Negation of an implication

Thm $\neg(A \Rightarrow B)$

Pf: Truth Table

A	B	$\neg(A \Rightarrow B)$	$A \wedge \neg B$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	F	F

Proposition: If $A, B \subset \mathcal{U}$, then
$A - B = A \cap B^c$

Pf: $(\subset)$ Let $x \in A - B$. Then
$x \in \{x \in A \mid x \notin B\}$. So
$x \in A$ and $x \in B^c$. Thus
$x \in A \cap B^c$. $\blacktriangle$

$(\supset)$ ________ ________

________________ $\blacksquare$

Def: __Power Set__ : $\mathcal{P}(A)$ is the set of all subsets of A.

Ex) If $A = \{0, 1\}$, then

$$\mathcal{P}(A) = \{\emptyset, \{0\}, \{1\}, A\}$$

Def: __Cardinality__ : $|A| = \#(A)$ is the number of elements in A.

Ex) Same A as above

$$|A| = 2, \quad |\mathcal{P}(A)| = 4$$

In general, for finite sets,

$$|\mathcal{P}(A)| = 2^{|A|}$$

What is $|\mathbb{Z}|$?

Notice that $\mathbb{Z}$ can be put into 1-1 correspondence with $\mathbb{Z}^+$.

$\mathbb{Z}^+$	$\mathbb{Z}$
1	0
2	1
3	-1
4	2
5	-2

$$\boxed{|\mathbb{Z}| = \aleph_0 \text{ "aleph null"}}$$

Def: Any set that can be put into 1-1 correspondence with $\mathbb{Z}^+$ is __countable__ or __countably infinite__.

Before 1900, Naive Set Theory

Russell Paradox (1901)

Let $S := \{ X \mid X \notin X \}$

Notice that $S \in S$ and $S \notin S$. ($\rightarrow\leftarrow$)

This problem led to Axiomatic Set Theory.

What is $|\mathbb{R}|$?

Thm: $\mathbb{R}$ is not countable.

Def: $|\mathbb{R}| = \mathfrak{c} \leq \aleph_1 := |\mathcal{P}(\mathbb{Z})|$

"Continuum"

$= 2^{\aleph_0}$

Note: $|\mathbb{Z}| = |\mathbb{Z}^+| = |\mathbb{Q}| = \aleph_0$