HW6

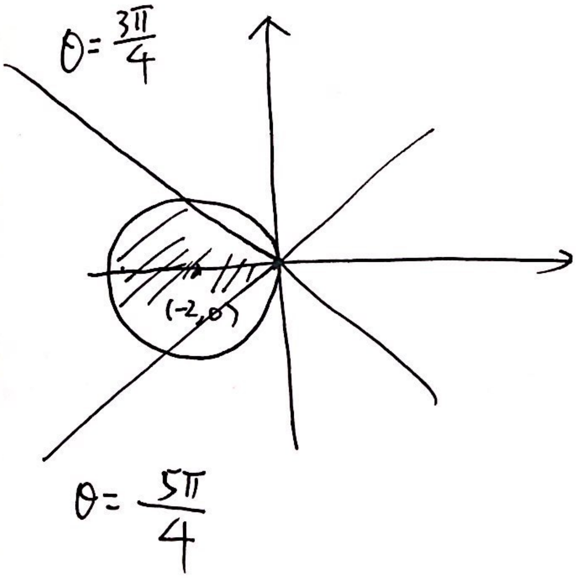
1. $r = -4\cos\theta$

$r^2 = -4r\cos\theta$

$x^2 + y^2 = -4x \Rightarrow x^2 + 4x + 4 + y^2 = 4$

$\Rightarrow (x+2)^2 + y^2 = 4$

a circle centered at $(-2, 0)$ with radius 2.




$$A = \int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}} \frac{1}{2} \cdot (-4\cos\theta)^2 \, d\theta$$

$$= 8 \int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}} \cos^2\theta \, d\theta$$

$$= 8 \int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}} \frac{\cos 2\theta + 1}{2} \, d\theta$$

$$= 4 \cdot \left(\frac{1}{2}\sin 2\theta + \frac{\theta}{2}\right) \Big|_{\frac{3\pi}{4}}^{\frac{5\pi}{4}}$$

$$= 2\sin\left(\frac{5\pi}{2}\right) - \sin\frac{3\pi}{2}) + 2 \cdot \frac{\pi}{2}$$

$$= 4 + \pi$$

HW6

2.. $r=\theta^2$ $\qquad \frac{dr}{d\theta}=2\theta$

$$L=\int_0^{\sqrt{5}} \sqrt{(\theta^2)^2+(2\theta)^2}\, d\theta$$

$$=\int_0^{\sqrt{5}} \sqrt{\theta^2(\theta^2+4)}\, d\theta$$

$$=\int_0^{\sqrt{5}} \theta\cdot\sqrt{\theta^2+4}\, d\theta$$

$u=\theta^2+4$

$du=2\theta\, d\theta.$

$$=\int \frac{1}{2}\sqrt{u}\, du$$

$$=\frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}} = \frac{1}{3}(\theta^2+4)^{\frac{3}{2}}\Big|_0^{\sqrt{5}}$$

$$=\frac{1}{3}\cdot\left[(9)^{\frac{3}{2}}-4^{\frac{3}{2}}\right]$$

$$=\frac{1}{3}(3^3-2^2) = \frac{27-4}{3} = \frac{23}{3}$$

3.

$r = 2 + 2\cos\theta \qquad 0 \le \theta \le \pi.$

$$\frac{dr}{d\theta} = -2\sin\theta$$

$$L = \int_0^{\pi} \sqrt{(2+2\cos\theta)^2 + (-2\sin\theta)^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{4 + 8\cos\theta + 4\cos^2\theta + 4\sin^2\theta}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{8(1+\cos\theta)}\, d\theta \quad \leftarrow \cos\theta = 2\cos^2\frac{\theta}{2} -$$

$$= \int_0^{2\pi} \sqrt{8\left(1+2\cos^2\frac{\theta}{2}-1\right)}\, d\theta$$

$0 \le \theta \le 2\pi$

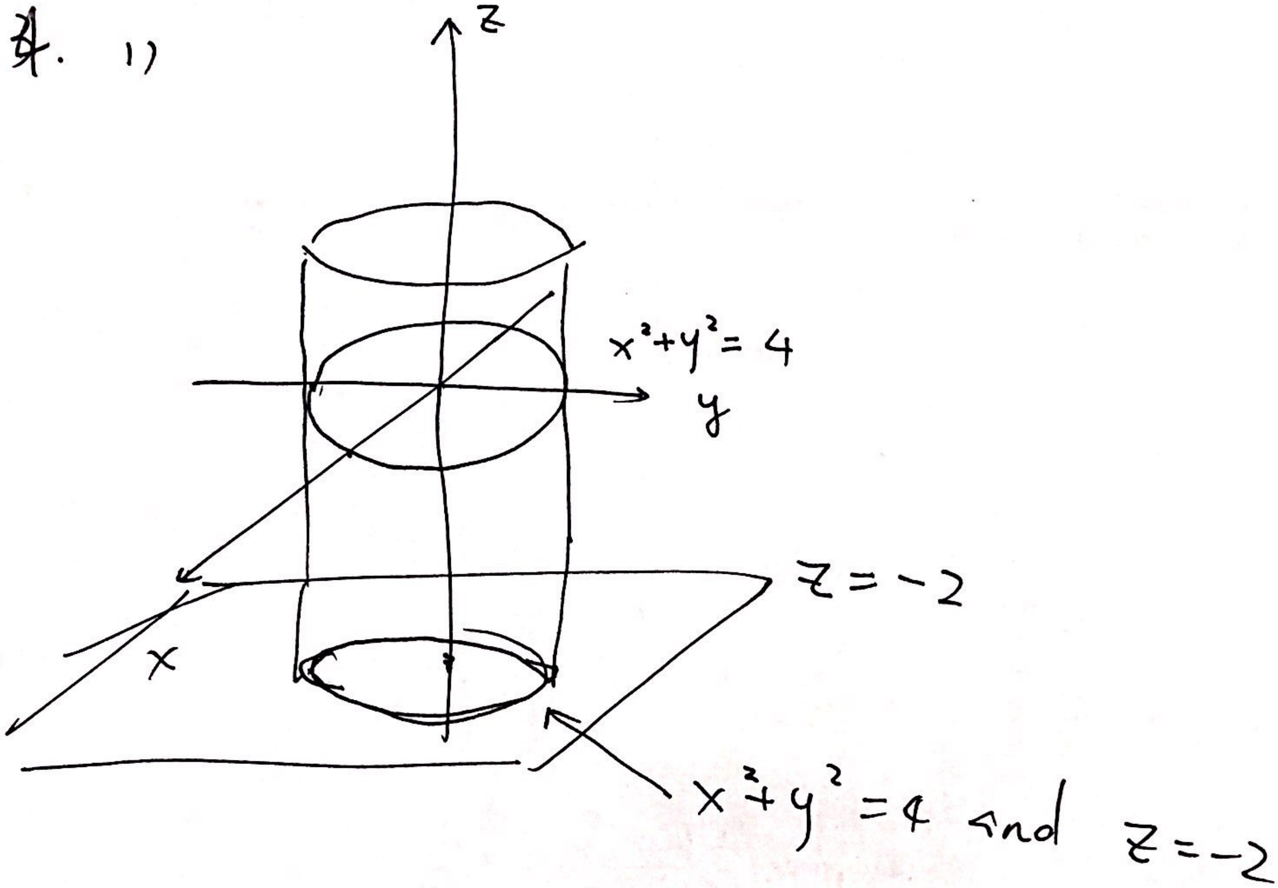
$$= \int_0^{2\pi} \sqrt{8\cdot 2}\sqrt{\cos^2\frac{\theta}{2}}\, d\theta$$

$0 \le \frac{\theta}{2} \le \frac{\pi}{2} \qquad \cos\frac{\theta}{2} \ge$

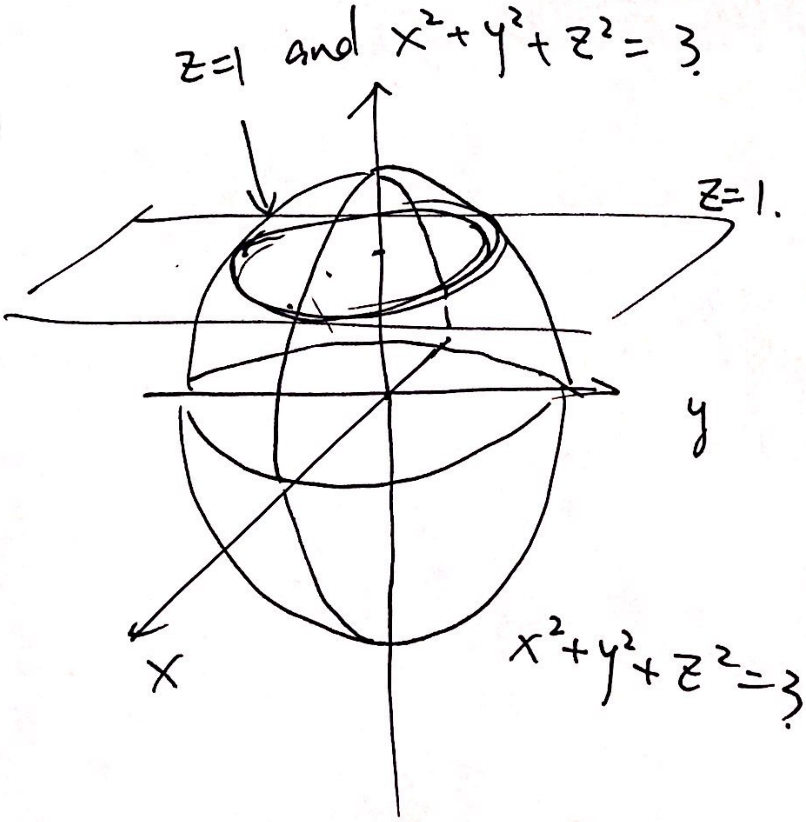
$$= \int_0^{2\pi} 4\cdot\left|\cos\frac{\theta}{2}\right| d\theta = 4\int_0^{\pi} \cos\frac{\theta}{2}\, d\theta = 8\sin\frac{\theta}{2}\Big|$$

$$= 8$$

4. 1)




2)



5.

$$\vec{PQ} = \langle -5-1,\ 2-(-2),\ 2-3 \rangle$$

$$= \langle -6,\ 4,\ -1 \rangle$$

$$|\vec{PQ}| = \sqrt{(-6)^2 + 4^2 + (-1)^2} = \sqrt{36+16+1}$$

$$= \sqrt{53}$$

6. 1) $\vec{u} + \vec{v} + \vec{w}$

$$= \langle 3+2-1,\ -2-4+2,\ 1-3+2 \rangle$$

$$= \langle 4,\ -4,\ 0 \rangle$$

$$\sqrt{4^2 + (-4)^2} = 4\sqrt{2}$$

2)

$2\vec{u} - 3\vec{v} - 5\vec{w}$

$= \langle 6, -4, 2 \rangle - \langle 6, -12, -9 \rangle - \langle -5, 10, 10 \rangle$

$= \langle 5, -4+12-10, 2+9-10 \rangle$

$= \langle 5, -2, 1 \rangle$

$\sqrt{5^2 + (-2)^2 + 1^2} = \sqrt{25 + 4 + 1} = \sqrt{30}.$

[illegible].7 $\vec{u} + \vec{v} = \langle 2, 4, -5 \rangle + \langle 1, 2, 3 \rangle$

$= \langle 3, 6, -2 \rangle$

$|\vec{u} + \vec{v}| = \sqrt{3^2 + 6^2 + (-2)^2} = \sqrt{9 + 36 + 4} = 7$

unit vector $\vec{w} = \frac{1}{7} \langle 3, 6, -2 \rangle = \langle \frac{3}{7}, \frac{6}{7}, -\frac{2}{7} \rangle$

or $-\vec{w}$.

8. $\vec{u} \cdot \vec{v}$

$= 4 \cdot 2 - 2\cancel{x} - 2 = 0$

$8 - 2 - 2a = 6 - 2a = 0$

$\cancel{x}\, a = 3$.