Example 1

Calculate with and without using l'Hopital's rule.

$$\lim_{y \to 0} \quad \frac{\sin(y)\cos(y)}{\sin(2y)}$$

THEOREM 10—Limits of Continuous Functions If g is continuous at the point b and $\lim_{x \to c} f(x) = b$, then

$$\lim_{x \to c} g(f(x)) = g(b) = g(\lim_{x \to c} f(x)).$$

Example 2

Calculate the limit. Note that $\ln(8x^2)$ and $\ln(1-\cos x)$ are undefined at $x=0$ but that this limit may exist nevertheless.

$$\lim_{x\to 0} \quad \ln(8x^2) - \ln(1-\cos x)$$

Example 3

Calculate the limit.

$$\lim_{y \to 1^+} \quad y^{\frac{1}{1-y}}$$

Example 4

Calculate the limit or explain why the limit does not exist

$$\lim_{x \to 0} \quad \frac{\tan(x)}{1 - \cos(x)}$$

Example 5

Calculate the limit or explain why the limit does not exist

$$\lim_{x \to 0+} \quad \frac{x-1}{x^3}$$

Problem 1

Calculate with and without using l'Hopital's rule.

$$\lim_{y \to 2\pi} \quad \frac{\sin(6y)}{\sin(3y)}$$

Hint: $\sin(6y) = \sin(2(3y))$ and follow example 1.

Problem 2

Calculate the limit.

$$\lim_{x \to \infty} \quad (1 + 2x)^{\frac{1}{2\ln(x)}}$$

Problem 3

Calculate the limit.

$$\lim_{x \to 0^+} \quad \frac{1}{x^4} - \frac{1}{x^5}$$