HW 3, MA 1023
Due 2/7

In exercise 1-4, determine the convergence or divergence of the series. If the series is convergent, find the sum value.

1.

$$\sum_{n=1}^{\infty} \frac{5}{2^n} + \frac{(-1)^{n+1}}{3^n} = \sum_{n=1}^{+\infty} \frac{5}{2^n} + \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{3^n} = \frac{\frac{5}{2}}{1-\frac{1}{2}} + \frac{\frac{1}{3}}{1+\frac{1}{3}} = \frac{5}{1} + \frac{1}{4} = \frac{21}{4}$$

2. -

$$\sum_{n=1}^{\infty} \underbrace{3^{n+1}}_{\text{diverges.}} - \underbrace{\frac{(-2)^n}{4^n}}_{\text{converges.}}$$

diverges

Test for divergence.

3.

$\lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = 1 \neq 0$

1) $\sum_{n=1}^{\infty} \frac{n}{n+1}$ diverges

2) $\sum_{n=1}^{\infty} \cos(\frac{2}{n})$ $\lim_{n\to\infty} \cos(\frac{2}{n}) = \cos 0 = 1$ diverges

4.

$\lim_{n\to\infty} n \to +\infty$

1) $\sum_{n=1}^{\infty} \underbrace{n}_{\text{diverges}} + \underbrace{\frac{(-2)^n}{3^n}}_{\text{converges.}}$ diverges

2) $\sum_{n=1}^{\infty} \frac{4^{n+1}}{5^n} = \frac{\frac{4^2}{5}}{1-\frac{4}{5}} = \frac{16}{5-4} = 16$

In exercise 5-6, determine the convergence or divergence of the series. Give reasons for your answer.

5.

1) $\sum_{n=1}^{\infty} \frac{3}{n^{\frac{3}{2}}}$ $p = \frac{3}{2} > 1$ converges. by p-series test

2) $\sum_{n=1}^{\infty} \frac{2n}{3n^{2}} = \sum_{n=1}^{+\infty} \frac{2}{3n}$ $p = 1.$ diverges by p-series test.

6.

1) $\sum_{n=1}^{\infty} \frac{5}{\sqrt{n}+1}$ limit comparison with $\sum \frac{5}{\sqrt{n}}$.

$\lim_{n\to\infty} \frac{\frac{5}{\sqrt{n}+1}}{\frac{5}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n}+1} = 1$.

Since $\sum \frac{1}{\sqrt{n}}$ diverges,

$\sum_{n=1}^{+\infty} \frac{5}{\sqrt{n}+1}$ diverges.

2) $\sum_{n=1}^{\infty} \frac{n+1}{n^3+1}$ $\sum \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{\frac{n+1}{n^3+1}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^3+n^2}{n^3+1} = 1$

Since $\sum_{n=1}^{+\infty} \frac{1}{n^2}$ converges

$\sum_{n=1}^{+\infty} \frac{n+1}{n^3+1}$ converges by limit comparison

In exercise 7-8, use ratio test to determine the convergence or divergence of the series.

7.

$$\sum_{n=1}^{\infty} \frac{2^{n+1}}{n3^{n-1}}$$

7. $\left|\frac{a_{n+1}}{a_n}\right|$

$= \left|\frac{2^{n+2}}{(n+1)3^n} \cdot \frac{n \cdot 3^{n-1}}{2^{n+1}}\right|$

$= \left|2 \cdot \frac{n}{n+1} \cdot \frac{1}{3}\right|$

$= \frac{2}{3} \cdot \frac{n}{n+1}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{2}{3} < 1$.

converges.

8.

$$\sum_{n=1}^{\infty} \frac{n^{2}}{(-4)^n}$$

8. $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(n+1)^2}{(-4)^{n+1}} \cdot \frac{(-4)^n}{n^2}\right|$

$= \left|\left(\frac{n+1}{n}\right)^2 \cdot \frac{1}{-4}\right|$.

$\lim_{n\to\infty} = \frac{1}{4} < 1$.

converges.