1 Distributions

BERNOULLI DISTRIBUTION

$P(X=1)=p, P(X=0)=1-p$
pmf- $p^x(1-p)^{1-x}$
$\mu = p$
$\sigma^2 = p(1-p)$

BINOMIAL DISTRIBUTION

X $\sim Bin(n,p)$
$\mu = np$
$\sigma^2 = np(1-p)$
$P(k)=P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$

POISSON DISTRIBUTION

X $\sim P(\lambda) \rightarrow p(k) = e^{-\lambda}\frac{\lambda^k}{k!}$
$\mu = \lambda$
$\sigma^2 = \lambda$
P$(np) \approx$ Bin(n,p)

GEOMETRIC DISTRIBUTION

X $\sim$ Geometric$(p)p(k)=(1-p)^{k-1}p$
$\mu = \frac{1}{p}$
$\sigma^2 = \frac{1-p}{p^2}$
·Geometric distribution is memoryless

NEGATIVE BINOMIAL DISTIBUTION

$X \sim \mathrm{NB}(r,p) \rightarrow p(k) = \binom{k-1}{r-1}(1-p)^{k-r}p^r$
$X \sim$ Geometric$(p) = X \sim$ NB$(1,p)$
$\mu = \frac{r}{p}$
$\sigma^2 = \frac{r(1-p)}{p^2}$

2 Continuous Random Variables

F(X) = CDF, f(x) = pdf
For a pdf to be valid,
$\int_{-\infty}^{\infty} f(x)dx = 1$
$F(a) = P(X \leq a) = \int_{-\infty}^{a} f(x)dx$
$P(a \leq X \leq b) = F(b) - F(a) = P(a < X < b)$
$f(x) = \frac{d}{dx}F(x)$
Expected value
$E[g(x)] = \sum g(x)f(x) = \int_{-\infty}^{\infty} g(x)f(x)dx$
$E[X+Y] = E[X] + E[Y]$
$E[aX+b] = aE[x] + b$
$Var(aX+b) = a^2Var(x)$
For even function, E[x] = 0

UNIFORM DISTRIBUTION

$f(x) = c \quad \alpha \leq x \leq \beta$
$f(x) = 0 \quad$ otherwise
$c = \frac{1}{\beta - \alpha}$
CDF
0 $\quad x \leq \alpha$
$\frac{x-\alpha}{\beta-\alpha} \quad \alpha < x < \beta$
1 $\quad x \geq \beta$
$\mu = \frac{\alpha+\beta}{2}$
$\sigma^2 = \frac{(\beta-\alpha)^2}{12}$

NORMAL APPROX. TO BINOMIAL

P(X = i) $\approx P(i - 0.5 < Y < i + 0.5)$

EXPONENTIAL DISTRIBUTION

$X \sim \mathrm{Exp}(\lambda) \rightarrow f(x) = \lambda e^{-\lambda x} \quad x \geq 0$
$F(X) = 1 - e^{-\lambda x}$
$\mu = \frac{1}{\lambda}$
$\sigma^2 = \frac{1}{\lambda^2}$
Distribution is memoryless
$E(\max(\lambda)) = \frac{3}{2\lambda}$
Min- Exp$(\lambda_1 + \lambda_2)$

GAMMA DISTRIBUTION

$X \sim G(\alpha, \lambda) \rightarrow \frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda x} \quad x \geq 0$
Exp$(\lambda) = G(1,\lambda)$
$\mu = \frac{\alpha}{\lambda}$
$\sigma^2 = \frac{\alpha}{\lambda^2}$