MA 2631 Conference 5

September 29, 2021

The exponential random variable with parameter $\lambda > 0$:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

1. Let X be an exponential random variable with parameter λ. Calculate $\text{Var}[X]$ in two ways:

   (a) By looking up $E[X]$ in the lecture notes, calculating $E[X^2]$ directly using the definition of expectation, and the formula $\text{Var}[X] = E[X^2] - (E[X])^2$;

   (b) By deriving the moment generating function $M_X(t) = E[e^{tX}]$ (a challenge problem).

2. Suppose that the length of a phone call in minutes is an exponential random variable with parameter $\lambda = \frac{1}{10}$. If someone arrives immediately ahead of you at public telephone booth, find the probability you will have to wait

   (a) more than 10 minutes;

   (b) not more than one standard deviation away from the mean.

3. We say that a nonnegative random variable X is memoryless if

$$P[X > s + t | X > t] = P[X > s] \quad \text{for all } s, t \geq 0.$$

   Show that an exponential random variable X with parameter λ is memoryless.

4. Suppose that the number of miles that a car can run before its battery wears out is exponentially distributed with an average value of $10,000$ miles. If a person desires to take a $5,000$ mile trip, what is the probability that he or she will be able to complete the trip without having to replace the car battery?