Example 1

Find and sketch a graph of the (natural) domain of the function

$$f(x, y) = \ln(\sqrt{5} - \sqrt{x^2 + y^2 - 4}) .$$

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The argument of a square root must be nonnegative. So we need:

$$\begin{aligned} x^2 + y^2 - 4 &\geq 0 \\ x^2 + y^2 &\geq 4 \\ x^2 + y^2 &\geq 2^2 \qquad (1) \end{aligned}$$

This is the region in the Cartesian plane along the circle of radius 2 centered at the origin as well as the region outside of that same circle.

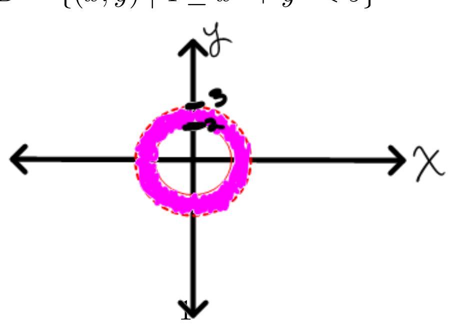
The argument of the natural logarithm must be strictly positive. So we also need:

$$\begin{aligned} &\sqrt{5} - \sqrt{x^2 + y^2 - 4} > 0 \\ &\sqrt{5} > \sqrt{x^2 + y^2 - 4} \\ &5 > x^2 + y^2 - 4 \\ &3^2 = 9 > x^2 + y^2 \qquad (2) \end{aligned}$$

This is the region in the Cartesian plane within the circle of radius 3 centered at the origin but not including the set of points that lie on the circle.

Both equation (1) and equation (2) need to hold for the function to be defined. Imposing both requirements gives an inequality to describe the natural domain of f, D, which is:

$$D = \{(x, y) \mid 4 \leq x^2 + y^2 < 9\}$$



Example 2

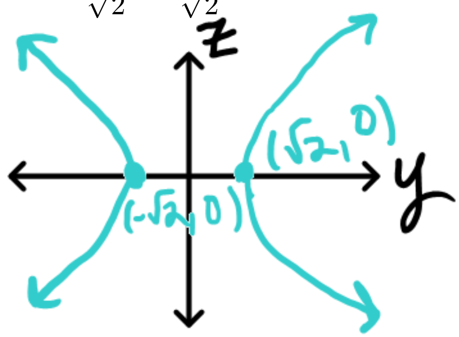
Consider the level surface given by $x^2 - y^2 + z^2 = 2$. Make a plot for:

(a) the slice $x = 2$.

This will give a hyperbola in the $y - z$ plane. To see what the graph will look like, it's helpful to put the equation into standard hyperbola form:

$$x = 2 \implies 2^2 - y^2 + z^2 = 2$$

$$\frac{y^2}{\sqrt{2}^2} - \frac{z^2}{\sqrt{2}^2} = 1$$

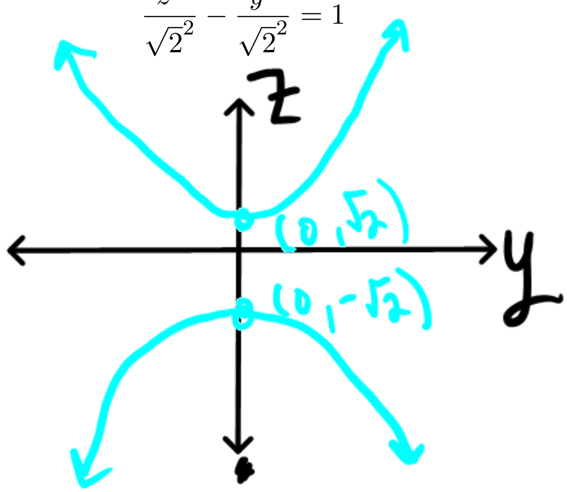


(b) the slice $x = 0$.

This will also give a hyperbola in the $y - z$ plane so we can put the equation in standard form again:

$$x = 0 \implies -y^2 + z^2 = 2$$

$$\frac{z^2}{\sqrt{2}^2} - \frac{y^2}{\sqrt{2}^2} = 1$$



Group Work

Describe the domain of the function $f(x, y) = \sqrt{x} + \sqrt{y}$.

We need the argument of the square root function to be nonnegative in order to have real valued outputs, meaning that both $x \geq 0$ and $y \geq 0$ must hold. The domain, D of f is then:

$$D = \{(x, y) \mid x \geq 0, y \geq 0\} \ .$$

This is quadrant 1 of the $x-y$ plane (including the portions of the axes that lie in the first quadrant).