§ Poles and Amplitude Response

Part I Problems

Problem 1: For each of the following functions $f(t)$, find the pole diagram of $F(s)$.

a) $f(t) = 1$

b) $f(t) = e^{-t} + 3e^{-3t}$

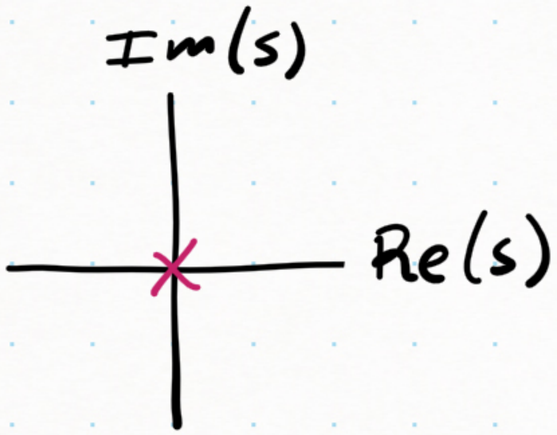
c) $f(t) = \cos(2t) + e^{-t}\sin(t)$

$s+1 = \pm\sqrt{-1}$

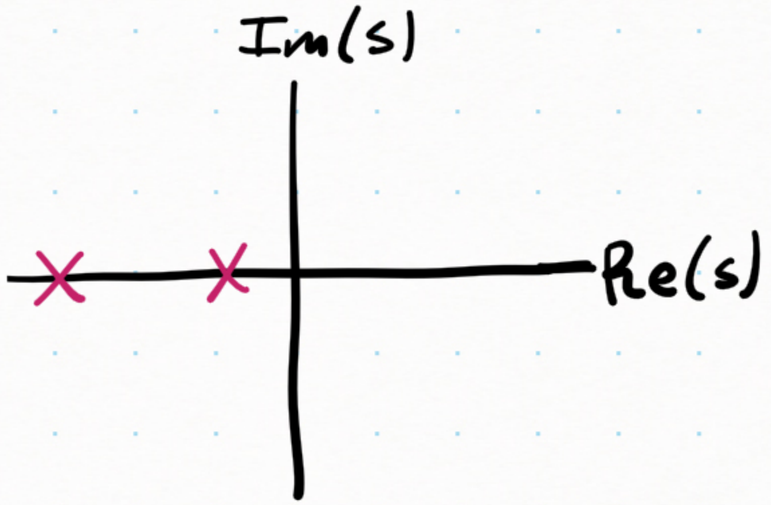
$s+1 = -1 \pm i$

Answer:

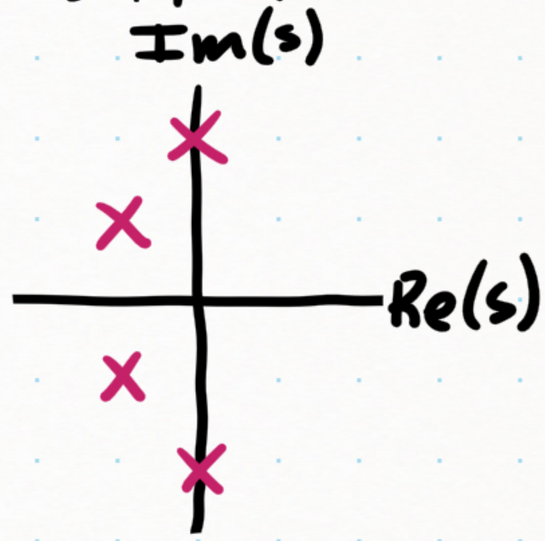
a) $F(s) = \frac{1}{s}$



b) $F(s) = \frac{1}{s+1} + \frac{3}{s+3}$



c) $F(s) = \frac{s}{s^2+4} + \frac{1}{(s+1)^2+1}$



Part II Problems

Problem 1: [Poles] **(a)** For each of the pole diagrams below:
(i) Describe common features of all functions $f(t)$ whose Laplace transforms have the given pole diagram.
(ii) Write down two examples of such $f(t)$ and $F(s)$.

The diagrams are: (1) $\{1, i, -i\}$. (2) $\{-1+4i, -1-4i\}$. (3) $\{-1\}$. (4) The empty diagram.

(b) A mechanical system is discovered during an archaeological dig in Ethiopia. Rather than break it open, the investigators subjected it to a unit impulse. It was found that the motion of the system in response to the unit impulse is given by $w(t) = u(t)e^{-t/2}\sin(3t/2)$.

(i) What is the characteristic polynomial of the system? What is the transfer function $W(s)$?

(ii) Sketch the pole diagram of the system.

(ii) The team wants to transport this artifact to a museum. They know that vibrations from the truck that moves it result in vibrations of the system. They hope to avoid circular frequencies to which the system response has the greatest amplitude. What frequency should they avoid?

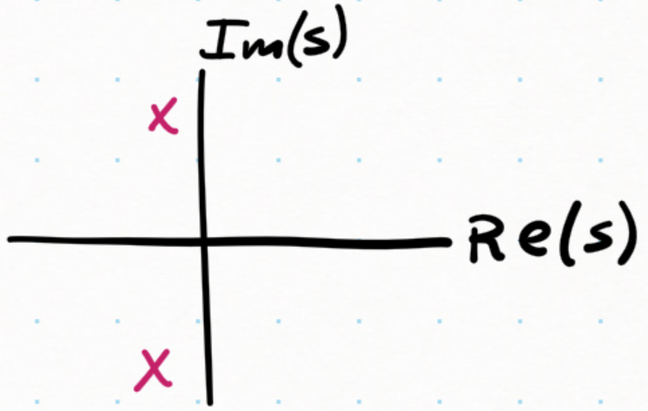
Answer:

b) i. $\frac{1}{P(s)} = W(s) = \frac{3/2}{(s+1/2)^2 + 9/4} = \frac{1}{\frac{2}{3}\left[s^2 + s + \frac{10}{4}\right]}$

$P(D) = \frac{2}{3}D^2 + \frac{2}{3}D + \frac{5}{3}$

ii. $(s+1/2)^2 = -9/4$

$s = -\frac{1}{2} \pm \frac{3}{2}i$



iii. $W(i\omega) = \dfrac{3/2}{(5/2 - \omega^2) + i\omega}$

$$|W(i\omega)| = \frac{3/2}{\sqrt{(5/2-\omega^2)^2 + \omega^2}}$$ (complex gain)

maximized for $\omega > 0$ by $\omega = \sqrt{2}$.

Avoid this frequency.