MA 1971 Bridge to Higher Mathematics Name: Solutions

Quiz 5 D Term, 2021

I affirm that I have not consulted my text, notes or any reference, paper or electronic, or any person once I opened and/or looked at this quiz.

Signature: _____

Show all work needed to reach your answers.

1. (5 points) Please give the graph theory definition of a *tree*.

(+1) A tree is a connected graph containing no cycles. (+4)

2. (5 points) Suppose that you have a friend who has taken a graph theory course and who claims to have drawn a tree with 15 vertices and 12 edges. In one sentence, please explain why is your friend mistaken.

(+5) For any tree, $|V| - |E| = 1$; since $15 - 12 = 3$ this graph can not be a tree.

3. (10 points) An edge in a connected graph G is a *bridge* iff its removal will disconnect the graph. Please show that if G is a cycle, then none of its edges is a bridge. Hint: Start with the definition of a cycle.

A cycle is a graph of the form (+3) $\{v_0, e_1, v_1, e_2, \ldots e_{n-1}, v_{n-1}, e_n, v_0\}$ where all edges and (+2) vertices are distinct (v_0 is on both ends)

(+5) If the edge e_k is removed from this graph, then one can still walk between any two vertices of the cycle, going through v_0 if necessary.