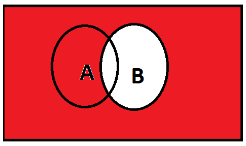
CARNEGIE MELLON UNIVERSITY
APPLIED STOCHASTIC PROCESSES
(COURSE 18-751)
HOMEWORK 1

Daniel Fekadu

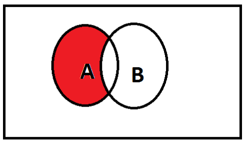
August 31, 2017

1 prove with Venn diagrams

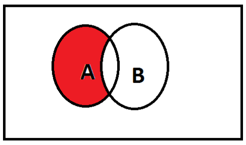
(a) $A \cap B^c = A - B$



(a) B^c



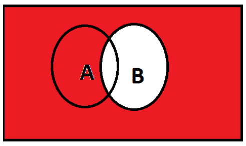
(b) $A \cap B^c$



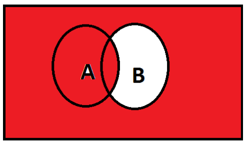
(c) $A - B$

this implies $A \cap B^c = A - B$ is true

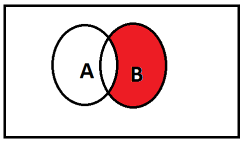
(b) $A \cup B^c = (A^c \cap B)^c$



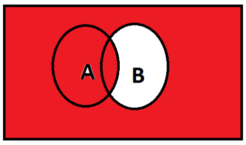
(a) B^c



(b) $A \cup B^c$



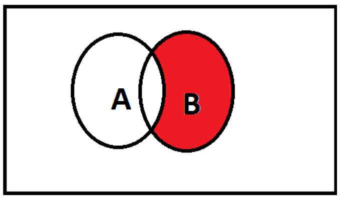
(c) $A^c \cap B$



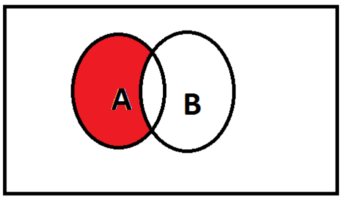
(d) $(A^c \cap B)^c$

this implies $A \cup B^c = (A^c \cap B)^c$ is true

(c) $B - A \neq A - B$



(a) $B - A$



(b) $A - B$

this implies $B - A \neq A - B$ is true

2

(a) Irreduciblity and periodicity

The Markov Chain is irreducible because we can go from any of the states to the other states in finit number of steps It is also aperiodic[Justfy]

(b) invariant distribution π

$$\pi_j = \sum_{n=0}^{n} \pi_k p_{kj} \tag{1}$$

$$\begin{align}
\pi_0 &= \pi_0 P_{00} + \pi_1 P_{10} + \pi_2 P_{20} \tag{2}\\
\pi_1 &= \pi_0 P_{01} + \pi_1 P_{11} + \pi_2 P_{21} \tag{3}\\
\pi_2 &= \pi_0 P_{02} + \pi_1 P_{12} + \pi_2 P_{22} \tag{4}\\
& \tag{5}
\end{align}$$

$$P = \begin{bmatrix} 0.3 & 0.7 & 0 \\ 0.1 & 0.4 & 0.5 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{align}
\pi_0 = 0.3\pi_0 + 0.1\pi_1 + \pi_2 \tag{6}\\
\pi_1 = 0.7\pi_0 + 0.4\pi_1 \tag{7}\\
\pi_2 = 0.5\pi_1 \tag{8}\\
\pi_0 + \pi_1 + \pi_2 = 1 \tag{9}
\end{align}$$

lets write everything interms of π_0

$$\begin{align}
\pi_1 = \pi_1 \tag{10}\\
\pi_2 = 0.5\pi_1 \tag{11}\\
\pi_0 = 0.3\pi_0 + 0.1\pi_1 + 0.5\pi_1 \tag{12}\\
0.7\pi_0 = 0.6\pi_1 \tag{13}\\
\pi_0 = \frac{6}{7}\pi_1 \tag{14}\\
\pi_0 + \pi_1 + \pi_2 = \frac{6}{7}\pi_1 + \pi_1 + 0.5\pi_1 = 1 \tag{15}
\end{align}$$

$$\frac{33}{14}\pi_1 = 1 \tag{16}$$

$$\pi_1 = \frac{14}{33} \tag{17}$$

$$\pi_0 = \frac{6}{7}.\frac{14}{33} = \frac{4}{11} \tag{18}$$

$$\pi_2 = 0.5.\frac{14}{33} = \frac{7}{33} \tag{19}$$

Ans.

$$\pi = \begin{bmatrix} \frac{4}{11} & \frac{14}{33} & \frac{7}{33} \end{bmatrix}$$

$$\pi \approx \begin{bmatrix} 0.3636 & 0.4242 & 0.2121 \end{bmatrix}$$

(c) Expected Time from 0 to 2

$$\beta(2) = 0$$

$$\beta(0) = 1 + 0.7\beta(1) + 0.3\beta(0)$$

$$\beta(1) = 1 + 0.1\beta(1) + 0.4\beta(1) + 0.5\beta(2)$$

since $\beta(2) = 0$

$$\beta(1) = 1 + 0.1\beta(0) + 0.4\beta(1)$$

$$\beta(0) = 1 + 0.3\beta(0) + 0.7\beta(1)$$

$$0.7\beta(0) - 0.7\beta(1) = 1$$

$$0.6\beta(1) - 0.1\beta(0) = 1$$

solving the eqn n1 and n2 we get $\beta(0) = 3.708$ and $\beta(1) = 2.286$ so the expected time from 0 to 2 is 3.708

(d) Probability that starting from 0, the MC has reached 2 after n-steps vs n

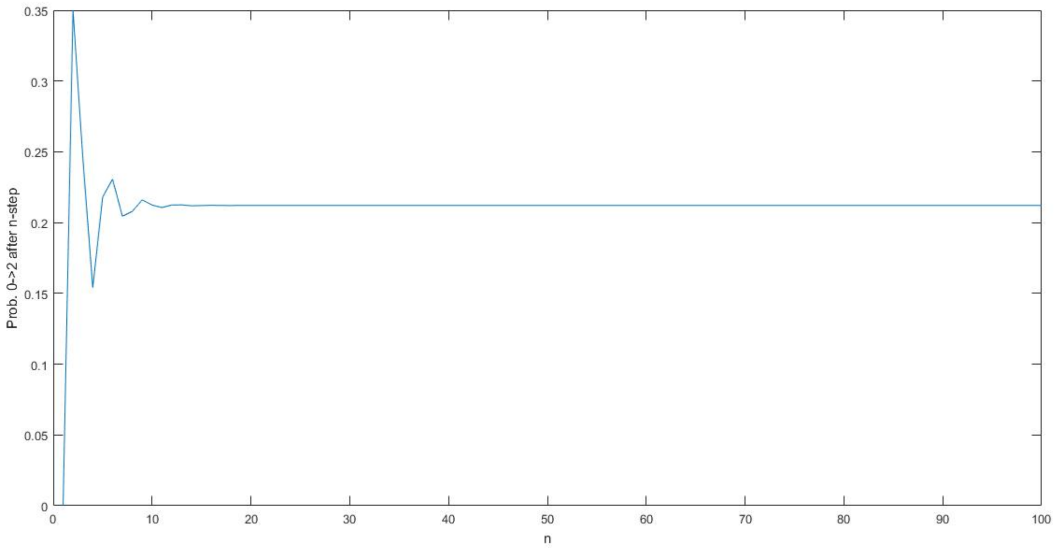


Figure 4: Probability that starting from 0, the MC has reached 2 after n-steps vs n

(e) Expected Time from 0 to 2

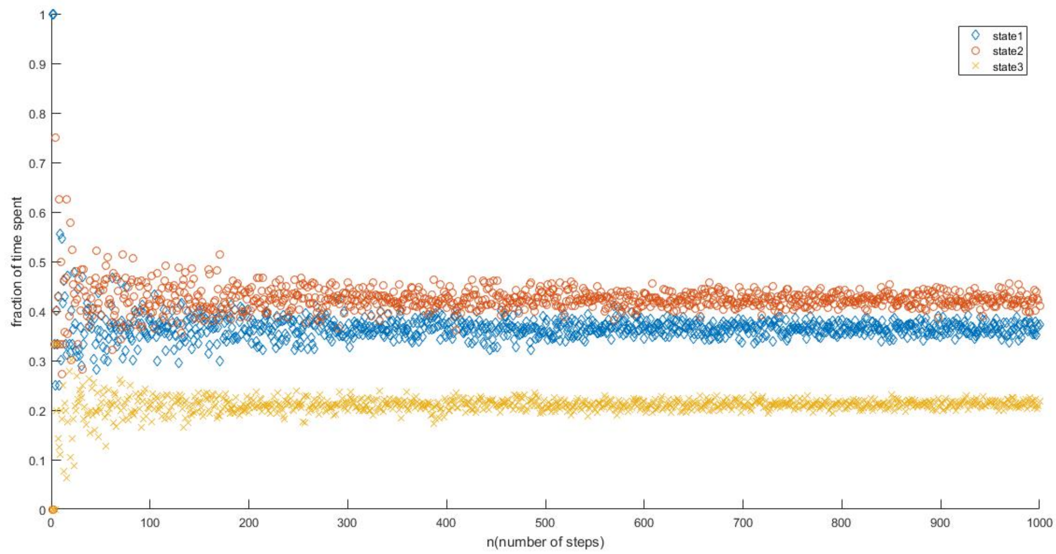


Figure 5: Probability that starting from 0, the MC has reached 2 after n-steps vs n

(f) π_n vs n

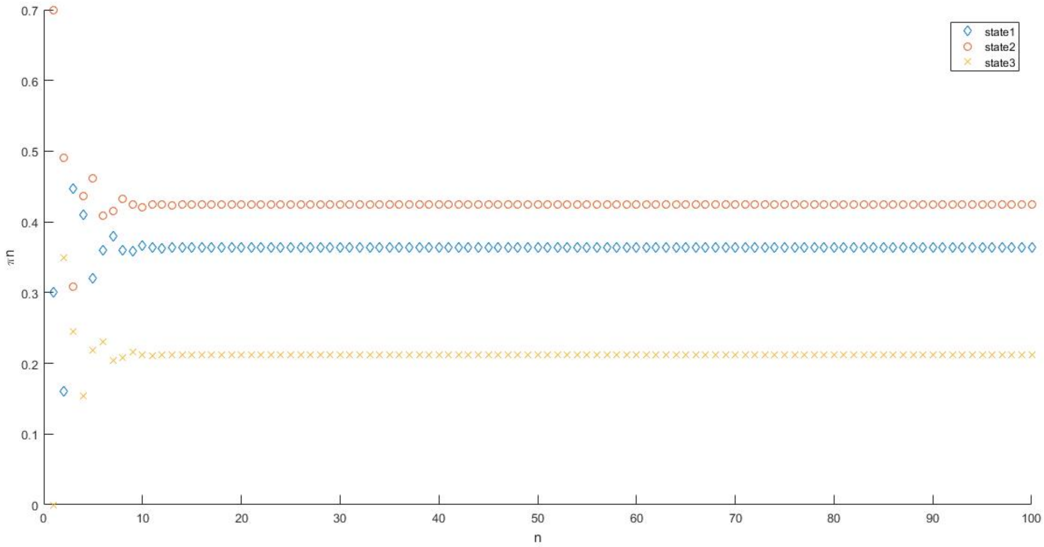


Figure 6: π_n vs n

Q3