Example 1 (Higher Order Partial Derivatives)

Verify that the function $f(x, y, z) = 1/\sqrt{x^2 + y^2 + z^2}$ is a solution to the Laplace equation $f_{xx} + f_{yy} + f_{zz} = 0$.

Example 2 (Higher Order Partial Derivatives)

Verify that the function $u(t, x) = e^{-k^2 t}\sin(kx)$ is a solution to the heat conduction equation $u_t = u_{xx}$.

Example 3 (Multivariable Chain Rule)

Use the chain rule to find $\frac{dz}{dt}$.

$$z = \arctan(y/x), \quad x = e^t, \quad y = e^{-t} .$$

Example 4 (Multivariable Chain Rule)

Use the chain rule to find $\frac{dw}{dt}$.

$$w = xe^{yz}, \quad x = t^2, \quad y = 1 - t, \quad z = 1 + 2t .$$

Example 5 (Multivariable Chain Rule)

The length x, width y, and height z of a box change with time. At a certain instant the dimensions are and $x = 1$ m, and $y = z = 2$ m. At the same time, x and y are increasing at a rate of 2 m/s while z is increasing at a rate of 3 m/s. At this instance, find the rate at which the volume of the box is changing.

Example 6 (Multivariable Chain Rule)

Use the chain rule to find $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$.

$$z = x^2 y^3, \quad x = s \text{ cos}t, \quad y = s \text{ sin}(t) .$$

Example 7 (Multivariable Chain Rule)

Use the chain rule to find $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$.

$$z = e^r \cos(\theta), \quad r = st, \quad \theta = \sqrt{s^2 + t^2}\ .$$

Example 8 (Implicit Function Theorem)

Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $F(x, y, z) = x^3 + y^3 + z^3 + 6xyz = 1$.