MA 3475 Exam II, 3/18/2021　　　　　　　　NAME _____

This is an open book open notes exam but you are not allowed to access the Internet or ask anyone for help. The answers must be 100% your work. Show your work as unsupported answers may receive no credit. One point will be deducted if you email your exam to me. One point for every five minutes late.

1. (5 points) Find the extremal of the functional

$$J[y, z] = \int_0^{\pi/2} \left((y')^2 + (z')^2 + 2yz \right) \, dx,$$

subject to the boundary conditions $y(0) = 1, z(0) = 1, y\left(\frac{\pi}{2}\right) = 0, z\left(\frac{\pi}{2}\right) = 0.$

2. (5 points) Find the extremal of the functional

$$J[y] = \int_a^b \left((y')^2 + (y'')^2 \right) dx$$

How many boundary conditions do you need to specify at the two endpoints to determine the extremal uniquely?

3. (5 points) Find the extremal of the functional

$$J[y] = \int_0^1 (y')^2 \, dx$$

subject to the conditions $y(0) = 0, y(1) = 0$ and the constraint $\int_0^1 y(x) \, dx = A$, where A is a given constant. You need to find the Lagrange multiplier as a function of A and cannot leave your answer in terms of the Lagrange multiplier.