Section 6.5 Notes

November 16, 2019

6.5.6

If a force of 90 N stretches a spring 1 m beyond its natural length, how much work does it take to stretch the spring 5 m beyond its natural length?

By Hooke's Law the force, F (here in Newton's) required to stretch a spring x meters beyond its natural length is given by:

$$F(x) = kx$$

where the proportionality constant k is specific to the spring we are working with. By the question statement, this gives us

$$90\,\mathrm{N} = k(1\,\mathrm{m}) \implies k = 90\,\mathrm{N/m}\,.$$

Since we are asked to find the work required to stretch the spring from 0 meters beyond its natural length to 5 meters beyond its natural length, we integrate force from $x = 0$ to $x = 5$ to find the work, W, done:

$$W = \int_0^5 F(x)\,dx = 90x\,dx = 90(\frac{5^2}{2}) = 1125\,\mathrm{Nm}\,.$$

6.5.13

This is an assigned problem so the answer will not be provided here. In order to find the work done you need to integrate the force function $F(x) = m(x)\,g$, where the mass m, in kilograms, is linear function of distance x, in meters, the water has been lifted above the ground. The function $m(x)$ (this does not denote $m \times x$ but instead indicates that m is a function of x) should attain the values $m(0) = 5$ and $m(20) = 0$ according to the story.

Pumping Liquids from Containers

See Example 4 from this link for a good example on how to think about these types of work problems:

http://tutorial.math.lamar.edu/Classes/CalcI/Work.aspx