The first three exercises are calculus and linear algebra review. If you are unable to do these exercises

1. Let

$$f(x) = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + b$$

where $a > 0$ and b are constants, log denotes the natural logarithm, and $|\cdot|$ denotes the absolute value.

a) What is the domain of $f(x)$?

The largest possible domain for f is $D = \mathbb{R} \backslash \{-a, a\}$ since $|\cdot| \geq 0 \ \forall x \in \mathbb{R}$ and the real valued log function $log(y)$ may be defined sensibly only for $y > 0$.

b) Compute $f'(x)$.

First we show for $u : D \subset \mathbb{R} \longrightarrow \mathbb{R}$, that wherever $|u(x)|$ is differentiable, its derivative is given by $\frac{u(x)u'(x)}{|u(x)|}$:

$$\frac{d}{dx}|u(x)| = \frac{d}{dx}\sqrt{(u(x))^2} = \frac{1}{2\sqrt{(u(x))^2}} 2u(x)u'(x) = \frac{u(x)u'(x)}{|u(x)|}.$$

Using this we have

$$f'(x) = \frac{1}{2a} \left| \frac{x+a}{x-a} \right| \frac{\frac{x-a}{x+a} \frac{x+a-(x-a)}{(x+a)^2}}{\left| \frac{x-a}{x+a} \right|} = \frac{1}{2a} \frac{|(x+a)^2|}{|(x-a)^2|} \frac{2a(x-a)}{(x+a)^3} = \frac{1}{x^2-a^2} \ (x \neq \pm a).$$

c) Evaluate the indefinite integral

$$\int \frac{1}{x^2 - 2x} \, dx$$

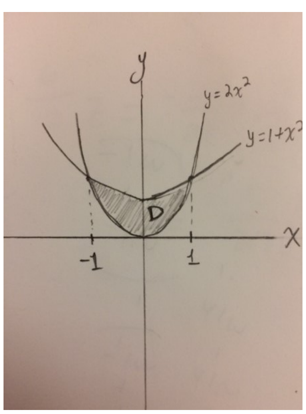
by completing the square.

$$
\begin{aligned}
\int \frac{dx}{x^2-2x+1-1} &= \int \frac{dx}{(x-1)^2-1} \qquad (x-1 = sec\theta \implies dx = sec\theta tan\theta d\theta)\\
&= \int \frac{sec\theta tan\theta d\theta}{sec^2\theta - 1}\\
&= \int \frac{sec\theta d\theta}{tan\theta}\\
&= \int csc\theta d\theta\\
&= -log|csc\theta + cot\theta| + c \quad (*)\\
&= -log\left|\frac{x-1}{\sqrt{x^2-2x}} + \frac{1}{\sqrt{x^2-2x}}\right| + c\\
&= log\left|\frac{\sqrt{x(x-2)}}{x}\right| + c\\
&= log|\sqrt{x(x-2)}| - log|x| + c\\
&= \frac{1}{2}(log|x| + log|x-2|) - log|x| + c\\
&= \frac{1}{2}(log|x-2| - log|x|) + c.
\end{aligned}
$$

(*) Here we used a standard integral list found at https://en.m.wikipedia.org/wiki/ .

2. Let D be the region in the xy-plane bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$ and satisfying $|x| < 1$.

a) Sketch the region D.



b) Evaluate the definite integral

$$\iint_D x^2\, dA = \int_{-1}^{1}\int_{2x^2}^{1+x^2} x^2\, dy\, dx = \int_{-1}^{1} x^2 y\Big|_{2x^2}^{1+x^2}\, dx = \int_{-1}^{1} x^2 - x^4\, dx = \frac{x^3}{3} - \frac{x^5}{5}\Big|_{-1}^{1} = \frac{4}{15}\,.$$

3. ** For problems 3 and 4 I have included in this document snippets of R code used for each problem. However, I have also attached all the code collected together in a seperate document submitted as well to possibly assist in grading.**

   Let

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \qquad \text{and} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

   a) Is the sum $\mathbf{A} + \mathbf{b}$ defined? If so, what is it?

      The sum $\mathbf{A} + \mathbf{b}$ is defined in the R programming language (but we note that this sum is not defined in standard linear algebra or in some other programming languages such as Matlab). In R the sum is:

$$\mathbf{A} + \mathbf{b} = \begin{bmatrix} 2 & 5 \\ 4 & 7 \\ 6 & 9 \end{bmatrix}.$$

   b) Write one line of R code that uses the `cbind` function to create the matrix $\mathbf{A}$ and assigns it to a variable named `A`.

      This is accomplished with the command : `A <- cbind(c(1,2,3),c(4,5,6))`

   c) Create the vector $\mathbf{b}$ using the command `b <- 1:3` and compute the sum `C <- A + b`. Give an expression for `C[i, j]` in terms of `A[i, j]` and `b[i]`.

      Creating `b` as described and using `A` as before to compute the sum `C <- A + b` we have `C[i,j] = A[i,j] + b[i]`.

4. R exercises: these exercises are meant to give you some practice subsetting vectors, reading R documentation files, and loading R packages.

   a) Among R's built-in constants is a vector named `letters` that contains 26 lowercase letters in alphabetical order. Spell your last name by subsetting `letters` (spaces, if any, should be omitted).

   ```
   my_last_name <- letters[c(10,15,8,14,19,15,14)]
   ```

   b) Read the documentation for `letters`. Use the `c` function and one or more components from another built-in constants vector to capitalize your last name appropriately.

   ```
   my_last_name_capitalized <- c(LETTERS[10],letters[c(15,8,14,19,15,14)])
   ```

   c) Repeat part ii for your first name.

   ```
   my_first_name_capitalized <- c(LETTERS[4],letters[c(1,14,5)])
   ```

   d) Use the `paste` function to write your first and last name (correctly capitalized) as a single character string.

   ```
   my_full_name <- paste(c(paste(my_first_name_capitalized,collapse=""),
   paste(my_last_name_capitalized,collapse = "")),collapse = " ")
   ```

   e) Use a logical vector to extract the first and last 5 letters from `letters`.

   ```
   x <- 1:26
   letters_subset <- letters[x[x < 6 | x > 21]]
   ```

   f) The MASS package contains a vector named `chem`. Write one line of R code that returns the number of components of `chem` that are in the interval $(3, 4)$.

   ```
   chem_components_in_range <- length(chem[chem > 3 & chem < 4])
   ```