1. There are two urns. In the first urn there are 6 red and 2 black balls, in the second one 3 red and 7 black. We roll a (fair) die, and if the die shows a 1 or 2 we draw at random a ball from the first urn and in the case that the die shows a 3, 4, 5, or 6 we draw a ball at random from the second urn.

a) What is the probability that the drawn ball is red?

b) If the drawn ball is red, what is the probability that it was from the first urn?

2. Two students participate in a quiz show where they are asked a true-false question. Both know, independently, the correct answer with probability p. Which of the following strategies is better for the team?

i) Choose a priori one of the two who will give the answer.

ii) Give the common answer if the answers agree, and if not flip a coin to decide which answer is given.

3. Let Ω be a sample space and E, F, G be independent events such that $\mathbb{P}[G] > 0$. Show that the events E and F are independent when using instead of $\mathbb{P}$ the probability $\mathbb{Q}$ defined as $\mathbb{Q}[A] = \mathbb{P}[A \,|G]$ for events A.

4. Assume that you have three four-sided dice with number 1, 2, 3, and 4 on the four sides and let denote by X the sum of the numbers shown on their bottom side. Write down and sketch the probability mass function of X.

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$U_1 : \{6R, 2B\} \quad U_2 : \{3R, 7B\}$

a) $P(R) = P(R|U_1)P(U_1) + P(R|U_2)P(U_2)$

$= 6/8 \cdot 1/3 + 3/10 \cdot 2/3 = 1/4 + 1/5 = 9/20$

b) $P(U_1|R) = P(U_1, R) / P(R) = 1/4 \,/\, 9/20 = 5/9$

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i) $P(C) = P(C|S_1)P(S_1) + P(C|S_2)P(S_2) = p/2 + p/2 = p$

ii) $p^2 + p(1-p) = p^2 + p - p^2 = p$

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$$Q(EF) = P(EF|G) = P(EFG)/P(G)$$

$$= P(E)P(F)P(G)/P(G) = P(E)P(F)$$

$$= \frac{P(E)P(G)}{P(G)} \frac{P(F)P(G)}{P(G)} = \frac{P(EG)}{P(G)} \frac{P(FG)}{P(G)}$$

$$= P(E|G) \cdot P(F|G) = Q(E)Q(F)$$

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$p_2 = 1/16$ $p_3 = 2/16$ $p_4 = 3/16$ $p_5 = 4/16$

$p_6 = 3/16$ $p_7 = 2/16$ $p_8 = 1/16$