MA 1024 Conference 6

Ex 1) Double Integrals Over Rectangular Regions

Ex 2) Double Integrals Over General Regions

Ex 3) Double Integrals Over General Regions

Ex 4) Reversing the Order of Integration for Double Integrals

Ex 5) Finding Volume Under the Surface

Ex 6) Using Properties of Double Integrals

Example 1 (Double Integrals Over Rectangles)

Find $\int\int_R f(x,y)\,dA$, where $f(x,y) = (2x+3y)^{-2}$ and $R = [0,1] \times [1,2]$.

THEOREM 1—Fubini's Theorem (First Form) If $f(x, y)$ is continuous throughout the rectangular region R: $a \leq x \leq b$, $c \leq y \leq d$, then

$$\iint_R f(x, y)\, dA = \int_c^d \int_a^b f(x, y)\, dx\, dy = \int_a^b \int_c^d f(x, y)\, dy\, dx.$$

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Example 2 (Double Integrals Over General Regions)

Find $\int\int_R f(x,y)\,dA$, where $f(x,y) = e^{\frac{x}{y}}$ and R is the region given by the set $R = \{(x,y) \mid 1 \le y \le 2,\ y \le x \le y^3\}$.

THEOREM 2—Fubini's Theorem (Stronger Form) Let $f(x, y)$ be continuous on a region R.

1. If R is defined by $a \le x \le b$, $g_1(x) \le y \le g_2(x)$, with g_1 and g_2 continuous on $[a, b]$, then

$$\iint_R f(x, y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\,dy\,dx.$$

2. If R is defined by $c \le y \le d$, $h_1(y) \le x \le h_2(y)$, with h_1 and h_2 continuous on $[c, d]$, then

$$\iint_R f(x, y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y)\,dx\,dy.$$

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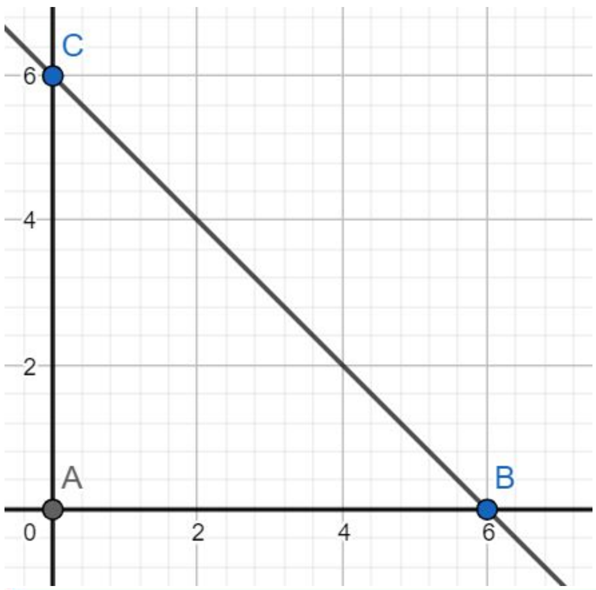
Example 3 (Double Integrals Over General Regions)

Find $\int\int_R xy$, where R is the triangular region in the xy plane with vertices $(0,0)$, $(6,0)$, and $(0,6)$.



THEOREM 2—Fubini's Theorem (Stronger Form) Let $f(x, y)$ be continuous on a region R.

1. If R is defined by $a \leq x \leq b$, $g_1(x) \leq y \leq g_2(x)$, with g_1 and g_2 continuous on $[a, b]$, then

$$\iint_R f(x, y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dy\, dx.$$

2. If R is defined by $c \leq y \leq d$, $h_1(y) \leq x \leq h_2(y)$, with h_1 and h_2 continuous on $[c, d]$, then

$$\iint_R f(x, y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y)\, dx\, dy.$$

(Problem repeated): Find $\int\int_R xy$, where R is the triangular region in the xy plane with vertices $(0,0)$, $(6,0)$, and $(0,6)$.

Example 4 (Reversing the Order of Integration)

Evaluate the integral $I = \int_0^8 \int_{\sqrt[3]{y}}^2 \sqrt{x^4+1}\,dx\,dy$ by reversing the order of integration (using Fubini's Theorem).

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Example 5 (Double Integrals as Volume)

Set up an integral to find the volume of the solid bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$.

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1. If R is defined by $a \leq x \leq b$, $g_1(x) \leq y \leq g_2(x)$, with g_1 and g_2 continuous on $[a, b]$, then

$$\iint_R f(x, y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dy\, dx.$$

2. If R is defined by $c \leq y \leq d$, $h_1(y) \leq x \leq h_2(y)$, with h_1 and h_2 continuous on $[c, d]$, then

$$\iint_R f(x, y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y)\, dx\, dy.$$

Example 6 (Properties of Double Integrals)

Using the maxima of the function over the region R, produce an upper estimate of the integral

$$I = \int\int_R e^{x^2+y^2}\, dA$$

Where R is the disk in the xy plane with $x^2 + y^2 \leq 1$.

If $f(x, y)$ and $g(x, y)$ are continuous on the bounded region R, then the following properties hold.

1. *Constant Multiple:* $\iint_R cf(x, y)\, dA = c \iint_R f(x, y)\, dA$ (any number c)
2. *Sum and Difference:*
$$\iint_R (f(x, y) \pm g(x, y))\, dA = \iint_R f(x, y)\, dA \pm \iint_R g(x, y)\, dA$$
3. *Domination:*
   (a) $\iint_R f(x, y)\, dA \geq 0$ if $f(x, y) \geq 0$ on R
   (b) $\iint_R f(x, y)\, dA \geq \iint_R g(x, y)\, dA$ if $f(x, y) \geq g(x, y)$ on R
4. *Additivity:* $\iint_R f(x, y)\, dA = \iint_{R_1} f(x, y)\, dA + \iint_{R_2} f(x, y)\, dA$
   if R is the union of two nonoverlapping regions R_1 and R_2

DEFINITION The **area** of a closed, bounded plane region R is

$$A = \iint_R dA.$$

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