MA502 Fall 2019 – Homework 6. Due October 31st, 2019

Write down detailed proofs of every statement you make

1. A 4×4 matrix A has eigenvalues $\lambda_1 = 1, \lambda_2 = 0, \lambda_3 = 2, \lambda_4 = -1$.
   - Is A invertible? Why or why not?
   - Is A diagonalizable? Why or why not?
   - Find the the characteristic polynomial, the trace and determinant of A.

2. Find a relation between the eigenvalues of a non-singular matrix A and those of its inverse A^{-1}

3. Find a relation between the eigenvalues of a matrix A and those of its square $A^2 = AA$

4. For the following either find an example or prove that such example cannot exist:
   (a) A 4×4 matrix with eigenvalues $\lambda_1 = 1$ with algebraic multiplicity 2 and geometric multiplicity 1; $\lambda_2 = 2$ with algebraic multiplicity 1 and geometric multiplicity 1 and $\lambda_3 = 3$ with algebraic multiplicity 1 and geometric multiplicity 1.
   (b) A 4×4 matrix with eigenvalues $\lambda_1 = 1$ with algebraic multiplicity 1 and geometric multiplicity 2; $\lambda_2 = 2$ with algebraic multiplicity 2 and geometric multiplicity 1 and $\lambda_3 = 3$ with algebraic multiplicity 1 and geometric multiplicity 1.
   (c) A 4×4 matrix with eigenvalues $\lambda_1 = 1$ with algebraic multiplicity 2 and geometric multiplicity 1; $\lambda_2 = 2$ with algebraic multiplicity 2 and geometric multiplicity 1 and $\lambda_3 = 3$ with algebraic multiplicity 1 and geometric multiplicity 1.
   (d) A 4×4 matrix with one eigenvalue $\lambda_1 = \pi$ with algebraic multiplicity 4 and geometric multiplicity 1;

5. Construct a 3×3 matrix A with eigenvalues π, π^2, π^3 and corresponding eigenvectors $(1, 0, 1),\ (1, 1, 0),\ (0, 0, 1)$. Is such matrix unique?