AMATH 301 - Spring 2018

Homework #9

Due on Sunday, June 3, 2018

Note: The first problem requires the file `arnold01.jpg` and the second problem requires the file `NoisyImage.mat`. These files are included with the homework assignment. You should download the files into the same directory as your script file and access them by using the `imread` and `load` commands, respectively. However, you should **not** upload them to Scorelator. Scorelator has its own copy of both files.

In the instructions below, I ask you to plot things. Plotting is not required to get full credit for the assignment, but I highly recommend it so that you can get a sense of what is being done. However, you should remove all plots before submitting to Scorelator.

1. Read the image `arnold01.jpg` into Matlab and convert it to a grayscale image with double precision numbers by using the following commands.

   ```
   A = imread('arnold01.jpg');
   A = double(rgb2gray(A));
   ```

   Plot the image using the commands:

   ```
   imagesc(A)
   colormap gray
   ```

   (a) Perform a singular value decomposition on the matrix A that was generated by the image `arnold01.jpg`. Create a 10×1 column vector that contains the ten largest singular values of A. Store this vector in `A1.dat`. The cumulative energy in the first r modes is the sum of the first r (i.e. r largest) singular values divided by the sum of all of the singular values. Calculate the cumulative energy in the first 10 modes and save it in `A2.dat`.

   (b) Find the smallest value of r such that the cumulative energy in the first r modes is greater than 0.5. Save the value of r in `A3.dat`. Approximate the matrix A by using the first r modes (i.e. the first r columns of U, the first r singular values, and the first r rows of V^T, where U and V come from the SVD). Subtract this approximation from the original matrix A and then calculate the size of the error by using the `norm` command (use the default matrix norm and NOT the infinity norm). Save the error in `A4.dat`. Plot the approximation using `imagesc` and compare with the original image.

(c) Now find the smallest value of r such that the cumulative energy in the first r modes is greater than 0.9. Save the value of r in `A5.dat`. Just like in part (b), approximate the matrix A by using the first r modes, calculate the error, and save the error in `A6.dat`. Plot the approximation using `imagesc` and compare with the original image.

Things to think about: What were the values of r that you found in parts (b) and (c)? How much less information needs to be stored if you just keep the truncated versions of U, Σ, and V? How good do the approximations look? How much cumulative energy do you think is needed for a "good" approximation?

2. Use the `load` command to load the data in `NoisyImage.mat`. Two matrices should appear in your workspace. The matrix `A` is the true version of some image and the matrix `A_noise` is a noisy version of the same image. Plot both images using the `imshow` command and compare them. The goal of this problem is to denoise the noisy image.

   (a) Calculate the error between the matrix `A` and the matrix `A_noise` by subtracting and taking the norm. Save the error in `A7.dat`. Perform a singular value decomposition on the matrix `A_noise`. Save the largest singular value in `A8.dat`.

   (b) Calculate the cumulative energy in the first 2 modes and save it in `A9.dat`. Then approximate the matrix `A_noise` by using the first 2 modes. Calculate the error between the matrix `A` which represents the true image and this approximation. Save the error in `A10.dat`. Plot this approximation and compare to both the true image and the noisy image.

Things to think about: Look at all of the singular values of `A_noise` (or at least the first 20). Do a singular value decomposition on `A` and look at the singular values.

How do they compare? Does this explain why using the first 2 modes of `A_noise` was so effective? Do you think the denoising process would be more difficult on a different image?