Worcester Polytechnic Institute
Department of Mathematical Sciences
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Fall 2021 - A Term

MA 2631

Probability Theory

Section AL01 / AD01

Assignment 7

due on Friday, September 24

based on Lectures of Chapter 4.1–4.3

1. (continuing problem 2, Assignment 6): Consider the random variable X with the probability mass distribution
$$P[X = 1] = 0.3, \qquad P[X = 4] = 0.25, \qquad P[X = 7] = 0.4, \qquad P[X = 10] = 0.05.$$
Calculate the variance of X and Y with $Y = 3X + 2$.

2. Suppose we pick a month at random from a non leap-year calendar and let X be the number of days in that month. Find the mean and the variance of X.

3. Let Y be a binomial distributed random variable with n trials of success probability p. Show that
$$\mathbb{V}ar[Y] = np(1-p).$$

4. Let Z be a geometric distributed random variable with success probability p. Calculate $\mathbb{V}ar[Z]$.

5. Assume that X is a random variable taking values on the non-negative integers that satisfies
$$\mathbb{P}[X \geq n + i \,|\, X \geq n] = \mathbb{P}[X \geq i].$$
Show that X is a geometric distributed random variable.

6. The number of errors on a book page follow a Poisson distribution. It has been determined that on 10% of the pages there is at least one error.

   a) Determine the parameter of the Poisson distribution.

   b) What is the expected number of errors on a page?

8 points per problems

Standard Carlton and Devore, Section 2.3: 32, 39 44; Section 2.4: 50, 54, 59, 61, 66, Section 2.5: 76, 81; Section 2.6: 98, 99, 102

Extra *Random Walk*: Consider the whole numbers and start at 0. You consecutively flip a coin, and if it shows 'heads' you move one to the right (+1) while for 'tails' you move one to the left (−1). This models a random movement, e.g., approximately the behavior of a (very) drunk person. Denote by X_n the random variable that marks your position after n steps.

- What are expectation and variance of X_n? What happens if you consider for them the limit $n \to \infty$?
- After k steps, you are at some point x. How likely is it that you will return to the starting point zero before time n? How does this probability behave for $n \to \infty$?