MA 3475, Homework 4, due midnight Sunday, 03/07/2021

Show your steps as unsupported answers may receive no credit.

1. (4 points) Find the extremals of the functional

$$J[y,z] = \int_a^b ((y')^2 + (z')^2 + y'z')\,dx\,.$$

2. (4 points) Find the extremals of the functional

$$J[y,z] = \int_a^b (2yz - 2y^2 + (y')^2 - (z')^2))\,dx\,.$$

3. (4 points) Find the extremals of the functional

$$J[y] = \int_0^1 (1 + (y'')^2)\,dx$$

subject to the boundary conditions $y(0) = 0, y'(0) = 1, y(1) = 1, y'(1) = 1$.

4. (4 points) Find the extremals of the functional

$$J[y] = \int_0^{\frac{\pi}{2}} ((y'')^2 - y^2 + x^2)\,dx$$

subject to the boundary conditions $y(0) = 1, y'(0) = 0,\ y(\frac{\pi}{2}) = 0, y'(\frac{\pi}{2}) = 1$.