Unit III Fourier Series and Laplace Transform

§ Fourier Series Basics

Problem 1: Find the smallest period for each of the following:
a) $\sin \pi t/3$
b) $|\sin t|$
c) $\cos^2 3t$

Answer:

a) $\frac{2\pi}{B} = \frac{\pi}{3} \longrightarrow B = 6$

b) $|\sin t| = |-\sin t|$, so $|\sin t|$ has a period half that of the period of $\sin t$. So $|\sin t|$ has a period of π.

c) By similar reasoning as in part b, $\cos^2(3t)$ has a period of $\frac{1}{2}\frac{2\pi}{3} = \frac{\pi}{3}$.

Problem 2:

Find the Fourier series of the function $f(t)$ of period 2π which is given over the interval $-\pi < t \leq \pi$ by

$$f(t) = \begin{cases} 0, & -\pi < t \leq 0 \\ 1, & 0 < t \leq \pi \end{cases}$$

Answer :

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,dt = \frac{1}{\pi}\pi = 1$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(nt)\,dt$$

$$= \frac{1}{\pi}\int_{0}^{\pi} \cos(nt)\,dt = \frac{1}{n\pi}\left[\sin(n\pi) - \sin(0)\right] = 0, \quad n \geq 1.$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(nt)\,dt = \frac{1}{\pi}\int_{0}^{\pi} \sin(nt)\,dt$$

$$= \frac{-1}{n\pi}\left[\cos(n\pi) - \cos(0)\right] = \begin{cases} \frac{2}{n\pi}, & n \text{ odd} \\ 0, & n \text{ even} \end{cases}$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nt) + \sum_{n=1}^{\infty} b_n \sin(nt)$$

$$= \frac{1}{2} + \sum_{K=1}^{\infty} \frac{2}{(2K-1)\pi} \sin((2K-1)t).$$