Worcester Polytechnic Institute
Department of Mathematical Sciences
Professor: Stephan Sturm
Teaching Assistant: Dane Johnson

Summer 2021 - E2 Term

MA 3231

Linear Programming

Section E162

Assignment 2

Content: up to Section 2.12

1. Find all the values of the parameter α such that the following linear program has a finite optimal solution:

   $\max z = \alpha x_1 + 2x_2 - x_3$
   subject to
   $2x_1 - x_2 + 3x_3 \leq 4$
   $-x_1 + x_2 - 2x_3 \leq 8$
   $3x_1 - 3x_3 \leq 2$
   $x_1,\, x_2,\, x_3 \geq 0$

2. Solve the following linear program using the simplex algorithm and a suitable auxiliary program:

   $\max z = x_1 + 3x_2$
   subject to
   $-x_1 - x_2 \leq -3$
   $-x_1 + x_2 \leq -1$
   $x_1 + 2x_2 \leq 2$
   $x_1,\, x_2 \geq 0$

   Draw the region of feasible solution to this problem and indicate the solution you get at each step of the simplex algorithm (only for the original problem, i.e., Phase II).

3. Solve the following linear program using the simplex algorithm and a suitable auxiliary program:

   $\max z = 2x_1 + 3x_2 + 4x_3$
   subject to
   $-2x_2 - 3x_3 \leq -5$
   $x_1 + x_2 + 2x_3 \leq 4$
   $x_1 + 2x_2 + 3x_3 \leq 7$
   $x_1,\ x_2,\ x_3 \geq 0$

4. Show that the following dictionary cannot be the optimal dictionary for any linear programming problem in which w_1 and w_2 are the initial slack variables:

$$\begin{array}{rcrrr} z & = & 4 & -w_1 & -2x_2 \\ \hline x_1 & = & 3 & & -2x_2 \\ w_2 & = & 1 & +w_1 & -2x_2 \end{array}$$

   Hint: If it could, what was the original problem from whence it came?

5. Consider the following linear programming problem:

   $\max z = 2x_1 - 3x_2 + 2x_3 + 12x_4$
   subject to
   $4x_1 + 5x_2 + 2x_3 \leq 10$
   $2x_1 - x_3 + x_4 \leq 30$
   $4x_2 + 2x_3 + x_4 \leq 20$
   $x_1,\ x_2,\ x_3,\ x_4 \geq 0$

   Solve it numerically using the built-in simplex algorithm of a spreadsheet program (such as MS Excel). Please just submit the .xlsx file as submission comment to your homework on Canvas.

6. Reconsider the degenerate dictionary of Lecture 2.8 that led to the cycling issue:

$$\begin{array}{rcrrrrrrrr} z & = & & & x_1 & - & 2x_2 & & & - & 2x_4 \\ \hline w_1 & = & & - & 0.5x_1 & + & 3.5x_2 & + & 2x_3 & - & 4x_4 \\ w_2 & = & & - & 0.5x_1 & + & x_2 & + & 0.5x_3 & - & 0.5x_4 \\ w_3 & = & 1 & - & x_1 & & & & & & \end{array}$$

Write down the corresponding linear programming problem and find the optimal solution by using

a) the lexicographic method (comment on the choice of the entering variable).

b) Bland's rule

8 points per problems

Problems to be discussed in the Office Hours

1. Solve the following linear program using the lexicographic method to avoid degeneracy:

   $\max z = 2x_1 + 3x_2 + 4x_3$
   subject to
   $4x_1 - x_2 \leq 0$
   $-x_1 + x_3 \leq 0$
   $2x_2 - 3x_3 \leq 0$
   $x_1,\ x_2,\ x_3 \geq 0$

   How about Bland's rule?

2. Solve the following linear programming problem by initializing it with an auxiliary program.

   $\max z = 3x_1 + 2x_2$
   subject to
   $x_1 - x_2 \leq -1$
   $x_1 + x_2 \leq 2$
   $x_1,\ x_2 \geq 0$