Quiz 5, Math 1023 Name: ____________ Conference: Mon. Fr.

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$\frac{d^2y}{dx^2}$

1. (5 points) Given a parametric curve $x = 2t^2 + 3$, $y = t^3$. Find ~~an equation of the tangent line to the curve~~ at $t = 1$.

Sol. $\frac{dx}{dt} = 4t$ pt $\frac{dy}{dt} = 3t^2$ pt $\frac{dy}{dx} = \frac{3t^2}{4t} = \frac{3}{4}t$ 1pt

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{3}{4}}{4t} = \frac{3}{16t}$$ 1pt

$t=1$ $\frac{d^2y}{dx^2} = \frac{3}{16}$ 1pt

2. (5 points) Find the length of the curves $x = e^{t} + e^{-t}$, $y = 1 - 2t$ $0 \le t \le 2$.

Sol. $\frac{dx}{dt} = e^{t} - e^{-t}$ 0.5 pt , $\frac{dy}{dt} = -2$ 0.5 pt

$$L = \int_0^2 \sqrt{(e^{t} - e^{-t})^2 + (-2)^2}\, dt$$ 1pt

$$= \int_0^2 \sqrt{e^{2t} - 2 + e^{-2t} + 4}\, dt$$

$$= \int_0^2 \sqrt{(e^{t})^2 + 2 + (e^{-t})^2}\, dt$$ 1pt

$$= \int_0^2 \sqrt{(e^{t} + e^{-t})^2}\, dt$$

$$= \int_0^2 e^{t} + e^{-t}\, dt = e^{t} - e^{-t}\Big|_0^2 = e^2 - e^{-2}$$ 1pt 1pt