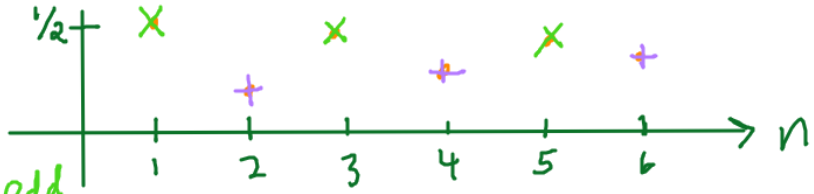
Ex 2 $\quad \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\dots}}}$

This continued fraction is the limit of the sequence

$$\left\{ \frac{1}{2},\ \boxed{\frac{1}{2+\frac{1}{2}}},\ \boxed{\frac{1}{2+\boxed{\frac{1}{2+\frac{1}{2}}}}},\ \frac{1}{2+\boxed{\frac{1}{2+\frac{1}{2+1/2}}}},\ \dots \right\}$$

$$= \left\{ \frac{1}{2} \searrow \frac{2}{5} \nearrow \frac{5}{12} \searrow \frac{12}{29} \nearrow, \dots \right\}$$



odd

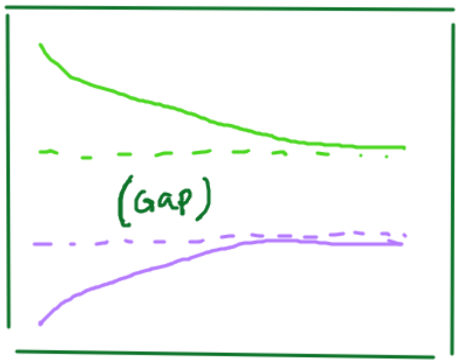
decreasing & bounded below

even increasing sequence bounded above.

So the green subsequence converges.

Also the even subsequences converges

Does the entire sequence converge?



consider

$$|a_{n+1} - a_n| = \left| \frac{1}{2+a_n} - \frac{1}{2+a_{n-1}} \right|$$

recurrence formula.

$$= \left| \frac{(2+a_{n-1}) - (2+a_n)}{(2+a_n)(2+a_{n-1})} \right|$$

$$= \left| \frac{(a_n - a_{n-1})}{(2+a_n)(2+a_{n-1})} \right|$$

Thus $\displaystyle |a_{n+1} - a_n| = \frac{|a_n - a_{n-1}|}{(2+a_n)(2+a_{n-1})}$

$$< |a_n - a_{n-1}| / 4$$

$$< \frac{1}{4}\left(\frac{|a_{n-1} - a_{n-2}|}{4} \right)$$

$$= \frac{|a_{n-1} - a_{n-2}|}{4^2}$$

$$< \frac{|a_2 - a_1|}{4^{n-1}}$$

Thus $|a_{2n} - a_{2n+1}| \to 0$ as $n \to +\infty$ No Gap

To find L, consider $L = \dfrac{1}{2+L}$

Ex 3 $\sqrt{x+\sqrt{x+\sqrt{x+\cdots}}}$ for $x \in [0,1]$

sequence: $\left\{ \underbrace{\sqrt{x}}_{f_1(x)}, \underbrace{\sqrt{x+\sqrt{x}}}_{f_2(x)}, \underbrace{\sqrt{x+\sqrt{x+\sqrt{x}}}}_{f_3(x)}, \ldots \right\}$

Two Questions:

1. Does this sequence converge?
2. If so, to what?

Notice that this sequence is increasing.
Is it bounded above? Yes.

Pf (Induction):

1. ~~$\sqrt{x} \le \sqrt{x+\sqrt{x}}$ for $x \in [0,1]$~~
2. Suppose

We will work on this tomorrow

Do as much as possible for HW4 #3

Watch for (possible) email.