

VI. Probability

Suppose we have a 365 day year. Suppose that P_0 (person zero) was born on d_0 (day 0). Consider P_1 .

$$P(d_1 = d_0) = \frac{1}{365} \quad \text{and} \quad P(d_1 \neq d_0) = 1 - \frac{1}{365} \quad (\text{complementarity})$$

$$\begin{aligned} P(d_2 \neq d_1, d_1 \neq d_0, d_2 \neq d_0) &= P(d_1 \neq d_0) P(d_2 \neq d_1, d_2 \neq d_0 \mid d_1 \neq d_0) \\ &\quad (\text{Independence}) \\ &= \left(1 - \frac{1}{365}\right) \left(1 - \frac{2}{365}\right). \end{aligned}$$

$$\text{In general} \quad P(n) = \prod_{k=1}^n \left(1 - \frac{k}{365}\right)$$

$$P(1) = 1 - \frac{1}{365} = \frac{364}{365}$$

$$P(2) = \frac{364}{365} \frac{363}{365}$$