Quiz 3, Math 1023 Name: ________________ Conference: 1 PM 9 AM

Show **all steps** to earn credits.

state the test

1. (6 points) Determine the convergence or divergence of the series. **Give reasons for your answer.**

$$1)\sum_{n=1}^{\infty}\cos\left(\frac{\pi}{n+1}\right) \qquad 2)\sum_{n=1}^{\infty}4n^{-2}$$

Sol 1) $\lim_{n\to\infty}\cos\left(\frac{\pi}{n+1}\right)$

$= \cos\left(\lim_{n\to\infty}\frac{\pi}{n+1}\right) = \cos 0 = 1 \neq 0$ 1 pt

By test for divergence, 1 pt $\sum_{n=1}^{+\infty} a_n$ diverges. 1 pt

2) $\sum_{n=1}^{+\infty}4n^{-2} = \sum_{n=1}^{+\infty}\frac{4}{n^2}$ 1 pt

$p=2>1$, by p-series test 1 pt

$\sum_{n=1}^{+\infty} a_n$ converges. 1 pt

2. (4 points) Determine the convergence or divergence of the series. **Give reasons for your answer.**

state the test

$$~~1)\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+1}~~ \qquad \sum_{n=1}^{\infty}\frac{n2^n}{3^{n+1}}$$

Sol. $a_n = \frac{n2^n}{3^{n+1}}$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(n+1)\cdot 2^{n+1}}{3^{n+2}}\cdot\frac{3^{n+1}}{n\cdot 2^n}\right| = \left|\frac{n+1}{n}\cdot\frac{3^{n+1}}{3^{n+2}}\cdot\frac{2^{n+1}}{2^n}\right|$$

$$= \left|\frac{n+1}{n}\cdot\frac{1}{3}\cdot 2\right| = \frac{2}{3}\cdot\left(1+\frac{1}{n}\right)$$

1 pt

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{2}{3}\left(1+\frac{1}{n}\right) = \frac{2}{3} < 1.$$ 1 pt

By ratio test, since $L=\frac{2}{3}<1$, $\sum a_n$ converges.

1 pt 1 pt