

Show all steps to earn credits.

1. (6 points) Determine the convergence or divergence of the series. Give reasons for your answer.

state the test

$$1) \sum_{n=1}^{\infty} \frac{n}{n+1}$$

$$2) \sum_{n=1}^{\infty} \frac{4}{\sqrt{n}}$$

Sol 1)  $\lim_{n \rightarrow \infty} \frac{n}{n+1} = \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{1}{n}} = 1 \neq 0$  1pt

2)  $p = \frac{1}{2} \leq 1$ , by p-series test,  $\sum a_n$  diverges. 1pt

By test for divergence,  $\sum a_n$  diverges. 1pt

2. (4 points) Determine the convergence or divergence of the series. Give reasons for your answer.

~~$$1) \sum_{n=1}^{\infty} \frac{n+1}{n^3+1}$$~~

~~$$2) \sum_{n=1}^{\infty} \frac{n+1}{4^n}$$~~

state the test

Sol  $a_n = \frac{n+1}{4^n}$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{n+2}{4^{n+1}} \cdot \frac{4^n}{n+1} \right| = \left| \frac{4^n}{4^{n+1}} \cdot \frac{n+2}{n+1} \right| = \frac{1}{4} \cdot \frac{n+2}{n+1}$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{1}{4} \cdot \frac{n+2}{n+1} = \frac{1}{4} \lim_{n \rightarrow \infty} \frac{1 + \frac{2}{n}}{1 + \frac{1}{n}} = \frac{1}{4} < 1$$

By ratio test, since  $L = \frac{1}{4} < 1$ ,  $\sum a_n$  converges. 1pt