MA 1971 Bridge to Higher Mathematics **Name:** Solutions

Quiz 3 **D Term, 2021**

I affirm that I have not consulted my text, notes or any reference, paper or electronic, or any person once I opened and/or looked at this quiz.

Signature: ______________________

Show all work needed to reach your answers.

1. (1 points)

either

$\mathbb{N} =$ $\{0, 1, 2, 3, \ldots\}$

or

$\{1, 2, 3, \ldots\}$ or $= \mathbb{Z}^+$

2. (12 points) Please complete the following multiplication table for $\mathbb{Z}_7$ (multiplication mod 7).

$\times$	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

exactly these entries / no partial credit 1 point each

3. (12 points) Please complete the following proof that the prime numbers ($\mathbb{P}$) are *countably infinite*.

Proof (Contradiction):
Suppose that the primes are finite, that is, suppose that $\mathbb{P} = \{p_1, p_2, ..., p_n\}$ for some finite $n \in \mathbb{Z}^+$.

Consider (+3) $m = p_1 p_2 p_3 \cdots p_n + 1$. Notice that (+2) $p_k \mid p_1 p_2 \cdots p_n$ $\forall k, 1 \leq k \leq n$, but (+1) $p_k \nmid 1$. So $p_k \nmid m$ (+3) $\Rightarrow m$ is prime. But $m > p_k$ $\forall k$, (+2) so m is a prime not on our list. Hence the primes (+1) are not finite. Since $\mathbb{P} \subset \mathbb{Z}^+$, $\mathbb{P}$ must be countably infinite.

$\boxed{\text{A different proof might be given.}}$