§ Exponential Input

Problem 1: a) Find a solution of $\dot{x} + 2x = e^{3t}$ of the form Be^{3t}. Then find the general solution.

b) Now do the same for the complex-valued differential equation $\dot{x} + 2x = e^{3it}$.

Answer:

a)
$$x(t) = Be^{3t}$$
$$\dot{x}(t) = 3Be^{3t}$$
$$e^{3t} = 3Be^{3t} + 2Be^{3t}$$
$$B = 1/5$$
$$x_p(t) = \frac{1}{5}e^{3t}$$
$$\dot{x} + 2x = 0 \rightarrow x_h(t) = Ce^{-2t}$$
$$x(t) = \frac{1}{5}e^{3t} + Ce^{-2t}$$

b)
$$x(t) = Be^{3it}$$
$$\dot{x}(t) = 3iBe^{3it}$$
$$e^{3it} = 3iBe^{3it} + 2Be^{3it}$$
$$1 = 3iB + 2B$$
$$B = \frac{1}{2+3i} = \frac{2-3i}{13}$$
$$x_p(t) = \left(\frac{2}{13} - \frac{3}{13}i\right)e^{3it}$$
$$x(t) = \left(\frac{2}{13} - \frac{3}{13}i\right)e^{3it} + Ce^{-2t}$$