§Nonlinear Systems

Problem 1: Find the critical points of the non-linear autonomous system

$$x' = 1 - x + y$$
$$y' = y + 2x^2$$

Answer:

$$0 = f(x,y) = 1 - x + y \longrightarrow y = x - 1$$
$$0 = g(x,y) = y + 2x^2$$

$$0 = x - 1 + 2x^2$$
$$x = \frac{-1 \pm 3}{4}, \quad y = \frac{-5 \pm 3}{4}$$

$$(x_0, y_0) = (-1, -2), \left(\frac{1}{2}, -\frac{1}{2}\right)$$

Problem 2: Write as equivalent first-order system and find the critical points:

$$x'' - x' + 1 - x^2 = 0$$

Answer: Let $y = x'$.

$$x' = f(x, y) = y$$
$$y' = g(x, y) = y + x^2 - 1$$

$$0 = f(x, y)$$
$$0 = g(x, y) \longrightarrow y = 0,\ x = \pm 1$$

$$(x_0, y_0) = (\pm 1, 0)$$

Problem 3: In general, what can you say about the relation between the trajectories and the critical points of the system on the left below, and those of the two systems on the right?

$$x' = f(x,y) \qquad a)\ x' = -f(x,y) \qquad b)\ x' = g(x,y)$$
$$y' = g(x,y) \qquad\quad y' = -g(x,y) \qquad\quad y' = -f(x,y)$$

Answer:

a) $(-f, -g)$ trajectories the same but traversed in opposite direction as (f, g).

$-f(x,y) = 0$, $-g(x,y) = 0$ iff $f(x,y) = 0$, $g(x,y) = 0$.

Same critical points.

b) $(g, -f)$ trajectories perpendicular to (g, f).

$f(x,y), -g(x,y) = 0$ iff $f(x,y), g(x,y) = 0$.

Same critical points.