I. Logic

B. Induction

Pf Types (7) Proof by Induction

To prove $P(n)$:

1) Prove $P(1)$ (or $P(0)$)

Inductive step 2) Prove that $P(k) \Rightarrow P(k+1)$

Related Forms:

1) Prove $P(0)$ and $P(1)$

2) Prove $P(k) \Rightarrow P(k+2)$

Thm: If $|A| = n \in \mathbb{Z}^+$, then $|\mathcal{P}(A)| = 2^n$.

Pf (Induction):

1) $P(1)$: If $|A| = 1$, then $|\mathcal{P}(A)| = 2$.

2) Suppose $P(k)$: If $|A| = k$, then $|\mathcal{P}(A)| = 2^k$. Now assume $|A| = k+1$. Then,

$A = \{a_1, a_2, \ldots, a_k, a_{k+1}\}$. So,

$A = \{a_1, a_2, \ldots, a_k\} \cup \{a_{k+1}\}$

$=: \tilde{A} \cup \{a_{k+1}\}$. Note that $|\mathcal{P}(\tilde{A})| = 2^k$.

Now notice that all the subsets of A are the 2^k subsets of $\tilde{A}$ unioned with the same subsets with a_{k+1} unioned in. So $|\mathcal{P}(A)| = 2|\mathcal{P}(\tilde{A})|$

$= 2 \cdot 2^k$

$= 2^{k+1}$. QED

(switches)

Negation: $\exists \longleftrightarrow \forall$

$\wedge \longleftrightarrow \vee$

"and" "or"

"but"

$\neg(A \Rightarrow B) = A \wedge \neg B$

"For $x \in \mathbb{R}$" $\equiv$ "For all $x \in \mathbb{R}$".

"$A \Rightarrow B$" $\equiv$ "$\neg B \Rightarrow \neg A$"

not $\neg A \Rightarrow \neg B$

Countably Infinite $|\mathbb{Z}| = |\mathbb{Q}| = |\mathbb{Z}^+| = \aleph_0$

$\aleph_0 < |\mathbb{R}| = \mathfrak{C} = |\mathcal{P}(\mathbb{Z}^+)| = 2^{\aleph_0}$

continuum

Cantor

$\aleph_0 < \aleph_1 \leq \mathfrak{C}$

Cantor

$=$

Continuum hypothesis