

§ Exponential Input

Problem 1: a) Find a solution of $\dot{x} + 2x = e^{3t}$ of the form Be^{3t} . Then find the general solution.

b) Now do the same for the complex-valued differential equation $\dot{x} + 2x = e^{3it}$.

Answer:

$$\begin{aligned} \text{a) } x(t) &= Be^{3t} \\ \dot{x}(t) &= 3Be^{3t} \end{aligned}$$

$$\begin{aligned} e^{3t} &= 3Be^{3t} + 2Be^{3t} \\ B &= 1/5 \end{aligned}$$

$$x_p(t) = \frac{1}{5} e^{3t}$$

$$\begin{aligned} \dot{x} + 2x &= 0 \rightarrow x_h(t) = Ce^{-2t} \\ x(t) &= \frac{1}{5} e^{3t} + Ce^{-2t} \end{aligned}$$

$$\begin{aligned} \text{b) } x(t) &= Be^{3it} \\ \dot{x}(t) &= 3iBe^{3it} \end{aligned}$$

$$\begin{aligned} e^{3it} &= 3iBe^{3it} + 2Be^{3it} \\ 1 &= 3iB + 2B \end{aligned}$$

$$B = \frac{1}{2+3i} = \frac{2-3i}{13}$$

$$\begin{aligned} x_p(t) &= \left(\frac{2}{13} - \frac{3}{13}i \right) e^{3it} \\ x(t) &= \left(\frac{2}{13} - \frac{3}{13}i \right) e^{3it} + Ce^{-2t} \end{aligned}$$