§Gain and Phase Lag

Problem 1: a) Find the periodic solution to

$$x''' + x = 2\cos t$$

in amplitude-phase form.
b) What is the gain and the phase lag ?

Answer:

a) $P(D)x = B\cos\omega t$ has the unique periodic solution $x_p = \frac{B}{|p(i\omega)|}\cos(\omega t - \phi)$, $\phi = Arg(p(i\omega))$, $\frac{1}{|p(i\omega)|}$ is the complex gain and ϕ is the phase lag.

$p(D) = D^3 + 1$, $B = 2$, $\omega = 1 \rightarrow p(i\omega) = 1 - i$, $|p(i\omega)| = \sqrt{2}$, $\phi = -\pi/4$.

$$x_p(t) = \frac{2}{\sqrt{2}}\cos(t + \frac{\pi}{4}) = \sqrt{2}\cos(t + \pi/4)$$

b) gain $= 1/\sqrt{2}$, phase lag $= \phi = -\pi/4$.