MA 2631 Conference 7

October 5, 2022

1. In a small town, there are 50 births a year. Assume that the probability that a newborn is a girl is 50%. How likely is it that in a given year, there are at least 25 and at most 27 girls born. Calculate this probability

   a) exactly.

   b) by an approximation with the normal distribution.

2. Consider a biased coin that shows *heads* in $\frac{2}{3}$ of all cases and *tails* only in $\frac{1}{3}$ of all cases. The coin is flipped consecutively (and independently) 200 times.

   a) What is the probability that *tails* shows up the first time at the 10th flip?

   b) Calculate the probability *heads* shows up more than 150 times (using a suitable approximation).

3. Assume that the joint probability mass distribution $p_{X,Y}$ of the random variable X and Y is given by

$$\begin{aligned} p_{X,Y}(0,0) &= \frac{1}{3} & p_{X,Y}(0,1) &= \frac{1}{4}; \\ p_{X,Y}(1,0) &= \frac{1}{4} & p_{X,Y}(1,1) &= \frac{1}{6}. \end{aligned}$$

   a) Calculate the marginal probability mass distributions p_X and p_Y.

   b) What is the probability mass distribution of the random variable $Z = X^2 + Y$?

4. Assume that X and Y are jointly distributed random variables with joint density

$$f_{X,Y}(x,y) = \begin{cases} cye^{-x} & \text{if } 0 \leq x < \infty,\ 0 \leq y \leq 1; \\ 0 & \text{else.} \end{cases}$$

   for some $c \in \mathbb{R}$.

   a) Calculate c.

   b) Calculate the marginal probability density functions f_X and f_Y.

   c) What is the probability $P[3X + Y^2 > 4]$?