MA 2631

Probability Theory

Section AL01 / AD01

Assignment 3

due on Friday, September 10

based on Lectures of Chapter 2.1–2.4

1. Let Ω be a sample space, $\mathbb{P}$ a probability and E, F events. Prove that

$$\mathbb{P}[E^c \cup F^c] \leq 2 - \big(\mathbb{P}[E] + \mathbb{P}[F]\big).$$

2. A person picks 13 cards out of a standard deck of 52.

   i) What is the probability that he has at least one of the four aces in his hand?

   ii) What is the probability that he has exactly one ace in his hand?

3. You repeatedly toss a coin until you get a heads. What is the probability that you get the head on an even-numbered toss?

4. An urn contains twelve balls, four of which are white, three green and five black.

   a) We draw three balls, what is the probability that all three balls are of different color?

   b) We draw three balls consecutively, after each drawing placing back the ball in the earn before drawing the next, what is the probability that the color of all three drawn balls is different?

5. Consider the lottery "Mega Millions" (a sample slip you may find on page 4 and 5 – if you do not know about lotteries, please read the instructions carefully).

   i) For one play, what is the probability that you will win the jackpot?

   ii) For one play, what is the probability that you have three winning numbers plus the MEGABALL?

6. 25 WPI math majors are joining their classes. 14 go to *Probability*, 12 go to *Linear Programming* and 9 to *Discrete Mathematics*. 7 take both, *Probability* and *Linear Programming*, 4 both *Probability* and *Discrete Mathematics* and 5 *Linear Programming* and *Discrete Mathematics* and 3 all three classes.

   a) If a student is picked at random, what is the probability that she is not any of the three classes?

   b) If a student is chosen randomly, what is the chance that he takes exactly one of these three classes?

   c) If two students are chosen randomly, what is the chance that at least one is taken any of these three classes?

<div align="right">8 points per problems</div>

Additional practice problems (purely voluntary - no points, no credit, no grading):

Standard Carlton and Devore, Section 1.2: Exercises 14, 15, 16, 19, 20, 22, 25, 27, 28, 30

Hard Show that one cannot define a probability such that the set of natural numbers $\mathbb{N}$ forms a Laplacian sample space (i.e., all outcomes/natural numbers are equally likely)

MEGA MILLIONS
WITH MEGAPLIER

Play MEGA MILLIONS - Every Tuesday and Friday

PICK 5 - ONE PLAY $1

PICK 1

MEGABALL NUMBER

PICK 5 - TWO PLAYS $2

PICK 1

MEGABALL NUMBER

PICK 5 - THREE PLAYS $3

PICK 1

MEGABALL NUMBER

MEGAPLIER

Option Doubles Ticket Price

YES

NO. OF DRAWS

1 2 3 4 5 6 7

INSERT THIS END

MEGA MILLIONS

MULTI-BET SLIP

This betting slip allows you to make from one to three $1 bets by picking five (5) out of fifty-six (56) numbers and one (1) out of forty-six (46) numbers in each of the one to three betting panels on the reverse side of the slip.

HOW TO PLAY (For each $1 bet)

1. Pick five (5) numbers out of the field of fifty-six (56) by marking your selection like this [ ] in each of the five (5) boxes containing the numbers you want to play.
2. Pick one (1) number, (your MEGABALL Number), out of the field of forty-six (46) by marking your selection like this [ ] in the box containing the number you want to play.
3. How to play Megaplier. Multiply your non-jackpot prizes by 2, 3 or 4 with the Megaplier option. To play Megaplier mark the box [ ] Yes. Megaplier wagers must be equal to the Mega Millions wager on bet slip. Selecting Megaplier doubles the price of your ticket.
4. You may play the same numbers for more than one drawing. Mark the appropriate box for number of consecutive draws.
5. Give the completed betting slip to an ON-LINE Lottery Sales Agent.
6. The Sales Agent will give you a separate ticket as your receipt. **IMPORTANT: CHECK YOUR TICKET! BE SURE IT REPRESENTS THE NUMBERS AND DRAWING THAT YOU WANTED TO PLAY, AS IT IS YOUR ONLY PROOF OF PLAYING THESE NUMBERS.**
7. The Lottery suggests, for your own protection, that you sign the back of your ticket to identify ownership.

NOTE: Use only pencil or blue ball point pen to mark the betting slip.

HOW TO WIN A PRIZE

1. For each drawing, five (5) winning numbers out of a field of fifty-six (56) and one (1) winning MEGABALL Number out of a field of forty-six (46) will be randomly drawn.
2. For each $1 bet that you make, match your selections for that bet against those drawn by the Lottery and win as follows:

PRIZES				
Match	Prize			
5 + MB	Jackpot**			
			MEGAPLIER	
		4x	3x	2x
5 + 0*	$250,000	$1,000,000	$1,000,000	$1,000,000
4 + MB*	$10,000	$40,000	$30,000	$20,000
4 + 0*	$150	$600	$450	$300
3 + MB*	$150	$600	$450	$300
2 + MB	$10	$40	$30	$20
3 + 0	$7	$28	$21	$14
1 + MB	$3	$12	$9	$6
0 + MB	$2	$8	$6	$4

THE OVERALL ODDS OF WINNING A PRIZE IN MEGA MILLIONS ARE 1:39.89

3. Approximately 50% of the amount of money wagered on each drawing goes into the prize pool.
* Players purchasing the Megaplier option and matching the first five (5) numbers without the MEGABALL win a $1 million prize. Prizes matching less than the first five (5) numbers with Megaplier will be multiplied by the Megaplier number drawn. In some cases, these prizes may be paid on a pari-mutuel basis and will be lower than these published prize levels. Refer to the official game rules for a detailed explanation of the pari-mutuel prize payment.
Jackpot Payment Options - Jackpot winners have 60 days beginning the day after the winning numbers were drawn to select a payment option. An **Annuity Option means winners can choose equal annual payments over a twenty-six (26) year period. A **Cash Option** means winners can choose a one-time cash payment which will be (approximately) the cost of the annuity divided by the number of winning tickets. Note: If a winner fails to claim the jackpot and select a jackpot payment option within 60 days, the prize will be automatically paid as an annuity. All prizes must be claimed within one year of the drawing.

HOW TO CLAIM YOUR PRIZE

1. For prizes of $600.00 and less, take your winning ticket to any ON-LINE Lottery Sales Agent to receive payment. For larger prizes, present your winning ticket to an official Lottery office.
2. Proof of positive identification is required for all claims of more than $600.00.
3. All prizes must be claimed within one year of the drawing date.

RULES AND REGULATIONS

1. By placing a MEGA MILLIONS Bet, bettors agree to abide by Massachusetts State Lottery Commission Rules and Regulations. Applicable rules are available for inspection by bettors by contacting the Lottery directly at the address shown below.
2. BETS ARE NOT EFFECTIVE UNTIL ACCEPTED BY THE LOTTERY'S ON-LINE COMPUTER PROCESSING SYSTEM PRIOR TO THE DRAWING OF THE WINNING NUMBERS AND IN ACCORDANCE WITH LOTTERY RULES. BETTORS ACKNOWLEDGE THAT ON-LINE LOTTERY SALES AGENTS ARE ACTING ON BEHALF OF THE BETTORS IN ENTERING BETS INTO THE LOTTERY'S ON-LINE COMPUTER PROCESSING SYSTEM. The liability of the Lottery and Lottery's ON-LINE Sales Agents whether arising through breach of contract, negligence, or otherwise, is limited to a refund of the amount wagered if the bet(s) is (are) not accepted by the Lottery's ON-LINE Computer Processing System, or if betting slips are marked incorrectly, entered incorrectly into the system, or are otherwise invalid. The placing of MEGA MILLIONS bets is done at the player's own risk through the ON-LINE Lottery Sales Agent who is acting on behalf of the player in entering the bet(s) into the Lottery's ON-LINE Computer Processing System.
3. By placing a MEGA MILLIONS bet, bettors agree, as their sole and exclusive remedy, that claims arising out of the participation in the game shall be pursued only against the state where the ticket was purchased or was attempted to be purchased.

IF YOU HAVE A PROBLEM WITH GAMBLING, HELP IS AVAILABLE. CALL 1-800-426-1234.
YOU MUST BE 18 YEARS OR OLDER TO PLAY.
MASSACHUSETTS STATE LOTTERY, 60 Columbian Street, Braintree, MA 02184

STEVEN GROSSMAN STATE TREASURER   PAUL R. STERNBURG EXECUTIVE DIRECTOR