§8.1 Introduction to Linear Programming

8.1.1 Find the vertices of the feasible set $x_1 + 2x_2 + 3x_3 = 6$, $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$.

Maximize and minimize $cx = 2x_1 + 7x_2 + 5x_3$ over this set.

$x = (6, 0, 0)$ $cx = 12$

$x = (0, 3, 0)$ $cx = 21$ maximum

$x = (0, 0, 2)$ $cx = 10$ minimum

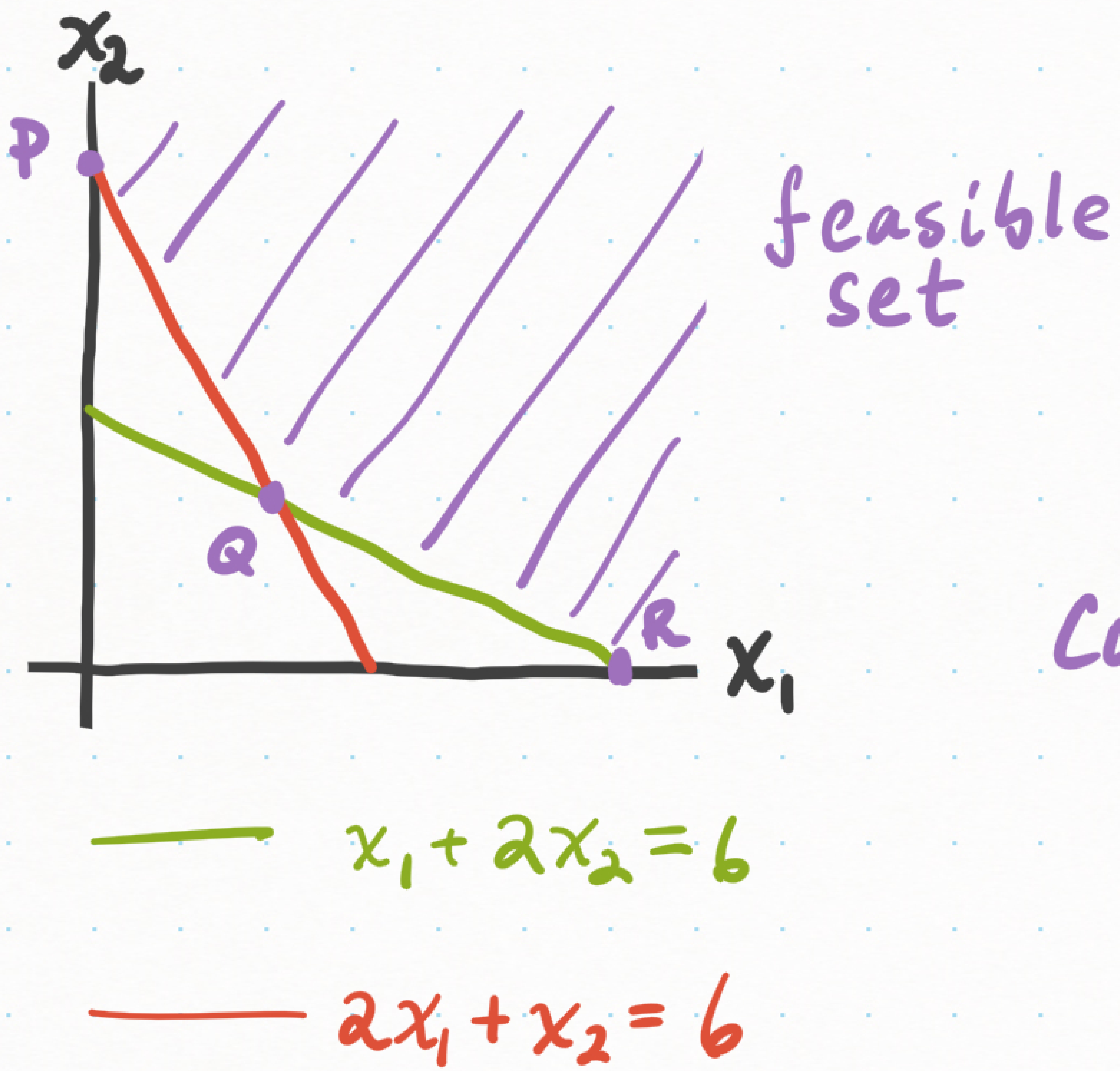
8.1.3 For what values of a does $x_1 + a x_2 = -1$, $x_1 \geq 0$, $x_2 \geq 0$ give an empty feasible set? For what values of a is the set unbounded?

The feasible set is empty if $a \geq 0$. If $a = 0$, then we must satisfy $x_1 = -1$. This is not possible since $x_1 \geq 0$. If $a > 0$, then $x_1 + a x_2 = -1$ implies $x_2 = -1/a - x_1/a < 0$ since $x_1 \geq 0$. But $x_2 < 0$ is not feasible.

The feasible set is nonempty if $a < 0$. For any $x_1 \geq 0$, $x_2 = -(1 + x_1)/a$ satisfies $x_1 + a x_2 = -1$ and since $-(1 + x_1) < 0$ and $a < 0$, $x_2 > 0$.

The set is unbounded for any $a < 0$. Since $x_2 = -(1 + x_1)/a$ has positive slope $-x_1/a$, x_2 can be made arbitrarily large by increasing $x_1 \geq 0$.

8.1.6 Sketch the feasible set for $x_1+2x_2 \geqslant 6$, $2x_1+x_2 \geqslant 6$, $x_1 \geqslant 0$, $x_2 \geqslant 0$. What points lie at the corners?



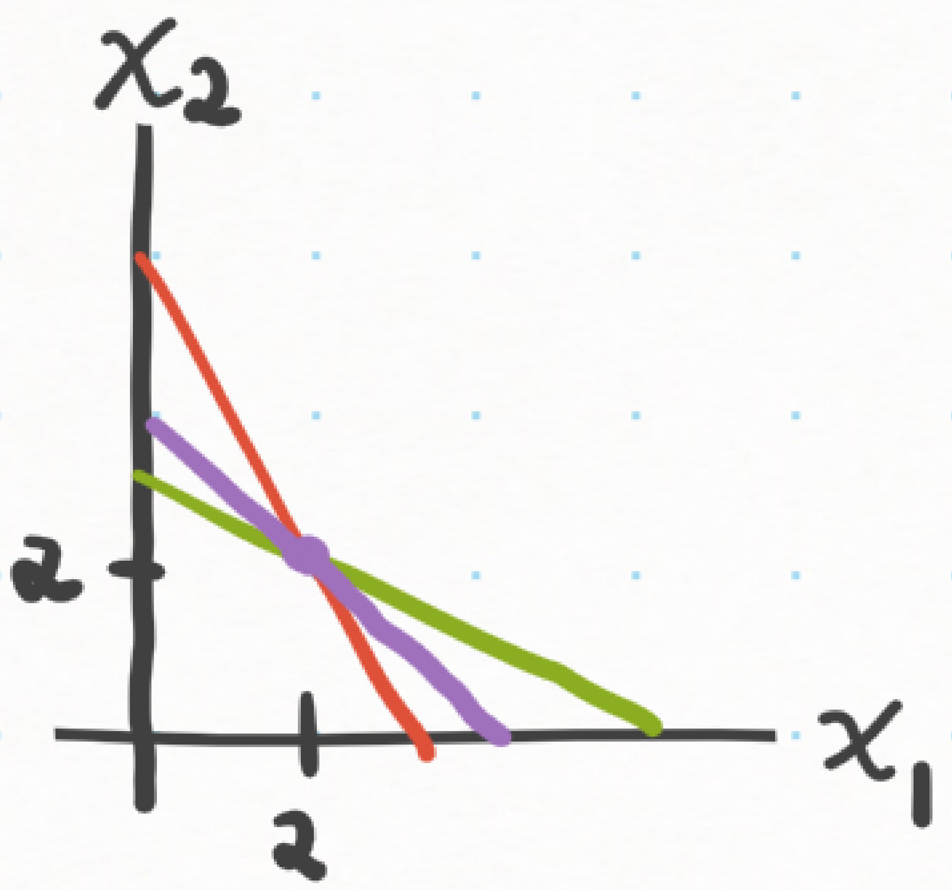
Corners:

$P=(6,0)$
$Q=(2,2)$
$R=(0,6)$

8.1.7 On the preceding feasible set from 8.1.6, minimize the cost $cx = x_1 + x_2$. Draw the line $x_1 + x_2$ that first touches the feasible set. How about the cost functions $3x_1 + x_2$ and $x_1 - x_2$?

cx is minimized at one of the corners.

At P, $cx = 6$. At Q, $cx = 4$. At R, $cx = 6$.
So cx is minimized at Q: $x = (2, 2)$.



The line that first touches the feasible set is $x_1 + x_2 = 4$

By checking cx at the corners we find $cx = 3x_1 + x_2$ has the minimum value 6 in the feasible set at $(0, 6)$.

Since x_2 has a negative coefficient, $cx = x_1 - x_2$ can be made arbitrarily negative. Minimum $-\infty$.