## Comments

• If you would like to contact me, send a message through canvas: inbox → compose new message → select calculus III (either option) → type "Dane Johnson". Or come to office hours. In particular if you have questions after grading has been completed.

• If you want me to grade your homework by annotating directly on your pdf submission, either mention this on the first page of your work or make a comment while / right after submitting on canvas. The same rules for submission apply but also you need to provide space next to your work that gives me sufficient room to make notes next to your work.

• Homework grading follows a rubric provided by your professors. When writing up your work, please read the directions carefully and compare your presentation to examples from videos lectures, active learning, and conference.

### Example 1

Calculate the following improper integral or show that the integral diverges.

$$\int_1^\infty \frac{(\ln(w))^2}{w}\,dw$$

$$\int_1^\infty \frac{(\ln w)^2}{w}\,dw$$

by definition if this limit exists

$$= \lim_{b\to\infty} \int_1^b \frac{(\ln w)^2}{w}\,dw$$

$u(w) = \ln w$

$du = \frac{1}{w}\,dw$

$u(1) = \ln 1 = 0$

$u(b) = \ln b$

$$= \lim_{b\to\infty} \int_0^{\ln b} u^2\,dw$$

$$= \lim_{b\to\infty} \frac{1}{3}u^3 \Big|_0^{\ln b}$$

$$= \lim_{b\to\infty} \frac{1}{3}(\ln b)^3 - \frac{1}{3}0^3$$

$$= \lim_{b\to\infty} \frac{1}{3}(\ln b)^3 = \infty$$

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$$\int_1^b \frac{(\ln w)^2}{w}\,dw = \underset{\dots}{\text{calculations}} = \frac{1}{3}(\ln b)^3$$

$$\int_1^\infty \frac{(\ln w)^2}{w}\,dw = \lim_{b\to\infty}\int_1^b \frac{(\ln w)^2}{w}\,dw = \lim_{b\to\infty}\frac{1}{3}(\ln b)^3 = \infty$$

## Example 2

Does the following integral converge or diverge? Note that you do not have to calculate the integrals to answer this question.

$$\int_1^\infty \frac{1}{x^{1/2}}\,dx$$

$$= \lim_{b\to\infty} \int_1^b \frac{1}{x^{1/2}}\,dx = \lim_{b\to\infty} 2x^{1/2}\Big|_1^b$$

$$= \lim_{b\to\infty} 2b^{1/2} - 2 = \infty$$

**Example 3**

Does the following integral converge or diverge? Note that you do not have to calculate the integrals to answer this question.

$$\int_1^\infty \frac{(\arctan(x))^3}{x^2}\,dx$$

For $1 \le x < \infty$,

$$0 \le \arctan x < \pi/2$$

$$0 \le (\arctan x)^3 < \pi^3/8$$

$$0 \le \int \frac{(\arctan x)^3}{x^2}\,dx < \int \frac{\pi^3}{8}\frac{1}{x^2}\,dx = \frac{\pi^3}{8}$$

$$\int_1^\infty \frac{\pi^3}{8}\frac{1}{x^2}\,dx = \lim_{b\to\infty}\int_1^b \frac{\pi^3}{8}\frac{1}{x^2}\,dx$$

$$= \lim_{b\to\infty}\frac{\pi^3}{8}\left(-\frac{1}{x}\right)\Big|_1^b$$

$$= \lim_{b\to\infty}\frac{\pi^3}{8}\left(-\frac{1}{b}+\frac{1}{1}\right)$$

$$= \lim_{b\to\infty}\frac{\pi^3}{8}\left(1-\frac{1}{b}\right) = \frac{\pi^3}{8}$$

$\therefore \int_1^\infty \frac{(\arctan x)^3}{x^2}\,dx$ converges.

## Problem 1

Calculate the following improper integral or show that the integral diverges. See example 1 if you're unsure where to start.

$$\int_{2}^{\infty} \frac{1}{x\ln(x)}\,dx$$

Try u-substitution, $u(x) = \ln x$.

## Problem 2

Does the following integral converge or diverge? Note that you do not have to calculate the integrals to answer this question. See examples 2 and 3.

$$\int_{1}^{\infty} \frac{2+\cos(x)}{x}\,dx$$

For $1 \le x < \infty$, $-1 \le \cos x \le 1$. Manipulate this inequality to get bounds on $\frac{2+\cos x}{x}$.

## Problem 3

Calculate the following improper integral or show that the integral diverges for the two different values of $p$ given.

$$\int_{17}^{\infty} z^{1-p^2}\,\cancel{dx}\,dz$$

a) $p = -1$. This means the integrand should be $z^{1-(-1)^2} = z^0 = 1$.
b) $p = 2$. This means the integrand should be $z^{1-2^2} = z^{-3} = \frac{1}{z^3}$

a) $\int_{17}^{\infty} z^{1-(-1)^2}\,dz = \lim_{b\to\infty} \int_{17}^{b} 1\,dz = \lim_{b\to\infty} b - 17 = \infty$
(Diverges)

b) $\int_{17}^{\infty} z^{1-2^2}\,dz = \lim_{b\to\infty} \int_{1}^{b} \frac{1}{z^3}\,dz = \lim_{b\to\infty} -\frac{1}{2}\frac{1}{z^2}\Big|_{1}^{b}$
$= \lim_{b\to\infty} -\frac{1}{2}\left(\frac{1}{b^2} - 1\right) = \frac{1}{2}$. (Integral converges)

Scratch Work : $\int_1^\infty z^{1-p^2}\,dz$ general case

$$\int_1^b z^{1-p^2}\,dz$$

$$= \frac{1}{1-p^2+1} z^{1-p^2+1} \Big|_1^b$$

$$= \frac{1}{2-p^2} z^{2-p^2} \Big|_1^b$$

$$= \frac{1}{2-p^2}\left[b^{2-p^2} - 1\right]$$

This is incorrect if $2-p^2=0$. Can you see why? Also, if $2-p^2=0$ then $1-p^2=-1$. What does the integral look like in this case?

$$\int_1^\infty z^{1-p^2}\,dz = \lim_{b\to\infty} \frac{1}{2-p^2}\left[b^{2-p^2} - 1\right] = ?$$

scratch work tells me to look at 3 cases:

1) $2-p^2=0$
2) $2-p^2>0$
3) $2-p^2<0$

Explore all 3 cases and use the results to explain in detail what happens and why.