Example 1 (Directional Derivatives)

Find the derivative of $f(x,y) = \frac{x-y}{xy+2}$ at $P_0 = (1,-1)$ in the direction of $\mathbf{v} = \langle 12, 5\rangle$.

1. Find $\nabla f = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\rangle$.
2. Calculate $(\nabla f)_{P_0}$.
3. The directional derivative is $D_{\mathbf{u}} f = (\nabla f)_{P_0} \cdot \mathbf{u}$, where $\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|}$.

Example 2 (Directional Derivatives)

Find the direction in which $f(x,y) = \frac{x-y}{xy+2}$ increases most rapidly at the point $P_0 = (1,-1)$. What is this maximum rate of change?

Example 3 (Directional Derivatives)

Use the contour diagram to estimate the directional derivative of $f(x,y)$ at the point $P_0 = (2,2)$ in the direction of $\mathbf{v} = \hat{\imath}$. (Note that $\hat{\imath} = \langle 1, 0\rangle$) is already a unit length vector).

Example 4 (Tangent Lines to Level Curves)

Let $f(x,y) = x^2 + y^2$. Sketch the level curve $f(x,y) = 4$ together with ∇f at the point $P_0 = (\sqrt{2}, \sqrt{2})$. Then write an equation for the tangent line.

Example 5 (Tangent Lines to Level Curves)

Determine if the level curves of $f(x,y) = 2x + 4y$ and $g(x,y) = 4x - 2y$ intersect at right angles.

Example 6 (Critical Points and the Second Derivative Test)

Find the local maxima, minima, and saddle points of the function $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$.