Homework groups:

You will complete each of twelve homework assignment as part of a three- or four-person group. Group members are assigned randomly from the class and will remain the same for the duration of the quarter. Each group turns in one homework, and each *participating* group member receives the same grade on the assignment. One member of the group is responsible for writing the homework (**the writer**), and this writer rotates for every assignment.

Homework groups work best if: Each member of the homework group finishes (or honestly attempts) the homework independently. At some appointed time, well before the due date, the group meets and everyone compares answers. Any discrepancies are discussed until a consensus is achieved. The writer notes the group consensus and makes sure she or he understands how to do the problem. After the meeting, but before class, the writer neatly and clearly writes the homework according to the Homework guidelines.

Homework groups don't work if: One or more of the members skips meetings; each group member does not honestly attempt the homework prior to the meeting; a consensus in not reached for each assigned problem. *If a group member does not adequately participate in the homework, write a note on the homework and alert your PLA. That person will not receive credit.*

Homework guidelines for writers:

(Adapted from the website of Professor Andy Ruina). To get full credit, please do these things on each homework.

1. As a group writer, upload your homework **as a single PDF** on the day it is due. Homework is available via Canvas Wednesday, and is due the following week by 5pm Eastern Time (unless stated otherwise). At my discretion, late homework may or may not be accepted for reduced credit.

2. On the first page of your homework, please do the following. On the top left corner, please put the course information, homework number and date, e.g.:

   MA 508
   HW 3
   Due September 22, 2021.

   On the top right corner, please put your group number, the names of your group members, with the writer at the top and clearly indicated. Also indicate any non-participating group members, e.g.:

   Group 3
   Jaromir Jagr (writer)
   Sarah Jessica Parker
   Michelle Wie
   James Van der Beek (did not participate)

3. **CITE YOUR HELP.** At the top of each problem, clearly acknowledge all help you got from faculty, students or any other source (with exceptions for lecture and the text, which need not be cited). You could write, for example: "Mary Jones pointed out to me that I had forgotten to divide by three in problem 2," or "Nadia Chow showed me how to do problem 3 from start to finish," or "I copied this solution word for word from Jane Lewenstein" or "I found a problem just like this one, number 9, at

cheatonyourhomework.com, and copied it," etc. You will not lose credit for getting and citing such help. Don't violate academic integrity rules: be clear about which parts of your presentation you did not do on your own. Violations of this policy are violations of the WPI Code of Academic Conduct.

4. Your work should be laid out neatly enough to be read by someone who does not know how to do the problem. For most jobs, it is not sufficient to know how to do a problem, you must convince others that you know how to do it. Your job on the homework is to practice this. **Box your answers.**

DUE: September 22, 2021. Your answers must be uploaded on Canvas, as a single PDF, by 5pm Eastern Time.

This homework covers 1. Transcritical bifurcations; 2. Pitchfork bifurcations (super- and subcritical); 3. Hysteresis; and 4. Imperfect bifurcations.

These topic are covered in §3.2-3.6 in Strogatz.

For your bifurcation diagrams:
i. Indicate stable fixed points with a solid line and unstable fixed points with a dashed line
ii. Show your calculations for how you determined the fixed points
iii. Explain how you determined stability and/or show your calculations
iv. Clearly indicate any bifurcation(s) (if they exist)
v. Clearly identify and label bifurcation(s) (saddle-node, transcritical, pitchfork, if they exist)

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1. Consider the equation

$$\dot{x} = ax - x(1-x)^2 \tag{1}$$

a) Draw a bifurcation diagram for this equation as a varies.
b) In the neighborhood of all bifurcation(s), if they exist, transform Eq. 1 into the normal form.

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2. Consider the following bifurcation diagram, showing fixed points (x^*) as a function of parameter, p, for an equation of the form $\dot{x} = f(x)$. Note that there are three different parameter values indicated, p_1, p_2, and p_3.

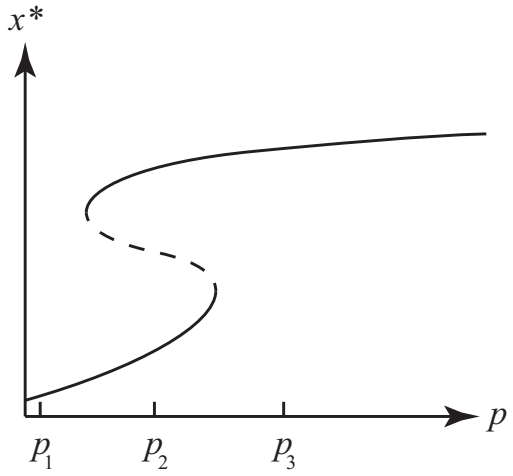

Figure 1: Bifurcation diagram; stable fixed points are drawn as a solid line, unstable as a dashed line.

a) Label and identify all bifurcations in the figure.
b) Draw phase portraits consistent with the bifurcation curve at each of the parameter values, p_1, p_2 and p_3. (You should draw one plot for each parameter value, a total of three phase portraits).
c) Can this system exhibit hysteresis (according to the definition used in class)?
d) How would your answer to c) change if the stability in Fig. 1 were flipped (i.e., each stable fixed point were unstable, and each unstable fixed point were stable)? Explain.

3. Here is the equation for an imperfect transcritical bifurcation

$$\dot{x} = ax + x^2 + \varepsilon \tag{2}$$

a) Sketch bifurcation diagrams for 1) $\varepsilon = 0$, 2) $\varepsilon > 0$, 3) $\varepsilon < 0$. You may assume ε is small in the latter two cases. On each of the three bifurcation diagrams, indicate stable fixed points with a solid line, unstable fixed points with a dashed line, and label all bifurcations.
b) Sketch a stability diagram (Recall that a stability diagram will have a and ε as axes, and will indicated regions where there are differing numbers of fixed points).

4. Turn in a completed version of worksheet 3, which you worked on during class on September 15.