Example 1 : N-th Term Test

Using the n-th term test, what can you say about convergence or divergence of the following series?

$$\sum_{n=1}^{\infty} \frac{n^3 - 2n + 4}{n^2 + n - 1}$$

The nth-Term Test for Divergence

$\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n$ fails to exist or is different from zero.

Example 2 : N-th Term Test

Using the n-th term test, what can you say about convergence or divergence of the following series?

$$\sum_{n=1}^{\infty} \frac{1-(-1)^n}{n}$$

The nth-Term Test for Divergence

$\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n$ fails to exist or is different from zero.

Example 3 : Geometric Series

Determine if the series converges or diverges. If the series converges calculate the sum.

$$\sum_{n=1}^{\infty} \frac{(-9)^n}{3^{2n}}$$

If $|r| < 1$, the geometric series $a + ar + ar^2 + \cdots + ar^{n-1} + \cdots$ converges to $a/(1 - r)$:

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1 - r}, \qquad |r| < 1.$$

If $|r| \geq 1$, the series diverges.

Example 4 : Geometric Series

Determine if the series converges or diverges. If the series converges calculate the sum.

$$\sum_{n=2}^{\infty}(-1)^{n+1}\frac{(4)^{2n}}{3^{5n-2}}$$

If $|r| < 1$, the geometric series $a + ar + ar^2 + \cdots + ar^{n-1} + \cdots$ converges to $a/(1 - r)$:

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}, \qquad |r| < 1.$$

If $|r| \geq 1$, the series diverges.

Example 5 : Integral Test

Use the integral test to show that the series converges.

$$\sum_{n=2}^{\infty} \frac{5n^4}{(n^5+17)^2}$$

THEOREM 9—The Integral Test Let $\{a_n\}$ be a sequence of positive terms. Suppose that $a_n = f(n)$, where f is a continuous, positive, decreasing function of x for all $x \geq N$ (N a positive integer). Then the series $\sum_{n=N}^{\infty} a_n$ and the integral $\int_N^{\infty} f(x)\,dx$ both converge or both diverge.

Example 6 : Integral Test

Use the integral test to show that the series diverges.

$$\sum_{n=1}^{\infty} \frac{(\ln n)^2}{\sqrt{n}}$$

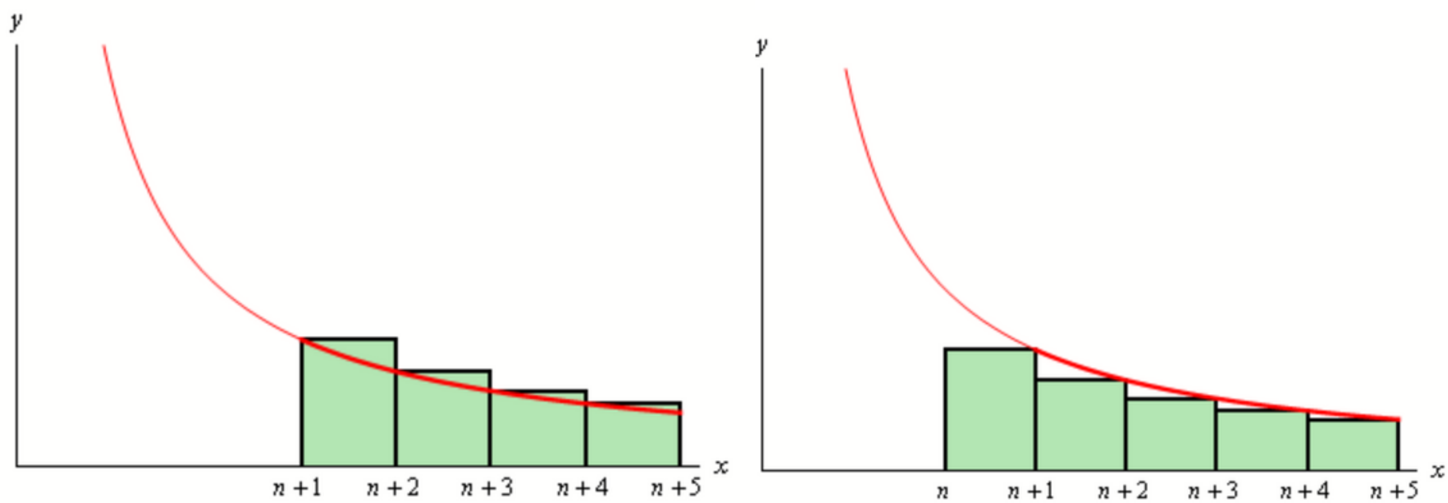
THEOREM 9—The Integral Test Let $\{a_n\}$ be a sequence of positive terms. Suppose that $a_n = f(n)$, where f is a continuous, positive, decreasing function of x for all $x \geq N$ (N a positive integer). Then the series $\sum_{n=N}^{\infty} a_n$ and the integral $\int_N^{\infty} f(x)\, dx$ both converge or both diverge.

Example 7 : Estimating a Sum with the Integral Test

You want to calculate the sum

$$\sum_{n=1}^{\infty} \frac{1}{n^8}\ .$$

Use the integral test to determine how many terms you need to add in order to estimate the sum with a guaranteed accuracy of four decimal places.



Source: Paul's Online Math Notes Estimating The Value Of A Series

Bounds for the Remainder in the Integral Test

Suppose $\{a_k\}$ is a sequence of positive terms with $a_k = f(k)$, where f is a continuous positive decreasing function of x for all $x \geq n$, and that Σa_n converges to S. Then the remainder $R_n = S - s_n$ satisfies the inequalities

$$\int_{n+1}^{\infty} f(x)\, dx \leq R_n \leq \int_{n}^{\infty} f(x)\, dx. \tag{1}$$

You want to calculate the sum

$$\sum_{n=1}^{\infty} \frac{1}{n^8}\,.$$

Use the integral test to determine how many terms you need to add in order to estimate the sum with a guaranteed accuracy of four decimal places.