1. Find the general solution using the method of integrating factors for the following problem (list of 4 potential problems)

$$\begin{aligned} y'(x) - 3\,y(x) &= e^{2x} \\ y'(x) + 2x\,y(x) &= x \\ 2\,y'(x) + 10\,y(x) &= 1 \\ y'(x) - y(x) &= 2\,e^{x} \end{aligned}$$

$y' - 3y = e^{2x}$

$\mu = e^{-3x}$

$ye^{-3x} = \int e^{-x}dx$

$ye^{-3x} = -e^{-x} + c$

$\boxed{y(x) = -e^{2x} + ce^{3x}}$

$y' + 2xy = x$

$\mu = e^{x^2}$

$ye^{x^2} = \int xe^{x^2}dx$

$ye^{x^2} = \frac{1}{2}e^{x^2} + c$

$\boxed{y(x) = \frac{1}{2} + ce^{-x^2}}$

$y' + 5y = 1/2$

$\mu = e^{5x}$

$ye^{5x} = \int \frac{1}{2}e^{5x}dx$

$ye^{5x} = \frac{1}{10}e^{5x} + c$

$\boxed{y(x) = \frac{1}{10} + ce^{-5x}}$

$y' - y = 2e^{x}$

$\mu = e^{-x}$

$ye^{-x} = \int 2dx$

$ye^{-x} = 2x + C$

$\boxed{y(x) = e^{x}(2x+C)}$

2. Solve the following initial-value problems using the method of integrating factors

$$\begin{aligned} \left(x^2+9\right)\,y'(x) + x\,y(x) &= 0\,,\ y(0)=3 \\ x\,y'(x) - 4\,y(x) = x^5\,e^{x} &= 0\,,\ y(1)=2 \\ x\,y'(x) + y &= 2\,x\,,\ y(0)=0 \\ t\,y'(t) + 2\,y(t) &= t^2 - t + 1\,,\ y(1) = \frac{1}{2} \end{aligned}$$

$y' + \frac{x}{x^2+9}y = 0$

$\int \frac{x}{x^2+9}dx = \int \frac{1}{2}\frac{1}{u}du = \frac{1}{2}\ln(x^2+9) + c$

$y\sqrt{x^2+9} = C$

$\mu = \sqrt{x^2+9}$

$y(x) = C/\sqrt{x^2+9}$ $3 = y(0) = C/3 \rightarrow 9 = C$

$\boxed{y(x) = 9/\sqrt{x^2+9}}$

$y' - 4/x\, y = x^4e^{x}$

$\mu = e^{-4\ln|x|} = x^{-4}$

$y/x^4 = \int e^{x}dx$

$y(x) = x^4e^{x} + cx^4$ $2 = y(1) = e + C$

$\boxed{y(x) = x^4e^{x} + (2-e)x^4 = x^4(e^{x} + 2 - e)}$

$y' + y/x = 2$

$\mu = e^{\ln|x|} = x$

$yx = \int 2x\,dx$

$yx = x^2 + c$ $0 = y(0) \Rightarrow c = 0$

$\boxed{y(x) = x}$

$y' + 2y/t = t - 1 + 1/t$

$\mu = e^{2\ln t} = t^2$

$(yt^2)' = t^3 - t^2 + t$

$yt^2 = t^4/4 - t^3/3 + t^2/2 + c$

$y(t) = t^2/4 - t/3 + 1/2 + c/t^2$

$1/2 = 1/4 - 1/3 + 1/2 + C \rightarrow C = 1/12$

$\boxed{y(t) = t^2/4 - t/3 + 1/2 + 1/12t^2}$