Unit 11 : Second Order Constant Coefficient Linear Equations

§ Characteristic Equation

Problem 5: Find a DE of the form $ay'' + by' + c = 0$ for the given family of solutions.

$$y = c_1 + c_2 x$$

Answer: we want a double root at $r = 0$.

$$1y'' + 0y' + 0y = 0$$

$$r^2 = 0$$
$$r = 0 \quad \text{(double root)}$$

$$y(x) = c_1 e^{0x} + c_2 x e^{0x}$$
$$= c_1 + c_2 x .$$

Problem 9: Find the general solution to the DE.

$$y^{(4)} - 8y'' + 16y = 0$$

Answer:

$$r^4 - 8r^2 + 16 = 0$$
$$(r^2 - 4)^2 = 0$$
$$(r-2)^2(r+2)^2 = 0$$
$$r = 2, -2 \text{ (both double roots).}$$

$$y(x) = c_1 e^{2x} + c_2 e^{-2x} + x\left(c_3 e^{2x} + c_4 e^{-2x}\right)$$