Homework groups:

You will complete each of twelve homework assignment as part of a three- or four-person group. Group members are assigned randomly from the class and will remain the same for the duration of the quarter. Each group turns in one homework, and each *participating* group member receives the same grade on the assignment. One member of the group is responsible for writing the homework (**the writer**), and this writer rotates for every assignment.

Homework groups work best if: Each member of the homework group finishes (or honestly attempts) the homework independently. At some appointed time, well before the due date, the group meets and everyone compares answers. Any discrepancies are discussed until a consensus is achieved. The writer notes the group consensus and makes sure she or he understands how to do the problem. After the meeting, but before class, the writer neatly and clearly writes the homework according to the Homework guidelines.

Homework groups don't work if: One or more of the members skips meetings; each group member does not honestly attempt the homework prior to the meeting; a consensus in not reached for each assigned problem. *If a group member does not adequately participate in the homework, write a note on the homework and alert your PLA. That person will not receive credit.*

Homework guidelines for writers:

(Adapted from the website of Professor Andy Ruina). To get full credit, please do these things on each homework.

1. As a group writer, upload your homework **as a single PDF** on the day it is due. Homework is available via Canvas Wednesday, and is due the following week by 5pm Eastern Time (unless stated otherwise). At my discretion, late homework may or may not be accepted for reduced credit.

2. On the first page of your homework, please do the following. On the top left corner, please put the course information, homework number and date, e.g.:

   MA 508
   HW 1
   Due September 8, 2021.

   On the top right corner, please put your group number, the names of your group members, with the writer at the top and clearly indicated. Also indicate any non-participating group members, e.g.:

   Group 3
   Jaromir Jagr (writer)
   Sarah Jessica Parker
   Michelle Wie
   James Van der Beek (did not participate)

3. **CITE YOUR HELP.** At the top of each problem, clearly acknowledge all help you got from faculty, students or any other source (with exceptions for lecture and the text, which need not be cited). You could write, for example: "Mary Jones pointed out to me that I had forgotten to divide by three in problem 2," or "Nadia Chow showed me how to do problem 3 from start to finish," or "I copied this solution word for word from Jane Lewenstein" or "I found a problem just like this one, number 9, at

cheatonyourhomework.com, and copied it," etc. You will not lose credit for getting and citing such help. Don't violate academic integrity rules: be clear about which parts of your presentation you did not do on your own. Violations of this policy are violations of the WPI Code of Academic Conduct.

4. Your work should be laid out neatly enough to be read by someone who does not know how to do the problem. For most jobs, it is not sufficient to know how to do a problem, you must convince others that you know how to do it. Your job on the homework is to practice this. **Box your answers.**

DUE: September 8, 2021. Your answers must be uploaded on Canvas, as a single PDF, by 5pm Eastern Time.

This homework covers 1. Linear ODEs (review); 2. Getting familiar with Matlab; 3. The phase line and trajectory sketching; and 4. Linear stability analysis.

These topic are covered in §2.0-2.4 in Strogatz.

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1. Consider the following chemical reaction, where one chemical (A) turns into a different chemical (B) and vice versa. Suppose that the total amount of chemical is constant, that is $A(t) + B(t) = C$, where C is a positive constant. This reaction can be represented schematically in the following way:

$$A \underset{k^-}{\overset{k^+}{\rightleftharpoons}} B$$

where the two positive constants k^+ and k^- are called rate constants.

The following differential equation describes how A changes with time

$$\frac{dA}{dt} = -k^+ A + k^- B \tag{1}$$

Recall that, in addition to this differential equation, we also have the conservation constraint $A(t)+B(t) = C$.

a) Solve for $A(t)$, given $A(0) = A_0$, with A_0 being a positive constant such that $A_0 < C$.
b) Use Matlab to check your answer for a few choices of A_0, C, k^+, and k^-. (I have provided code that will assist you).

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2. The position of an object moving in 1D ($x(t)$) on a damped, linear spring obeys the following differential equation

$$m\ddot{x} = -b\dot{x} - kx \tag{2}$$

where m, b, and k are positive constant representing the mass of the object, the damping coefficient and the stiffness of the spring, respectively.

a) Solve for $x(t)$, given $x(0) = x_0$, and $\dot{x}(0) = v_0$.
b) Use Matlab to check your answer for a few choices of x_0, v_0, m, b, and k. (I have provided code that will assist you).

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3. The following equation describes the velocity, $v(t)$, of a relatively large object falling through a relatively inviscid medium (e.g., a baseball falling through the air)

$$m\dot{v} = -cv|v| + mg \tag{3}$$

where m, c and g are positive constants representing the mass of the object, the drag of the medium, and the pull of gravity. a) draw a plot of $\dot{v}$ vs. v. Label any equilibrium point(s) and indicate the stability of each. On the horizontal axis, indicate the flow direction.
b) without solving the equation, sketch $v(t)$ as a function of t for several different initial conditions.
c) solve the equation for $v(t)$, given $v(0) = 0$. (It will simplify your life to assume that $v \geq 0$ to get rid of the

absolute value sign. Once you have a solution, you can determine whether this is a reasonable assumption). d) Use Matlab to check your solution. I have not provided code, but you should be able to modify the code for problem 1.

4. Turn in a completed version of worksheet 1, which you worked on during class on September 1.