CFRM 405: Mathematical Methods for Quantitative Finance Homework 1

Instructions: Solve each of the exercises both by hand (must show all work for credit).

1. Compute the following limits.

   (a) $\lim\limits_{h\to 0} \dfrac{4(x+h-3)^2 - 4(x-3)^2}{h}$

   (b) $\lim\limits_{x\to\infty} \dfrac{1}{\sqrt{4x^2-2x-10}+2x}$

2. Compute the derivatives of the following functions.

   (a) $f(x) = \dfrac{1}{1-x}$

   (b) $f(x) = \displaystyle\sum_{k=1}^{7} ke^{-a_k x^3}$ (the $\{a_k\}$ are constants)

   (c) $f(x) = \dfrac{\log\left(\frac{x}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$ ($K > 0$, r, q, $\sigma > 0$, and $T > t$ constant)

   (d) $f(x) = \dfrac{\log\left(\frac{S}{K}\right) + \left(r - q + \frac{x^2}{2}\right)(T-t)}{x\sqrt{T-t}}$ ($S > 0$, $K > 0$, r, q, and $T > t$ constant)

   (e) $f(x) = \dfrac{\log\left(\frac{S}{K}\right) + \left(x - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$ ($S > 0$, $K > 0$, q, $\sigma > 0$, and $T > t$ constant)

3. Recall that

$$d_+(\cdot) = \frac{\log\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

   (a) Parts (c), (d), and (e) of Problem 2 correspond to partial derivatives of d_+. What partial derivative does each correspond to?

   (b) Compute the partial derivative of d_+ with respect to t.

4. Compute the following antiderivatives.

   (a) $\displaystyle\int x^2 \log(x)\, dx$

   (b) $\displaystyle\int x^2 e^x\, dx$

   (c) $\displaystyle\int \big[\log(x)\big]^2\, dx$

5. Evaluate the following definite integrals.

   (a) $\displaystyle\int_4^7 x^2 \log(x)\, dx$

   (b) $\displaystyle\int_0^2 \frac{1}{(1+x)^2}\, dx$

6. If you are an online student, please write down the proctor you are using for the exams in this class. They must be approved (or pending approval) at the time of submission of this assignment. The guidelines and procedure for proctor approval can be found at: http://depts.washington.edu/compfin/info/exam-proctors/