Quiz 1, Math 1023 Name: _____

Show **ALL YOUR WORK** to earn credits.

1. (6 points) Find the limits of the following functions

1) $\lim_{x\to 0} \frac{x^2}{\sin x}$ 2) $\lim_{x\to\infty} \frac{e^x + x^2}{e^x + x}$

Sol. $\frac{0}{0}$

1) $\lim_{x\to 0} \frac{x^2}{\sin x}$

$= \lim_{x\to 0} \frac{2x}{\cos x}$ ← 1pt, ← 1pt

$= \frac{0}{1} = 0$ 1pt

$\frac{\infty}{\infty}$

2) $\lim_{x\to\infty} \frac{e^x + x^2}{e^x + x}$

$= \lim_{x\to\infty} \frac{e^x + 2x}{e^x + 1}$ $\frac{\infty}{\infty}$ 1pt

$= \lim_{x\to\infty} \frac{e^x + 2}{e^x}$ $\frac{\infty}{\infty}$ $= \lim_{x\to\infty} \frac{e^x}{e^x} = 1$

1pt 1pt

2. (4 points) Evaluate the improper integral

$$\int_0^\infty \frac{2x}{(x^2+4)^2} dx.$$

or

$\left(= \lim_{x\to\infty} 1 + \frac{2}{e^x} = 1 + 0 = 1 \right)$

Sol. Let $A(t) = \int_0^t \frac{2x}{(x^2+4)^2} dx$

1pt

(substitution $u = x^2+4$, $du = 2x\,dx$)

$= \int u^{-2} du = \frac{1}{-2+1} u^{-2+1} = -(x^2+4)^{-1} \Big|_0^t$

1pt

$= \frac{-1}{t^2+4} + \frac{1}{4}$ 1pt

$$\int_0^\infty \frac{2x}{(x^2+4)^2} dx = \lim_{t\to\infty} A(t) = \lim_{t\to\infty} \left(\frac{1}{4} - \frac{1}{t^2+4}\right) = \frac{1}{4}$$

1pt