Quiz 2, Math 1023 Name: ____________________ Conference: Mon. Fr.

Show **all steps** to earn credits.

1. (6 points) Determine the convergence or divergence of the sequences. If the sequence is convergent, find the limit.

$$1)\ a_n = \frac{\sin(2n+1)}{n^2} \qquad 2)\ a_n = \frac{n+1}{2\sqrt{n}+n^2}$$

Sol. 1) $-1 \le \sin(2n+1) \le 1$

$\Rightarrow -\frac{1}{n^2} \le \frac{\sin(2n+1)}{n^2} \le \frac{1}{n^2}$ 1pt

Since $\lim\limits_{n\to\infty} -\frac{1}{n^2} = \lim\limits_{n\to\infty} \frac{1}{n^2} = 0$, by 1pt

Squeeze Thm, $\lim\limits_{n\to\infty} a_n = 0$. 1pt

2) $a_n = \dfrac{\frac{n+1}{n^2}}{\frac{2\sqrt{n}+n^2}{n^2}} = \dfrac{\frac{1}{n}+\frac{1}{n^2}}{\frac{2}{n^{\frac{3}{2}}}+1}$ 1pt

$\lim\limits_{n\to\infty} a_n = \dfrac{\lim\limits_{n\to\infty}\left(\frac{1}{n}+\frac{1}{n^2}\right)}{\lim\limits_{n\to\infty}\left(\frac{2}{n^{\frac{3}{2}}}+1\right)}$

$= \dfrac{0+0}{0+1} = 0$ 1pt

1pt

2. (4 points) Determine if the geometric series converges or diverges. If the series converges, find the value.

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{3^{n+1}}$$

Sol $a_n = \dfrac{(-2)^n}{3^{n+1}}$

$r = \dfrac{a_{n+1}}{a_n} = \dfrac{(-2)^{n+1}}{3^{n+2}} \cdot \dfrac{3^{n+1}}{(-2)^n} = \dfrac{1}{3}\cdot(-2) = -\dfrac{2}{3}$. 1pt.

Since $-1 < -\frac{2}{3} < 1$, $\sum\limits_{n=1}^{+\infty} a_n = \dfrac{a_1}{1-r} = \dfrac{(-2)^1}{3^{1+1}} \cdot \dfrac{1}{1-(-\frac{2}{3})}$

1pt converges 1pt

$= \dfrac{-2}{9} \cdot \dfrac{1}{1+\frac{2}{3}} = -\dfrac{2}{9}\cdot\dfrac{3}{5} = -$