AMATH 301 - Spring 2018

Homework #1

Due on Thursday, April 5, 2018

Note: All questions in your homework are rhetorical (i.e. you do not need to answer them). They are there to guide your thinking and hopefully help you learn. You will only be graded on the contents of the `.dat` files created by your Matlab script.

1. Define the following matrices and vectors.

$$A = \begin{pmatrix} 4 & 2 \\ 1 & -2 \end{pmatrix}, \ B = \begin{pmatrix} 3 & -1 \\ 3 & -2 \end{pmatrix}, \ C = \begin{pmatrix} 1 & 2 \\ 1 & -3 \\ -2 & 3 \end{pmatrix}, \ D = \begin{pmatrix} 1 & -3 & -2 \\ 1 & 9 & 4 \end{pmatrix},$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ \mathbf{y} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \ \mathbf{z} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}.$$

Calculate the following:

(a) $A + B$,

(b) $3\mathbf{x} - 2\mathbf{y}$,

(c) $A\mathbf{x}$,

(d) $A(\mathbf{y} - \mathbf{x})$,

(e) $C\mathbf{x}$,

(f) $C\mathbf{y} + \mathbf{z}$,

(g) AB,

(h) BA.

Save the answers in eight separate .dat files, named `A1.dat`, `A2.dat`, . . . , `A8.dat`. Are the answers in (g) and (h) the same?

(i) Now try to calculate both AD and DA. One is a valid matrix multiplication and the other will give you an error. Save the one that is valid in `A9.dat`.

Next, access and save the following elements:

(j) The second column of D,

(k) Both columns of the last two rows of C,

(l) The first two columns of the second row of D.

Save the answers in three separate .dat files, named `A10.dat`, `A11.dat` and `A12.dat`.

2. The following four expressions are exactly equal to zero:

$$x_1 = \left|2000 - \sum_{k=1}^{20{,}000} 0.1\right|, \quad x_2 = \left|2000 - \sum_{k=1}^{16{,}000} 0.125\right|,$$
$$x_3 = \left|2000 - \sum_{k=1}^{10{,}000} 0.2\right|, \quad x_4 = \left|2000 - \sum_{k=1}^{8{,}000} 0.25\right|$$

However, computers store floating-point numbers with a binary representation and only finitely many digits, so most decimal representations have a small truncation error. This error is usually too small to be noticeable, but it accumulates if you add up many copies of that number. As a result, x_1, x_2, x_3, and x_4 might not be exactly zero if you compute them with a computer. To verify this effect, use Matlab to compute x_1, x_2, x_3, and x_4 and save them in `A13.dat` - `A16.dat`, respectively. Can you explain the differences in these values?

3. The **Logistic Map** is a function that is often used to model population growth. It is defined by

$$P(t+1) = rP(t)\Big(1 - P(t)\Big),$$

where $P(t)$ represents the population at year t as a proportion of the maximum population. Therefore, $P(t) = 1$ means the population is at its maximum value and $P(t) = 0.5$ means that the population is half of the maximum value. The parameter r is the growth rate. Using this equation, if we know the population for some year $P(t)$, we can plug it into the right-hand side of the equation to find the population for the next year, $P(t+1)$. For example, let's say that $t = 0$ represents the current year and we know the current population $P(0)$. Then we can calculate the population after 1 year, $P(1)$, using

$$P(1) = rP(0)\Big(1 - P(0)\Big),$$

Once we know the population after 1 year, we can calculate the population after 2 years using

$$P(2) = rP(1)\Big(1 - P(1)\Big),$$

Let $r = 3$ and let the current population be $P(0) = 0.4$. Find the population after 100 years, $P(100)$, and save it in `A17.dat`

Bonus (for fun): The Logistic map is famous for exhibiting some strange and interesting behavior. In particular, it exhibits chaos for some values of r (we will see more chaotic systems later in the course). If you would like to explore it further, try the following:

(a) Make a vector that contains all of the values from $P(0)$ to $P(100)$. That is, make the vector:

$$\begin{pmatrix} P(0) & P(1) & P(2) & \dots & P(100) \end{pmatrix}$$

Then plot the population versus time t.

(b) Now do the same with $r = 3.5$ and $r = 4$. Compare the behavior in each case.