Problem Set 9

5.2.17 $f(x,y) = x+y$, region $x+y=2$, $y^2-2y-x=0$.

$$\int_{-1}^{2}\int_{y^2-2y}^{2-y}(x+y)\,dx\,dy$$

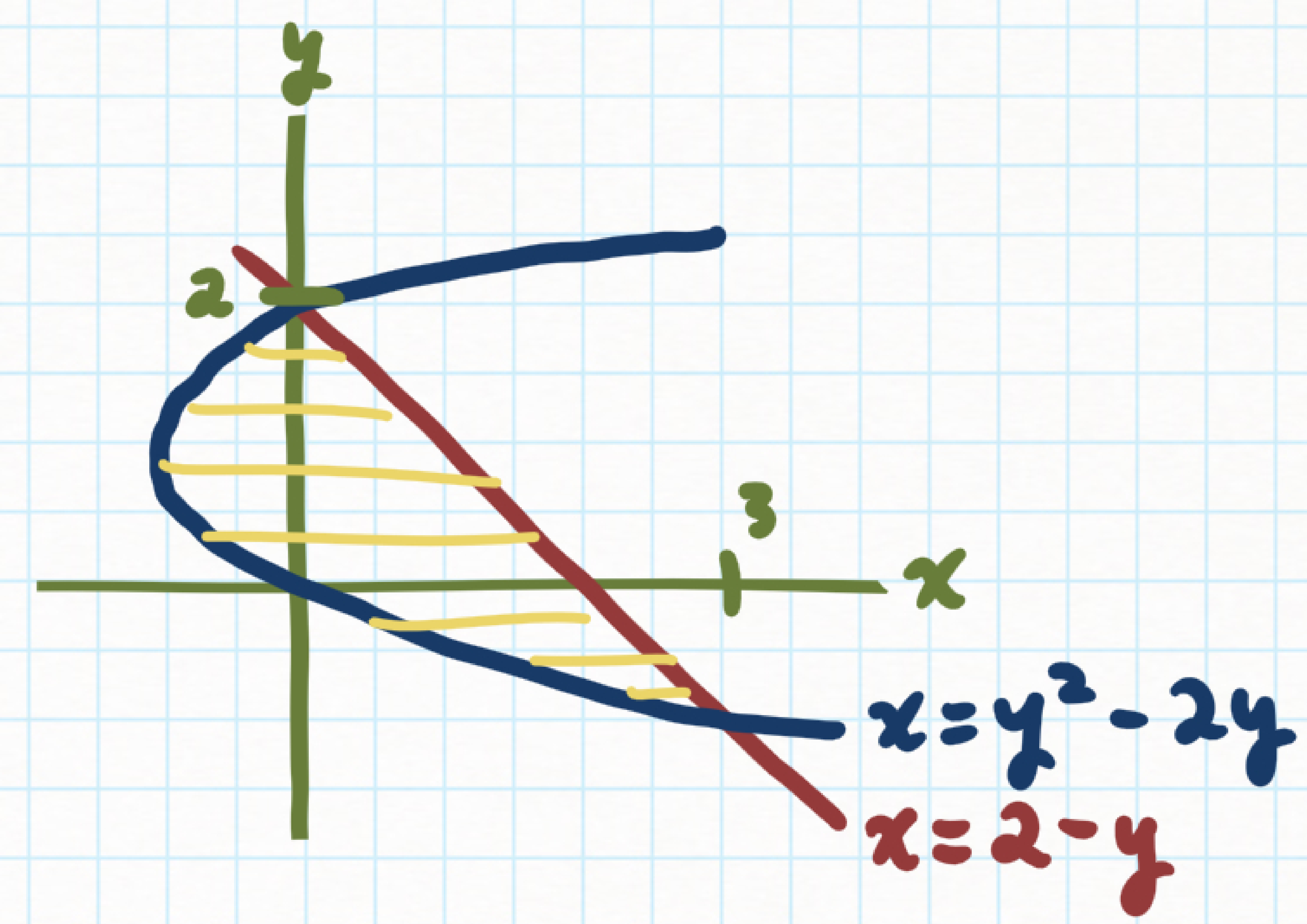
$$= \int_{-1}^{2}\left(\frac{(2-y)^2}{2} + (2-y)y - \frac{(y^2-2y)^2}{2} - (y^2-2y)y\right)dy$$

$$= \int_{-1}^{2}\left(2-2y+y^2/2 + 2y - y^2 - y^4/2 + 2y^3 - 2y^2 - y^3 + 2y^2\right)dy$$

$$= \int_{-1}^{2}\left(2 - y^2/2 + y^3 - y^4/2\right)dy$$

$$= 2y - y^3/6 + y^4/4 - y^5/10 \Big|_{-1}^{2}$$

$$= 99/20$$

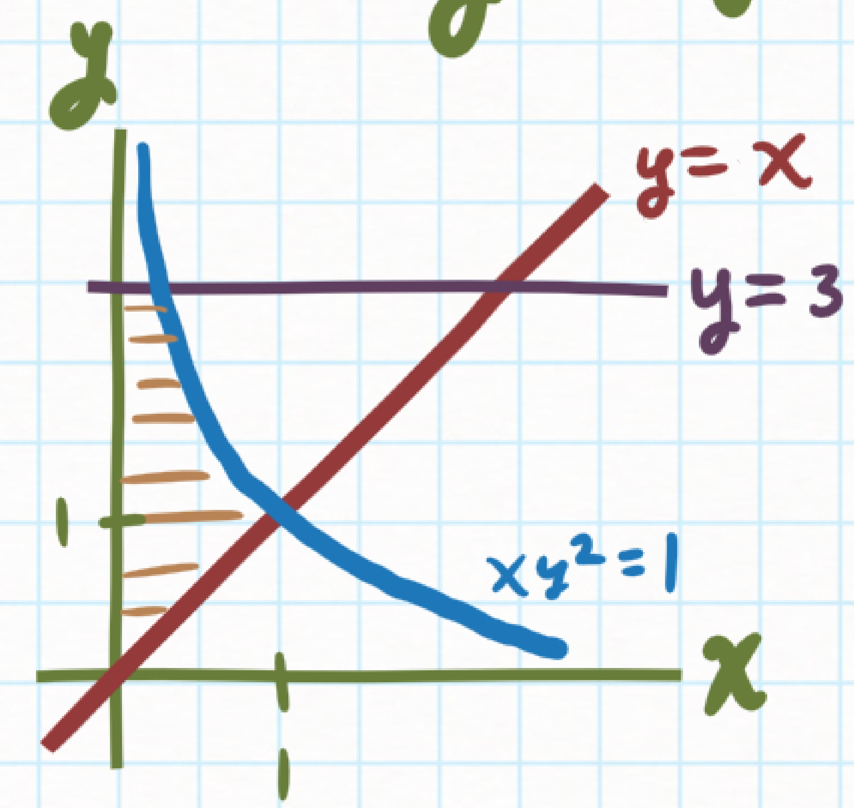


5.2.20 $\iint_D 3y\,dA$, D is the region bounded by $xy^2=1$, $y=x$, $x=0$, $y=3$.

$$\int_0^1\int_0^y 3y\,dx\,dy + \int_1^3\int_0^{1/y^2} 3y\,dx\,dy$$

$$= \int_0^1 3y^2\,dy + \int_1^3 3/y\,dy$$

$$= 1 + 3\ln 3$$



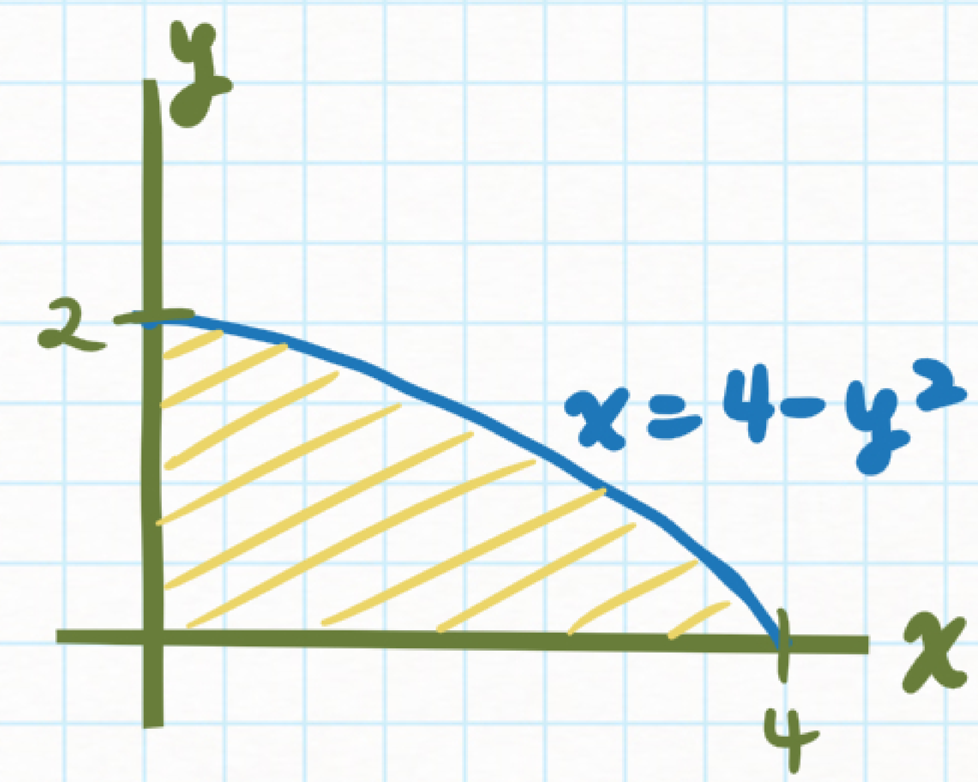
5.3.4

$$\int_0^2\int_0^{4-y^2} x\,dx\,dy = \int_0^2 (4-y^2)^2/2\,dy$$

$$= \int_0^2 \left(8 - 4y^2 + y^4/2\right)dy$$

$$= \left(8y - \tfrac{4}{3}y^3 + y^5/10\right)\Big|_0^2$$

$$= 128/15$$



$$\int_0^4\int_0^{\sqrt{4-x}} x\,dy\,dx = \int_0^4 x\sqrt{4-x}\,dx$$

$$= -\tfrac{2}{3}x(4-x)^{3/2}\Big|_0^4 + \int_0^4 \tfrac{2}{3}(4-x)^{3/2}\,dx$$

$$= 0 + \left(-\tfrac{4}{15}(4-x)^{5/2}\right)\Big|_0^4$$

$$= \frac{4}{15}4^{5/2}$$

$$= 128/15$$

5.3.13

$$\int_0^8\int_0^{\sqrt{y/3}} y\,dx\,dy + \int_8^{12}\int_{\sqrt{y-8}}^{\sqrt{y/3}} y\,dx\,dy$$

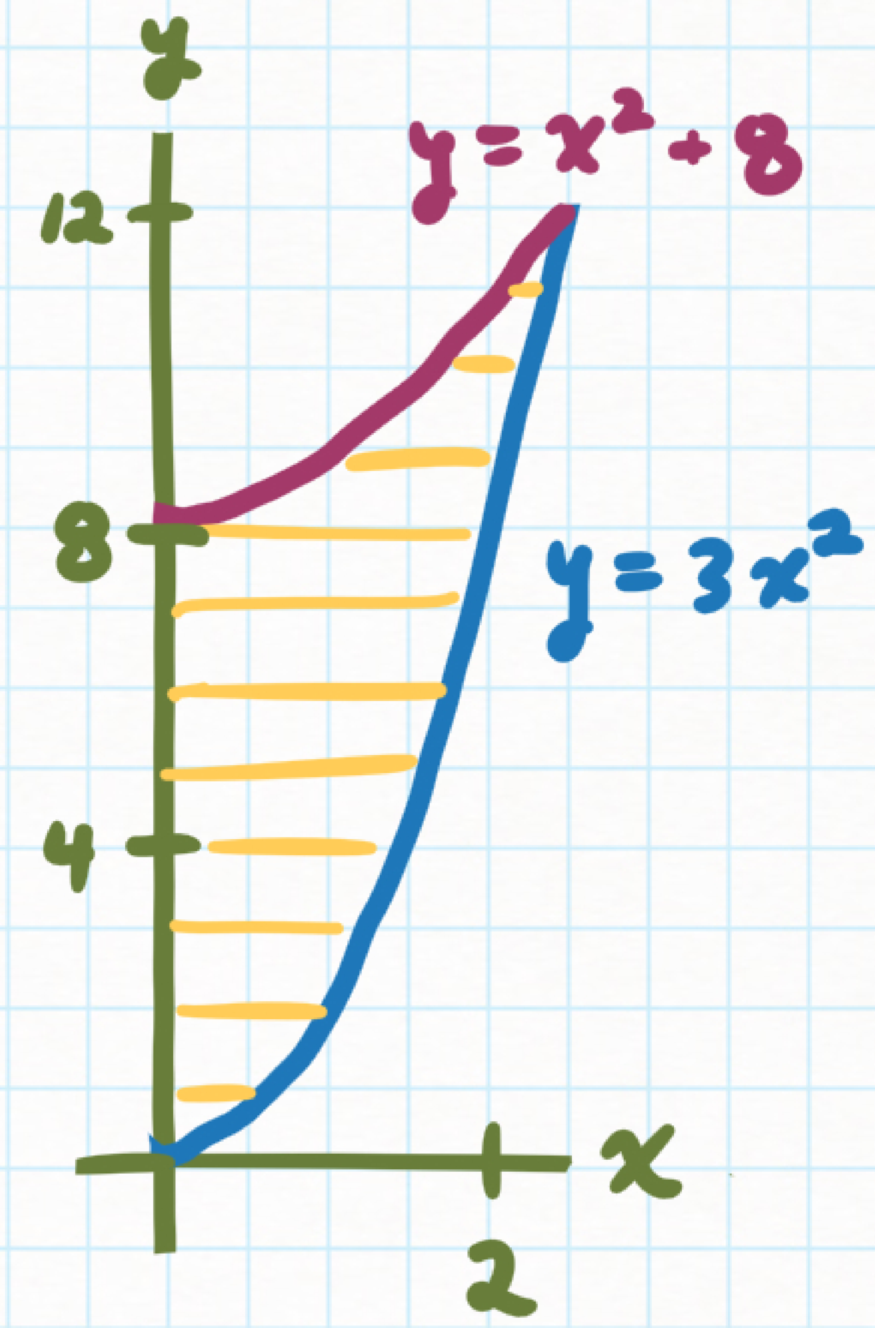
$$= \int_0^2\int_{3x^2}^{x^2+8} y\,dy\,dx$$

$$= \int_0^2 \left(x^4/2 + 8x^2 + 32 - 9x^4/2\right) dx$$

$$= 2^5/10 + 8\cdot 2^3/3 + 2^6 - 9\cdot 2^5/10$$

$$= 2^4/5 + 2^6/3 + 2^6 - 9\cdot 2^4/5$$

$$= 896/15$$



5.4.13 $f(x,y,z) = 8xyz$ W is bounded by $y = x^2$, $y + z = 9$, $z = 0$.

$$\iiint_W f(x,y,z)\,dV = \int_{-3}^{3}\int_{x^2}^{9}\int_0^{9-y} 8xyz\,dz\,dy\,dx$$

$$= \int_{-3}^{3}\int_{x^2}^{9} 4xy(9-y)^2\,dy\,dx$$

$$= 4\int_{-3}^{3} x \int_{x^2}^{9} (81y - 18y^2 + y^3)\,dy\,dx$$

$$= 4\int_{-3}^{3} \left[x\left(3^8/2 - 2\cdot 3^7 + 3^8/4\right) - 81x^5/2 + 6x^7 - x^9/2\right] dx$$

$= 0$ (since x, x^5, x^7, x^9 are odd and we integrate symmetrically about $x = 0$)

5.4.14 $f(x,y,z) = z$, W is the region in the first octant bounded by $y^2+z^2=9$, $y=x$, $x=0$, $y=0$

$$\iiint_W f(x,y,z)\,dV = \int_0^3 \int_0^y \int_0^{\sqrt{9-y^2}} z\,dz\,dx\,dy$$

$$= \int_0^3 \int_0^y (9-y^2)/2\,dx\,dy$$

$$= \int_0^3 (9y/2 - y^3/2)\,dy$$

$$= 81/8$$

5.4.25 $\int_{-1}^1 \int_{y^2}^1 \int_0^{1-x} f(x,y,z)\,dz\,dx\,dy$

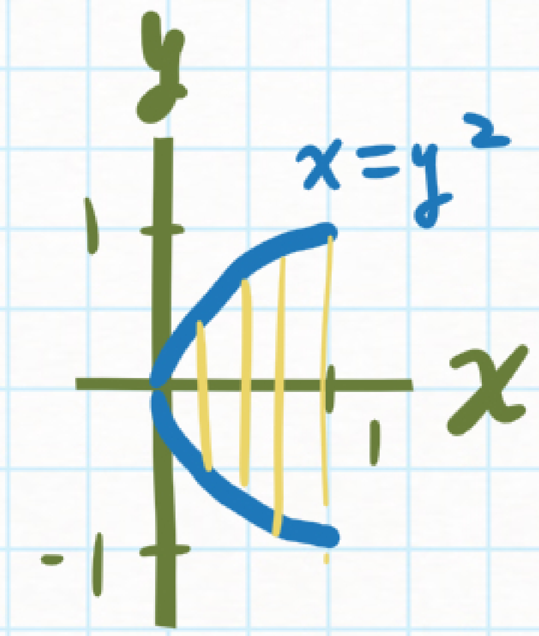
$$= \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} \int_0^{1-x} f(x,y,z)\,dz\,dy\,dx$$

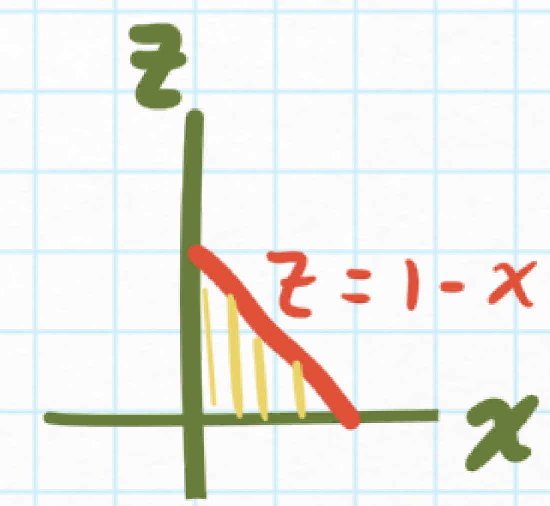
$$= \int_0^1 \int_0^{1-x} \int_{-\sqrt{x}}^{\sqrt{x}} f(x,y,z)\,dy\,dz\,dx$$

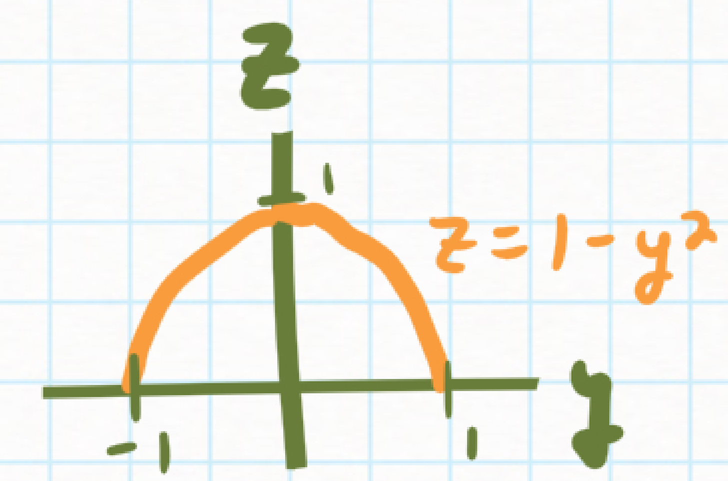
$$= \int_0^1 \int_0^{1-z} \int_{-\sqrt{x}}^{\sqrt{x}} f(x,y,z)\,dy\,dx\,dz$$

$$= \int_{-1}^1 \int_0^{1-y^2} \int_{y^2}^{1-z} f(x,y,z)\,dx\,dz\,dy$$

$$= \int_0^1 \int_{-\sqrt{1-z}}^{\sqrt{1-z}} \int_{y^2}^{1-z} f(x,y,z)\,dx\,dy\,dz$$







5.5.3 $T(u,v)=\begin{bmatrix}2&3\\-1&1\end{bmatrix}\begin{bmatrix}u\\v\end{bmatrix}$, D^* has vertices $(0,0)$, $(1,3)$, $(-1,2)$, $(0,5)$.

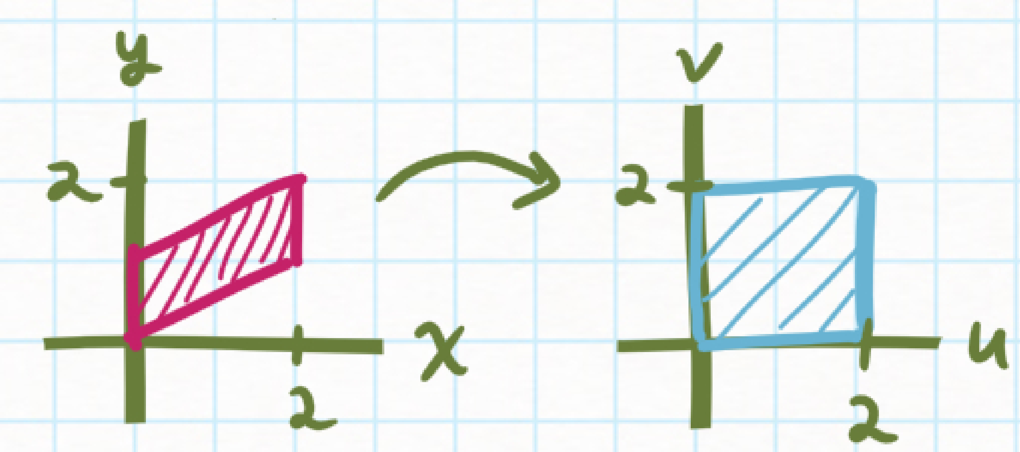
$D=T(D^*)$ has vertices $T(0,0)=(0,0)$, $T(1,3)=(11,2)$, $T(-1,2)=(4,3)$, and $(15,5)$.

5.5.4 D^* has vertices $(0,0)$, $(-1,3)$, $(1,2)$, $(0,5)$, $D=T(D^*)$ and D has vertices $(0,0)$, $(3,2)$, $(1,-1)$, $(4,1)$.

Since T is a linear transformation with the matrix $\begin{bmatrix}a&b\\c&d\end{bmatrix}$, $\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}a\\c\end{bmatrix}$, and $\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}b\\d\end{bmatrix}$, we want to calculate $T(1,0)$ and $T(0,1)$.

$$T(1,0)=-\tfrac{2}{5}T(-1,3)+\tfrac{3}{5}T(1,2)=-\tfrac{2}{5}(3,2)+\tfrac{3}{5}(1,-1)=(-3/5,-7/5)$$
$$T(0,1)=\tfrac{1}{5}T(0,5)=\tfrac{1}{5}(4,1)=(4/5,1/5)$$
$$\therefore\ T(u,v)=\tfrac{1}{5}\begin{bmatrix}-3&4\\-7&1\end{bmatrix}.$$

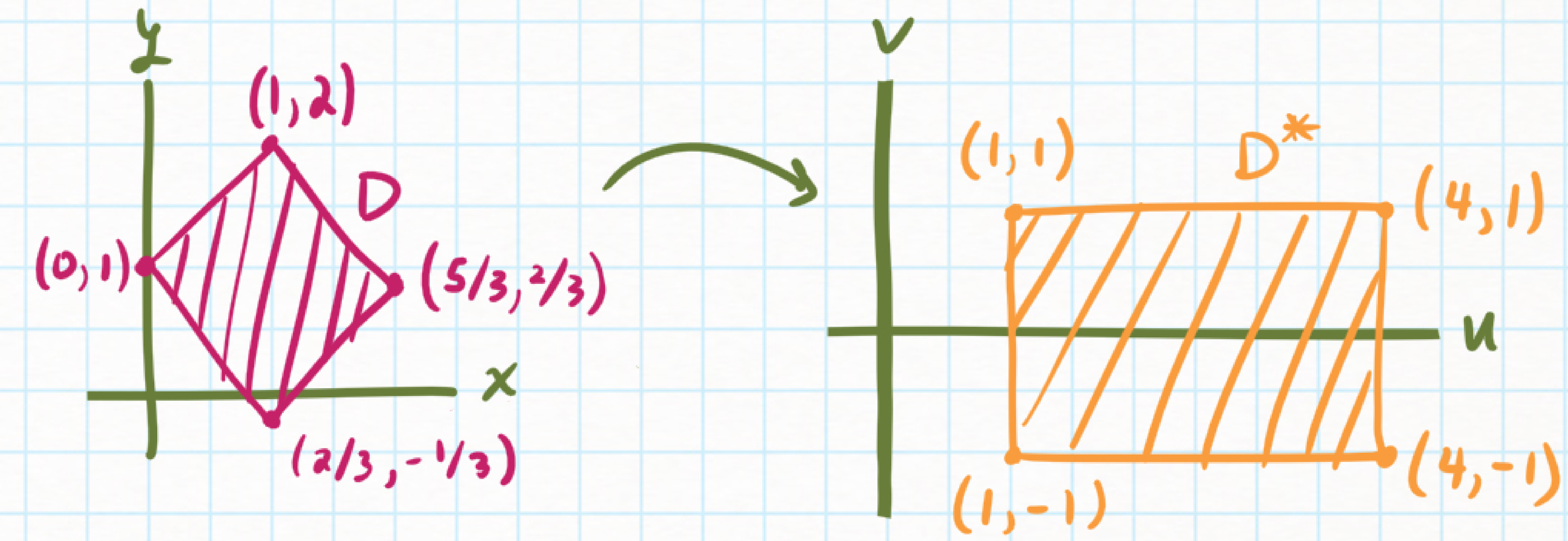
5.5.9 $$\int_0^2\int_{x/2}^{x/2+1}x^5(2y-x)e^{(2y-x)^2}\,dy\,dx \quad \left(\begin{matrix}u=x, & v=2y-x \\ x=u, & y=(u+v)/2\end{matrix}\quad \frac{\partial(x,y)}{\partial(u,v)}=\begin{vmatrix}1&0\\1/2&1/2\end{vmatrix}=1/2\right)$$
$$=\int_0^2\int_0^2 u^5ve^{v^2}\left|\tfrac{1}{2}\right|dv\,du$$
$$=\left(u^6/6\right)\Big|_0^2\int_0^4\tfrac{1}{4}e^w\,dw \quad \left(w=v^2,\ \tfrac{1}{4}dw=\tfrac{1}{2}v\,dv\right)$$
$$=(2^6/6)(e^4-e^0)/4$$
$$=8(e^4-1)/3$$



5.5.11 $\iint_D (2x+y)^2 e^{x-y}\,dA$, D is the region enclosed by $2x+y=1$, $2x+y=4$, $x-y=-1$ and $x-y=1$.

$u=2x+y,\ v=x-y \longrightarrow x=(u+v)/3,\ y=(u-v)/2 \qquad \dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1/3 & 1/3 \\ 1/2 & -1/2 \end{vmatrix} = -1/3$

$$\iint_D (2x+y)^2 e^{x-y}\,dA = \int_1^4 \int_{-1}^1 u^2 e^v \,|-1/3|\,dv\,du$$
$$= \left(\frac{u^3}{3}\right)\Bigg|_1^4 \left(\frac{e^v}{3}\right)\Bigg|_{-1}^1$$
$$= 7\,(e - 1/e)$$

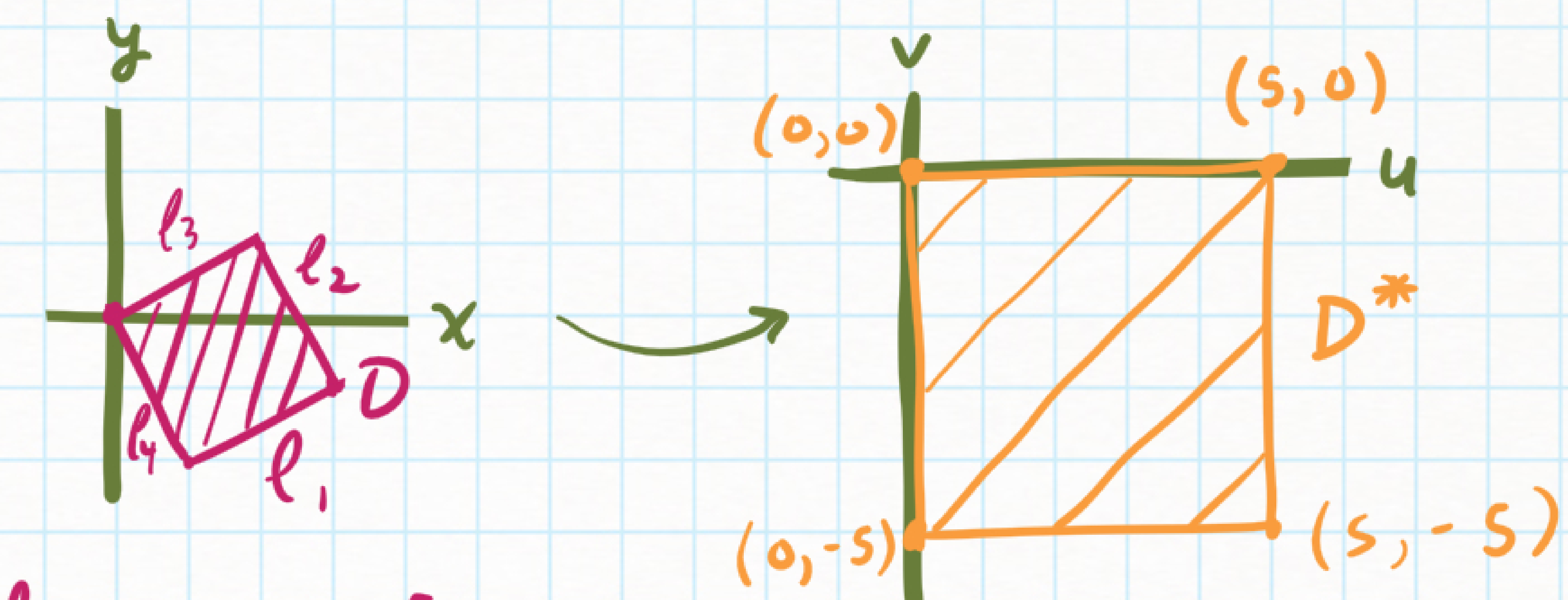


5.5.12 $\iint_D \dfrac{(2x+y-3)^2}{(2y-x+6)^2}\,dA$, D is the region bounded by $(0,0), (2,1), (3,-1), (1,-2)$.

$u=2x+y,\ v=2y-x \rightarrow x=(2u-v)/5,\ y=(u+2v)/5$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 2/5 & -1/5 \\ -1/5 & 2/5 \end{vmatrix} = 1/5$$

$$\iint_D \frac{(2x+y-3)^2}{(2y-x+6)^2}\,dA = \iint_{D^*} \frac{(u-3)^2}{(v+6)^2}\,|1/5|\,dA$$
$$= \int_0^5 \int_{-5}^0 \frac{(u-3)^2}{(v+6)^2}\,\frac{1}{5}\,dv\,du$$
$$= \left(\frac{(u-3)^3}{3}\right)\Bigg|_0^5 \left(-\frac{1}{v+6}\right)\Bigg|_{-5}^0 \frac{1}{5}$$
$$= 35/18$$



$\ell_1: 2y-x+5=0$

$\ell_2: 2x+y-5=0$

$\ell_3: 2y-x=0$

$\ell_4: 2x+y=0$