Quiz 5, Math 1023 Name: _____ Conference: 9 AM 1 PM

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1. Given a parametric curve $x = 2t^2 + 3$, $y = t^3$.
~~1) (4 points) Find an equation of the tangent line to the curve at $t = 1$.~~
2) (5 points) Find the value of $\frac{d^2y}{dx^2}$ at $t = 1$.

Sol $\frac{dx}{dt} = 4t$ 1pt $\quad \frac{dy}{dt} = 3t^2$ 1pt $\quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2}{4t} = \frac{3t}{4}$ 1pt

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{3}{4}}{4t} = \frac{3}{16t}$ 1pt $\quad t=1 \quad \frac{d^2y}{dx^2} = \frac{3}{16}$ 1pt

2. (5 points) Find the length of the curves $x = \frac{t^3}{3} + \frac{1}{t}$, $y = 1 - 2t$ $1 \le t \le 2$.

Sol $\frac{dx}{dt} = \frac{1}{3} \cdot 3t^2 - \frac{1}{t^2} = t^2 - \frac{1}{t^2}$ $\quad \frac{dy}{dt} = -2$ 1pt

$L = \int_1^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_1^2 \sqrt{\left(t^2 - \frac{1}{t^2}\right)^2 + (-2)^2}\, dt$ 1pt

$= \int_1^2 \sqrt{t^4 - 2 + \frac{1}{t^4} + 4}\, dt = \int_1^2 \sqrt{t^4 + 2 + \frac{1}{t^4}}\, dt$ 1pt

$= \int_1^2 \sqrt{\left(t^2 + \frac{1}{t^2}\right)^2}\, dt = \int_1^2 t^2 + \frac{1}{t^2}\, dt$ 1pt

$= \frac{1}{3}t^3 - \frac{1}{t}\Big|_1^2 = \frac{1}{3} \cdot 8 - \frac{1}{2} - \left(\frac{1}{3} - 1\right) = \frac{8-1}{3} + 1 - \frac{1}{2}$ 1pt

$= \frac{7}{3} + \frac{1}{2} = \frac{14+3}{6} = \frac{17}{6}$