Exercises on linear transformations and their matrices

Problem 30.1: Consider the transformation T that doubles the distance between each point and the origin without changing the direction from the origin to the points. In polar coordinates this is described by

$$T(r,\theta) = (2r,\theta).$$

a) Yes or no: is T a linear transformation?

b) Describe T using Cartesian (xy) coordinates. Check your work by confirming that the transformation doubles the lengths of vectors.

c) If your answer to (a) was "yes", find the matrix of T. If your answer to (a) was "no", explain why the T isn't linear.

Problem 30.2: Describe a transformation which leaves the zero vector fixed but which is not a linear transformation.

<u>30.1</u> Yes.

$$\begin{aligned} &T(r+r', \theta+\theta') \\ &= (2(r+r'), \theta+\theta') \\ &= (2r, \theta) + (2r', \theta') \end{aligned}$$

$$\begin{aligned} &T(cr, c\theta) \\ &= (2cr, c\theta) \\ &= c(2r, \theta) \\ &= cT(r,\theta) \end{aligned}$$

b) $T(x,y) = (2x, 2y)$

c) $T = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ cartesian

$T = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ polar

<u>30.2</u>

$$T(x,y) = (x, y^2)$$

$$T(0,0) = (0,0)$$

$$\begin{aligned} T(x+x', y+y') &= (x+x', y^2 + 2yy' + (y')^2) \\ &\neq (x+x', y^2+(y')^2) \\ &= T(x,y) + T(x',y'). \end{aligned}$$

Also,

$$\begin{aligned} T(cx, cy) &= (cx, c^2y^2) \\ &\neq c(x, y^2) \\ &= cT(x,y). \end{aligned}$$