

MA 3457 / CS 4033
HW #6
Due: Friday 12/4 by 11 pm

This assignment is due on **Friday 12/4 by 11 pm on Canvas**. Submit all of your documented Matlab code. Additional written work and discussion of problems should be a single pdf that is well-organized and either typed or neatly written. Matlab output should be discussed in the write-up.

1. (6 points) *Unique Solutions to IVPs*

Show that each of the following initial-value problems has a unique solution:

(a) $y' = \frac{4t^3 y}{1+t^4}, 0 \leq t \leq 1, y(0) = 1$

(b) $y' = \cos^2(y), 0 \leq t \leq 1, y(0) = \pi/4$

2. (14 points total) *Epidemic Model*

A community with L members contains $I(t)$ infected individuals at time t and $S(t)$ is the uninfected/susceptible individuals. Assuming L is a constant, we have $S(t) = L - I(t)$. For a mild illness, such as the common cold, everyone continues to be active and the epidemic spreads from those who are infected to those who are susceptible. Since there are $I(t)S(t)$ possible contacts between these two groups, the rate of change of $I(t)$ is proportional to $I(t)S(t)$. The model can be stated as the following IVP:

$$I' = kI(L - I), \quad I(0) = I_o, \quad 0 \leq t \leq 60$$

where t is in days.

(a) (8 points) Write your own Euler's method code with $h = 0.2$ to approximate the solution to above IVP on $[0,60]$ using the parameters $L = 25,000$, $k = 0.00003$, with initial condition $I_o = 250$.

(b) (4 points) Create a plot that includes the approximations at each mesh point using $h = 0.2, 0.1, 0.05$, and 0.025 . Looking at $I(30)$, what order of convergence are you seeing?

Note: There should be a different value for the infected population at $t = 30$ for the different values of h . When we did the integration error check for Trapezoid and Simpsons rule, we looked at $abs(NumericalSolution - Exact)/h^p$ where h^p was the expected theoretical error. The error is approximately ch^p , so when we observed a constant value c in the previous homework for different h values, we could say that the error was as we expected. Do the same analysis for Euler method using the solution to $h = 0.0125$ as the 'exact' solution in the calculation. Often, if we do not have an analytical solution, we can use a finer grid as a good estimate (as long as the method converges and approaches the true solution as the grid is refined).

(c) (2 points) Estimate the average number of individuals infected by finding the average solution value. Compare this to fitting a curve to the Euler solution and using mean value theorem for integrals.