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Summer 2021 - E2 Term

MA 3231

Linear Programming

Section E162

Assignment 1

Content: up to Section 2.5

Due: July 16, at noon

1. A steel company must decide how to allocate next week's time on a rolling mill, which is a machine that takes unfinished slabs of steel as input and can produce either of two semi-finished products: bands and coils. The mill's two products come off the rolling line at different rates: bands at 200 tons per hour, and coils at 140 tons per hour. They also produce different profits: Bands at \$25 per ton, and coils at \$30 per ton. Based on currently booked orders, the following upper bounds are placed on the amount of each product to produce: Bands at 6,000 tons, and coils at 4,000 tons. Given that there are 40 hours of production time available this week, the problem is to decide how many tons of bands and how many tons of coils should be produced to yield the greatest profit. Formulate this as a linear programming problem in standard form. Can you solve this problem by inspection?
2. A small airline flies between three cities: Ithaca, Newark, and Boston. They offer several flights but, for this problem, we focus on the Friday afternoon flight that departs from Ithaca, stops in Newark, and continues to Boston. There are three types of passengers:
 - (a) Those traveling from Ithaca to Newark.
 - (b) Those traveling from Newark to Boston.
 - (c) Those traveling from Ithaca to Boston.

The aircraft is a small commuter plane that seats 30 passengers. The airline offers three fare classes:

- (a) *Y* class: full coach.
- (b) *B* class: nonrefundable.
- (c) *M* class: nonrefundable, 3-week advanced purchase.

Ticket prices have been set and advertised as follows:

	Ithaca-Newark	Newark-Boston	Ithaca-Boston
Y	300	160	360
B	220	130	280
M	100	80	140

Based on past experience, demand forecasters at the airline have determined the following upper bounds on the number of potential customers in each of the 9 possible origin-destination/fare-class combinations:

	Ithaca-Newark	Newark-Boston	Ithaca-Boston
Y	4	8	3
B	8	13	10
M	22	20	18

The goal is to decide how many tickets from each of the 9 origin-destination/fare-class combinations to sell. The constraints are that the plane cannot be overbooked on either of the two legs of the flight and that the number of tickets made available cannot exceed the forecasted maximum demand. The objective is to maximize revenue. Formulate this problem as a linear programming problem in standard form.

3. Consider the following linear programming problem:

$$\min z = x_1 - 2x_2 - 3x_3$$

subject to

$$x_1 + 2x_2 + x_3 \leq 14$$

$$x_1 + 2x_2 + 4x_3 \geq 12$$

$$x_1 - x_2 + x_3 = 2$$

$$x_3 \geq 3$$

$$x_1, x_2, x_3 \geq 0$$

Reformulate this program so it is in standard form.

4. Consider the following linear programming problem:

$$\begin{aligned} \max z &= 2x_1 + 3x_2 \\ \text{subject to} \\ x_1 + 3x_2 &\leq 4 \\ 2x_1 + 2x_2 &\leq 2 \\ 3x_1 + 3x_2 &\leq 2 \\ 2x_1 + x_2 &\leq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Draw the set of feasible solution given the constraints. By drawing the line of the objective function for a particular value and shifting it parallel, determine approximately the optimal solution to this problem.

5. Solve the following linear program using the simplex algorithm:

$$\begin{aligned} \max z &= 5x_1 + 3x_2 + 2x_3 \\ \text{subject to} \\ 4x_1 + 5x_2 + 2x_3 + x_4 &\leq 20 \\ 3x_1 + 4x_2 - x_3 + x_4 &\leq 30 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

6. Solve the following linear program using the simplex algorithm:

$$\begin{aligned} \max z &= 7x_1 + 8x_2 \\ \text{subject to} \\ 4x_1 + x_2 &\leq 100 \\ x_1 + x_2 &\leq 80 \\ x_1 &\leq 40 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Draw the region of feasible solution to this problem and indicate the solution you get at each step of the simplex algorithm.

8 points per problems

Problems to be discussed in the Office Hours

1. A company is producing three goods: A, B, C. A costs to produce 20\$ a piece, and earns a profit of \$ 3 a piece if sold. B costs to produce 50\$ a piece, and earns a profit of \$ 5 a piece if sold. C costs to produce 100\$ a piece, and earns a profit of \$ 1 a piece if sold.

Moreover, for production constraints, there have to be produced at least as double the amount of goods from good A than from good C. On the other hand, good B cannot be produced more than the triple number of goods of C. Finally, not more than 100 goods can be produced overall.

Formulate this problem as a linear programming problem and bring it into standard form.

2. Consider the following linear programming problem:

$$\max z = 2(x_1 - 4x_2 + 2x_3)$$

subject to

$$2x_1 + 2x_2 - x_3 \geq 12$$

$$-x_1 + 2x_2 - 4x_3 \leq 1$$

$$x_1 + x_3 = 0$$

$$x_1 \leq 2$$

$$x_1, x_2, x_3 \geq 0$$

Reformulate this program so it is in standard form. Can you say if it is feasible? Is it bounded or unbounded?

3. Consider the following linear programming problem:

$$\max z = 3x_1 + x_2$$

subject to

$$x_1 + 2x_2 \leq 2$$

$$2x_1 + x_2 \leq 3$$

$$x_1 + x_2 \leq 1$$

$$x_1, x_2 \geq 0$$

Draw the set of feasible solution given the constraints. By drawing the line of the objective function for a particular value and shifting it parallel, determine approximately the optimal solution to this problem.