Quiz 2, Math 1023 Name: ________________ Conference: 1 PM 9 AM

Show **all steps** to earn credits.

1. (6 points) Determine the convergence or divergence of the sequences. If the sequence is convergent, find the limit.

$$1)\ a_n = \frac{\cos(n^2+2)}{3^n} \qquad 2)\ a_n = \frac{\sqrt{n}+2}{n+n^3}$$

Sol 1) $-1 \le \cos(n^2+2) \le 1$

$\Rightarrow -\frac{1}{3^n} \le \frac{\cos(n^2+2)}{3^n} \le \frac{1}{3^n}$ 1pt

Since $\lim_{n\to\infty} -\frac{1}{3^n} = \lim_{n\to\infty} \frac{1}{3^n} = 0$, by 1pt

Squeeze Thm, $\lim_{n\to\infty} a_n = 0$. 1pt

2) $a_n = \dfrac{\frac{\sqrt{n}+2}{n^3}}{\frac{n+n^3}{n^3}}$ 1pt

$= \dfrac{\frac{1}{n^{\frac{5}{2}}} + \frac{2}{n^3}}{\frac{1}{n^2}+1}$

$\lim_{n\to\infty} a_n = \dfrac{\lim_{n\to\infty}\left(\frac{1}{n^{\frac{5}{2}}} + \frac{2}{n^3}\right)}{\lim_{n\to\infty}\left(\frac{1}{n^2}+1\right)} = \dfrac{0+0}{0+1}$ 1pt

$= 0$ 1pt

2. (4 points) Determine if the geometric series converges or diverges. If the series converges, find the value.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2^{n+1}}$$

Sol. $a_n = \dfrac{(-1)^n}{2^{n+1}}$

$r = \dfrac{a_{n+1}}{a_n} = \dfrac{(-1)^{n+1}}{2^{n+2}} \cdot \dfrac{2^{n+1}}{(-1)^n} = \dfrac{1}{2} \cdot (-1) = -\dfrac{1}{2}$ 1pt

Since $-1 < -\frac{1}{2} < 1$, $\sum_{n=1}^{+\infty} a_n$ converges and 1pt

$\sum_{n=1}^{+\infty} a_n = \dfrac{a_1}{1-r} = \underline{\dfrac{(-1)^1}{2^{1+1}}}$ 1pt $\cdot \dfrac{1}{1-(-\frac{1}{2})} = -\dfrac{1}{4} \cdot \dfrac{1}{1+\frac{1}{2}} = -\dfrac{1}{4} \cdot \dfrac{2}{3} = -\dfrac{1}{6}$ 1pt