MA502 Fall 2019 – Homework 2. Due September 5th, 2019

1. Consider $T : \mathbb{P}_3 \to \mathbb{P}_2$ defined by differentiation, i.e., by $T(p) = p' \in \mathbb{P}_2$ for $p \in \mathbb{P}_3$. Find the matrix representation of T with respect to the bases
$\{1 + x, 1 - x, x + x^2, x^2 + x^3\}$ for $\mathbb{P}_3$ and $\{1, x, x^2\}$ for $\mathbb{P}_2$.

2. What is the dimension of $\mathbb{S} = \text{span}\{\, v_1, v_2, v_3\} \subseteq \mathbb{R}^3$, where

$$v_1 = (1, 0, 1), \quad v_2 = (1, 1, 0), \text{ and } v_3 = (1, -1, 2).$$

   If the dimension is less than three, find a subset of $\{v_1, v_2, v_3\}$ that is a basis for $\mathbb{S}$ and expand this basis to a basis for $\mathbb{R}^3$.

3. Consider the transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by the orthogonal projection onto the plane $x_2 = 0$. (1) Find a matrix representation for T in the coordinates induced by the canonical basis; (2) What is the kernel of T?; (3) Find a basis for the range of T.

4. Find the matrix of transformation of coordinates (back and forth) from the canonical basis in $\mathbb{R}^3$ to the basis

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \right\}$$

   (these vectors coordinates are with respect to the canonical basis).

5. Express the linear transformation given by a clockwise rotation of $\pi/4$ in the plane spanned by e_1, e_2 along the e_3 axis, both in terms of the canonical basis and the basis $\mathcal{B}$.