Problem Set 1

1.1.20

(a) If $a \in \mathbb{R}^3$, $0a = \begin{bmatrix} 0a_1 \\ 0a_2 \\ 0a_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Similar if $a \in \mathbb{R}^2$.

(b) If $a \in \mathbb{R}^3$, $1a = \begin{bmatrix} 1a_1 \\ 1a_2 \\ 1a_3 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = a$. Similar if $a \in \mathbb{R}^2$.

1.1.22

The set $P = \{ x + ta + sb \mid 0 \leq t, s \leq 1 \}$, $x = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$.

1.1.24

(a) 4 mph

(b) 1/10 hr = 6 minutes

(c) $(4\ mi/hr)(\frac{1}{10}\ hr)(5280\ ft/mi) - 1250\ ft = 862\ ft$

1.2.3

$(3, \pi, -7) = 3\hat{\imath} + \pi\hat{\jmath} - 7\hat{k}$

1.2.10

$\pi\hat{\imath} - \hat{\jmath} = (\pi, -1, 0)$

1.2.11

(a) $b = (3,1) = 2a_1 + 1a_2 = 2(1,1) + 1(1,-1)$.

(b) $b = (3,-5) = -1a_1 + 4a_2$

(c) $\begin{cases} b_1 = c_1 + c_2 \\ b_2 = c_1 - c_2 \end{cases} \rightarrow \begin{cases} c_1 = (b_1 + b_2)/2 \\ c_2 = (b_1 - b_2)/2 \end{cases}$, $(b_1, b_2) = c_1 a_1 + c_2 a_2$

1.2.14

$$\begin{cases} x = 12 + 5t \\ y = -2 - 12t \\ z = t \end{cases}, \quad t \in \mathbb{R}$$

1.2.16

$$\begin{cases} x = 2 + 1t \\ y = 1 - 2t \\ z = 2 + 3t \end{cases}, \quad t \in \mathbb{R}$$

1.2.24

$$\frac{x-2}{5} = \frac{y-3}{-2} = \frac{z+1}{4} = t \longrightarrow \begin{cases} x = 2 + 5t \\ y = 3 - 2t \\ z = -1 + 4t \end{cases}$$

1.2.28

Setting $t=0$ shows $(-5,2,1)$ is on ℓ_1.
Setting $t=3$ shows $(-5,2,1)$ is also on ℓ_2.

Setting $t=1$ shows $(-3,5,-5)$ is on ℓ_1.
Setting $t=2$ shows $(-3,5,-5)$ is also on ℓ_2.

since ℓ_1 and ℓ_2 share two points, $\ell_1 = \ell_2$.

1.2.30

Yes since $t \mapsto t^3$ is a bijection from $\mathbb{R}$ to $\mathbb{R}$.
This is equivalent to the line $x = 3\lambda + 7$, $y = 2 - \lambda$, $z = 5\lambda + 1$, $\lambda \in \mathbb{R}$.

1.2.35

$x = 2t - 3$, $y = 3t + 2$, $z = 5 - t$

$x = 0 \to t = 3/2 \quad (x,y,z) = (0, 13/2, 7/2)$
$y = 0 \to t = -2/3 \quad (x,y,z) = (-13/3, 0, 17/3)$
$z = 0 \to t = 5 \quad (x,y,z) = (7, 17, 0)$

1.3.8

$$\theta = \cos^{-1}\left(\frac{(-1,2)\cdot(3,1)}{\|(-1,2)\|\,\|(3,1)\|}\right) = \cos^{-1}\left(-\frac{1}{5\sqrt{2}}\right) \approx 1.713 \text{ radians}$$

1.3.13

$$\text{proj}_a b = \|b\|\cos\theta\,\frac{a}{\|a\|} = \frac{\|a\|\,\|b\|\cos\theta}{\|a\|}\,\frac{a}{\|a\|} = \frac{a\cdot b}{\|a\|^2}\,a$$

$$= \frac{5/\sqrt{2}}{12}(\hat{\imath}+\hat{\jmath})/\sqrt{2} = \frac{5}{2}(\hat{\imath}+\hat{\jmath}).$$

1.3.17

$$\vec{u} = (2\hat{\imath}-\hat{\jmath}+\hat{k})/\sqrt{4+1+1} = (2\hat{\imath}-\hat{\jmath}+\hat{k})/\sqrt{6}.$$