MA502 Fall 2019 – Homework 8. Due November 17th, 2019

Write down detailed proofs of every statement you make

1. Let A be a $n \times n$ matrix with a eigenvalue $\alpha \in \mathbb{C}$. Set $d_i = dim(Ker(A-\alpha I)^i)$. Let $d_0 = 0$ and recall that $d_k - d_{k-1}$ is the number of Jordan blocks larger or equal than k.
   - If $n = 4$, and $d_1 = 2$, $d_2 = 4$ find the Jordan canonical form of A.
   - If $n = 6$ and $d_1 = 3$, $d_2 = 5$ and $d_3 = 6$, find the Jordan canonical form of A.
   - If $n = 5$ and there is one eigenvalue $\alpha = 0$ with $d_1 = 2, d_2 = 3, d_3 = 4$; and one eigenvalue $\alpha = 1$ with $d_1 = 1$. Find the Jordan canonical form of A.

2. Find all eigenvectors and the size of the Jordan blocks of

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}.$$

3. Prove that for any linear transformation $A : V \to V$, with eigenvalues $\lambda_1, ..., \lambda_n$ and any polynomial $f(t)$ the linear transformation $f(A)$ will have as eigenvalues $f(\lambda_1), ..., f(\lambda_n)$.

4. Show that if A is a square matrix with zero determinant, then there exists a polynomial $p(t)$ such that

$$A \cdot p(A) = 0.$$

5. Find four 4×4 matrices A_1, A_2, A_3, A_4 with minimal polynomial of degree $1, 2, 3, 4$ respectively.