MA 3475, Homework 2 due midnight 02/14/2021

Show your work as supported answers may receive no credit. Find the extremals of the following functionals.

1. (5 points) $J[y] = \displaystyle\int_a^b y(x)^2 - y'(x)^2\,dx.$
2. (5 points) $J[y] = \displaystyle\int_a^b xy'(x) + y'(x)^2\,dx.$
3. (5 points) $J[y] = \displaystyle\int_1^2 \frac{\sqrt{1+y'(x)^2}}{x}\,dx,\;\; y(1)=0, y(2)=1.$

Extra problems for practice. No need to turn in. May involve hyperbolic functions.

1. $J[y] = \displaystyle\int_a^b y(x)^2 + y(x)y'(x) + (y'(x))^2\,dx.$
2. $J[y] = \displaystyle\int_a^b (1+x)(y'(x))^2\,dx.$
3. Let $p(x), q(x)$ be positive continuous functions defined on $[a,b]$. Let

$$J[y] = \int_a^b p(x)y'(x)^2 + q(x)y(x)^2\,dx\,,$$

where $y(x) \in C^2[a,b]$, $y(a) = A, y(b) = B$. Describe the Euler-Lagrange equation for this functional.