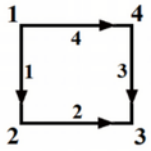
Problem 12.1: (8.2 #1. *Introduction to Linear Algebra:* Strang) Write down the four by four incidence matrix A for the square graph, shown below. (Hint: the first row has -1 in column 1 and +1 in column 2.) What vectors (x_1, x_2, x_3, x_4) are in the nullspace of A? How do you know that (1,0,0,0) is not in the row space of A?



$$A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix} \qquad Ax = 0 \longrightarrow \begin{array}{l} x_2 - x_1 = 0 \\ x_3 - x_2 = 0 \\ x_3 - x_4 = 0 \\ x_4 - x_1 = 0 \end{array} \longrightarrow x_1 = x_2 = x_3 = x_4$$

$(1,0,0,0)$ is not in the row space of A because only rows 1 and 4 have nonzero entries in the first component and the only other nonzero entries in each row do not belong in the same component - no combination can produce zeros in the last three components. Also, $(1,0,0,0)$ is not orthogonal to the nullspace. If $Ax = 0$, then x is orthogonal to each row of A. But then if this is the case x is orthogonal to any linear combination of the rows of A. If $(1,0,0,0)$ were a linear combination of rows of A, it would be orthogonal to vectors of the form (c,c,c,c), which is not the case.