

This is an open book open notes exam but you are not allowed to access the Internet or ask anyone for help. The answers must be 100% your work. Show your work as unsupported answers may receive no credit.

1. (5 points) Find the extremal of the functional

$$J[y] = \int_a^b 12xy + (y')^2 dx .$$

2. (5 points) Consider the functional

$$J[y] = \int_1^2 x^2 (y')^2 + 2y^2 \, dx.$$

- (2 points) Find the Euler-Lagrange equation of the above functional.
- (1 point) Find r such that $y(x) = x^r$ solves your Euler-Lagrange equation.
- (2 points) Find the extremal of $J[y]$ satisfying the boundary conditions $y(1) = 0$, $y(2) = -\frac{7}{4}$.

3. (5 points) Determine the extremal of the functional

$$J[y] = \int_0^1 (y')^2 - 2\alpha yy' - 2\beta y' dx$$

where α, β are nonzero constants for each of the following boundary conditions:

(a) $y(0) = 0, y(1) = 1$.

(b) $y(0) = 0, y(1)$ is free.

(c) $y(0)$ and $y(1)$ are both free.