Conference 5

Milne's Method

Time Stepping Techniques for an IVP

1. Milne's Method

Use the quadratic interpolant to $y'(t) = f(t, y(t))$ at t_n, t_{n-1}, t_{n-2} to obtain the formula (for $n \geq 3$)

$$y_{n+1} = y_{n-3} + \frac{4h}{3}\left(2y'_n - y'_{n-1} + 2y'_{n-2}\right) + \mathcal{O}(h^5)\ .$$

Time Stepping Techniques for an IVP

The ODE $y' = 2y/t + t^2e^t$ on $1 \le t \le 2$ with $y(1) = 0$ has a solution $y(t) = t^2(e^t - e)$. Approximate the solution to this ODE with $h = 0.2$ using:

(a) Taylor's method of order two

(b) Runge-Kutta order 2

(c) Implicit Euler method

Create a plot (or plots) that include each of the approximations at each mesh point and the exact solution at each mesh point. Comment on: (1) error for each approximation, (2) computation time for each method, (3) sensitivity to time steps. In the third part, continue to increase and decrease time step h to understand conditional stability of explicit methods and unconditional stability of the Implicit (forward) Euler method.

	Total Error (sum(exact-approx))			Elapsed Time (seconds)		
step size h	Taylor Method Order 2	Runge Kutta Order 2	Implicit Euler Method	Taylor Method Order 2	Runge Kutta Order 2	Implicit Euler Method
0.05	0.415094	0.325391	13.73329	0.000074	0.000059	0.000053
0.1	0.841109	0.655126	15.77001	0.000056	0.000044	0.000039
0.2	1.720148	1.323737	20.84934	0.000043	0.000037	0.000032
0.5	4.492491	3.365712	52.28256	0.000044	0.000031	0.000027

Table 1: Error and Computation Time using different step size and methods.

	Approx. Value at t = 1.4				Error at t = 1.4		
step size h	Taylor Method Order 2	Runge Kutta Order 2	Implicit Euler Method	real value	Taylor Method Order 2	Runge Kutta Order 2	Implicit Euler Method
0.05	2.6099	2.6119	2.9404	2.62036	0.01046	0.00846	0.32004
0.1	2.581	2.5888	3.3084	2.62036	0.03936	0.03156	0.68804
0.2	2.4805	2.5096	4.2335	2.62036	0.13986	0.11076	1.61314

Table 2: Approximated Values at t =1.4 and corresponding errors.