

Quiz 2

D Term, 2021

I affirm that I have not consulted my text, notes or any reference, paper or electronic, or any person once I opened and/or looked at this quiz.

Signature: _____

Show all work needed to reach your answers.

1. (5 points) If $S = \{1, \pi, \sqrt{3}\}$, please find the power set of S .

1 point for each two correct subsets; -1 point for any incorrect entries; 5 points for exactly these sets

$$\mathcal{P}(S) = \{\emptyset, \{1\}, \{\pi\}, \{\sqrt{3}\}, \{1, \pi\}, \{\pi, \sqrt{3}\}, \{\sqrt{3}, 1\}, S\}$$

2. (10 points) Suppose that A , B and C are all subsets of some universe U . Please show that

$$A \cap (B \cup C) \subset (A \cap B) \cup (A \cap C)$$

Suppose that $x \in A \cap (B \cup C)$. Then $x \in A$ and $x \in B \cup C$. So $x \in A$ and $x \in B$, or $x \in A$ and $x \in C$. Thus $x \in A \cap B$, or $x \in A \cap C$, which implies that $x \in (A \cap B) \cup (A \cap C)$. QED

3. (10 points) Please use induction to show that $2 + 4 + 6 + \dots + 2n = n(n+1)$.

Let $P(n) = "2 + 4 + 6 + \dots + 2n = n(n+1)"$. Then $P(1) = "2 = 1(1+1)"$ is true. Now let's assume that $2 + 4 + 6 + \dots + 2k = k(k+1)$, and then use this $P(k)$ to show $P(k+1)$:

$$\begin{aligned} P(k+1): 2 + 4 + 6 + \dots + 2k + 2(k+1) &= k(k+1) + 2(k+1) \\ &= (k+1)(k+2) = (k+1)(k+1+1) \end{aligned}$$

Thus $P(k) \Rightarrow P(k+1)$, so by induction, the formula holds.

QED