§Frequency Response

Problem 1: A driven spring-mass-dashpot system is modeled by the DE

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t$$

with $m = 1$, $c = 6$, and $k = 45$. $F_0 = 50$. Find the amplitude $A(\omega)$ of the response as a function of the input frequency ω and find the frequency which gives the largest system response. Is this a system for which 'practical resonance' occurs?

Answer:
$$A(\omega) = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + c^2 \omega^2}} = \frac{50}{\sqrt{(45 - \omega^2)^2 + 36\omega^2}}.$$

$$\omega_{max} = \left(\frac{k}{m} - \frac{1}{2}\left(\frac{c}{m}\right)^2 \right)^{1/2} = \left(45 - \frac{1}{2} \cdot 36 \right)^{1/2} = 3\sqrt{3} \ \frac{rad}{sec}.$$