c. 1740

Conjecture (Goldbach): Every even integer greater than 4 can be written as the sum of two primes.

$(6 = 3+3 \quad 8 = 3+5 \quad 10 = 3+7)$

$(4 = 1+1+1+1,\ 4 = 1+1+2,\ 4 = 2+2,\ 4 = 1+3)$

IV. Analysis

A. Uncountability of $\mathbb{R}$.

Thm: $\mathbb{Q}^+$ is countable (countably infinite).

p/q	1	2	3	4	5	
1	1 $\pm 1/1$ 2,3	2 $\pm 2/1$ 4,5	4 $3/1$	6 $4/1$	10 $5/1$	$\cdots$
2	3 $\pm 1/2$ 6,7	$2/2$	7 $3/2$	$4/2$	$5/2$	$\cdots$
3	5 $1/3$	8 $2/3$	$3/3$	$4/3 \cdots$		
4	9 $1/4$	$2/4$				

$\mathbb{Z}^+$	1	2	3	4	5	6	7	8	$\cdots$
$\mathbb{Q}$	0	$1/1$	$-1/1$	$2/1$	$-2/1$	$1/2$	$-1/2$	$3/1$	$\cdots$

Thm (Cantor): $\mathbb{R}$ is not countable.

Pf: Suppose $\mathbb{R}$ is countable. Consider $[0,1] \subset \mathbb{R}$, and suppose we have a count of this interval. For $a_{ij} \in \{0, 1, \ldots, 9\}$, Consider $x = 0.x_1 x_2 x_3 x_4 \ldots$ where

$$x_i = \begin{cases} 5, & a_{ii} \neq 5 \\ 7, & a_{ii} = 5 \end{cases}.$$

Cantor Diagonalization

$$\begin{aligned}
x_1 &= 0.a_{11} a_{12} a_{13} a_{14} \ldots \\
x_2 &= 0.a_{21} a_{22} a_{23} a_{24} \ldots \\
x_3 &= 0.a_{31} a_{32} a_{33} a_{34} \ldots \\
x_4 &= 0.a_{41} a_{42} a_{43} a_{44} \ldots \\
x_5 &= 0.a_{51} a_{52} a_{53} a_{54} \ldots \\
&\vdots \qquad\qquad \vdots
\end{aligned}$$

Notice x is not in our count.

Note: $0.999\ldots = 1.000\ldots$

$0.\bar{9} = 1.\bar{0}$

$3\left(\frac{1}{3}\right) = (0.3333\ldots)3$

$1 = 0.999\ldots$, or

$9 \sum_{i=1}^{\infty} \left(\frac{1}{10}\right)^i = 1$