HW 2, MA 1023
Due 1/31

In exercise 1, find a defining formula $a_n = f(n)$ for the sequence.

1.

$a_n = n-5$ 1) $-4, -3, -2, -1, 0, \cdots$ 2) $\frac{1}{9}, -\frac{2}{12}, \frac{2^2}{15}, -\frac{2^3}{18}, \frac{2^4}{21}, \cdots$ $a_n = (-1)^{n+1}\frac{2^{n-1}}{3(n+2)}$

In exercise 2-6, determine the convergence or divergence of the sequences. If the sequence is convergent, find the limit.

2.

diverges. (1) $a_n = 1 + (-1)^n$ (2) $a_n = \left(\frac{n+1}{2n}\right)\left(1 - \frac{1}{n}\right)$ $\frac{1}{2}$

3.

Use Squeeze Thm. (1) $a_n = \frac{\sin^2(2n+1)}{n^2}$ $0$ (2) $a_n = \frac{\cos(2n+3)}{2^n}$ $0$

4.

Squeeze. $\frac{1}{2}$ (1) $a_n = \frac{n+(-1)^{n+1}}{2n} = \frac{1}{2} + \frac{(-1)^{n+1}}{2n}$ (2) $a_n = \frac{2n+1}{1-3\sqrt{n}} = \frac{2+\frac{1}{n}}{\frac{1}{n} - \frac{3}{\sqrt{n}}} \to \infty$ diverges

5.

L'Hôpital (1) $a_n = \frac{\ln(2n+1)}{\sqrt{n}}$ $0$ (2) $a_n = \cos(2\pi + \frac{1}{n^2})$ $\to \cos(2\pi) = 1.$

6.

Use. $\lim_{n\to\infty} \frac{x^n}{n!} = 0 \to$ (1) $a_n = \frac{(-4)^n}{n!}$ $0$ (2) $a_n = 2 + (\frac{1}{2})^{2n}$ $= 2 + (\frac{1}{4})^n \to 2+0 = 2.$

7. Determine if the geometric series converges or diverges. If the series converges, find the value.

7.1) converges. $r = -\frac{1}{4}$

(1) $\sum_{n=1}^{\infty} \frac{(-1)^n}{4^{n+1}}$ (2) $\sum_{n=1}^{\infty} \frac{(-3)^n}{2^n}$ diverges

$a_1 = \frac{-1}{4^2} = -\frac{1}{16}$

$-\frac{1}{16} \cdot \frac{1}{1+\frac{1}{4}} = -\frac{1}{16} \cdot \frac{4}{5} = -\frac{1}{20}$

$a_1 = \frac{-3}{2}$ $r = \frac{a_{n+1}}{a_n} = (\frac{-3}{2})^{n+1} \cdot (\frac{2}{-3})^n = -\frac{3}{2} < -1$

8. Find a formula for the $n$-th partial sume of the series and use it to determine if the series converges or diverges. If a series converges, find its value.

$a_n = \frac{(-1)^n}{4^{n+1}}$ (1) $\sum_{n=1}^{\infty}\left(\frac{3}{n^2} - \frac{3}{(n+1)^2}\right)$ (2) $\sum_{n=1}^{\infty}\left(\sqrt{n+4} - \sqrt{n+3}\right)$

$a_{n+1} = \frac{(-1)^{n+1}}{4^{n+2}}$

$r = \frac{a_{n+1}}{a_n} = \frac{(-1)^{n+1}}{4^{n+2}} \cdot \frac{4^{n+1}}{(-1)^n} = \frac{(-1)}{4}$

2) $S_n = a_1 + \cdots + a_n$

$= \sqrt{n+4} - \sqrt{4}$

diverges

8. 1) $S_n = a_1 + a_2 + \cdots + a_n = 3 - \frac{3}{(n+1)^2}$

$\lim_{n\to\infty} S_n = 3$