Section 6.4 Notes

November 16, 2019

6.4.8a

Set up an integral for the area of the surface generated by revolving the given curve about the indicated axis.

$$y = \int_1^x \sqrt{t^2-1}\,dt, \quad 1 \le x \le \sqrt{5}; \quad x - \text{axis}$$

The appropriate formula for the surface area, S, is:

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx\,.$$

In this case, $a = 1$, $b = \sqrt{5}$, $y = \int_1^x \sqrt{t^2-1}\,dt$, and so

$$\frac{dy}{dx} = \frac{d}{dx}\int_1^x \sqrt{t^2-1}\,dt = \sqrt{x^2-1}\,.$$

Therefore the integral we use to calculate S is:

$$S = \int_1^{\sqrt{5}} 2\pi \left(\int_1^x \sqrt{t^2-1}\,dt\right)\sqrt{1 + \left(\sqrt{x^2-1}\right)^2}\,dx$$

$$= \int_1^{\sqrt{5}} 2\pi \left(\int_1^x \sqrt{t^2-1}\,dt\right)\sqrt{1+|x^2-1|}\,dx = \int_1^{\sqrt{5}} 2\pi \left(\int_1^x \sqrt{t^2-1}\,dt\right) x\,dx\,.$$

Note that we used the assumption that $1 \le x \le \sqrt{5}$ to conclude that $|x^2-1| = x^2-1$ and $\sqrt{x^2} = |x| = x$ in order to simplify in the last step. This is something you do need to check before removing absolute value bars.

6.4.10

Find the lateral surface area of the cone generated by revolving the line segement $y = x/2$, $0 \le x \le 4$ abouth the $y-$axis. Check your answer with the geometry formula

Lateral surface area $= \frac{1}{2} \times$ base circumference $\times$ slant height.

Since we are revolving our curve about the y-axis, the appropriate formula to calculate surface area S, is

$$S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy\,.$$

So we'll need to write x as a function of y and determine the bounds c and d for y such that $c \le y \le d$. Here $x = 2y$ and $0 \le y \le 2$ (graph this curve if it's not clear why those are our bounds). So $\frac{dx}{dy} = 2$ and the lateral surface area is:

$$S = \int_0^2 2\pi(2y)\sqrt{1+2^2}\, dy \ = 4\sqrt{5}\pi \int_0^2 y\, dy = 2\sqrt{5}\pi \left(y^2\Big|_0^2 \right) = 8\sqrt{5}\pi \text{ units}^2.$$

Using the given geometry formula

Lateral surface area $= \frac{1}{2} \times (2\pi(4)) \times \sqrt{4^2+2^2} = 4\pi \times \sqrt{20} = 8\sqrt{5}\pi$ units^2.

The slant height is the length of the diagonal line segment connect the points $(0,0)$ and $(4,2)$, which are the endpoints of the portion of the curve that is revolved about the y-axis.

6.4.16

Find the area of the surface generated by revolving the given curve about the indicated axis.

$$y = \sqrt{x+1}, \quad 1 \le x \le 5; \quad x - \text{axis}.$$

The appropriate formula for the surface area, S, is:

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx\,.$$

Here $a = 1$, $b = 5$, and $\frac{dy}{dx} = \frac{1}{2}\frac{1}{\sqrt{x+1}}$. So the surface area, S, is given by:

$$\begin{aligned}
S &= \int_1^5 2\pi\sqrt{x+1}\sqrt{1 + \left(\frac{1}{2}\frac{1}{\sqrt{x+1}}\right)^2}\, dx && (1)\\
&= \int_1^5 2\pi\sqrt{x+1}\sqrt{1 + \frac{1}{4}\frac{1}{|x+1|}}\, dx && (2)\\
&= \int_1^5 2\pi\sqrt{x+1}\sqrt{1 + \frac{1}{4}\frac{1}{x+1}}\, dx && (3)\\
&= \int_1^5 2\pi\sqrt{x + 1 + \frac{1}{4}\frac{x+1}{x+1}}\, dx && (4)\\
&= \int_1^5 2\pi\sqrt{x + \frac{5}{4}}\, dx && (5)\\
&= 2\pi\frac{2}{3}\left((x + \frac{5}{4})^{3/2}\Big|_1^5\right) && (6)\\
&= \frac{4\pi}{3}\left((25/4)^{3/2} - (9/4)^{3/2}\right) && (7)\\
&= \frac{4\pi}{3}\left(\frac{125}{8} - \frac{27}{8}\right) && (8)\\
&= \frac{49\pi}{3}\,. && (9)
\end{aligned}$$

To get from line 3 to line 4 we used the fact that $\sqrt{a}\sqrt{b} = \sqrt{ab}$ for $a, b \geq 0$.