



A Probability and Statistics Primer for Quantitative Finance

Week 1: Introduction

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Slides originally produced by Kjell Konis

Course Information

- ▶ Format:
 - ▶ CFRM 410 has both an on-campus section (A) and an online section (B)
 - ▶ Due to limited classroom size, all online students must take proctored exams

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- ▶ Evaluation:
 - ▶ Mastery-based grading

Course Information

- ▶ There are **fourteen** key skills students should demonstrate to succeed in this course
- ▶ Each skill will be either “mastered,” “improving,” or “not satisfactory”
- ▶ Your final grade depends only upon the number of skills you have mastered

Number of Mastered Skills	Grade Point
14	4.0
13	3.8
12	3.6
11	3.4
10	3.2
9	3.0
8	2.5
7	2.0
6 and below	0.0

Course Information

Non-examination Key Skills:

- (I) Attendance and participation
- (II) Homework completion and reflection
 - ▶ Mastery requires every assignment to be turned in and at least three assignments to be revised to full credit or annotated if already full credit
- (III) Proficiency in R programming
 - ▶ Student has turned in well-documented, well-commented, and well-written code for three code-based homework questions (revision allowed)

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Examination Key Skills:

- ▶ During each of four in-class exam periods, students may complete problems demonstrating their mastery of the other eleven **key skills**
- ▶ Each skill may be attempted during any examination period after its in-class introduction, as many times as desired until mastery is achieved
- ▶ There is no penalty associated with *when* the skill is mastered
- ▶ Prior to re-attempting a demonstration of mastery, students must revise their previous attempts

Course Information

Exams:

During each in-class exam, students can attempt to demonstrate mastery of key skills

- ▶ Midterm 1: Wednesday, January 24th
- ▶ Midterm 2: Wednesday, February 21
- ▶ Midterm 3: Friday, March 9
- ▶ Final Exam: Thursday, March 15

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- ▶ Students may also schedule **one** additional examination period before the final examination of 50 minutes with the instructor, TA, or a proctor to attempt to demonstrate mastery of key skills

Organization

- Textbook
- ▶ Diez, David M., Christopher D. Barr, and Mine Cetinkaya-Rundel. OpenIntro statistics. CreateSpace, 2012.
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Office Hours

- ▶ Professor: Tuesdays and Thursdays 10:00-11:00 Pacific Time Over Skype or in person at Lewis Hall Room 128 (in the basement)
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- ▶ Reading assignments and lecture slides
- ▶ Distribution and collection of assigned work

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- Piazza
- ▶ Course discussion board

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- ▶ Presentation will count towards your total score (poorly presented correct results will only receive partial credit), this applies to exams too

Homework Policy

$$1) \quad 7071.53 = \frac{2500}{1+r} + \frac{2500}{(1+r)^2} + \frac{2500}{(1+r)^3}$$

$$\dots 7071.53 (1+r)^3 = 2500 (1+r)^2 + 2500 (1+r) + 2500$$

$$\dots 7071.53 (1+r)^3 - 2500 (1+r)^2 - 2500 (1+r) - 2500 = 0$$

$$\dots \text{Solve with Wolfram Alpha} \quad (1+r) \approx 1.03$$

$$\dots \Rightarrow r = 3\%$$

Homework Policy

$$\begin{aligned}
 \frac{\partial P}{\partial T} &= K\phi(d_2)\frac{\partial}{\partial t}(d_2)e^{-r(T-t)} + Kr\phi(d_2)e^{-r(T-t)} - S\phi(d_1)\frac{\partial}{\partial t}(d_1)e^{-q(T-t)} - Sq\phi(d_1)e^{-q(T-t)} \\
 &= K\phi(d_2)e^{-r(T-t)}\left(\frac{\partial}{\partial t}(d_2) + r\right) - S\phi(d_1)e^{-q(T-t)}\left(\frac{\partial}{\partial t}(d_1) + q\right) \\
 &= K\phi(d_2)e^{-r(T-t)}\left(\frac{\partial}{\partial t}\left(-\frac{(T-t)^{\frac{1}{2}}}{\sigma}\sqrt{\frac{S}{K}} + \left(r - q - \frac{\sigma^2}{2}\right)(T-t)\right)\right) - S\phi(d_1)e^{-q(T-t)}\left(\frac{\partial}{\partial t}(d_1) + q\right) \\
 &= K\phi(d_2)e^{-r(T-t)}\left(\frac{\partial}{\partial t}\left(\frac{-1}{\sigma}\sqrt{\frac{S}{K}}(T-t)^{\frac{1}{2}} + \left(r - q - \frac{\sigma^2}{2}\right)(T-t)\right)\right) - S\phi(d_1)e^{-q(T-t)}\left(\frac{\partial}{\partial t}(d_1) + q\right) \\
 &= K\phi(d_2)e^{-r(T-t)}\left(\frac{-1}{\sigma}\sqrt{\frac{S}{K}}\frac{1}{2}(T-t)^{-\frac{1}{2}} + \left(r - q - \frac{\sigma^2}{2}\right)\right) - S\phi(d_1)e^{-q(T-t)}\left(\frac{\partial}{\partial t}(d_1) + q\right) \\
 &= K\phi(d_2)e^{-r(T-t)}\left(\frac{-1}{2\sigma}\sqrt{\frac{S}{K}}(T-t)^{-\frac{3}{2}} + \frac{1}{2}\right) - S\phi(d_1)e^{-q(T-t)}\left(\frac{\partial}{\partial t}(d_1) + q\right)
 \end{aligned}$$

Exam Policy

These instructions are on the cover of the exams

During the exam, students are permitted writing utensils, blank paper and a copy of the exam. No study aids (cheat sheets, etc.) are allowed. Students are not allowed to access their mobile phone or any internet connected device. Answers must be justified to receive full credit (i.e., show your work).

Questions are not allowed during the exam. If you find something to be unclear,

- i) explain why you think the question is unclear,*
- ii) precisely state your interpretation of the question,*
- iii) provide an answer to your interpretation of the question.*

Students are not be permitted to leave the room during the exam.

Calculator Policy

- ▶ Allowed calculator: Texas Instruments TI-30Xa Scientific Calculator

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- ▶ Mathematics provides a framework for
 1. precisely expressing a wide variety of complex ideas, and
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- ▶ What does statistics mean?

Statistics is the use of mathematics to extract information from data in the presence of uncertainty

Probability

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 - ▶ Probability is the field of mathematics that studies random (stochastic) phenomena
 - ▶ Probability provides a mathematical foundation for building models that include uncertainty
- ▶ Probability gives us a means to understand and quantify the effect of uncertainty on information extracted from data

Syllabus

Module 1: Exploratory Data Analysis and R Programming (Weeks 1-4)

- ▶ Introduction
- ▶ R Programming and financial datasets
- ▶ Exploratory financial data analysis

(IV) Describing univariate data

- ▶ Student can present a financial data set, describe key features, and produce the appropriate representation: boxplots, histograms, or charts of categorical data

(V) Characterizing sampled data and distributions

- ▶ Student is able to characterize sample data in terms of center, spread, skewness, kurtosis, and modality both qualitatively and quantitatively
- ▶ Student can use z-scores and quantile-quantile plots to compare financial data to normal distributions

(VI) Comparing multiple sets of data

- ▶ Student can compare two financial data sets through scatterplots, side-by-side box plots and histograms, covariance, and correlation and can convincingly interpret the results

Syllabus

Module 2: Probability theory, random variables, and distributions (Weeks 5-8)

- ▶ Probability theory
- ▶ Distributions and their use in finance
- ▶ Multivariate distributions and sampling

(VII) Interpreting and analyzing discrete probability problems

- ▶ Student is able to use venn diagrams, conditional probability, and counting to solve discrete probability problems

(VIII) Computing quantities of interest for discrete probability distributions

- ▶ Student is able to state definitions of expected value, variance, and other key quantities of discrete probability distributions and compute them in financial contexts

(IX) Integrating concepts from calculus to study continuous probability distributions

- ▶ Student is able to use concepts from calculus to solve problems relating to continuous probability distributions

Syllabus

Module 2: Probability theory, random variables, and distributions (Weeks 5-8)

- ▶ Probability theory
- ▶ Distributions and their use in finance
- ▶ Multivariate distributions and sampling

(X) Manipulating multivariate probability distributions

- ▶ Students can compute conditional distributions, expected values, and related quantities for discrete and continuous multivariate distributions

(XI) Familiarity with important limit theorems in probability theory

- ▶ Student is able to state the law of large numbers and the central limit theorem, and use them to draw conclusions about financial scenarios

Syllabus

Module 3: Estimation and hypothesis testing (Weeks 9-10)

- ▶ Estimation
- ▶ Financial hypothesis testing

(XII) Ability to derive estimators

- ▶ Student is able to derive maximum likelihood estimators for unknown financial quantities

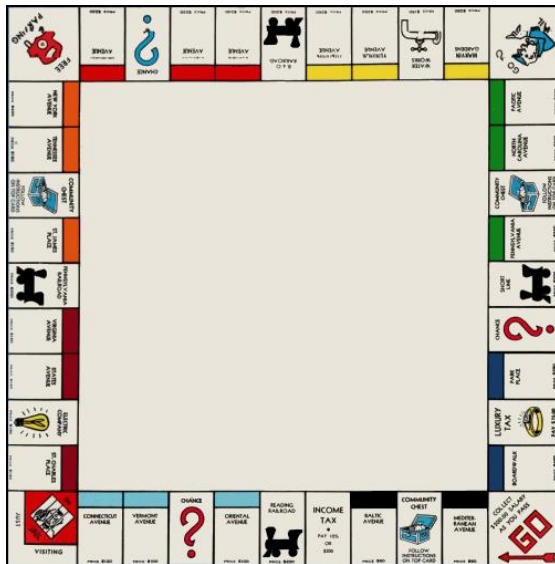
(XIII) Interpretation of estimators

- ▶ Student is able to compute the bias and variance of estimators, interpret the results, and discuss confidence intervals

(XIV) Proficiency with hypothesis testing

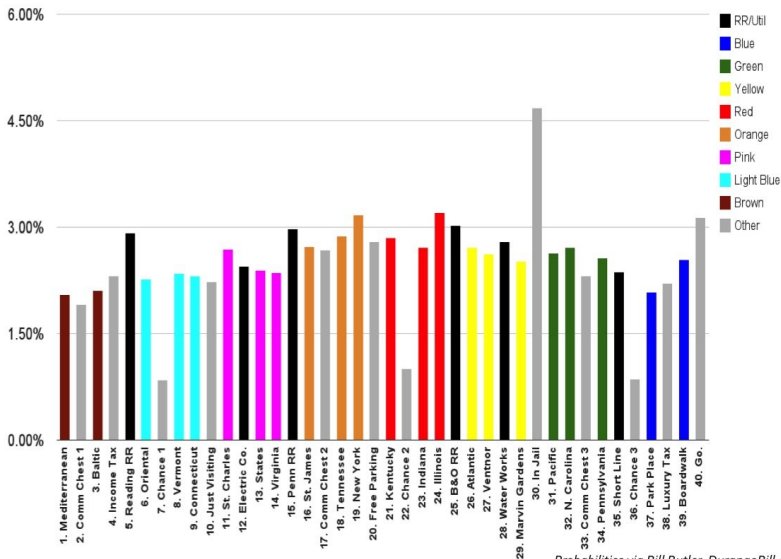
- ▶ Student is able to use statistical techniques to test a hypothesis about a financial scenario, interpret their results, and explain them clearly

Example: Monopoly



Example

End of turn steady state Monopoly space probabilities



Probabilities via Bill Butler, DurangoBill.com



COMPUTATIONAL FINANCE & RISK MANAGEMENT

UNIVERSITY *of* WASHINGTON

Department of Applied Mathematics

<http://computational-finance.uw.edu>