Applied Stochastic Processes, 18-751, TX Brown, Fall 2017
Homework #7

Due 5pm Monday October 16.

Read ch 7 in JW and 9.1 - 9.3 in G&Y. On your own do Quizzes 9.1, 9.2.

Read Sec. 1 and 3 of G. Welch, G. Bishop, *An introduction to the Kalman filter*, Dept. of CS TR 95-041, UNC, July 24, 2006. `https://www.cs.unc.edu/~welch/media/pdf/kalman_intro.pdf`

1. Given a 1-d Kalman filter where $f_k = 1, b_k = u_k = q_k = 0$, and $r_k^2 = r^2 \quad \forall k$. Let $z_i = \mu + v_k$ and $\hat{x}_1 = z_1$. Prove that $\hat{x}_n \sim N(\mu, \frac{r^2}{n})$ and as a result $\lim_{n\to\infty} \hat{x}_n = \mu$.

   This shows the Kalman filter with unity gain ($f_k = 1$) tends to average over measurements.

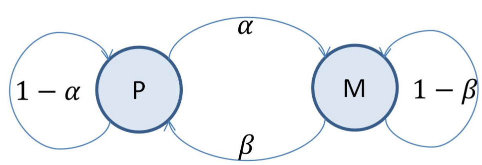
2. Consider a room temperature monitoring system. The system tracks both the interior and exterior temperatures, $x = (t^I, t^E)$, according to the following model:

$$
\begin{aligned}
t_k^I &= t_{k-1}^I + 0.05(t_{k-1}^E - t_{k-1}^I) + w_k^I, \quad w_k^I \sim N(0, 0.04) && (1)\\
t_k^E &= t_{k-1}^E + 0.1\sin(\frac{2\pi k}{300}) + w_k^E, \quad w_k^E \sim N(0, 0.01) && (2)\\
z_k^I &= t^I k + v_k^I, v_k^I \sim N(0, 4) && (3)\\
z_k^E &= t^E k + v_k^E, v_k^E \sim N(0, 1) && (4)\\
B_k &= (-1, 0), u_k = 0 && (5)\\
x_0 &= (25, 25) && (6)
\end{aligned}
$$

   (a) Derive the Kalman filter for the above model. Define all vectors and matrices.

   (b) Simulate the model for at least one daily cycle (300 steps). Plot the internal temperature, the the internal temperature estimate, and $\pm$ one standard deviation error bars around the internal temperature estimate.

   (c) Let u_k represent the control signal for an air conditioner. On and off are represented by $u_k = 1$ and $u_k = 0$. At the end of each step compute the probability that the internal temperature is more than 28 degrees. If this exceeds 10%, then in the next step let $u_k = 1$. Simulate the model and plot as in (b). Plot u_k on the same graph.



3. Consider a sensor network where the timing of new measurements may be highly variable or bursty due to network errors or other faults. We model with the above Markov chain. Entering the state P means that time is advanced and there is a prediction step. Entering the state M means there is a new measurement and there should be a fusion step. In problem 2, $\alpha = \beta = 1$. Simulate the model in the previous problem 2.c (with cooling) under the following conditions.

   (a) $\alpha = 0.2, \beta = 1$ (this allows missing measurements)

   (b) $\alpha = 0.2, \beta = 0.2$ (if there are multiple measurements in a row, they should be processed in order and only the last estimate plotted. Time is only advanced on a prediction step.