Exercises on solving $Ax = 0$: pivot variables, special solutions

Problem 7.1:

a) Find the row reduced form of:

$$A = \begin{bmatrix} 1 & 5 & 7 & 9 \\ 0 & 4 & 1 & 7 \\ 2 & -2 & 11 & -3 \end{bmatrix}$$

b) What is the rank of this matrix?

c) Find any special solutions to the equation $Ax = \mathbf{0}$.

a) $A \sim \begin{bmatrix} 1 & 5 & 7 & 9 \\ 0 & 4 & 1 & 7 \\ 0 & -12 & -3 & -21 \end{bmatrix} \sim \begin{bmatrix} 1 & 5 & 7 & 9 \\ 0 & 4 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 23/4 & 1/4 \\ 0 & 1 & 1/4 & 7/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

b) rank$(A) = 2$ since rref(A) has 2 pivots.

c) $x_1 + \frac{23}{4}x_3 + \frac{1}{4}x_4 = 0$ $\longrightarrow$ $x_1 = -\frac{23}{4}x_3 - \frac{1}{4}x_4$

$x_2 + \frac{1}{4}x_3 + \frac{7}{4}x_4 = 0$ $\quad$ $x_2 = -\frac{1}{4}x_3 - \frac{7}{4}x_4$

Set $x_3 = -4$, $x_4 = -4$. Then $x_1 = 24$, $x_2 = 8$.

$s_1 = \begin{bmatrix} 23 \\ 1 \\ -4 \\ 0 \end{bmatrix}$, $s_2 = \begin{bmatrix} 1 \\ 7 \\ 0 \\ -4 \end{bmatrix}$ are 2 special soln's to $Ax = 0$. There are many choices, but $N(A) = \text{span}\{s_1, s_2\}$.

Problem 7.2: (3.3 #17.b *Introduction to Linear Algebra:* Strang) Find A_1 and A_2 so that rank$(A_1B) = 1$ and rank$(A_2B) = 0$ for $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

Quick choices are $A_1 = I_{2\times 2}$, $A_2 = O_{2\times 2}$ since B has rank 1 already and then so does $A_1 B = B$. Of course if $A_2 = O_{2\times 2}$ then $A_2 B = O_{2\times 2}$, which has 0 pivots and thus rank 0.