Section 8.5 Notes

December 7, 2019

8.5.2

Expand the quotient by partial fractions:

$$\frac{5x-7}{x^2-3x+2} = \frac{5x-7}{(x-2)(x-1)}$$

Before doing anything else, check that the degree of the numerator is strictly less than the degree of the denominator. Otherwise you'll end up doing a lot of algebra only to find out that partial fractions 'breaks'...yes the degree of the numerator is 1 and the degree of the denominator is 2.

$$\frac{5x-7}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{x-1}$$

$$5x-7 = A(x-1) + B(x-2)$$

$$5x-7 = (A+B)x + (-A-2B)$$

Matching the $'x'$ coefficients on both sides and matching the constants on both sides gives two equations:

$$5 = A+B \quad -7 = -A-2B$$

Solve this system of equations however you prefer. The result is $A = 3, B = 2$. Conclude that:

$$\frac{5x-7}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{x-1} = \frac{3}{x-2} + \frac{2}{x-1}$$

This means that if we were asked to evaluate $\int \frac{5x-7}{(x-2)(x-1)}\,dx$, it would be the same thing as evaluate $\int (\frac{3}{x-2} + \frac{2}{x-1})\,dx$. The second integral is easier to find.

8.5.8

Expand the quotient by partial fractions:

$$\frac{t^4+9}{t^4+9t^2}$$

The degree of the numerator is not less than the degree of the denominator, so first long division:

$$\begin{array}{r|l} & \quad 1 \\ \hline t^4+9t^2 & t^4+9 \\ & -(t^4+9t^2) \\ \hline & \quad -9t^2+9 \end{array}$$

$$\frac{t^4+9}{t^4+9t^2} = 1 + \frac{-9t^2+9}{t^4+9t^2} = 1 + \frac{-9t^2+9}{t^2(t^2+9)}$$

Just work with the fraction part for now.

$$\frac{-9t^2+9}{t^2(t^2+9)} = \frac{A}{t} + \frac{B}{t^2} + \frac{Ct+D}{t^2+9}$$

$$-9t^2+9 = At(t^2+9) + B(t^2+9) + (Ct+D)t^2$$

$$-9t^2+9 = (A+C)t^3 + (B+D)t^2 + (9A)t + 9B$$

Equating coefficients gives the four equations:

$$\begin{aligned} 0 &= A + C \\ -9 &= B + D \\ 0 &= A \\ 9 &= 9B \end{aligned}$$

This system has the solution $A=0, B=1, C=0, D=-10$. Conclude that

$$\frac{t^4+9}{t^4+9t^2} = 1 + \frac{-9t^2+9}{t^2(t^2+9)} = \frac{A}{t} + \frac{B}{t^2} + \frac{Ct+D}{t^2+9} = 1 + \frac{1}{t^2} - \frac{10}{t^2+9}$$