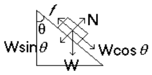
FYI

- $Deg = Rad \times \frac{180}{\pi}$ F=ma M=F(rsinθ)=F($d_{\perp}$)



- Cartesian to polar: $r = \sqrt{x^2+y^2}$ | $\theta = \tan^{-1}\left(\frac{y}{x}\right)$
- Polar to Cartesian: $x = r\cos\theta$ | $y = r\sin\theta$
- Gravity $9.81\frac{m}{s^2} = 32.2\frac{ft}{s^2}$

Kinematic for S, V, A as f(t) of time

$s = s(t) \rightarrow v = \frac{ds}{dt}$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

$v = v(t) \rightarrow s = s_0 + \int_{t_0}^{t} v(t)\,dt$

$a = a(t) \rightarrow v = v_0 + \int_{t_0}^{t} a(t)\,dt$

Kinematic for V, A, T as f(s) of position

$t = t(s) \rightarrow v(s) = 1/(\frac{dt}{ds})$

$v = v(s) \rightarrow a(s) = v(s)\frac{dv}{ds}$

$t(s) = t_0 + \int_{s_0}^{s} \frac{ds}{v(s)}$

$a = a(s) \rightarrow v(s) = \pm\sqrt{{v_0}^2 + 2\int_{s_0}^{s} a(s)\,ds}$

Kinematic for S, A, T as f(v) of speed

$t = t(v) \rightarrow a(v) = 1/(\frac{dt}{dv})$

$a = a(v) \rightarrow s(v) = s_0 + \int_{v_0}^{v} \frac{v}{a(v)}\,dv$

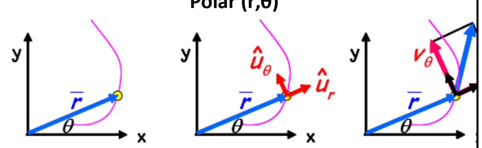
$t(v) = t_0 + \int_{v_0}^{v} \frac{dv}{a(v)}$

$s = s(v) \rightarrow a(v) = v/(\frac{ds}{dv})$

Normal Acceleration

$a_n = \frac{v^2}{\rho} = v\dot{\theta}$ | $\rho(rofcurv) = [1 + \left(\frac{dy}{dx}\right)^2]^{3/2}/\left|\frac{d^2y}{dx^2}\right|$

Polar (r,θ)



Path Definition Radial and Transverse Unit Vectors Velocity

$\bar{r} = ru_r$

$\bar{v} = \frac{dr}{dt} = \dot{r}\,u_r + r\dot{\theta}u_\theta = v_r u_r + v_\theta u_\theta$

$\bar{a} = \frac{dv}{dt} = (\ddot{r} - r\dot{\theta}^2)u_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})u_\theta$

$= a_r u_r + a_\theta u_\theta$

Chain Rule: $\dot{r} = \dot{\theta}\frac{dr}{d\theta}$ $\ddot{r} = \dot{\theta}^2\frac{d^2r}{d\theta^2} + \ddot{\theta}\frac{dr}{d\theta}$

|Convert polar|| $u_r = (\cos\theta)i + (\sin\theta)j$

|to Cartesian || $u_\theta = -(\sin\theta)i + (\cos\theta)j$

$\bar{F} = F_r u_r + F_\theta u_\theta$

Cartesian (n,t)

$\bar{v} = v\bar{\boldsymbol{u}}_t = \dot{s}\bar{\boldsymbol{u}}_t$

$\bar{\boldsymbol{a}} = a_t\bar{\boldsymbol{u}}_t + a_n\bar{\boldsymbol{u}}_n$

$\bar{\boldsymbol{a}} = \dot{v}u_t + \left(\frac{v^2}{\rho}\right)u_n$

Polar & Cartesian

$\tan\Psi = r/\frac{dr}{d\theta}$

$\Psi = \measuredangle\frac{b}{w}u_t$ & $u_r = \measuredangle b/w\ a_r$ & a_t

Moment = $\circlearrowleft\sum M = Fd + Fd$

u_n = Unit Normal

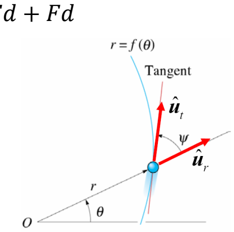
u_t = Unit Tangential (Cartesian)

u_r = Unit Radial (Polar)

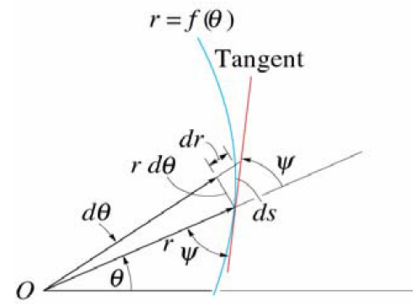
u_θ = Unit Transverse

$\sum F = 0$ If Static

$\uparrow\sum F_y = F_y + g$



Relating (n,t) and (r,θ)



$v_r = v\cos\theta = \dot{r}$ $v_\theta = -v\sin\theta = r\dot{\theta}$

$a_r = a\sin\theta = \ddot{r} - r\dot{\theta}^2$ $a_\theta = a\cos\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$\tan\Psi = \frac{rd\theta}{dr} = r/\frac{dr}{d\theta}$

Friction Forces

$F_f \le F_{f(max)} = \mu_s F_n$ | $F_{f(dynamic)} = \mu_k F_n$

$F_{static} = -F_{applied}$

Types of Problems

- Member with Weight / Pulley / Pin
  - $\rightarrow\sum F_x$ & $\uparrow\sum F_y$
  - $\circlearrowleft or \circlearrowright \sum M_{pin}$
- Objects Moving Together / Friction
  - Split Objects, FBD / KD
  - Compare F_{static} & F_{max}
  - Check for Acceleration
- S/V/A : $a = \dot{v} = \ddot{s}$ [Use Kinematic Eqns, ∫ & plug in]

$\boldsymbol{Arclength_{semicircle}} = \theta * \boldsymbol{r}$

Centroids:

$dA = (top\ curve - bottom\ curve)dx;\ x_{el} = x$

$y_{el} = top - \left(\frac{top - bottom}{2}\right) = \frac{top + bottom}{2}$

$A = \int dA = \int(height)dx$

$V = \int dV = \int Adx$

Area (curve): $X_G = \frac{\int X_{el}dA}{A}$ | $Y_G = \frac{\int Y_{el}dA}{A}$

Area (figure): $A = A_1 + A_2 + A_3 + \cdots$

x-coord of centroid:

$X_G = \frac{(C_{G1x})(A_1) + (C_{G2x})(A_2) + (C_{G3x})(A_3) + \cdots}{A}$

Volume (figure): $X_G = \frac{\int X_{el}dV}{V}$ | $Y_G = \frac{\int Y_{el}dV}{V}$

Center of Mass(Homogenous Bar):

$CG_{semi} = \frac{2r}{\pi}$; $L = L_1 + L_2 + L_3$

$X_G = \frac{(C_{G1x})(L_1) + (C_{G2x})(L_2) + (C_{G3x})(L_3)}{L}$

$Y_G = \frac{(C_{G1y})(L_1) + (C_{G2y})(L_2) + (C_{G3y})(L_3)}{L}$

$Z_G = \frac{(C_{G1z})(L_1) + (C_{G2z})(L_2) + (C_{G3z})(L_3)}{L}$

Center of Mass(Homogenous Object):

Substitute V for L in previous eqns

Moments of Inertia:

$I_O = I_G + (mass)(dist\ O \rightarrow CG)^2$

$I_G = \frac{1}{12}ml^2$

$m = \frac{W}{g} = \rho V = \rho\int V = \rho\int yt\,dx$

$dm = \rho ty\,dx$ | $I_x = \int_A x^2 dA$

Under Curve: (about y-axis) → $I_y = \int x_{el}^2\,dA$

About y-axis thru centroid: $I_{y_G} = I_y - A{x_G}^2$

About x-axis: $I_x = \int\frac{1}{12}y^3dx + \int y_{el}^2\,dA$

Thin Plates: $I_y = \int x_{el}^2\,dm$; $\boldsymbol{CG_{semicircle}} = \frac{\boldsymbol{4r}}{\boldsymbol{3\pi}}$

$I_x = \int\frac{1}{12}y^2dm + \int y_{el}^2\,dm$

Homogenous Bars: $I_O = I_{O1} + I_{O2}$

CG:$X_G = \frac{(m_1)(dist\ O \rightarrow CG_1) + (m_2)(dist\ O \rightarrow CG_2)}{m_{total}}$

| | Axis Thm: Mass $I_O = I_G + (mass)(X_G)^2$

Area $(I_P)_x = (I_G)_x + A(y_G)^2$

Homogenous 3-D Part: find dist CG → O

$m = \rho V$ $I_{G_{rect}} = \frac{m}{12}(a^2 + b^2)$

$\frac{mr^2}{2} = I_{half_{cyl}} = I_G + (m)(\frac{4r}{3\pi})$

Area radius of gyration: $k_x = \sqrt{\frac{I_x}{A}}$ $k_y = \sqrt{\frac{I_y}{A}}$

Complex Figure:

$I_{x_{rect}} = \frac{1}{12}(base)(height)^3 + A(dist\ O \rightarrow CG)^2$

$I_{x_{sphere}} = \frac{1}{4}\pi r^4 + A(dist\ O \rightarrow CG)^2$

Forces and Moments:

$F_{Tot} = F_1 + F_2 + F_3 \ldots$

$\bar{F} = Fcos\theta_x i + Fcos\theta_y j + Fcos\theta_z k$

$1 = cos^2\theta_x + cos^2\theta_y + cos^2\theta_z$

$F_{xz} = Fcos(90 - \theta_y) = Fsin\theta_y$

$F_x = F_{xz}cos\theta_{xz}$ | $F_y = Fcos\theta_y$ | $F_z = F_{xz}sin\theta_{xz}$

$\lambda_{1\rightarrow 2} = \frac{((x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$

$\bar{F}_{12} = F\ \ \lambda_{1\rightarrow 2}$ | $F_{1\rightarrow 2} = \bar{F}\cdot\lambda_{1\rightarrow 2} = Fcos\theta$

$\sum\bar{M}_O = \bar{r}_{OF} \times \bar{F}$

$\sum M_{line_{1\rightarrow 2}} = 0 = \sum\bar{M}_1\cdot\lambda_{1\rightarrow 2} = M_1 cos\theta$

Trusses:

Sum reaction forces

Find angles

Sum forces at joints to find unknowns

3-D Rigid Body:

CG: $X_G = \frac{(C_{G1x})(W_1) + (C_{G2x})(W_2) + (C_{G3x})(W_3)}{W}$

$Y_G = \frac{(C_{G1y})(W_1) + (C_{G2y})(W_2) + (C_{G3y})(W_3)}{W}$

$Z_G = \frac{(C_{G1z})(W_1) + (C_{G2z})(W_2) + (C_{G3z})(W_3)}{W}$

Mass Moment of Inertia: (x-axis)

$\boldsymbol{I_{x_{rect}} = m\left[\frac{1}{12}(z^2 + y^2) + ({CG_z}^2 + {CG_y}^2)\right]}$

$\boldsymbol{I_{x_{cyl}} = m\left[\left(\frac{1}{4}r^2 + \frac{1}{12}h^2\right) + ({CG_z}^2 + {CG_y}^2)\right]}$

If length of cylinder is along x-axis:

$I_{x_{cyl}} = m\left[\frac{1}{2}r^2 + ({CG_z}^2 + {CG_y}^2)\right]$

Thru CG: $I_x = I_{x_G} + m({y_G}^2 + {z_G}^2)$

General:

$F_f = \mu_s F_n$ at limit of no slip

With friction there is normal force

Forces at a pin on one bar are = and opposite of the same pin on the other bar

Pulley w/ rope over it, $T_{rope} = F_{px}$ & F_{py}

To find α, have to have a_{CG} values

Kinetics:

$\rightarrow\sum\boldsymbol{F_x} = \boldsymbol{F_x} = \boldsymbol{ma_{gx}}$

$\uparrow\sum\boldsymbol{F_y} = \boldsymbol{F_y} = \boldsymbol{ma_{gy}}$

$\circlearrowleft\sum\boldsymbol{M_A} = \boldsymbol{M_A} = \boldsymbol{I_A\alpha}$

3-D:

$\sum M_O = 0 = M_{Ox} + M_{Oy} + M_{Oz} + (F \times r)$

Kinematics:

Fixed axis rotation:

$V_A = V_B + V_{A/B}$

$a_A = a_B + a_{A/B}$

Not fixed axis: $I_G = \frac{1}{12}ml^2$

| | Axis Thm:

$I_O = I_G + (mass)(dist\ O \rightarrow CG)^2$

$I_A = I_G + m(dist\ O \rightarrow CG)^2$

$\bar{V}_B = \bar{V}_A + (\bar{\omega} \times r_{B/A})$

$\bar{a}_B = \bar{a}_A + (\alpha \times \bar{r}_{B/A}) - \omega^2(\bar{r}_{B/A})$

Pt B on circle origin O: $a_B = -\frac{v^2}{\rho}$ (ρ=r)

2Disks w/ no slip: $v_{contact}; a_{contact}$ *are equal*

$\omega = \omega_o + \int_0^t \alpha\, dt$

Fixed Pt: $\sum M_{O_{Bar}} = I_O \alpha_{Bar} = \frac{1}{3} ml^2 \alpha_{Bar}$

CG: $\sum M_{G_{Bar}} = I_{G_{Bar}} \alpha_{Bar} = \frac{1}{12} ml^2 \alpha_{Bar}$

$\sum M_{G_{Disk}} = I_{Disk} \alpha_{Disk} = \frac{1}{2} mr^2 \alpha_{Disk}$

Shear and Moment Equations:

FBD & External reactions;

Cut at right, V↓ ,M ↺ , F_{axial} ⟶

Distance is x; Plot data

For constant applied force, F=#*distance and is located half way down beam

Two Force Members:

No mass or weight

End forces are equal and opposite

Disks w/ Rope: $\omega = \frac{v}{R}$

$\beta = angle\ btw\ ropes\ on\ top\ of\ pulley\ (rad)$

$T_2 = T_1 e^{\mu_s \beta}$ T_2 is in direction of motion

Either may be F_A or W ^ slip or verge of

$\circlearrowleft \sum M_{pin} = 0 = T_2 r + T_1 r + M$

$I_{G_{Disk}} = m(radius\ of\ gyration)^2$

Pulleys: $L_{rope} = S_A + S_B + constants$

$\dot{L} = V_A + V_B = 0$; $\ddot{L} = a_A + a_B = 0$

arnd || axis & radofgyr(k_r): $I_{G_{BAR}} = m \cdot (k_r)^2$

Raising winch → $\dot{L} = -\omega r$

No rope slip: $\alpha = \frac{a}{R}$

Disks no slip: $a_G = \alpha R$

General:

When you are given a radius of gyration and rotating about parallel axis, the moment of inertia is:

$I_G = mk^2$; $k = radius\ of\ gyration$

Fixed Pt A: $I_A = \frac{1}{3} ml^2$

Pt not CG: $I_A = \frac{1}{12} ml^2 + m(dist \to CG)^2$

$v = \omega r$; $a = \omega^2 r$; $a = \alpha r$

Object displacing – work energy

Use kinematics to relate v and ω

$\%\ Energy\ Loss = \frac{T_1 - T_2}{T_1}$

$F > F_f =$ *object moves*

Weight is non-impulsive

$I_{ball\ rolling\ no\ slip} = \frac{2}{5} mr^2$

W-E: U[work] ; T[kE] ; V[pE] {position/speed/force/moments}

$T_1 + U_{1\to2} = T_2$; $\theta[rad] = \frac{S}{R}$; $U_{1\to2} = Fd$

$T = \frac{1}{2} mv^2 + \frac{1}{2} I_G \omega^2$; $U_{1\to2} = M_G \theta$

energy cons. $V_1 + T_1 = V_2 + T_2$

V=W=Mθ $V_{spring} = \frac{k_s}{2} s^2 = \frac{k_\theta}{2} \theta^2$

Power: $P = Fv = M\omega$

Spring: Uon spring = ½ ks(s_2^2-s_1^2)

$U_{on\ body} = -\frac{k_s}{2}({s_2}^2 - {s_1}^2)$

$U_{on\ body} = -\frac{k_\theta}{2}({\theta_2}^2 - {\theta_1}^2)$

I-M: {Δvelocity / Δtime} while collision

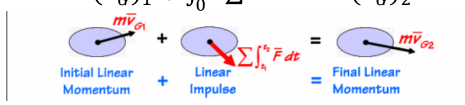
Impulsive – magnitude is fn of t impulse

Non-Impulsive – magnitude not fn of t impulse

Impulse can be: $I = \frac{1}{2} t * \sum F = mv$

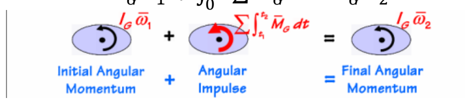
Linear: internal forces only, impulse = 0

$$m(\bar{v}_G)_1 + \int_0^{\Delta t} \sum \bar{F} dt = m(\bar{v}_G)_2$$



Angular:

About CG: $I_G \bar{\omega}_1 + \int_0^{\Delta t} \sum \bar{M}_G dt = I_G \bar{\omega}_2$



Translation w/ fixed pt:

- $(\bar{H}_p)_1 + \int_0^{\Delta t} \sum \bar{M}_p dt = (\bar{H}_p)_2$

$(\bar{H}_p)_1 = I_G \bar{\omega}_1 + r_{G/p1} \times m\bar{v}_{G1}$

$(\bar{H}_p)_2 = I_G \bar{\omega}_2 + r_{G/p2} \times m\bar{v}_{G2}$

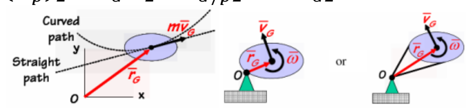


Figure 10.3 Angular momentum associated with translation.

Figure 10.4 Angular momentum when rotation is about a fixed axis.

For ω_{max} find $\lim_{t\to\infty} fn$

Impacts w/ Rebound:

LOI - line of impact – perpendicular to TP

Central – centered – LOI through both CGs

Eccentric – off-center – LOI not through both CGs

Direct – V & LOI vectors are parallel

Oblique – one or both V vectors not parallel to LOI

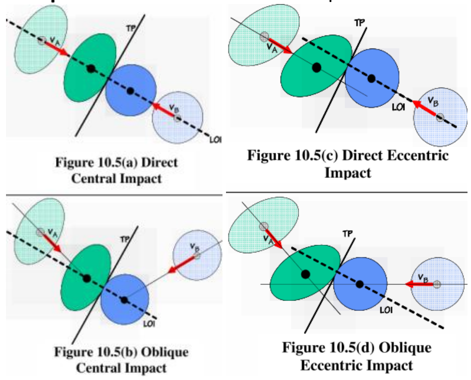


Figure 10.5(a) Direct Central Impact

Figure 10.5(c) Direct Eccentric Impact

Figure 10.5(b) Oblique Central Impact

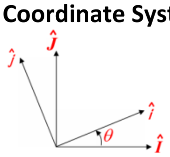
Figure 10.5(d) Oblique Eccentric Impact

$$e = \frac{V_{BN2} - V_{AN2}}{V_{AN1} - V_{BN1}}$$

If no external impulses:

$m_A v_{AN1} + m_B v_{BN1} = m_A v_{AN2} + m_B v_{BN2}$

Coordinate Systems: xyz → XYZ



$\hat{\imath} = (cos\theta)\hat{I} + (sin\theta)\hat{J}$

$\hat{\jmath} = -(sin\theta)\hat{I} + (cos\theta)\hat{J}$

- $(\bar{\boldsymbol{V}}_B)_{XYZ} = (\bar{\boldsymbol{V}}_A)_{XYZ} + \left(\bar{\boldsymbol{V}}_{B/A}\right)_{xyz} + \bar{\boldsymbol{\Omega}} \times \left(\boldsymbol{r}_{B/A}\right)_{xyz}$

- $(\bar{\boldsymbol{a}}_B)_{XYZ} = (\bar{\boldsymbol{a}}_A)_{XYZ} + \left(\bar{\boldsymbol{a}}_{B/A}\right)_{xyz} + \dot{\bar{\boldsymbol{\Omega}}} \times \left(\bar{\boldsymbol{r}}_{B/A}\right)_{xyz} - \boldsymbol{\Omega}^2 \left(\boldsymbol{r}_{B/A}\right)_{xyz} + 2\bar{\boldsymbol{\Omega}} \times (\bar{\boldsymbol{v}}_{B/A})_{xyz}$

Translation: movement of a pt or mass moving on a body along a fixed path (track, slot) & that path itself moves and rotates w/ the body.

Tangent (no friction): v's sign conv. || to LOI

$(v_{BT})_1 = (v_{BT})_2$ & $(v_{AT})_1 = (v_{AT})_2$

Notes:

S=rθ ; v=ωr ; a=αr

RadOfGyr=$k_G = \sqrt{I_g/m}$

Inertia of anything = $I_P = I_G + md^2$

Inertia of disc end of bar (non rotate disk):

$I_P = I_{Bar} + m_D d^2$

Inertia of bar = $I_G = {}^1/_{12}\, ml^2$

$I_{End} = {}^1/_3\, ml^2$

Inertia of disk = $I_G = {}^1/_2\, mr^2$

$I_G = mk^2$

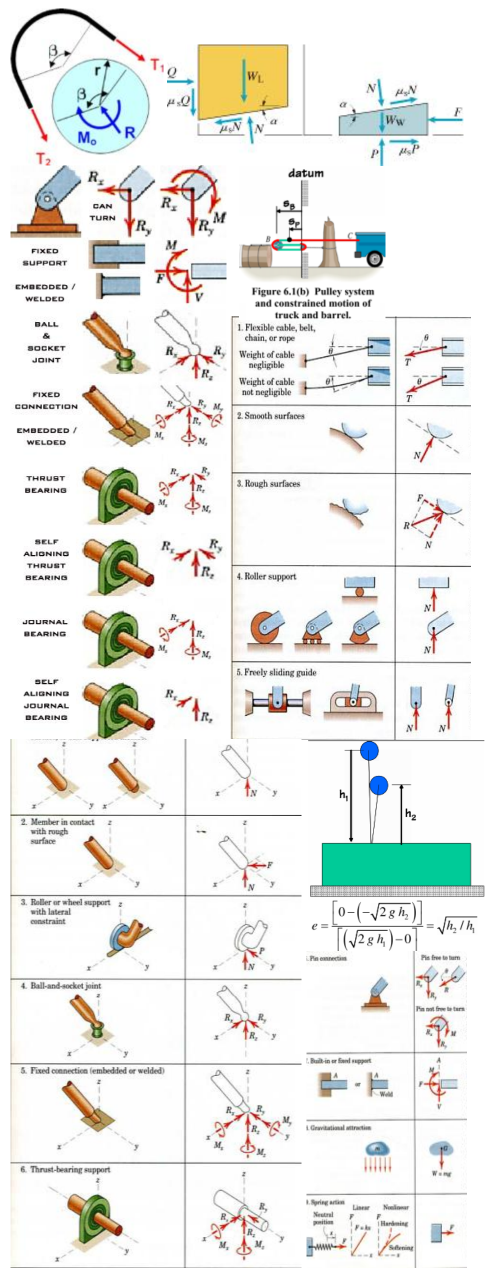


Figure 6.1(b) Pulley system and constrained motion of truck and barrel.