- If $P(A) > 0, P(B) > 0, P(A \cap B^c) = P(B \cap A^c)$, show $P(A|B) = P(B|A)$
  - $P(A) = P(A \cap B^c) + P(A \cap B) = P(B \cap A^c) + P(A \cap B) = P(B) > 0$
  - $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)} = P(B|A)$

- Random variable X $\sim$ Geometric(p), show that $E(X) = 1/p$
  - $E(X) = \sum\limits_{k=1}^{\infty} k \cdot p(k) = \sum\limits_{k=1}^{\infty} k(1-p)^{k-1}p$
  - $(1-p)E(X) = \sum\limits_{k=1}^{\infty} k(1-p)^k p$, let $k = j-1$
  - $= \sum\limits_{j=2}^{\infty} (j-1)(1-p)^{j-1}p$
  - $E(X) - (1-p)E(X) = 1 \cdot (1-p)^{1-1} \cdot p + \sum\limits_{k=2}^{\infty} (1-p)^{k-1}p = 1$
  - $pE(X) = 1 \rightarrow E(X) = 1/p$

- X is a random variable, show that $E(X^2) \geq [E(X)]^2$
  - $Var(X) = E(X^2) - [E(X)]^2 \geq 0$

- Random variable X $\sim$ Bin(n,p), show that $E(X) = np$
  - $E(X) = \sum\limits_{k=0}^{n} k \cdot \binom{n}{k} p^k (1-p)^{n-k}$
  - $= \sum\limits_{k=1}^{n} k \binom{n-1}{k-1} \frac{n}{k} \cdot p^{k-1} \cdot p \cdot (1-p)^{(n-1)-(k-1)}$, let $j = k-1$
  - $= np \sum\limits_{j=0}^{n-1} \binom{n-1}{k-1} p^{k-1} (1-p)^{(n-1)-(k-1)} = np \cdot [p + (1-p)]^{n-1} = np$
- Drawing n distinct items, number of ways to draw r of them
  - Without replacement & order matters- $n!/(n-r)!$
  - With replacement & order matters- n^r
  - Without replacement & order doesn't matter- $\binom{n}{r} = n!/(n-r)!r!$

- If $E \subset F$ and $F \subset E$ then $E = F$
- If $E \subset F$, then $P(E) \leq P(F)$
- $F = E \cup E^c F, P(F) = P(E) + P(E^c F)$ since $P(E^c F) \geq 0$

- Prove that $P(E \cup F) = P(E) + P(F) - P(EF)$
  - $P(E \cup F) = P(E \cup E^c F) = P(E) + P(E^c F)$
  - $F = EF \cup E^c F, P(F) = P(EF) + P(E^c F), P(E^c F) = P(F) - P(EF)$