Bridge to Higher Mathematics **MA 1971**

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Course Summary

I. Logic

  A. Statements and Predicates (quantifiers: $\exists$, $\forall$; theorem, proposition, lemma, corollary)
  B. If-Then Implications
  C. Truth Tables
  D. Statement Operations (direct, negation, converse, contrapositive, contradiction)
  E. Counterexample

II. Set Theory

  A. Basics (empty set, set equality, complements, set difference $A - B$, power set)
  B. Set Containment
  C. Induction
  D. Russell Paradox: If $S := \{x | x \notin x\}$, then $S \in S \Leftrightarrow S \notin S$
  E. Axiomatic Set Theory (existence, equality, specification, pairing, union, power set, induction, choice, substitution)

III. Number Theory

  A. Natural Numbers (Peano Axioms)
  B. Primes Are Countably Infinite
    1. Euclid Proof
    2. Goldbach Proof (Fermat numbers)
    3. Euler Proof
  C. When Can a Positive Integer Be Written as the Sum of Two Natural Squares? (modular arithmetic, finite fields)

IV. Analysis

  A. $\mathbb{Q}$ is Countable, $\mathbb{R}$ is Uncountable (cardinality, $\forall n \in \mathbb{Z}$, $n < \aleph_0 < \aleph_1 \leq \mathfrak{c}$)
  B. Continued Fractions, Continued Roots (completeness, greatest lower bound property)
  C. Harmonic, Geometric and Arithmetic Means
  D. Cauchy Sequences

V. Graph Theory

  A. Basics (vertices, edges, order, degree, planar, faces, cycle, tree)
  B. Tree $\implies |V| - |E| = 1$
  C. Euler Formula
  D. Four Color Problem (unavoidable set, Kempe chain, reducible configurations)

VI. Probability (Birthday Paradox)