant distribution, π, of this Markov chain.
   (c) Calculate the expected time from 0 to 2.
   (d) Use Matlab to plot the probability that starting from 0, the MC has reached 2 after n steps versus n.
   (e) Use Matlab to simulate the MC and plot the fraction of time it spends in the different states versus n the number of steps.
   (f) Use Matlab to plot π_n versus n.

3. Consider the previous problem, but let the states corresponded to web pages with the corresponding transition ('click') probabilities.

   (a) Order the states from highest to lowest Page rank (Hint: use your answers from the previous problem).
   (b) In reality clicks that reflect back to the same page distort the page ranking. Remove the two state self connections and normalize the total exit probability to be 1.
   (c) Try to trick the page rank by creating two new states, 0a and 1a. State 0 transitions to state 0a with probability 0.3 and state 0a transitions to state 0 with probability 1. State 1 transitions to state 1a with probability 0.4 and state 1a transitions to state 1 with probability 1. Draw the state diagram and compute the order of page ranks for states 0, 1, and 2. How does this compare to parts (a) and (b)?