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Summer 2021 - E2 Term

MA 3231

Linear Programming

Section E162

Assignment 4

Content: up to Section 4

1. Solve the following linear program using

   a) the perturbation (lexicographic) method

   b) Bland's rule

   $\max z = -\frac{3}{4}x_1 + 150x_2 - \frac{1}{50}x_3 + 6x_4$
   subject to
   $\frac{1}{4}x_1 + 150x_2 - \frac{1}{25}x_3 + 9x_4 \leq 0$
   $\frac{1}{2}x_1 - 90x_2 - \frac{1}{50}x_3 + 3x_4 \leq 0$
   $x_3 \leq 1$
   $x_1, x_2, x_3, x_4 \geq 0.$

2. Consider the following linear programming problem:

   $\max z = x_1 + 2x_2$
   subject to
   $-2x_1 - x_2 + x_3 \leq 1$
   $x_1 + x_2 \leq 2$
   $x_1 + x_3 \leq 3$
   $x_1,\ x_2,\ x_3 \geq 0$

   a) Solve the linear program.

   b) Find the dual program.

   c) Solve the dual program.

   d) Compare the solutions of primal and dual program.

3. Consider the following linear programming problem:

   $\max z = x_1 + 2x_2 + x_3 + x_4$
   subject to
   $2x_1 + x_2 + 5x_3 + x_4 \leq 8$
   $2x_1 + 2x_2 + 4x_4 \leq 12$
   $3x_1 + x_2 + 2x_3 \leq 18$
   $x_1,\ x_2,\ x_3,\ x_4 \geq 0$

   You know that the final dictionary for this program is given by

$$\begin{array}{rcrcrcrcr}
z & = & 12.4 & - & 1.2x_1 & - & 0.2x_5 & - & 0.9x_6 & - & 2.8x_4 \\
\hline
x_2 & = & 6 & - & x_1 & & & - & 0.5x_6 & - & 2x_4 \\
x_3 & = & 0.4 & - & 0.2x_1 & - & 0.2x_5 & + & 0.1x_6 & + & 0.2x_4 \\
x_7 & = & 11.2 & - & 1.6x_1 & + & 0.4x_5 & + & 0.3x_6 & + & 1.6x_4
\end{array}$$

   (where x_5, x_6, x_7 are slack variables)

   a) What will be the optimal solution to the problem if the objective function is changed to
   $$3x_1 + 2x_2 + x_3 + x_4$$

   b) What will be the optimal solution to the problem if the objective function is changed to
   $$x_1 + 2x_2 + 0.5x_3 + x_4$$

   c) For each of the three objective functions above, find the range of values for which the final dictionary will remain optimal.

4. Use the parametric self-dual simplex method to solve the following problem

   $\max z = 3x_1 - x_2$
   subject to
   $x_1 - x_2 \leq 1$
   $-x_1 + x_2 \leq -4$
   $x_1,\ x_2 \geq 0$