MA 1971 Bridge to Higher Mathematics Name: Solutions

Quiz 4 D Term, 2021

I affirm that I have not consulted my text, notes or any reference, paper or electronic, or any person once I opened and/or looked at this quiz.

Signature:

Show all work needed to reach your answers.

Consider the continued root

$$\sqrt{5+\sqrt{5+\sqrt{5+\dots}}}$$

1. (2 points) Which sequence is equivalent to this continued root?

$$\left\{\sqrt{5},\ \sqrt{5+\sqrt{5}},\ \sqrt{5+\sqrt{5+\sqrt{5}}},\ \dots\right\}$$

2. (3 points) Call this sequence $\{a_n\} = \{a_1, a_2, a_3, ...\}$ (just to give it and its terms a name). Please write down the recurrence formula giving a_{n+1} in terms of a_n.

$$a_{n+1} = \sqrt{5+a_n}$$

Small variations might be acceptable.

3. (12 points) This sequence can be shown to be increasing, so to guarantee convergence, **please use the recurrence formula to show the sequence is bounded above.** Hint: Induction.

A different bound could be used.

Let $P(n)$ be the statement "$a_n \le 4$" +2

Step 1: $P(1)$ is true: $a_1 = \sqrt{5} < \sqrt{9} = 3 < 4$. +2

Step 2 (Inductive Step): Suppose +2 $a_n \le 4$. Then by the recurrence formula, $a_{n+1} = \sqrt{5+a_n} \le \sqrt{5+4} = 3 < 4$. Thus +4 $P(n) \overset{+1}{\Rightarrow} P(n+1)$.

Hence by induction, +1 a_n is bounded above $\forall n$.

These last details could be implicit.

4. (8 points) Please compute the value of this continued root (the limit of the sequence).

Since the sequence converges, $a_n \to L$ and $a_{n+1} \to L$.

So $L \overset{+2}{=} \sqrt{5+L} \Rightarrow L^2 - L - 5 = 0$ +2 $\Leftrightarrow L = $ +2 $\dfrac{1 \pm \sqrt{1+20}}{2}$

Since $L > 0$, +2 $L = \dfrac{1}{2} + \dfrac{\sqrt{21}}{2} \simeq 2.8$