Calculus III
Practice Exam II, Math 1023 Name: _____

Show **all work clearly and in order**.

1. 1) (8 points) Find an equation of the tangent line to the given curve

$$x = t^2 + 3t, \quad y = t^3$$

at $t = -1$.

2) (8 points) Find the value of $\frac{d^2y}{dx^2}$ at $t = -1$.

2. (14 points) Find the length of the curves

$$x = \ln t - \frac{t^2}{2}, \quad y = 1 - 2t \qquad\qquad 1 \leq t \leq 3$$

3. (18 points) 1) (5 points) Find the Cartesian equation of the polar curve $r = 2\cos\theta$.

2) (5 points) Sketch the region bounded by this polar curve $r = 2\cos\theta$ and $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$.

2) (8 points) Find the area of the above region.

4. (16 points) Give $\vec{u} = \langle 3, -2, 1\rangle$, $\vec{v} = \langle -2, -1, -3\rangle$, $\vec{w} = \langle -1, 2, a\rangle$

(1) (6 points) Find the magnitude of $\vec{u} + \vec{v}$;

(2) (4 points) Find a unit vector parallel to the sum of $\vec{u} + \vec{v}$;

(3) (6 points) Find a parametric equation of a line passing through $(1, 1, -1)$ and parallel to $\vec{v}$.

5. Given three points $P(0, 2, 1)$, $Q(2, 0, -1)$ and $R(1, -1, 2)$.
1) (16 points) Find the area of the triangle with vertices P, Q and R.

2) (4 points) Find an equation for plane passing through P, Q and R.

6. Let $\vec{r}(t) = \langle \cos 2t, \sin t, \sin(t^2) \rangle$ be the position of a particle in space.

1) (12 points) Find the particle's velocity $\vec{v}(t)$ and acceleration vector $\vec{a}(t)$ at $t = 0$.

2) (4 points) Determine if the above two vectors $\vec{v}(0)$ and $\vec{a}(0)$ are parallel to each other.

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