Conference 4

Numerical Differentiation: Optimal Step Size

Numerical Integration: Trapezoid Rule

Numerical Differentiation: Optimal Step Size

In class, the optimal step size h for the centered difference approximation of $f'(x_0)$ with $\mathcal{O}(h^2)$ error was derived. Write a matlab program and create a graph that shows this theoretically optimal h matches with the computations for the function $f(x) = x^2 \ln(x)$ at $x_0 = 2$.

Numerical Differentiation: Optimal Step Size

a. Derive the five-point midpoint approximation of $f'(x_0)$.

b. Find the optimal h that minimizes both the computational and truncation (Taylor) error in the five-point midpoint approximation of $f'(x_0)$.

c. For the function $f(x) = x^2 \ln(x)$ evaluated at the point $x_0 = 2$, show that this theoretically optimal h actually matches with the computations.

Numerical Integration: Trapezoid Rule

a. Evaluate using the trapezoid rule with $x_0 = -1/4$, $x_1 = 1/4$.

$$\int_{-1/4}^{1/4} \cos^2(x)\, dx\ .$$

b. What is the actual error of the approximation in part a?

c. What is the theoretical upper bound on the error of the approximation in part a?

Numerical Integration: Trapezoid Rule

Assuming that the interval $[a, b]$ is divided evenly by the points $a = x_0 < x_1 < ... < x_N = b$ with step size h, develop a composite trapezoid rule for approximating $\int_a^b f(x)\, dx$.