Exercises on diagonalization and powers of A

Problem 22.1: (6.2 #6. *Introduction to Linear Algebra:* Strang) Describe all matrices S that diagonalize this matrix A (find all eigenvectors):

$$A = \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}.$$

Then describe all matrices that diagonalize A^{-1}.

Problem 22.2: (6.2 #16.) Find Λ and S to diagonalize A :

$$A = \begin{bmatrix} .6 & .9 \\ .4 & .1 \end{bmatrix}.$$

What is the limit of Λ^k as $k \to \infty$? What is the limit matrix of $S\Lambda^k S^{-1}$? In the columns of this matrix you see the ________.

22.1

$\lambda = 2, 4$

$$\left[\begin{array}{cc|c} 2 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\vec{s}_1 = \begin{bmatrix} 0 \\ c_1 \end{bmatrix}, \quad c_1 \neq 0$

$$\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & -2 & 0 \end{array}\right] \quad \vec{s}_2 = \begin{bmatrix} 2c_2 \\ c_2 \end{bmatrix}, \quad c_2 \neq 0$$

$$S = \begin{bmatrix} 0 & 2c_2 \\ c_1 & c_2 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} -\frac{c_1}{2c_1 c_2} & \frac{1}{c_1} \\ \frac{1}{2c_2} & 0 \end{bmatrix}, \quad A = S\Lambda S^{-1}$$

Ex) set $c_1 = c_2 = 1$.

$$A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} -1/2 & 1 \\ 1/2 & 0 \end{bmatrix}$$

$$= S\Lambda S^{-1}.$$

Since $A^{-1} = S\Lambda^{-1}S^{-1}$, the same S, S^{-1} pair diagonalizes A, but with Λ^{-1} instead of Λ

22.2

$$A = \begin{bmatrix} .6 & .9 \\ .4 & .1 \end{bmatrix}$$

$$\begin{aligned} 0 &= (.6-\lambda)(.1-\lambda) - .36 \\ &= \lambda^2 - .7\lambda - .3 \\ &= 10\lambda^2 - 7\lambda - 3 \\ &= 10\lambda^2 - 10\lambda + 3\lambda - 3 \\ &= (10\lambda + 3)(\lambda - 1) \end{aligned}$$

$\lambda = 1, -3/10.$

$$\left[\begin{array}{cc|c} -.4 & .9 & 0 \\ .4 & -.9 & 0 \end{array}\right] \rightarrow \vec{s}_1 = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} .9 & .9 & 0 \\ .4 & .4 & 0 \end{array}\right] \rightarrow \vec{s}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$S = \begin{bmatrix} 9 & 1 \\ 4 & -1 \end{bmatrix}, \quad S^{-1} = \begin{bmatrix} 1/13 & 1/13 \\ 4/13 & -9/13 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & -3/10 \end{bmatrix}$$

$$A = S \Lambda S^{-1}$$

$$\Lambda^k \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{ as } k \to \infty.$$

$$A^k = S \Lambda^k S^{-1}$$

$$\rightarrow \begin{bmatrix} 9/13 & 9/13 \\ 4/13 & 4/13 \end{bmatrix} \text{ as } k \to \infty.$$

In this matrix you see the <u>steady state vector</u>.