1 Kirchoff's Current Law

- $\sum_{n=1}^{N} i = 0$
- Current pointing into a node is negative, pointing out is positive

2 Kirchoff's Voltage Law

- $\sum^{N} \pm v_n = 0$
- $\Delta V = IR$
- Voltage rise (+V) means entering from - and going to +
- Voltage drop (-V) enters at + and leaves at -

3 Resistors

Series

- $R_{series} = R_1 + R_2 + R_3 + \cdots$
- $Q = Q_1 = Q_2 = Q_3 = \cdots$
- $\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \cdots$
- Current same everywhere, voltage breaks up

Parallel

- $R_{parallel} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots\right)^{-1}$
- $Q = Q_1 + Q_2 + Q_3 + \cdots$
- $\Delta V = \Delta V_1 = \Delta V_2 = \Delta V_3 = \cdots$
- Voltage same everywhere, current breaks up

4 Power

- $P = I\Delta V = RI^2 = \frac{\Delta V^2}{R}$
- When current flows into +, power is dissipated (P = IV)
- When current flows into -, power is supplied (P = -IV)
- 1 W = 1 V·A

5 Energy

- $E = vi\Delta t$(Watt·seconds)
- $V \cdot Ah$ approx. = Ah when voltage is implied

6 Nodal Analysis

- $i_{node} = \frac{v_{node} - v_{adjacent}}{R}$
- $\sum v_n = (i_{mesh} - i_{adjacent})R$ (mesh current)
- Node voltages uses KCL, mesh current uses KVL

7 Equivalent Circuits

- Thevenin is a single voltage source and a resistor in series
- Norton is a single current source and a resistor in parallel
- Max resistance= $P_{max} = \frac{V_{th}^2}{4R_{th}}$

8 Misc.

- Number of equations $= n - m - 1$
- $I = \frac{dq}{dt}$
- $I_{avg} = \frac{\Delta Q}{\Delta t}$
- $R = \frac{\rho l}{A} = \frac{L}{\sigma A}$
- $\Delta V_{bat} = \epsilon - Ir$
- $\sum_{loop} \Delta V = 0$
- $\sum I_{in} = \sum I_{out}$
- $V_1 = \frac{R_n}{R_1 + R_2 + \ldots} V_{tot}$ (voltage divider)
- $I_n = \frac{\left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots\right)^{-1}}{R_n} I_{tot}$ (current divider)
- If current comes from + terminal, $V = -iR$, from -, $V = iR$
- No current flows through open circuit or switch
- Short circuit- $R = 0$
- **Constant voltage source**- produces a constant voltage not affected by other components, current determined by connections to other components and can supply any current (car battery)
- **Constant current source**- produces a constant current not affected by connections to other components, voltage determined by connections to other components and can supply any voltage (cell phone charger)
- **Terminal**- point where a component or part of the circuit connects to other components of the circuit
- **Node**- connection point
- **Branch**- portion of a circuit with only two external terminals
- **Loop**- a closed connection of branches
- **Mesh**- a loop that does not contain other loops
- **Voltage**- work done to move charge between two points
- Mesh is a loop but a loop is not necessarily a mesh

9 Equivalent Circuits

- Any part of the circuit with two terminals can be replaced by a single voltage source and a resistor in series for Thevenin
- Find R_{eq} and find open circuit voltage for Thevenin
- Any part of the circuit with two terminals can be replaced by a single current source and a resistor in parallel for Norton

10 Loops

- For analysis, $n-m-1$ equations where m is the number of dependent loops
- $i_{node} = \frac{V_{node} - V_{adjacent}}{R}$
- For mesh current, $\sum V_n$ around mesh
- $(i_{mesh} - i_{adjacent})R$

11 Short and Open Circuits

- Voltage source is a short circuit
- Current source is an open circuit

12 Misc.

- $P_{max} = \frac{V_{th}^2}{4R_{th}}$

13 Blank Space

14 Capacitors

- $C = \frac{\epsilon_0 \epsilon_r A}{D}$ Farad (F) (D = plate distance, A = plate area, $\epsilon_0 \epsilon_r$ = property of material between plates)
- Capacitors combined opposite of resistors

Time Domain

- $i_c(t) = C\frac{dv_c(t)}{dt}$
- $p(t) = v(t)C\frac{dv(t)}{dt}$
- $W = \frac{1}{2}Cv^2(t)$
- $v_s(t) = V_{max}\cos(\omega t)$
- $i = CV_{max}\omega\cos(\omega t + \frac{\pi}{2})$

Frequency Domain

- $V_C(j\omega) = V_S(j\omega) = V_{max}\angle 0°$
- $I_C(j\omega) = CV_{max}\omega\angle 90°$
- $\frac{V_C(j\omega)}{I_C(j\omega)} = \frac{1}{C\omega}\angle -90°\Omega$

15 Inductors

- $L = \frac{\mu_0 \mu_r N^2 A}{l}$ Henry (H) (A = coil area, l = coil height, $\mu_0 \mu_r$ = property of core material, N = number of coil turns)
- Inductors combined same as resistors

Time Domain

- $v_L(t) = L\frac{di_L(t)}{dt}$
- $p(t) = L\frac{di(t)}{dt}i(t)$
- $W = \frac{1}{2}Li^2(t)$
- $v_s(t) = V_{max}\cos(\omega t)$
- $i = \frac{V_{max}}{\omega L}\cos(\omega t - \frac{\pi}{2})$

Frequency Domain

- $V_L(j\omega) = V_S(j\omega) = V_{max}\angle 0°$
- $I_L(j\omega) = \frac{V_{max}}{\omega L}\angle -90°$
- $\frac{V_L(j\omega}{I_L(j\omega)} = \omega L\angle 90°\Omega$

16 Resistors

Time Domain

- $v_s(t) = V_{max}\cos(\omega t)$
- $i = \frac{V_{max}}{R}\cos(\omega t)$

Frequency Domain

- $V_R(j\omega) = V_S(j\omega) = V_{max}\angle 0°$
- $I_R(j\omega) = \frac{V_{max}}{R}\angle 0°$
- $\frac{V_R(j\omega)}{I_R(j\omega)} = \frac{V_{max}\angle 0°}{\frac{V_{max}}{R}\angle 0°} = R\angle 0°\Omega$

17 Power & Energy

- Power can be absorbed or supplied for both
- All energy dissipated as heat for resistor, stored in magnetic field for inductor (0 if i = 0), electric field for capacitor (0 if v = 0)

18 AC Sources

- $f = \frac{1}{T}$ Hz (frequency)
- $\omega = 2\pi f = \frac{2\pi}{T}$ rad/s
- $\phi = 2\pi\frac{\Delta t}{T}$ rad $= 360\frac{\Delta t}{T}$ deg (phase shift)
- $x(t) = A\cos(\omega t + \phi)$ (A = amplitude)
- $V_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt} = \frac{V_{max}}{\sqrt{2}}$

19 Phasors

- $v(t) = A\cos(\omega_0 t + \theta) \rightarrow V(j\omega) = Ae^{j\theta} = A\cos\theta + jA\sin\theta = A\angle\theta$

20 Impedance

- $Z(j\omega) = \frac{V(j\omega)}{I(j\omega)}$
- $V_T = I_N Z$
- Combined similar to resistors for total Z
- Thevenin equivalent combines a single voltage source and impedance in series
- Norton equivalent consists of a single current source and impedance in parallel
- For resistors, $Z_R = R = R\angle 0° = R\angle 0$
- For inductors, $Z_L = j\omega L = \omega L\angle 90° = \omega L\angle \frac{\pi}{2}$
- For capacitors, $Z_C = \frac{1}{j\omega C} = \frac{1}{C\omega}\angle -90° = \frac{1}{C\omega}\angle -\frac{\pi}{2}$

21 Misc.

- Complex rect. form - $x = A + jB$
- $C = \sqrt{A^2 + B^2}$
- $\theta = tan^{-1}(\frac{B}{A})$
- $x = Ce^{j\theta} = C\angle\theta$ (polar form)
- $-180° < \theta \leq 180°$ or $-\pi < \theta \leq \pi$
- $x = C\cos\theta + jC\sin\theta$ (rectangular form)
- $V_1 \cdot V_2 = C_1\angle\theta_1 \cdot C_2\angle\theta_2 = C_1C_2\angle(\theta_1 + \theta_2)$
- $\frac{V_1}{V_2} = \frac{C_1\angle\theta_1}{C_2\angle\theta_2} = \frac{C_1}{C_2}\angle(\theta_1 - \theta_2)$

22 Power

- Instantaneous power - $p(t) = \frac{v^2(t)}{R}$
- Average power - $\frac{1}{T}\int_0^T p(t)dt = \frac{1}{2}VI\cos(\theta_V - \theta_I) = V_{rms}I_{rms}\cos(\theta_V - \theta_I)$
- Placeholder
- Above image uses RMS values
- Complex power - $P + Qj$
- P is in watts, Q is in VAR, S is in VA
- $S = \tilde{V}I^*$
- S and Z (impedance) have the same angle
- Impedance angle is the phase shift between voltage and current

23 Voltage

- Mean square voltage - $v^2 = \frac{V^2}{2}$
- RMS voltage - $V_{rms} = \frac{V}{\sqrt{2}}$

24 Transformers

- $N = \frac{n_2}{n_1}$
- $\frac{V_2}{n_2} = \frac{V_1}{n_1}$
- $n_2I_2 = n_1I_1$
- $V_2 = NV_1$
- $I_2 = \frac{I_1}{N}$
- $Z_{ab} = \frac{1}{N^2}Z_L$
- $R_L = N^2R_s$
- $X_L = -N^2X_s$
- $P_{av_{max}} = \frac{V_s^2}{4R_s}$
- All above use RMS values

25 Resistors

- Absorb average power $(\frac{VI}{2})$
- No VAR (Q=0)
- $\phi = 0$
- $S = I_{rms}^2R = \frac{V_{rms}^2}{R}$
- $v(t) = V\cos(\omega t)$
- $i(t) = I\cos(\omega t)$
- $p(t) = \frac{VI}{2}(1 + \cos(2\omega t))$

26 Inductors

- No average power
- Absorbs VAR $(Q > 0)$
- $\phi = +90°$
- $S = I_{rms}^2 j\omega L = j\frac{V_{rms}^2}{\omega L}$
- $v(t) = V\cos(\omega t)$
- $i(t) = I\cos(\omega t - 90°)$
- $p(t) = \frac{VI}{2}(\sin(2\omega t))$

27 Capacitors

- No average power
- Generates VAR $(Q < 0)$
- $\phi = -90°$
- $S = -\frac{I_{rms}^2}{\omega C}j = -V_{rms}^2\omega Cj$
- $v(t) = V\cos(\omega t)$
- $i(t) = I\cos(\omega t + 90°)$
- $p(t) = -\frac{VI}{2}(\sin(2\omega t))$

28 Power Factor

- $pf = \frac{P_{av}}{VI} = \frac{P_{av}}{|S|} = \cos\theta$
- $pf = 0 \implies$ Purely inductive/capacitive, no avg. dissipated power
- $pf = 1 \implies$ Purely resistive load, all power is dissipated power
- $0 < pf < 1 \implies$ combination of resistive and reactive
- $\theta < 0 \implies$ leading and capacitive
- $\theta > 0 \implies$ lagging and inductive
- $Q_A = -Q_B$
- $C = \frac{-Q_B}{\omega \widetilde{V_L}^2}$
- For power factor corrections, load type must be opposite of the part you can observe
- $\theta < 0 \implies$ needs inductor and $\theta > 0 \implies$ needs capacitor
- $X_B = -\frac{R^2 + X_A^2}{X_A}$
- If $X_A = \omega L \implies X_B = -\frac{1}{\omega C}$
- If $X_A = \frac{-1}{\omega C} \implies X_B = \omega L$

29 Wye

- $V_1 = \tilde{V}\angle 0°, V_2 = \tilde{V}\angle -120°, V_3 = \tilde{V}\angle -240°$
- Voltages are at the same frequency and have the same magnitude (sum of all three equals 0)
- Capital letters are for load nodes
- $\widetilde{V}_{Line} = \widetilde{V}_{Phase} * \sqrt{3}\angle 30°$
- $V_p = \frac{V_{line}}{\sqrt{3}}$
- $P = 3V_p I_p \cos\theta = \sqrt{3} V_{line} I_{line} \cos\theta$

30 Delta

- $I_1 = \tilde{I}\angle 0°, I_2 = \tilde{I}\angle -120°, I_3 = \tilde{I}\angle -240°$
- $\widetilde{I}_{Line} = \widetilde{I}_{Phase} * \sqrt{3}\angle -30°$
- $I_{phase} = \frac{I_{line}}{\sqrt{3}}$
- $P = 3V_p I_p \cos\theta = \sqrt{3} V_{line} I_{line} \cos\theta$

31 Misc.

- I^* indicates complex conjugate
- $S = \frac{V^2}{Z^*}$
- $R_{eq} \frac{1}{N^2} R_L$
- If power is greater than 0 the load is absorbing power
- A negative value needs an inductors while a positive value needs a capacitor
- Power factor correction reduces current
- Imaginary power describes storage and return of energy in inductors and capacitors
- If secondary is $\frac{1}{4}$ primary voltage, the secondary current is $4\times$ primary current
- pf = 1 for real power = apparent power
- LVDT measures position by changing magnetic coupling between primary and secondary coils

32 Frequency Response

- Gain = amplitude of output divided by amplitude of input
- Inductor impedance increases with frequency
- Capacitor impedance decreases with frequency
- Resistor impedance stays the same
- Filters shape the frequency response to perform specific operations
- Four types - low pass (lets low through), band pass (lets middle through), high pass (lets high through), notch (blocks middle)
- Cutoff frequency is the frequency where power is reduced to half its max value and gain is 0.707 of max
- Gain $= \frac{|V_c|}{|V|} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$
- Convert to dB - $20log_{10}\frac{V_{out}}{V_{in}}$
- Resonant frequency - $\omega_0 = \frac{1}{\sqrt{LC}}$

33 Op Amps

- Negative sign is inverting input, positive sign is non-inverting input, other side is output
- Inverting amplifier - $v_{out} = -\frac{R_F}{R_S}v_s$
- Non-inverting amplifier - $v_{out} = \left(1 + \frac{R_F}{R_S}\right) v_s$
- Buffer - $v_{out} = v_s$
- Differential - $v_{out} = \frac{R_2}{R_1}(v_2 - v_1)$
- Summing amplifier - $v_{out} = \sum\limits_{k=1}^{n} -\frac{R_F}{R_{Si}}v_{Sk}$

34 Diodes & Semiconductors

- p-type semiconductor is doped so that the majority of the current is due to holes
- n-type semiconductor is doped so that majority of the current is due to electrons
- Zenor diode is breakdown, photo diode is reverse bias, LED is forward bias
- For p-type, hole moves in same direction of current
- Ideal diode dissipates no power

35 Digital/Binary Systems

- Analog signal can take any value in a range while digital signal can only take a finite number of values in a range
- To convert to binary, divide by two, add a 1 if there's a remainder and a 0 if clean division
- In logic tables, 0 is false and 1 is true

36 Logic Gates and Digital Computation

- Types of gates- AND, OR, NOT, NAND, NOR, XOR
- $A + \overline{A} = 1$
- $A \cdot \overline{A} = 0$
- $A \cdot A = A + A = A$
- $A + 1 = 1$
- $\overline{A + B} = \overline{A} \cdot \overline{B}$
- $\overline{A \cdot B} = \overline{A} + \overline{B}$

37 Blank Space