Applied Stochastic Processes, 18-751, TX Brown, Fall 2017
Homework #13 (Last One!)

Due 8pm Wednesday December 6.

Read G&Y 11.3,11.4

1. Several times we have mentioned complex numbers (numbers such as $c = 2 + j3$, where $j = \sqrt{-1}$).
   (a) Show that for any $c = u + jv$ we can rewrite it in radial coordinates as $c = ae^{j\phi}$ for some amplitude a and phase ϕ. What are a and ϕ as functions of u and v. Rewrite $c = 2 + j3$ in radial coordinates.
   (b) Show that $x(t) = A\cos(2\pi ft + \phi_0) = c\cos(2\pi ft)$ for some complex amplitude, c. What is a and ϕ for this complex amplitude in terms of A and ϕ_0? Compute a and ϕ for $x(t) = 5\cos(2\pi ft + 0.35\pi)$. Hint: $\cos x = \frac{e^{jx}+e^{-jx}}{2}$.
   (c) Show that $x(t) = a_i\cos(2\pi ft) + a_q\sin(2\pi ft) = c\cos(2\pi ft)$ for some complex amplitude c. Compute c for $x(t) = 0.5\cos(2\pi ft) + \sin(2\pi ft)$?

   This shows that we can equivalently think of any sinusoid signal as a cosine with a phase or as a cosine multiplied by a complex amplitude or as a combination of cosine and sine signals.

2. Consider a digital communication system that wants to be very efficient with bandwidth. To do this, the decides on the spectrum it wants to use first and then computes the resulting impulse response. To be concrete, the system sends data at 1Mbps and the baseband spectrum must have no frequencies higher than 0.5Mhz. The sampling rate is 10MHz.
   (a) Draw an ideal $f_c = 0.5$MHz low-pass filter for the continuous time $H(f)$. Note that the filter should pass frequencies from -0.5MHz to $+0.5$MHz and so has a 1 MHz bandwidth. What is the equivalent cutoff frequency $0 \le f_e \le 0.5$ for the discrete time system? Draw the corresponding low-pass filter, $H(\phi)$.
   (b) Consider the impulse response, $h_n = \frac{\sin(2\pi f_e n)}{\pi n}, h_0 = 2f_e$? Plot h_{-200} to h_{200}.
   (c) Since the impulse response has an infinite number of non-zero values, it is necessary to truncate the impulse response for a practical system. Choose a value of n that includes the first $c = 2$ zero crossings above and below h_0. Plot the Fourier transform of this truncated impulse response. Repeat for $c = 5$ and for $c = 10$. Comment on the apparent bandwidth of the system in all three cases.
   (d) Given binary data, map 1 and 0 to amplitudes 1 and -1 respectively. For a random stream of i.i.d. equally likely bits, plot the sequence of impulses after you map from bits to amplitude with pulse shape h_n and $c = 5$, plot a sample sequence of pulses. What is the spectrum of this, $S_{XX}(\phi)$. Draw the spectrum for $c = 2, 5, 10$.