2. $\vec{u} = <2, -3, -1>$ $\vec{v} = <1, 4, -2>$

1)

$\vec{u} \cdot \vec{v} = 2 \cdot 1 + (-3) 4 + (-1) \cdot (-2)$

$= 2 - 12 + 2 = -8$

2) $\vec{v} \times \vec{u} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 1 & 4 & -2 \\ 2 & -3 & -1 \end{vmatrix}$

$= \vec{\imath}(-4-6) - \vec{\jmath}(-1+4)$

$+ \vec{k}(-3-8)$

$= -10\vec{\imath} - 3\vec{\jmath} - 11\vec{k}$

$= <-10, -3, -11>$

3)

$$\vec{u} + \vec{v} = \langle 3, 1, -3 \rangle$$

$$\vec{u} - \vec{v} = \langle 1, -7, 1 \rangle$$

$$(\vec{u} + \vec{v}) \times (\vec{u} - \vec{v}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -3 \\ 1 & -7 & 1 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} 1 & -3 \\ -7 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} 3 & -3 \\ 1 & 1 \end{vmatrix}$$

$$+ \vec{k} \begin{vmatrix} 3 & 1 \\ 1 & -7 \end{vmatrix}$$

$$= \vec{i} (1-21) - \vec{j} (-21-1) + \vec{k} (-21-1)$$

$$= \langle -20, 22, -22 \rangle$$

3. $\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -2 & -4 \\ 1 & 2 & -1 \end{vmatrix}$

$= \vec{i}(-2+8) - \vec{j}(-4-(-4))$

$+ \vec{k}(8-(-2))$

$= 6\vec{i} - 0\vec{j} + 10\vec{k}$

$= \langle 6, 0, 10 \rangle$

$|\vec{u} \times \vec{v}| = \sqrt{36 + 100}$

$\vec{w} = \frac{1}{\sqrt{136}} \langle 6, 0, 10 \rangle = \langle \frac{6}{\sqrt{136}}, 0, \frac{10}{\sqrt{136}} \rangle$

4. 1) $\vec{PQ} = \langle 2-1, 0-(-1), -1-2\rangle$

$$= \langle 1, 1, -3\rangle$$

$\vec{PR} = \langle 0-1, 2-(-1), 1-2\rangle$

$$= \langle -1, 3, -1\rangle$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix}$$

$$= \langle -1-(-9), -(-1-3), 3-(-1)\rangle$$

$$= \langle 8, 4, 4\rangle$$

$A_{PQR} = \frac{1}{2}|\vec{PQ} \times \vec{PR}| = \frac{1}{2}\sqrt{8^2 + 4^2 + 4^2}$

$$= \frac{1}{2}\sqrt{64+16+16} = \frac{1}{2}\sqrt{96}.$$

2).

$$\vec{n} = \frac{1}{\sqrt{96}} < 8, 4, 4 >$$

$$\sqrt{16 \cdot (4+1+1)}$$

$$= \frac{1}{4\sqrt{6}} < 8, 4, 4 > = < \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} >$$

or $-\vec{n}$

4. 1)

$$x(t) = 3 + t$$

$$y(t) = 4 + t$$

$$z(t) = -1 + t \qquad -\infty < t < +\infty$$

2) $\vec{PQ} = < 2-1, \quad 0-(-1), \quad -1-2 >$

$$= < 1, 1, -3 >$$

$$x(t) = 1 + t$$

$$y(t) = -1 + t \qquad -\infty < t < +\infty$$

$$z(t) = 2 - 3t$$

5. $P(x, y, z)$.

$$\overrightarrow{P_0P} = \langle x - 0, y - 2, z + 1 \rangle$$

$$\overrightarrow{P_0P} \cdot \vec{n} = 0$$

$$3(x-0) - 2(y-2) - (z+1) = 0$$

6.

$$\vec{PQ} = \langle 1, -1, 3 \rangle$$

$$\vec{PR} = \langle -1, -3, 2 \rangle$$

$$\vec{n} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} 1 & -1 & 3 \\ -1 & -3 & 2 \end{vmatrix}$$

$$= \langle -2+9, -(2+3), -3-1 \rangle$$

$$= \langle 7, -5, -4 \rangle$$

$$7(x-2) - 5y - 4(z-2) = 0$$

7. 1)

$$\vec{r}'(t) = \langle -2\sin t, \ 3\cos t, \ 4 \rangle$$

$$\vec{r}''(t) = \langle -2\cos t, \ -3\sin t, \ 0 \rangle$$

2) $\quad |\vec{r}'(t)| = \sqrt{4\sin^2 t + 9\cos^2 t + 16}$

$t = \dfrac{\pi}{2} \qquad |\vec{r}'(t)| = \sqrt{4 + 0 + 16} = \sqrt{20}$

$$\frac{1}{\sqrt{20}} \langle -2\sin \tfrac{\pi}{2}, \ 3\cos \tfrac{\pi}{2}, \ 4 \rangle$$

$$= \langle -\frac{2}{\sqrt{20}}, \ 0, \ \frac{4}{\sqrt{20}} \rangle$$

8. 1) $\vec{r}'(t) = \langle -e^{-t}, -6\sin 3t, 6\cos 3t \rangle$

$\vec{r}''(t) = \langle e^{-t}, -18\cos 3t, -18\sin 3t \rangle$

2)

$$|\vec{r}'(t)| = \sqrt{e^{-2t} + 36}$$

$t = 0$

$$|\vec{r}'(t)| = \sqrt{37}$$

$$\frac{1}{\sqrt{37}} \langle -1, 0, 6 \rangle$$

$$= \langle -\frac{1}{\sqrt{37}}, 0, \frac{6}{\sqrt{37}} \rangle$$