MA 1024 Conference 7

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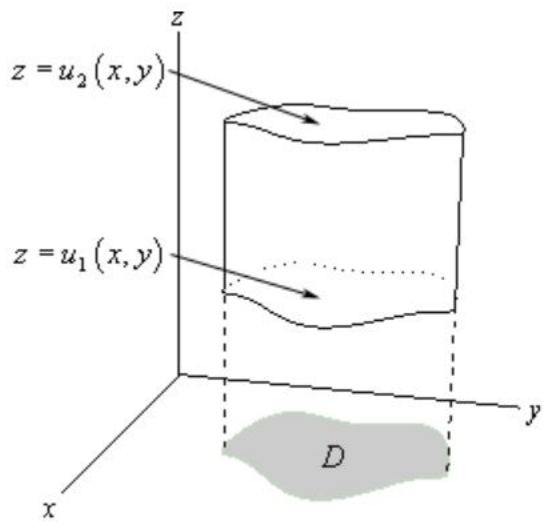
Example 1 (Triple Integral in Rectangular Coordinates, General Solids)

Evaluate $\int\int\int_E xyz\,dV$, where E is the solid for which $0 \leq z \leq 1$, $0 \leq y \leq z$, $0 \leq x \leq y$.

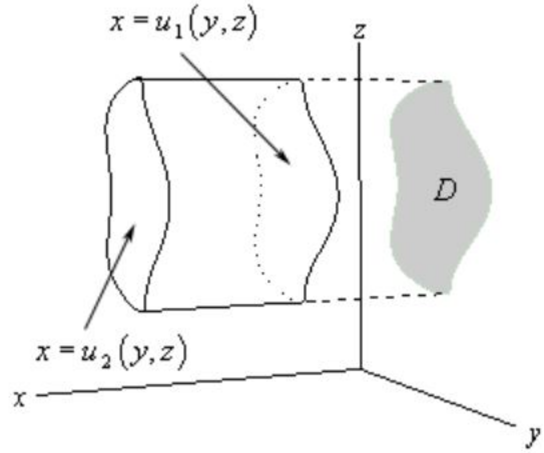


$$E = \{(x, y, z) \mid (x, y) \in D,\ \ u_1(x, y) \le z \le u_2(x, y)\}$$

$$\iiint\limits_E f(x, y, z)\ dV = \iint\limits_D \left[\int_{u_1(x,y)}^{u_2(x,y)} f(x, y, z)\ dz\right] dA$$

Example 2 (Triple Integral in Rectangular Coordinates, General Solids)

Evaluate $\int\int\int_E z\,dV$, where E is the solid tetrahedron bounded by the four planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$.

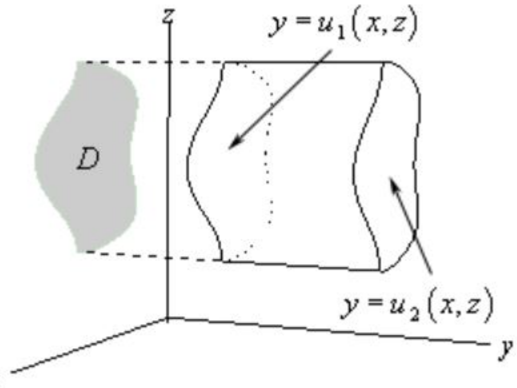


$$E = \{(x, y, z) \,|\, (y, z) \in D,\ \ u_1(y, z) \le x \le u_2(y, z)\}$$

$$\iiint\limits_E f(x, y, z)\, dV = \iint\limits_D \left[\int_{u_1(y,z)}^{u_2(y,z)} f(x, y, z)\, dx \right] dA$$

Example 3 (Triple Integral in Rectangular Coordinates, General Solids)

Set up the triple integral to find the mass of the tetrahedron with vertices $(0, 0, 0)$, $(1, 1, 0)$, $(1, 0, 0)$, and $(1, 0, 1)$ with the density function $\rho(x, y, z) = yx$.



$$E = \{(x, y, z) \mid (x, z) \in D,\ \ u_1(x, z) \le y \le u_2(x, z)\}$$

$$\iiint\limits_E f(x, y, z)\ dV = \iint\limits_D \left[\int_{u_1(x,z)}^{u_2(x,z)} f(x, y, z)\ dy\right] dA$$

Example 4 (Triple Integral in Rectangular Coordinates, General Solids)

Evaluate $\int\int\int_E \sqrt{x^2 + z^2}\, dV$, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.

Example 5 (Triple Integral in Cylindrical Coordinates)

Evaluate $\int\int\int_E x^2+y^2\,dV$, where $E = \{(x,y,z) \mid -2 \le x \le 2, -\sqrt{4-x^2} \le y \le \sqrt{4-x^2}, \sqrt{x^2+y^2} \le z \le 2\}$.

Equations Relating Rectangular (x, y, z) and Cylindrical (r, θ, z) Coordinates

$$x = r\cos\theta, \qquad y = r\sin\theta, \qquad z = z,$$
$$r^2 = x^2 + y^2, \qquad \tan\theta = y/x$$

Example 6 (Triple Integral in Spherical Coordinates)

Use spherical coordinates to evaluate $\int\int\int_E \frac{e^{-(x^2+y^2+z^2)}}{\sqrt{x^2+y^2+z^2}}\, dV$, where E is the the hemisphere bounded by $z = \sqrt{9 - x^2 - y^2}$ and $z = 0$.

Equations Relating Spherical Coordinates to Cartesian and Cylindrical Coordinates

$$\begin{aligned} r &= \rho \sin\phi, \qquad & x &= r\cos\theta = \rho\sin\phi\cos\theta, \\ z &= \rho\cos\phi, \qquad & y &= r\sin\theta = \rho\sin\phi\sin\theta, \end{aligned} \tag{1}$$
$$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2}.$$