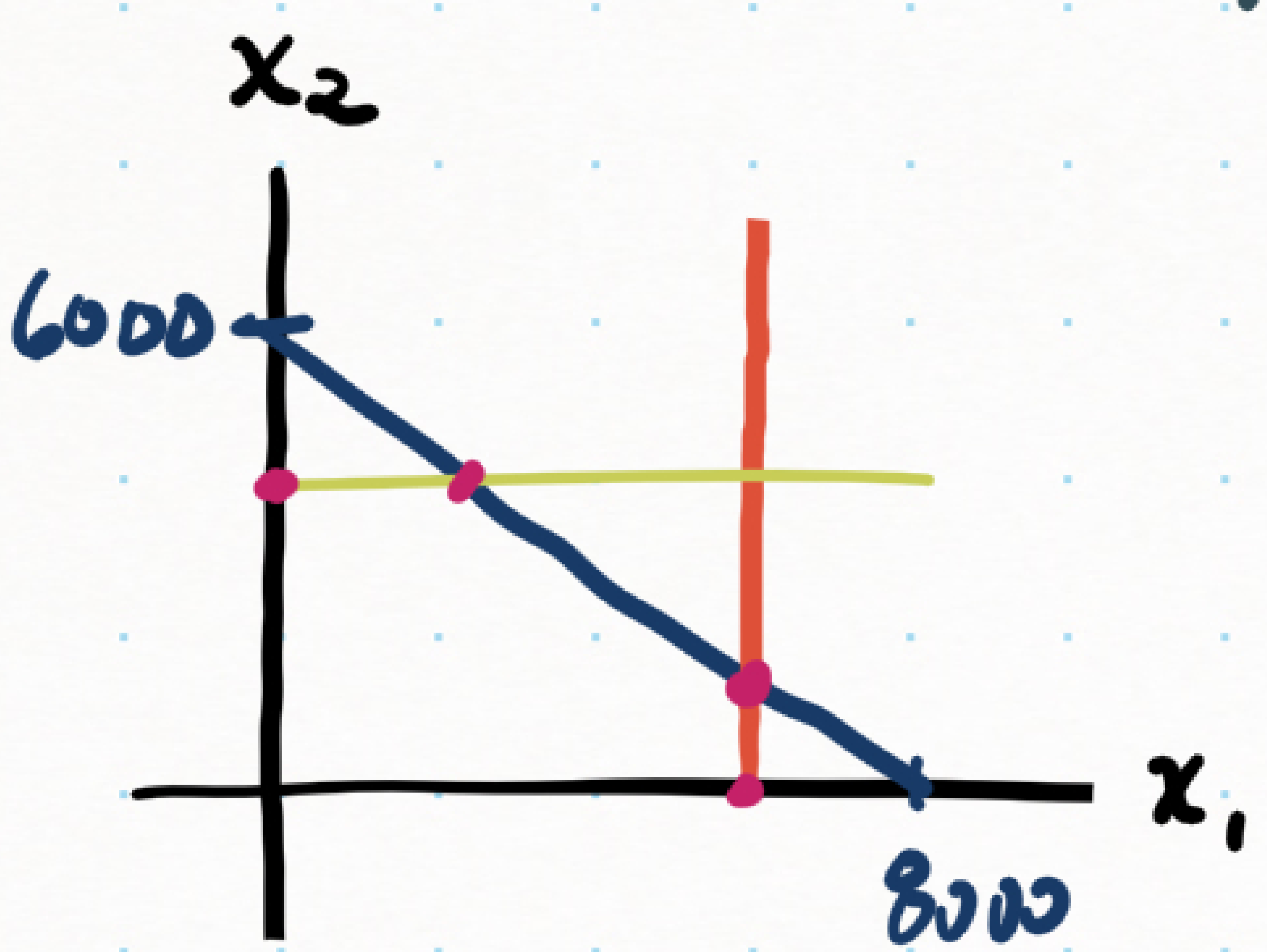
MA 3231 Assignment 1

1. Let x_1 be the number of tons of bands and x_1 the number of tons of coil. Let z be profit in dollars.

maximize $z = 25x_1 + 30x_2$

subject to
$$\tfrac{1}{200}x_1 + \tfrac{1}{140}x_2 \le 40$$
$$x_1 \le 6000$$
$$x_2 \le 4000$$
$$x_1, x_2 \geq 0$$

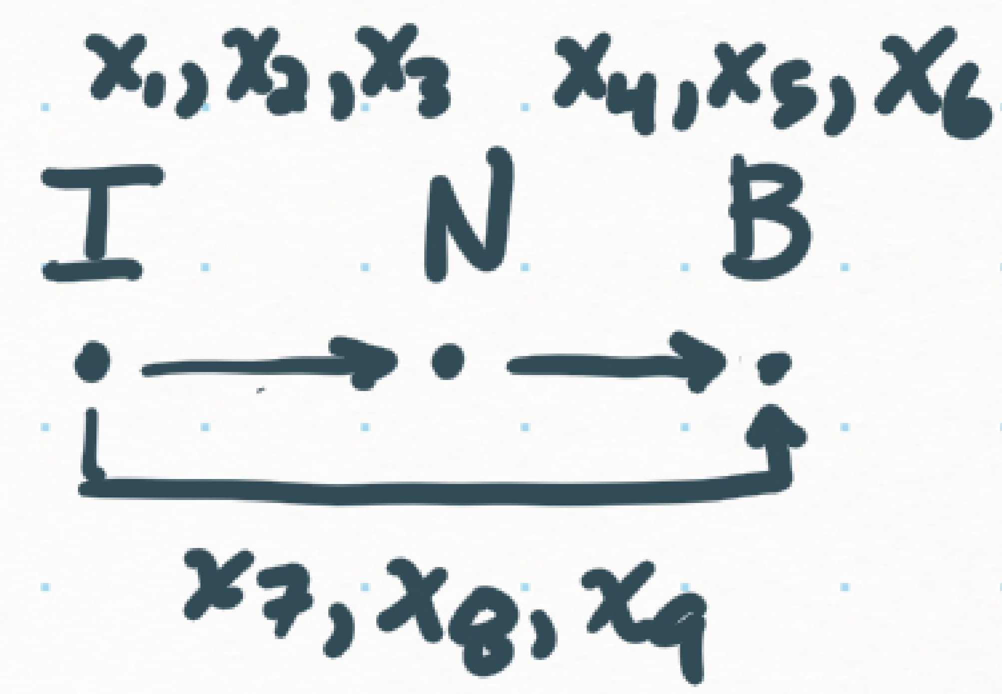
Notice:
x_1 yields $25 \cdot 200 = \$5000/\text{hr}$ profit
x_2 yields $30 \cdot 1500 = \$4500/\text{hr}$ profit
Exhaust x_1 production first and then switch to x_2.
$\therefore (x_1, x_2) = (6000, 1400)$
$z = \$192,000$



$z(6000, 0) = 150,000$
$z(0, 4000) = 120,000$
$z(6000, 1400) = 195,000$
$z(16000/7, 4000) \approx 125,714$

By checking z at the corners confirms z is maximized in the feasible region by $x_1 = 6000$, $x_2 = 1400$ with a maximum value of \$192,000.

2. Let x_1 : Ithaca – Newark , Y class
x_2 : 〃 , B class
x_3 : 〃 , C class
x_4 : Newark – Boston , Y class
x_5 : 〃 , B class
x_6 : 〃 , C class
x_7 : Ithaca – Boston , Y class
x_8 : 〃 , B class
x_9 : 〃 , C class
z : ticket revenue (\$)



$$\text{maximize} \quad z = 300x_1 + 200x_2 + 100x_3 + 160x_4 + 130x_5 + 80x_6 + 360x_7 + 280x_8 + 140x_9$$

subject to

$$x_1 \le 4, \quad x_2 \le 8, \quad x_3 \le 22,$$
$$x_4 \le 8, \quad x_5 \le 13, \quad x_6 \le 20,$$
$$x_7 \le 3, \quad x_8 \le 10, \quad x_9 \le 18,$$
$$x_1 + x_2 + x_3 + x_7 + x_8 + x_9 \le 30,$$
$$x_7 + x_8 + x_9 + x_4 + x_5 + x_6 \le 30,$$
$$x_i \ge 0 \text{ for } i = 1, 2, \ldots, 9$$

3. minimize $z = x_1 - 2x_2 - 3x_3$

subject to
$$\begin{aligned}
x_1 + 2x_2 + x_3 &\leq 14 \\
x_1 + 2x_2 + 4x_3 &\geq 12 \\
x_1 - x_2 + x_3 &= 2 \\
x_3 &\geq 3 \\
x_1, x_2, x_3 &\geq 0
\end{aligned}$$

To convert the equality constraint $x_1 - x_2 + x_3 = 2$ to an inequality constraint redefine $x_3 - x_4 := x_3$, $x_3 \geq 0$, $x_4 \geq 0$. Then,

$$x_1 - x_2 + x_3 - x_4 \leq 2$$
$$x_1 - x_2 + x_3 - x_4 \geq 2$$

Negate the second inequality.

Standard Form:

maximize $z = -x_1 + 2x_2 + 3x_3 - 3x_4$

subject to
$$\begin{aligned}
x_1 + 2x_2 + x_3 - x_4 &\leq 14 \\
-x_1 - 2x_2 - 4x_3 + 4x_4 &\leq -12 \\
x_1 - x_2 + x_3 - x_4 &\leq 2 \\
-x_1 + x_2 - x_3 + x_4 &\leq -2 \\
-x_3 + x_4 &\leq -3 \\
x_1, x_2, x_3, x_4 &\geq 0
\end{aligned}$$

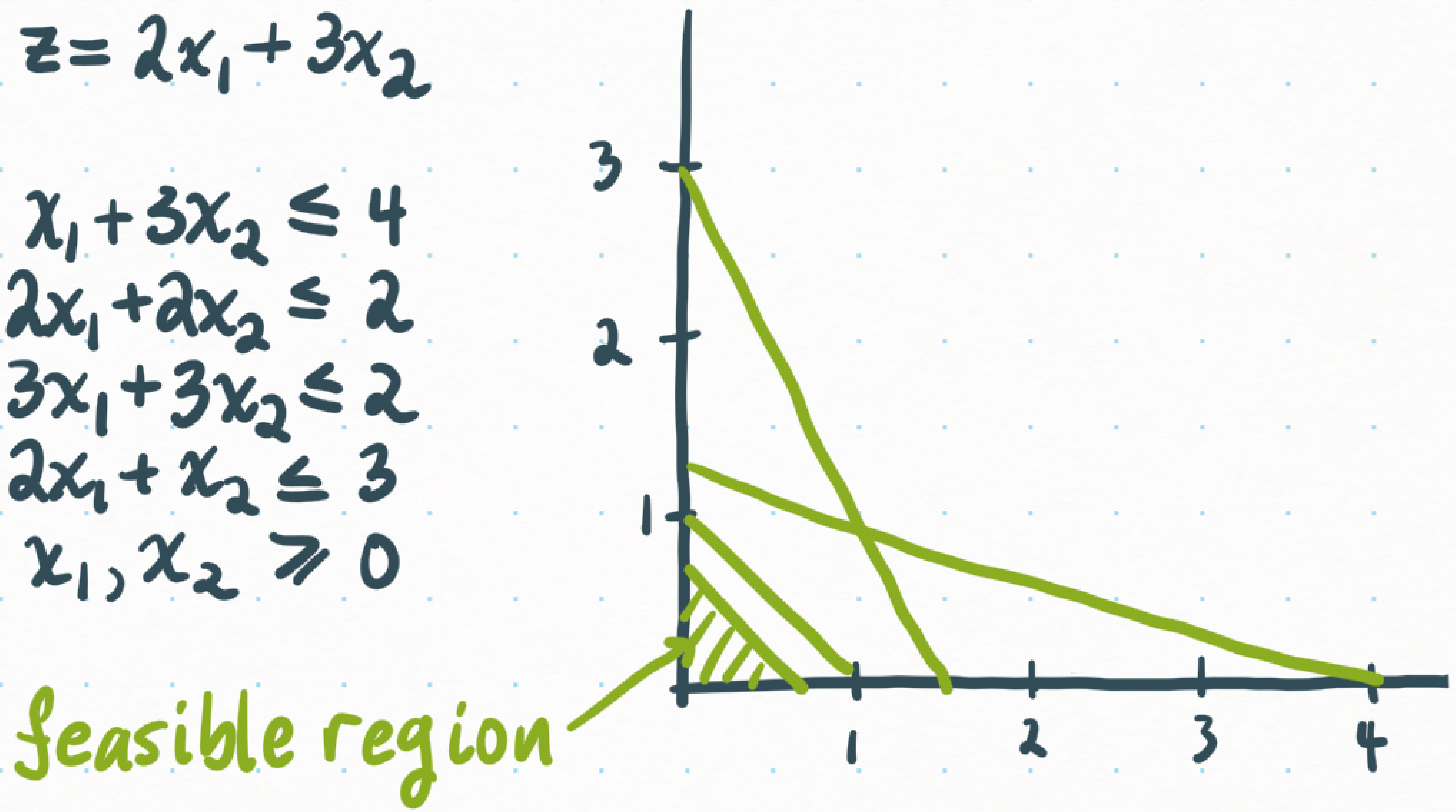
Standard Form (Alternative)

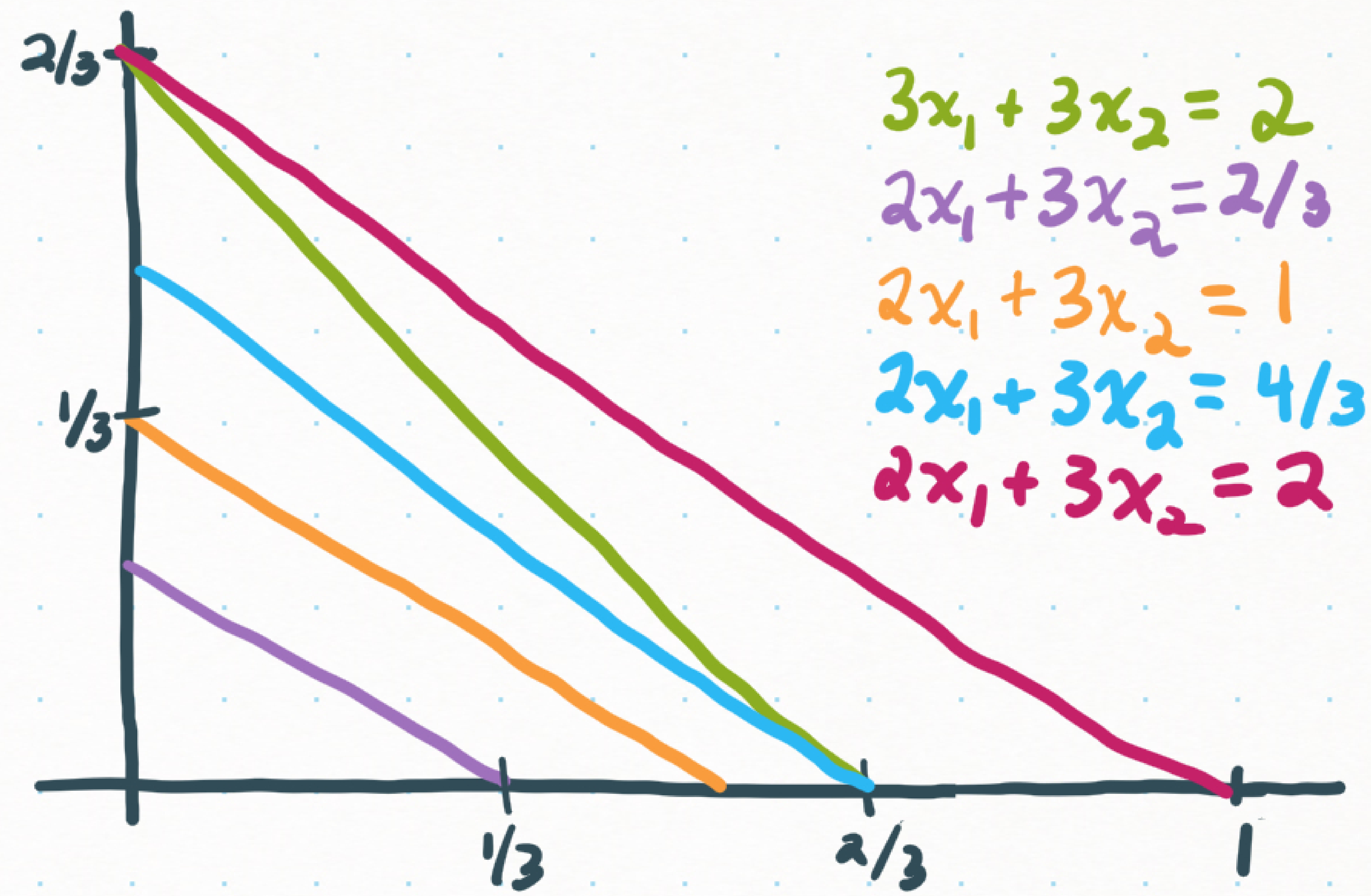
Maximize $z = -x_1 + 2x_2 + 3x_3$

subject to
$$x_1 + 2x_2 + x_3 \leq 14$$
$$-x_1 - 2x_2 - 4x_3 \leq -12$$
$$x_1 - x_2 + x_3 \leq 2$$
$$-x_1 + x_2 - x_3 \leq -2$$
$$-x_3 \leq -3$$
$$x_1, x_2, x_3 \geq 0$$

4. maximize $z = 2x_1 + 3x_2$

subject to
$$x_1 + 3x_2 \leq 4$$
$$2x_1 + 2x_2 \leq 2$$
$$3x_1 + 3x_2 \leq 2$$
$$2x_1 + x_2 \leq 3$$
$$x_1, x_2 \geqslant 0$$





z is maximized in the feasible region by $x_1 = 0$, $x_2 = 2/3$ and $z(0, 2/3) = 2$

5. Maximize $z = 5x_1 + 3x_2 + 2x_3$

Subject to
$$4x_1 + 5x_2 + 2x_3 + x_4 + x_5 = 20$$
$$3x_1 + 4x_2 - x_3 + x_4 + x_6 = 30$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Start with the initial feasible solution $x = (0, 0, 0, 0, 20, 30)$ and try x_1 as the entering variable.

$$x_5 = 20 - 4x_1$$
$$x_6 = 30 - 3x_1$$
$$z = 5x_1$$

$4 \cdot 4 + 2 \cdot 2$

$5 \cdot 4 =$

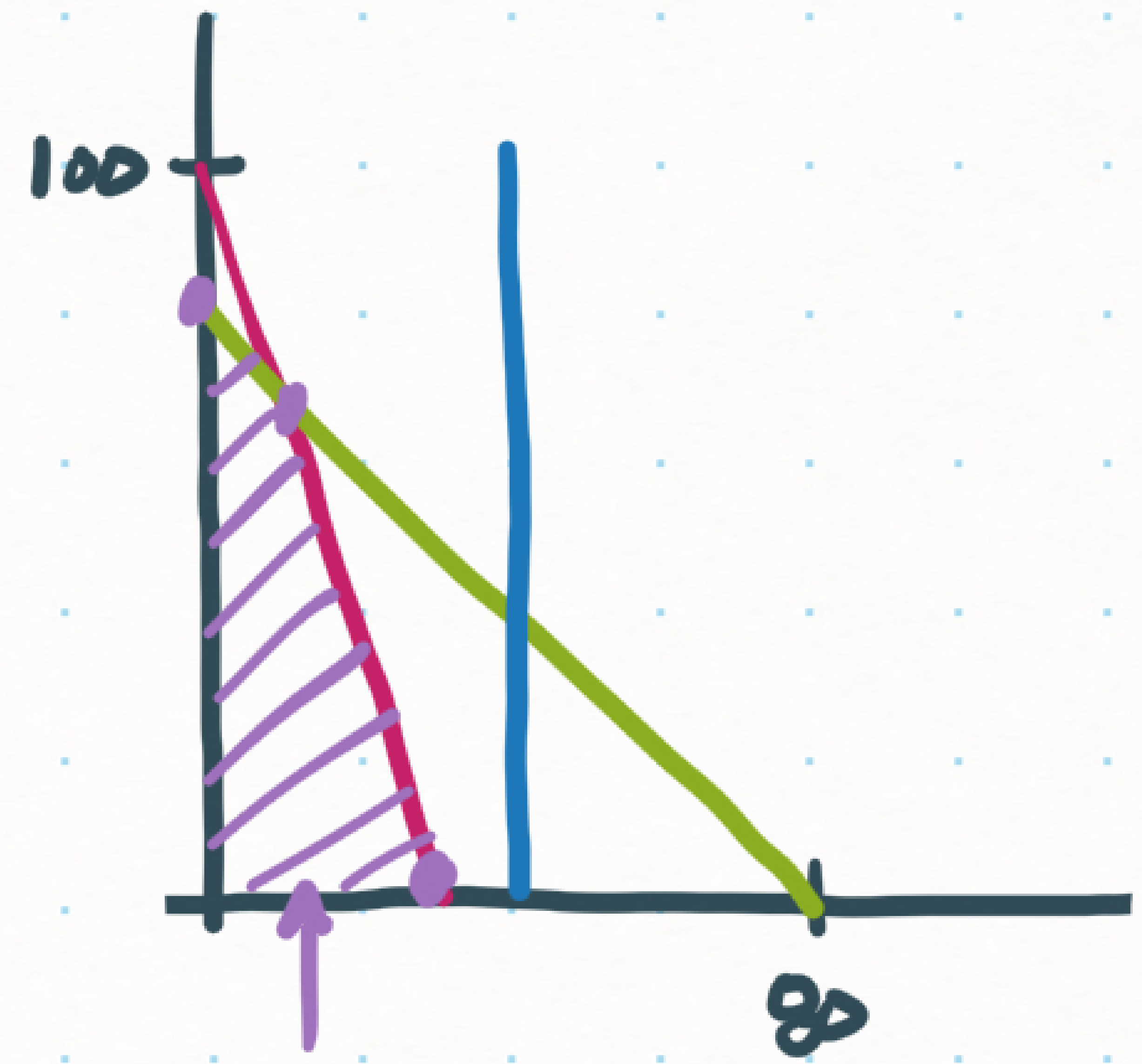
Set $x_1 = 20/4 = 5$ as x_5 leaves, $x = (5, 0, 0, 0, 0, 15)$, $z = 25$.
Using $x_1 = (20 - 5x_2 - 2x_3 - x_4 - x_5)/4$,

$$z = 25 - 13/4\, x_2 - 1/2\, x_3 - 5/4\, x_4 - 5/4\, x_5$$

Since all coefficients are negative, $x_1 = 5$, $x_2 = x_3 = x_4 = 0$ is the optimal solution and $z = 25$.

6. Maximize $z = 7x_1 + 8x_2$

Subject to
$$4x_1 + x_2 \le 100$$
$$x_1 + x_2 \le 80$$
$$x_1 \le 40$$
$$x_1, x_2 \geq 0$$



Let $x_3, x_4, x_5 \geq$ such that

$$4x_1 + x_2 + x_3 = 100$$
$$x_1 + x_2 + x_4 = 80$$
$$x_1 + x_5 = 40$$

Start at $x = (0, 0, 100, 80, 40)$, $z = 0$.
Enter x_2:

$$x_3 = 100 - x_2$$
$$x_4 = 80 - x_2$$
$$x_5 = 40$$

Then x_4 leaves and $x = (0, 80, 20, 0, 40)$, $z = 640$.

Using $x_2 = 80 - x_1 - x_4 = 80 - x_1$,

$z = 7x_1 + 640 - 8x_1 = 640 - x_1$.

Increasing x_1 decreases z.
Conclude $x_1 = 0$, $x_2 = 80$ is optimal with $z = 640$.