Applied Stochastic Processes, 18-751, TX Brown, Fall 2017
Homework #5

Due 5pm Monday October 2.

Read Chapter 5 of JW. On your own do Quizzes 1.9, 3.5, 8.2 (see back page). Note page 124.

1. An $[n, k, e]$ error correcting code takes k bits of information and then produces an n bit codeword where $n \geq k$. These n bits are transmitted to a receiver over a channel. The channel causes some of the n bits to be in error at the receiver. The receiver can correctly detect the codeword if there are e or fewer errors. If there are more than e errors, the decoder chooses a different n bit codeword which maps to a random k bits of information.

   (a) If p is the probability that a transmitted bit is in error and errors are independent, write an expression for the probability that the receiver receives a codeword in error.

   (b) In most system designs, p is a very small number (e.g. $p < 0.01$) while $n < 100$. Write an approximation to your expression in (a) that considers only the one most significant term.

   (c) The codeword error rate is not the same as information bit error rate. Compute the information bit error rate noting that a random bit is correct half the time.

   (d) Compute the information bit error rate for a $[1, 1, 0]$ code (i.e. sending an ordinary bit), and for a $[7, 4, 1]$ code when $p = 0.01, p = 0.001$, and $p = 10^{-6}$.

   (e) For the $[1, 1, 0]$ and $[7, 4, 1]$ codes find the value of p that will result in an information bit error rate of 10^{-6}.

2. A transmitter sends a signal $s = a, a > 0$ to represent a 1 with probability π and $s = -a$ to represent a 0 with probability $1 - \pi$. The received signal is $r = s + x$ where $x \sim \mathcal{N}(0, 1)$.

   (a) Compute the MAP and MLE decision boundaries.

   (b) For $a = 0.5$ and $\pi = 0.5$ draw on the same graph the pdf of r, $(r|1)$, and $(r|0)$ where $(r|b)$ means r given b was sent. Repeat on a second graph for $\pi = 0.1$. In both graphs mark the MAP and MLE decision regions.

   (c) Compute the expected bit error rates in part (b) for the MAP and MLE decision boundaries and for $\pi = 0.5$ and $\pi = 0.1$. (Hint: recall the $Q()$ function).

   (d) Draw the receiver operating charactaristics (ROC) for part (b) for $\pi = 0.5$ and $\pi = 0.1$.

3. Communication engineers concern themselves with how much energy is needed to send information because of batteries and the cost of energy. Use the scenario in the previous problem with $\pi = 0.5$. The energy to send each bit is βa^2 for some constant β.

   (a) For the $[1, 1, 0]$ code, compute the necessary a and average energy per bit to send equally likely bits so that the MAP decision boundary results in a bit error rate of 10^{-6}.

   (b) For the $[7, 4, 1]$ code, compute the necessary a and average energy per *information* bit to send equally likely bits so that the MAP decision boundary results in an information bit error rate of 10^{-6}. Note that every 4 bits of information requires 7 bits to be sent. So compute the energy to send one codeword and divide by the number of bits of information in the codeword to get the average energy per bit.

   (c) Which of the two codes is the most efficient and by what factor?

Study Help

Some have asked is there anything they can do to practice the concepts that we are covering in the class and how to study further. You will note on the front page of this homework, "Quizzes 3.1, ..." These refer to quizzes from the Goodman and Yates text. The end of each section has a quiz question. So, for example, Quiz 1.1 is found at the end of section 1.1. These quiz questions are purely for your self study and are not graded or turned in. If you have any question about the quiz questions, please come see me or ask in class. I am happy to talk with you about them.

The advantage of the quizzes is that there is a solution manual. The manual is posted on Canvas. The quizzes are open book, so feel free to use the book while answering the questions. DO NOT simply jump to the solutions manual without attempting to do the problem first. That turns the quiz problems into example problems and the book is already full of example problems. Forcing yourself to try to apply what you read is where you will really test your understanding and lock it in. Make an attempt at each one. Come up with a solution. Only after you have a solution check your solution against the solution found in the manual.

I have gone back through and collected all the quizzes that are relevant to the material we have covered so far (see below). It will take you about five hours to review all of these quizzes (assuming you have already done the reading). Don't let the five hours stop you. You are not going to do them in one sitting. That is called cramming! Divide the problems into groups of five questions. Review the sections covered by your five questions. When you have an hour, set a timer like a real quiz and take a shot at the five. If you finish early, check your answers on the ones you finished and then go do something else. If you reach one hour without solving all of them record any questions or trouble you are having and then send me an email or come to my office. Don't be concerned if you cannot finish all five in one hour. Some of the problems are more time consuming than others. Simply add the ones you didn't finish to your pool of remaining problems for future quizzes. If you spend one hour like this on different days over the next two weeks, you will be well on your way to a better and deeper understanding of the material.

Quizzes so far: 1.1, 1.2, 1.3, 1.4*, 1.5*, 1.6*, 1.7, 1.8, 1.9*, 1.10, 1.11, 2.1, 2.2, 2.3*, 2.4*, 2.5, 2.9, 2.10, 3.1*, 3.2*, 3.3, 3.4, 3.5*, 3.6, 3.8*

Quizzes with a "*" are considered especially relevant.