Unit IV First Order Systems

§ Linear Systems

Problem 3: Write the following equations as equivalent first-order systems.

a) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + tx^2 = 0$

b) $y'' - x^2y' + (1 - x^2)y = \sin x$

Answer:

a) $y = \dot{x}$

$\dot{y} = -5y - tx^2$

b) $z = y'$

$z' = \sin x + x^2 z + (x^2 - 1)y$

Problem 4: Solve the system $x' = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x$ in two ways:

a) Solve the second equation, substitute for y in the first equation, and solve it.

b) Eliminate y by solving the first equation for y, then substitute into the second equation, getting a second order equation for x. Solve it, and then find y from the first equation. Do your two methods give the same answer?

Answer: $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$, $\vec{x}' = \begin{bmatrix} x' \\ y' \end{bmatrix}$, $x = x(t)$, $y = y(t)$

a) $y' = y$

$y = c_1 e^t$

$x' = x + y = x + c_1 e^t$

$x' - x = c_1 e^t$

$x e^{-t} = c_1 t + c_2$

$x(t) = c_1 t e^t + c_2 e^t$

b) $y = x' - x$, $y' = x'' - x'$

$y' = y$

$x'' - 2x' + x = 0$

$r^2 - 2r + 1 = 0$

$r = 1$ (double root)

$x(t) = c_1 t e^t + c_2 e^t$