MA 3475 Exam II Review

1. Find the extremal of the functional

$$J[y,z] = \int_0^1 \left((y')^2 - (z')^2 - 8y'y - 4y^2\right)\,dx,$$

subject to the boundary conditions $y(0) = 1, y(1) = 0, z(0) = 0, z(1) = e$.

2. Find the extremal of the functional

$$J[y,z] = \int_a^b \left((y')^2 + (z')^2 + yz\right)\,dx$$

3. Find the extremal of the functional

$$J[y] = \int_0^{\pi/2} \left((y')^2 - (y'')^2\right)\,dx$$

subject to the boundary conditions $y(0) = 0, y'(0) = 0, y\left(\frac{\pi}{2}\right) = 1, y'\left(\frac{\pi}{2}\right) = 1$.

4. Find the extremal of the functional

$$J[y] = \int_a^b \left((y''')^2 + (y'')^2\right)\,dx$$

How many boundary conditions do you need to specify at the two endpoints to determine the extremal uniquely?

5. Find the extremal of the functional

$$J[y] = \int_0^1 \left(x^2 + (y')^2\right)\,dx$$

subject to the conditions $y(0) = 0, y(1) = 0$ and the constraint $\int_0^1 \left(x^2 + 2y(x)\right)\,dx = 1$.

6. Find the extremal of the functional

$$J[y] = \int_0^1 \frac{1}{2}(y')^2\,dx$$

subject to the conditions $y(0) = 0, y(1) = 1$ and the constraint $\int_0^1 \frac{1}{2}y^2\,dx = 1$. For this problem you may stop after expressing $y(x)$ in terms of the multiplier λ but this should be the only unknown constant. For a challenge, determine λ to finish solving the problem.