based on Lectures of Chapter 4.1–4.3

1. Consider the random variable X with the probability mass distribution

$$\mathbb{P}[X=-1]=0.3, \qquad \mathbb{P}[X=2]=0.5, \qquad \mathbb{P}[X=5]=0.1, \qquad \mathbb{P}[X=10]=0.1.$$

   Calculate the expected value and variance of X as well as the expectation of Y with $Y=e^{2X}$.

2. Remember that the geometric distribution is given by the probability mass distribution

$$p(n)=P[X=n]=(1-p)^n p$$

   for non-negative integers n and some $p\in(0,1)$.

   a) Prove that the probability mass distribution describes indeed a probability, i.e. show that

$$\sum_{n=0}^{\infty} p(n)=1.$$

   b) Calculate the probability of $\mathbb{P}[X\geq 3]$.

   c) Prove that it holds for non-negative integers n, i that

$$\mathbb{P}[X\geq n+i \,|\, X\geq n]=\mathbb{P}[X\geq i].$$

3. Let X be a Poisson distributed random variable with parameter $\lambda=2$.

   a) Calculate

$$\mathbb{P}\big[X\geq 2 \,|\, X\geq 1\big].$$

   b) Calculate

$$\mathbb{E}\left[\frac{1}{X+1}\right].$$

4. Let Y be a Poisson distributed random variable with parameter λ, where λ is a non-negative integer. Calculate

$$\mathbb{E}\big[|Y-\lambda|\big].$$