Applied Stochastic Processes, 18-751, TX Brown, Fall 2017
Homework #6

Due 5pm Monday October 9.

Read 2.6, 2.7, 2.8, 3.7, 4.6, 4.7, 5.5. On your own do Quizzes 2.6, 2.7, 3.7, 4.6.

1. Let $X \sim N(0,1)$ and let $Y = g(X)$ where

$$g(x) = \begin{cases} 0 & x \leq 0 \\ x & 0 < x \leq 1 \\ 1 & x > 1 \end{cases}$$

   (It may help to sketch $g(x)$.) Compute $F_Y(y)$ and $f_Y(y)$ from $f_X(x)$.

2. Let $X_1, X_2, \ldots, X_n$ be i.i.d. r.v.'s with distribution $F_X(x)$. Let $Z = \min\{X_1, X_2, \ldots, X_n\}$.
   (a) Compute $F_Z(z)$ and $f_Z(z)$ in terms of $F_X(x)$ and $f_X(x)$.
   (b) If $F_X(x) = 1 - e^{-x}$ for $x > 0$ and zero otherwise, compute and sketch $f_Z(z)$ for $n = 3$.
   (c) Design an $\alpha = 0.05$ one-sided significance test based on the distribution in (b).
   (d) Suppose the X_i are really chosen from distribution $F_Y(y) = 1 - e^{-\lambda y}$. If $\lambda < 1$ is your test in (c) based on $\lambda = 1$ more likely or less likely to reject the hypothesis than $\alpha = 0.05$.

3. Let

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} \exp\left[-\left(\frac{x^2 - 2\rho xy + y^2}{2\sigma^2(1-\rho^2)}\right)\right],$$

   where $|\rho| < 1$.
   (a) Show that $E[Y] = 0$ but $E[Y|X = x] = \rho x$.
   (b) Show that $\sigma_Y^2 = E[Y^2] - (E[Y]^2) = \sigma^2$, but $\sigma_{Y|X=x}^2 = \sigma^2(1-\rho^2)$.
   (c) With this distribution, does knowing X provide any information about Y?

4. Let $f_{XY}(x,y)$ be defined as in the previous problem. Compute the joint pdf of $f_{VW}(v,w)$ of

$$\begin{aligned} V &= \frac{1}{2}(X^2 + Y^2) \\ W &= \frac{1}{2}(X^2 - Y^2). \end{aligned}$$

   Hint: Solve for X and Y in terms of V and W. Also note $\sqrt{Z^2} = \pm Z$.