Problem Set 1 Answers

1.2.4 Convert to decimal (base-10) form.

a) 1.0110101_{two}

$= 2^0 + 2^{-2} + 2^{-3} + 2^{-5} + 2^{-7}$
$= 1.4140625.$

b) 11.0010010001_{two}

$= 2^1 + 2^0 + 2^{-3} + 2^{-6} + 2^{-10}$
$= 3.1416015625$

1.2.5 Find the errors between $\sqrt{2}$ and π and the answers to 1.2.4.

a) $|\sqrt{2} - 1.0110101_{two}| \approx 1.51062 \times 10^{-4}$ (absolute)

b) $|\pi - 11.0010010001_{two}| \approx 8.9089 \times 10^{-6}$ (absolute)

1.2.13 Determine what happens when a computer with a 4-bit mantissa tries to calculate the following:

b) $\left(\frac{1}{10} + \frac{1}{3}\right) + \frac{1}{5} \approx (.1015625 + .34375) + .203125$
$= .4453125 + .203125$
$\approx .4375 + .203125$
$= .640625$
$\approx .625$ (exact is $0.6\bar{3}$).

1.3.1 Find the error E_x and the relative error R_x. Also find the number of significant digits in the approximation.

b) $y = 98.350$, $\hat{y} = 98.000$

$E_y = |y - \hat{y}| = .350$

$R_y = \dfrac{E_x}{|y|} = .00355872$

$\hat{y}$ approximates y to d significant digits if d is the largest integer for which,

$$\frac{|y-\hat{y}|}{|y|} < \frac{10^{-d}}{2}$$

Since $\dfrac{10^{-3}}{2} < R_x < \dfrac{10^{-2}}{2}$, $\hat{y}$ approximates y to 2 significant digits.

c) $z = .000068$, $\hat{z} = .00006$

$E_z = .000008$

$R_z = .11764706$

Since $\dfrac{10^{-1}}{2} < R_z < \dfrac{10^{0}}{2}$, $\hat{z}$ approximates z to $d = 0$ significant digits.

1.3.12 Show that the roots x_1, x_2 of $ax^2+bx+c=0$ can be calculated using

$$x_1 = \frac{-2c}{b+\sqrt{b^2-4ac}}, \quad x_2 = \frac{-2c}{b-\sqrt{b^2-4ac}}.$$

$$x_1 = \left[\frac{-b+\sqrt{b^2-4ac}}{2a}\right]\left[\frac{-b-\sqrt{b^2-4ac}}{-b-\sqrt{b^2-4ac}}\right]$$

$$= \frac{b^2-(b^2-4ac)}{-2ab-2a\sqrt{b^2-4ac}}$$

$$= -\frac{4ac}{2ab+2a\sqrt{b^2-4ac}} = \frac{-2c}{b+\sqrt{b^2-4ac}}$$

$$x_2 = \left[\frac{-b-\sqrt{b^2-4ac}}{2a}\right]\left[\frac{-b+\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right]$$

$$= \frac{b^2-(b^2-4ac)}{-2ab+2a\sqrt{b^2-4ac}} = \frac{-2c}{b-\sqrt{b^2-4ac}}$$

1.3.13 Use the formulas derived in 1.3.12 to calculate x_1 and x_2 for:

a) $x^2 - 1000.001x + 1 = 0$

$x_1 = -2/[-1000.001 + \sqrt{(-1000.001)^2 - 4}]$

$= 1000$

$x_2 = 0.001$