MA502 Fall 2019 – Homework 9. Due December 3rd, 2019

Write down detailed proofs of every statement you make

1. Let A be a $n \times n$ matrix with eigenvalues $\lambda_i \in \mathbb{C}$ such that $|\lambda_i| < 1$. Give a direct proof that the matrix $I - A$ is invertible and write an expression for the inverse.

2. Show that for any $n \times n$ matrix $\sin A, \cos A$ are well defined and prove that $(\cos A)^2 + (\sin A)^2 = Identity$.

3. Find an example of two matrices A, B such that

$$\cos(A + B) \neq \cos A \cos B - \sin A \sin B$$

4. Show that for any square matrix A and any number $t \in \mathbb{R}$, the expression $\exp(tA)$ is well defined. Compute

$$\frac{d}{dt} \exp(tA) \quad \text{and} \quad \frac{d}{dt} \det(\exp(tA))$$

5. (a) Prove that the function

$$A \to \max_{|v|=1} |Av|$$

defines a norm (called the operator norm) on the space of square matrices with real coefficients.

(b) Prove that the function

$$A \to \sqrt{trace(AA^T)}$$

defines a norm (called the Frobenious or Hilbert-Schmidt norm) on the space of square matrices with real coefficients. Is this norm comparable with the norm defined in the previous exercise?