Worcester Polytechnic Institute
Department of Mathematical Sciences
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Fall 2021 - A Term

MA 2631

Probability Theory

Section AL01 / AD01

Assignment 1

due on Tuesday, August 31 based on Lectures of Chapter 1.1–1.3

1. Twenty workers are to be assigned to twenty different jobs. How many different assignments are possible?

2. A tourist wants to visit six out of thirteen American cities; seven of them are on the East Coast, three on the West Coast and three in the middle of the country. In how many ways can she do that if

   i) the order of the visits does not play a role;

   ii) the order of the cities is important;

   iii) the order is not important, but she wants to visit at least three cities on the East Coast and at least two on the West Coast.

3. How many words can you build from the letters

   ARRANGE

   if

   a) you have to use all the letters?

   b) you do not have to use all the letters (but every word has at least one letter)?

4. Two experiments are to be performed. The first one can result in r different outcomes. If the first experiment results in outcome j, then the second experiment can result in n_j possible outcomes, $j = 1, \ldots, r$. What is the total number of possibles outcomes of the two experiments?

5. Prove by induction that for all positive integers n it holds that

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n.$$

6. A soccer coach has 2 goalkeepers and 15 field players at his disposition: 5 defenders, 7 midfielders and 3 forwards.

   a) How many different soccer teams (consisting of 1 goalkeeper and 10 field players) can he build up from these players?

   b) If he wants additionally that there are at least 3 defenders, at leas 4 midfielders and at least 2 forwards in his team, how many teams he can form under this restriction?

8 points per problems

Additional practice problems (purely voluntary - no points, no credit, no grading):

Standard Carlton and Devore, Section 1.3: Exercises 31, 32, 36, 37, 43, 49.

Extra Remember that Pascal's triangle can be generated by filling in for each entry the number created by adding the two numbers in the line above (more formally, the k-th element in the n-th row is given as the sum of the $k-1$-st and the k-th element in the $n-1$-st row). Here all the left- and rightmost elements in each row (the 0-th and the n-th element) are 1. We try to generalize the triangle a bit by assuming that the 0-th element in each row is some (real) number a and the n-th element is some real number b (with the additional specification that the only element in the 0-th row is a). From here on, we can fill up the rest of the triangle (the interior) by the same rule as in the original Pascal's triangle (i.e., each element is the sum of the two elements above; the k-th element in the n-th row is given as the sum of the $k-1$st and the k-th element in the $n-1$st row). Can you derive an explicit (i.e., non-recursive) formula for the k-th element in the n-th row?