Section 7.1 Notes

November 16, 2019

7.1.12

Evaluate the integral

$$I = \int \frac{ln(ln(x))}{xln(x)}\, dx \, .$$

Let $u = ln(x)$ so that $du = \frac{1}{x}\, dx \implies x\, du = dx$. By substitution,

$$I = \int \frac{ln(u)}{xu} x\, du = \int \frac{ln(u)}{u}\, du \, .$$

Making a second substitution $w = ln(u)$, we get

$$I = \int \frac{ln(u)}{u}\, du \; = \int w\, dw = \frac{w^2}{2} + C \, .$$

We need to express our answer in terms of x since this is the variable the problem actually began with. Therefore,

$$I = \frac{w^2}{2} + C = \frac{(ln(u))^2}{2} + C = \frac{[ln(ln(x))]^2}{2} + C \, .$$

7.1.28

Evaluate the integral

$$I = \int_{-2}^{0} 5^{-\theta}\, d\theta \, .$$

We should consider the result given on page 431, which states that for $a > 0$,

$$\int a^u \, du = \frac{a^u}{\ln a} + C \, .$$

This tells us the indefinite integral, which can be used to find a similar definite integral. To fit this form, we can make the substition $u = -\theta$ or use properties of exponents to rewrite I as

$$I = \int_{-2}^{0} \left(\frac{1}{5}\right)^{\theta} \, d\theta \, .$$

Since $\frac{1}{5} > 0$, the result from page 431 applies and so

$$I = \int_{-2}^{0} \left(\frac{1}{5}\right)^{\theta} \, d\theta = \frac{(\frac{1}{5})^{\theta}}{\ln \frac{1}{5}}\Big|_{-2}^{0} = \frac{5^{-\theta}}{\ln \frac{1}{5}}\Big|_{-2}^{0} = -\frac{5^{-\theta}}{\ln 5}\Big|_{-2}^{0} = -\frac{1}{\ln 5}(5^0 - 5^2) = \frac{24}{\ln 5} \, .$$

7.1.46

Evaluate the integral

$$I = \int \frac{dx}{x(\log_8 x)^2} \, .$$

Using the logarithm definition, let $y = \log_8 x \implies 8^y = x \implies \ln 8^y = \ln x \implies y = \frac{1}{\ln 8} \ln x$. Rewrite I using this to get

$$I = \int \frac{dx}{x(\frac{1}{\ln 8} \ln x)^2} = \ln^2 8 \int \frac{dx}{x(\ln x)^2} \, .$$

Let $u = \ln x$. Then,

$$I = \ln^2 8 \int u^{-}2 \, du = -(\ln^2 8) u^{-1} + C = -\frac{\ln^2 8}{\ln x} + C \, .$$