§Sinusoidal Functions

Problem 1: Write each of the following functions $f(t)$ in the form $A\cos(\omega t - \phi)$. In each case, begin by drawing a right triangle.

a) $2\cos(3t) + 2\sin(3t)$

b) $\sqrt{3}\cos(\pi t) - \sin(\pi t)$

c) $\cos(t - \frac{\pi}{8}) + \sin(t - \frac{\pi}{8})$

Answer: $c_1\cos\omega t + c_2\sin\omega t = A\cos(\omega t - \phi)$, $A = \sqrt{c_1^2 + c_2^2}$

$\phi = \arctan\frac{c_2}{c_1}$

a) $\omega = 3$

$A = 2\sqrt{2}$

$\phi = \pi/4$

$f(t) = 2\sqrt{2}\cos(3t - \pi/4)$

b) $\omega = \pi$

$A = 2$

$\phi = -\pi/6$

$f(t) = 2\cos(\pi t + \pi/6)$

c) $u = t - \pi/8$

$f(u) = \cos u + \sin u$

$\omega = 1$

$A = \sqrt{2}$

$\phi = \pi/4$

$f(t) = \sqrt{2}\cos(t - \pi/8 - \pi/4)$

$= \sqrt{2}\cos(t - 3\pi/8)$

Problem 2: Use complex exponentials to find $\int e^{2x} \sin x \, dx$.

Answer:

$$\int e^{(a+bi)x} dx = \frac{1}{a+bi} e^{(a+bi)x} + c$$

$$\int e^{2x} \sin x \, dx = \mathrm{Im}\left[\int \left(e^{2x}(\cos x + i \sin x)\right) dx\right]$$

$$= \mathrm{Im}\left[\int e^{(2+i)x} dx\right]$$

$$= \mathrm{Im}\left[\frac{1}{2+i} e^{(2+i)x} + k\right], \quad k = c + di$$

$$= \mathrm{Im}\left[\frac{2-i}{5} e^{2x}(\cos x + i \sin x) + k\right]$$

$$= \left(\frac{2}{5}\sin x - \frac{1}{5}\cos x\right) e^{2x} + d$$