4.4.2 Find the real and imaginary parts of

(d) $i \log i \log \log i$

$z = i \log i \log \log i$

$\log i = w \rightarrow e^w = i = e^{i\pi/2} \rightarrow \log i = i\pi/2$

$z = i \cdot i\pi/2 \log i\pi/2$

$\log i\pi/2 = \log i + \log \pi/2 = i\pi/2 + \log \pi/2$

$z = i \cdot i\pi/2 (\log \pi/2 + i\pi/2)$
$= -\pi/2 (\log \pi/2 + i\pi/2)$

$= \boxed{\underbrace{-\pi/2 \log \pi/2}_{\text{real}} \underbrace{- i\pi^2/4}_{\text{imaginary}}}$

4.4.3 What can you say about

(c) the product of two numbers on the unit circle $z = e^{i\theta}$?

You can say the product is also on the unit circle.

$v = e^{i\theta},\ w = e^{i\phi} \rightarrow vw = e^{i(\theta+\phi)}$

(d) the sum of two numbers on the unit circle?

You can say the sum is on the disk $|z| \leq 2$.

$v = e^{i\theta},\ w = e^{i\phi} \rightarrow |v+w| \leq |v| + |w| = 1 + 1 = 2.$

You cannot say the sum is on $|z| = 1$ or $|z| = 2$ generally. Consider the examples $v = i, w = -i$ with $|v+w| = 0$ or $v = 1, w = i$ with $|v+w| = \sqrt{2}$.

4.4.4 Find the absolute value (or modulus) $|z|$ if

(a) $z = e^i$

$e^i = e^{i \cdot 1} = \cos 1 + i \sin 1 \rightarrow |e^i|^2 = \cos^2 1 + \sin^2 1 = 1$

(c) $z = \frac{3+i}{3-i}$

$\frac{3+i}{3-i} = \frac{(3+i)(3+i)}{(3-i)(3+i)} = \frac{9+6i+i^2}{9-i^2} = \frac{8+6i}{10} = 4/5 + 3/5\, i \rightarrow |z| = (16/25 + 9/25)^{1/2} = 1$

(e) $z = e^{3+4i}$

$e^{3+4i} = e^3 e^{4i} \rightarrow |z| = |e^3 e^{4i}| = |e^3||e^{4i}| = e^3 \cdot 1 = e^3$

4.4.7 Are the following functions analytic?

a, c, e
2, 3, 4, 7, 8, 10, 11, 13, 17, 18, 20, 21, 23
d c, d

(a) $f = |z|^2 = x^2 + y^2$

(b) $f = \mathrm{Re}\, z = x$

(c) $f = \sin z = \sin x \cosh y + i \cos x \sinh y$

Can a function satisfy Laplace's equation without being analytic?