Calculus III, [redacted]

Practice Exam I, Math 1023 Name: ______________________

Show **all work clearly and in order.**

1. (16 points) Find the limits of the following functions.

1) (8 points)

$$\lim_{x\to\infty} \frac{e^x + 2x^2}{e^x + 2x} \quad \leftarrow \quad \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{e^x+2x}{e^x+2} \quad 3\text{pts} \quad \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{e^x+2}{e^x} \quad 3\text{ pts} \quad = \lim_{x\to\infty} \frac{e^x}{e^x} = 1 \quad 2\text{pts.}$$

2) (8 points)

$$\lim_{x\to 0+} \sqrt{x}\ln(x^2) \quad (0\cdot(-\infty)) \quad x\ln(x^2)$$

$$= \lim_{x\to 0^+} \frac{\ln(x^2)}{x^{-1}} \quad \frac{-\infty}{\infty} \quad 2\text{ pt}$$

$$= \lim_{x\to 0^+} \frac{x^{-2}\cdot 2x}{-1\cdot x^{-2}} \quad 2\text{ pt} \quad 2\text{ pt}$$

$$= \lim_{x\to 0^+} -2x = 0 \quad 2\text{ pt.}$$

2. (12 points) Evaluate the following integrals

$$\int_{\cancel{-\infty}\ 0}^{\infty} \frac{2x}{(x^2+2)^3} dx$$

$$= \int_0^{+\infty} \frac{2x}{(x^2+2)^3} dx.$$

I.

$$\int_0^t \frac{2x}{(x^2+2)^3} dx$$ 2 pt.

$u = x^2 + 2.$

$du = 2x\, dx$ 2 pts

$$= \int \frac{du}{u^3} = \frac{1}{-3+1} u^{-3+1} = -\frac{1}{2} u^{-2}$$ 2 pts.

$$= -\frac{1}{2} \cdot \frac{1}{(x^2+2)^2} \Big|_0^t = -\frac{1}{2(t^2+2)^2} + \frac{1}{2} \cdot \frac{1}{2^2}$$

1 pt.

$$= \frac{1}{2} \cdot \frac{1}{4} - \frac{1}{2(t^2+2)^2}$$ 2 pts.

II

$$\lim_{t \to \infty} \int_0^t \frac{2x}{(x^2+2)^3} dx = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}.$$

2 pts

1 pt

20 pts

3. (20 points) Determine the convergence or divergence of the sequences. If the sequence is convergent, find the limit.

1)(8 points) ~~$a_n = (-2)^n$~~ $a_n = \left(\frac{n+1}{2n}\right)\left(1+\frac{1}{n^2}\right)$

or $\left(\frac{1+\frac{1}{n}}{2}\right)$

$$\lim_{n\to\infty}\left(\frac{n}{2n}+\frac{1}{2n}\right)\cdot\lim_{n\to\infty}\left(1+\frac{1}{n^2}\right)$$

2 pts 2 pts

2 pts

$$=\frac{1}{2}\cdot 1=\frac{1}{2}$$

2 pts

2)(8 points) $a_n = \dfrac{\cos(2n+1)}{2^n}$

4 pts

Sol. $-1 \le \cos(2n+1) \le 1$, $-\frac{1}{2^n} \le a_n \le \frac{1}{2^n}$.

$\lim_{n\to\infty}\frac{-1}{2^n} = \lim_{n\to\infty}\frac{1}{2^n} = 0$. By Squeeze Thm. 0.

2 pts

2 pts

3)(4 points) $a_n =$ ~~$\sqrt[n]{2^{3n+1}}$~~ $\frac{10^{n+1}}{n!}$

$$a_n = \frac{10^n}{n!}\cdot 10$$

2 pts

By commonly occurring limits, $\lim_{n\to\infty}\frac{10^n}{n!} = 0$. then

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{10^n}{n!}\cdot 10 = 10\cdot\lim_{n\to\infty}\frac{10^n}{n!} = 0.$$

2 pts

geometric.

4. (12 points) Determine if the series converges or diverges. Give reasons for your answer. **If the series converges, find the value.**

(1)(8 points) $\sum_{n=1}^{\infty} \frac{3^n}{4^{n+1}}$ $a_n = \frac{3^n}{4^{n+1}}$

$a_1 = \frac{3^1}{4^{1+1}} = \frac{3}{4^2} = \frac{3}{16}$ 2 pts $r = \frac{a_{n+1}}{a_n} = \frac{3^{n+1}}{4^{n+2}} \cdot \frac{4^{n+1}}{3^n} = \frac{3}{4}$ 3 pts

converges to $\frac{3}{16} \cdot \frac{1}{1-\frac{3}{4}} = \frac{3}{16} \cdot \frac{4}{4-3} = \frac{3}{4}$ 1 pt

2 pts

(2)(4 points) $\sum_{n=1}^{\infty} \frac{(-3)^n}{2^n}$

$a_n = \frac{(-3)^n}{2^n}$

$r = \frac{a_{n+1}}{a_n} = \frac{(-3)^{n+1}}{2^{n+1}} \cdot \frac{2^n}{(-3)^n} = \frac{-3}{2} \leq -1$, diverges. 2 pts

2 pts

5. (24 points) Determine the convergence or divergence of the series. Give reasons for your answer.

1)(8 points) $\sum_{n=1}^{\infty} \cos(\frac{2}{n^2})$

$\lim_{n\to\infty} \cos(\frac{2}{n^2}) = \cos(0) = 1 \neq 0$ 3 pts

Test for divergence. diverges. 3 pts

2 pts

2)(8 points) $\sum_{n=1}^{\infty} \frac{n+1}{2^n n!}$

2 pts

$\left|\frac{a_{n+1}}{a_n}\right| = \frac{n+2}{2^{n+1}(n+1)!} \cdot \frac{2^n n!}{n+1} = \frac{n+2}{n+1} \cdot \frac{1}{2} \cdot \frac{1}{n+1}$

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1 \cdot \frac{1}{2} \cdot 0 = 0 < 1$ 2 pts converges. by ratio test.

2 pts 2 pts

2)(8 points) $\sum_{n=1}^{\infty} \frac{n^2-1}{n^5+n^3}$

Define. $\sum b_n = \sum_{n=1}^{+\infty} \frac{1}{n^3}$ converges. by p-series test $p = 3 > 1$. 3 pts

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^2-1}{n^5+n^3} \cdot n^3 = 1 > 0$. 2 pts

Limit comparison test implies. $\sum a_n$. converges. since $\sum b_n$ converges.

3 pts.

6. (16 points) Find all the values of x such that the following power series converge

$$\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$$

$$a_n = \frac{x^n}{n3^n}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1}}{(n+1)3^{n+1}} \cdot \frac{n3^n}{x^n}\right| = |x| \cdot \frac{n}{n+1} \cdot \frac{1}{3}$$

1 pt 2 pts

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{3}|x| < 1.$$

1 pts

3 pts.

$-3 < x < 3$.

When $x=3$. $a_n = \frac{3^n}{n3^n} = \frac{1}{n}$. $\sum \frac{1}{n}$ diverges. 4 pts

p-series test $p=1$

$x=-3$ $a_n = \frac{(-1)^n}{n}$ converges. 4 pts.

alternating series test since $u_n = \frac{1}{n}$

$\lim_{n\to\infty} \frac{1}{n} = 0$ and $\frac{1}{n+1} \le \frac{1}{n}$.

When $-3 \le x < 3$. 1 pt

the power series converges.