

MA 2631 Conference 1

September 1, 2021

1. In how many ways can 3 science fiction books, 4 math books and 1 cooking book arranged on a bookshelf, if
 - (a) there are no restrictions on the arrangement?
 - (b) all the science fiction books have to be stored together and also all the math books have to be stored together?
 - (c) (only) the math books have to be stored together, the rest can be arranged without restriction?
2. Mike has nine friends and wants to throw a party. As his apartment is not big enough to invite all of them, he decides to invite only six.
 - (a) How many choices of invitations has he?
 - (b) Two of his friends of his are feuding and will not attend the party together. Accounting for this fact, how many possibilities has he?
 - (c) Two of his friends are very close and will attend the party only if invited together. Accounting for this fact, how many possibilities has he?
3. A coin is tossed repeatedly until the first time "heads" appears.
 - (a) Describe mathematically the sample space of this experiment.
 - (b) Describe mathematically the events

$E = \text{"there are no more than four tails"}$

$F = \text{"there are at least two tails"}$

- (c) Describe mathematically the events $E \cap F$ and $E \cup F^c$
4. Given a family of events $E_1, E_2, \dots, E_n, \dots$ on some sample space Ω , construct a new family $F_1, F_2, \dots, F_n, \dots$ on the same sample space Ω such that the events F_i are monotone, ($F_m \subseteq F_n$ for $m \leq n$) and

$$\bigcup_{k=1}^n F_k = \bigcup_{k=1}^n F_k \text{ for any positive integer } n.$$

Prove that the constructed family has the desired properties.