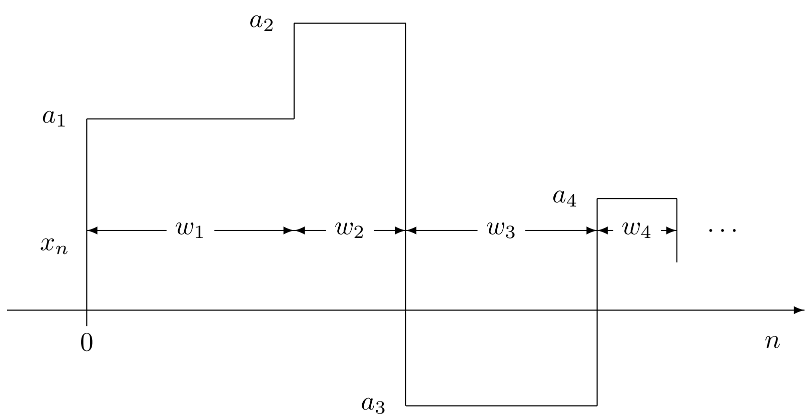
Applied Stochastic Processes, 18-751, TX Brown, Fall 2017
Homework #12

Due 8pm Monday November 27.

Read G&Y 11.1, 11.2, 11.5-11.9 (do as many quizzes as you can)

1. Consider $Z_m = X_m + N_m$ where X_m and N_m are WSS and mutually uncorrelated with power spectral densities $S_{XX}(\phi)$ and $S_{NN}(\phi)$ and zero means. Let $Y_m = L\{Z_m\}$ where L is a LTI system with impulse response h_m.
   (a) Compute the PSD of the output Y_m.
   (b) Compute the cross-power spectral density of X and Y, i.e. find $S_{XY}(\phi)$ and $S_{YX}(\phi)$.
   (c) Define the error $\eta_m \stackrel{\text{def}}{=} Y_m - X_m$ and compute the PSD of η_m.
   (d) Define $h_m = a\delta[m]$ and compute the value of a which minimizes $E[\eta_m^2] = R_{\eta\eta}[0]$.



2. The problems in this homework are based on the picture above. A random process x_n is constructed from a sequence of random length and random amplitude square pulses as shown above. The amplitude of pulse i is a_i. The amplitudes are i.i.d. and have mean, μ_a and standard deviation σ_a. The width of pulse i is w_i. The widths are i.i.d. and are geometrically distributed with mean 100 steps.
   (a) For $m \geq t_1 \gg 0$ compute $R_{XX}(t_1, m)$. Is x_n wide-sense stationary?
   (b) Compute the Power Spectral Density, $S_{XX}(\phi)$ of x_n.
   (c) Compute the power in the signal x_n in three different ways.
   (d) Let y_n be the output of a linear time invariant (LTI) filter with frequency response:
   $$H(\phi) = \frac{j\phi c}{1 + j\phi c},$$
   where $c = 10$ (for review you might want to compute the impulse response, h_n, of this function). Compute the output power spectral density, $S_{YY}(\phi)$. What is the power in the output signal, y_n.
   (e) Plot the original and filtered PSD on the same graph. What is the purpose of the filter?

Discrete Time function	Discrete Time Fourier Transform
$\delta[n] = \delta_n$	1
1	$\delta(\phi)$
$\delta[n-n_0] = \delta_{n-n_0}$	$e^{-j2\pi\phi n_0}$
$u[n]$	$\dfrac{1}{1-e^{-j2\pi\phi}} + \frac{1}{2}\sum_{k=-\infty}^{\infty}\delta(\phi+k)$
$e^{j2\pi\phi_0 n}$	$\sum_{k=-\infty}^{\infty}\delta(\phi-\phi_0-k)$
$\cos 2\pi\phi_0 n$	$\dfrac{1}{2}\delta(\phi-\phi_0) + \dfrac{1}{2}\delta(\phi+\phi_0)$
$\sin 2\pi\phi_0 n$	$\dfrac{1}{2j}\delta(\phi-\phi_0) - \dfrac{1}{2j}\delta(\phi+\phi_0)$
$a^n u[n]$	$\dfrac{1}{1-ae^{-j2\pi\phi}}$
$a^{\lvert n\rvert}$	$\dfrac{1-a^2}{1+a^2-2a\cos 2\pi\phi}$
g_{n-n_0}	$G(\phi)e^{-j2\pi\phi n_0}$
$g_n e^{j2\pi\phi_0 n}$	$G(\phi-\phi_0)$
g_{-n}	$G^*(\phi)$
$\sum_{k=-\infty}^{\infty} h_k g_{n-k}$	$G(\phi)H(\phi)$

Note that $\delta[n]$ is the discrete impulse, $u[n]$ is the discrete unit step, and a is a constant with magnitude $|a| < 1$.

Table 11.2 Discrete-Time Fourier transform pairs and properties.