

Department of Mathematical Sciences

**MA 3231**

# Linear Programming

## Assignment 2

1. Solve the following linear program using the simplex algorithm:

$$\begin{aligned} \max z &= 10x_1 + 6x_2 + 4x_3 \\ \text{subject to} \\ 4x_1 + 5x_2 + 2x_3 + x_4 &\leq 20 \\ 3x_1 + 4x_2 - x_3 + x_4 &\leq 30 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

2. Solve the following linear program using the simplex algorithm: (careful: is this linear program in standard form?)

$$\begin{aligned} \min z &= -7x_1 - 8x_2 \\ \text{subject to} \\ 4x_1 + x_2 &\leq 100 \\ -2x_1 - 2x_2 &\geq -160 \\ x_1 &\leq 40 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Draw the region of feasible solution to this problem and indicate the solution you get at each step of the simplex algorithm.

3. Solve the following linear program using the simplex algorithm and a suitable auxiliary program:

$$\begin{aligned} \max z &= 2x_1 + 6x_2 \\ \text{subject to} \\ -x_1 - x_2 &\leq -3 \\ -3x_1 + 3x_2 &\leq 3 \\ x_1 + 2x_2 &\leq 2 \\ x_1, x_2 &\geq 0 \end{aligned}$$

optional: Use the graphical method to find the region of feasible solutions.

4. Solve the following linear program using the simplex algorithm and a suitable auxiliary program: (careful: is this linear program in standard form?)

$$\begin{aligned} \min z &= -2x_1 - 3x_2 - 4x_3 \\ \text{subject to} \\ 2x_2 + 3x_3 &\geq 5 \\ x_1 + x_2 + 2x_3 &\leq 4 \\ x_1 + 2x_2 + 3x_3 &\leq 7 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

5. Explain why the following dictionary cannot be the optimal dictionary for any linear programming problem in which  $w_1$  and  $w_2$  are the initial slack variables:

$$\begin{array}{rclcl} z & = & 4 & -w_1 & -2x_2 \\ \hline x_1 & = & 3 & & -2x_2 \\ w_2 & = & 1 & +w_1 & -2x_2 \end{array}$$

*Hint:* If it could, what was the original problem from which it came?