MA 3475, Homework 1 due midnight February 3, 2021.

Show your work as unsupported answer may receive no credit.

1. (5 points) Find all local extrema of the function $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$ and classify them.

2. (5 points) Using the method of Lagrange multiplier, find the global maximum and minimum of the objective function $f(x, y) = x^2 y$ subject to the constraint $g(x, y) = x^2 + y^2 - 3 = 0$.

3. (5 points) Suppose that, in a population to be sampled used a stratified sampling, all of the H strata have the same variance. Show that the choice of strata sample sizes $n_1, \cdots, n_H$ that minimizes the variance of the stratified sampling estimator for the sample mean is given by proportional allocation. This is a problem in statistics and the problem reduces to:

$$\min_{n_h} \sum_{h=1}^{H} \left(1 - \frac{n_h}{N_h}\right) \left(\frac{N_h}{N}\right)^2 \frac{S_h}{n_h} \tag{1}$$

$$\text{subject to} \quad \sum_{h=1}^{H} n_h = n$$

In the above expression, H, N_h, N, S_h and n are constant and the expression in (1) is the variance of the sample mean under stratified sampling, The population has size N and it is divided into H strata. N_h is the size of the population in stratum h. Thus, $N_1 + N_2 + \cdots N_H = N$. We sample $n < N$ individuals in the population, n_h in stratum h, S_h is the variance of the sample mean in stratum h. Solve the minimization problem without assuming S_h are all equal. Show that if $S_h = S$ for all h, then $n_h = p_h n$, where p_h's sum to one.