MA502 Fall 2019 – Homework 3. Due September 19th, 2019

1. Given two vector basis $\mathbb{B}_1 = \{v_1, ..., v_n\}$ and $\mathbb{B}_2 = \{w_1, ..., w_n\}$ in a vector space V and a linear transformation $L : V \to V$, prove that

$$[L]_{\mathbb{B}_2 \to \mathbb{B}_1}[a]_{\mathbb{B}_2} = [\mathbb{B}_2 \to \mathbb{B}_1][L]_{\mathbb{B}_1 \to \mathbb{B}_2}[a]_{\mathbb{B}_1}$$

   for any $a \in V$. (Hint: show separately that each side is identical to $[L(a)]_{\mathbb{B}_1}$.)

2. Consider the linear map $L : \mathbb{R}^3 \to \mathbb{R}^3$ represented in canonical coordinates by the matrix

$$[L]_{\mathcal{C} \to \mathcal{C}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 4 \\ 2 & 1 & 3 \end{bmatrix}$$

   Find (1) The Null space; (2) The Range. Determine if the linear systems

$$Lv = (1, 2, 0)$$
$$Lv = (6, 8, 6)$$

   have a solution, if it is unique or not. If there exists at least a solution compute one.

3. Consider the operator $T(p) = \int p(x)dx$ from the space of all polynomials $\mathbb{P}$ to itself. Compute its Null space and its range. (Note: $\mathbb{P}$ is not a finite dimensional space)

4. Is it possible for a linear map from $\mathbb{R}^3 \to \mathbb{R}^{100}$ to be onto? Explain your answer in detail.

5. Is it possible for a linear map from $\mathbb{R}^{100} \to \mathbb{R}^3$ to be one to one? Explain your answer in detail. Is it possible for such a map to be onto? If your answer is yes do provide an example.