Review for Test 2

November 24, 2019

1

Compute the indefinite integrals.

(a) $\int \left(\sqrt[7]{x} - \frac{2}{\sqrt{1-x^2}} \right) dx$.

(b) $\int \frac{\sin^2(\ln(x))\cos(\ln(x))}{x} dx$.

(c) $\int \frac{x+\sqrt{x}}{x^3} dx$.

(d) $\int \tan^6(x)\sec^6(x)\, dx$.

(e) $\int \frac{2x^3+4x^2-5}{x+3} dx$.

(f) $\int \pi^{-x} dx$.

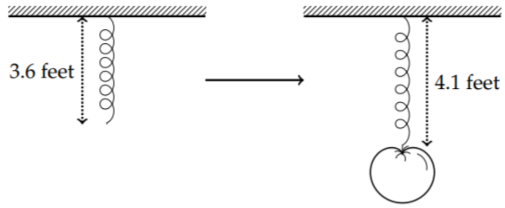
(g) Make sure to get this question 'completely squared away' before moving on to the next question. $\int \frac{dx}{\sqrt{8x-x^2}}$.

(h) Hold on, this is the last 'part' of question 1: $\int x^2 e^{2x}\, dx$.

2

Recall Hookes law, which says that the force required to compress or stretch a spring from its natural length by some distance is proportional to that distance.

A spring (of negligible mass) is suspended from the ceiling and has a natural length of 3.6 feet. When a 0.4-pound tomato is attached to the end



Mass.png

of the spring, it stretches to a length of 4.1 feet.

Compute the work required to stretch this same spring from a length of 5 feet to 6 feet.
(Express your answer in foot-pounds.)

3

Let $\mathcal{R}$ be the region in the $x-y$ plane below $y = \sec(x)\tan(x)$ and above $y = -2$ from $x = 0$ to $x = \pi/4$.

(a) Write an integral to compute the volume of the solid formed by revolving $\mathcal{R}$ around the line $y = -2$.

(b) Evaluate the integral from part (a).

4

Determine the surface area of the solid obtained by rotating $x = y^3$, $0 \le y \le 2$ about the y-axis.

5

Determine the center of mass for the plate of constant density δ on the region bounded by $y = x^3$ and $y = \sqrt{x}$.

6

A tank is formed by revolving the line $y = \frac{15}{4}x$, $0 \le x \le 4$ about the y-axis (assume the x and y axis are measured in meters). If the tank is filled with water with density 1000 kg/m^3, find the work done in pumping all of the water to the top of the tank.

Answers

1a. $\frac{7}{8}x^{8/7} - 2\text{arcsin}(x) + C$.
1b. $\frac{\sin(\ln(x))}{3} + C$.
1c. $-x^{-1} - \frac{2}{3}x^{-\frac{3}{2}} + C$.
1d. $\frac{1}{11}\tan^{11}(x) + \frac{2}{9}\tan^{9}(x) + \frac{1}{7}\tan^{7}(x) + C$.
1e. $\frac{2}{3}x^3 - x^2 + 6x + 13\ln|x+3| + C$.
1f. $-\frac{\pi^{-x}}{\ln(\pi)} + C$.
1g. $\text{arcsin}(\frac{x-4}{4}) + C$.
1h. $\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$.

2) 1.52 ft-lbs

3) $\pi\left(-11/3 + 4\sqrt{2} + \pi\right)$.

4) $\frac{\pi}{27}\left(145^{3/2} - 1\right)$.

5) $(\overline{x}, \overline{y}) = \left(\frac{12}{25}, \frac{3}{7}\right)$.

6) $W = \int_{y_0}^{y_1} \rho g A(y) d\, dy = \int_0^{15} (1000)(9.8)[\pi(\frac{4}{15}y)^2](15-y)\, dy \approx 9.23628 \times 10^6$J, where ρ is the density of the water, g is gravitational acceleration, $A(y)$ is the area of an arbitrary horizontal cross section, d is the distance an arbitrary cross section is pumped, and y_0, y_1 are the lowest and highest cross sectional elevations respectively.