MA508 – Worksheet 3

In class, I discussed the following mechanical example of a bead, constrained to move along a wire and connected to a spring, pictured below

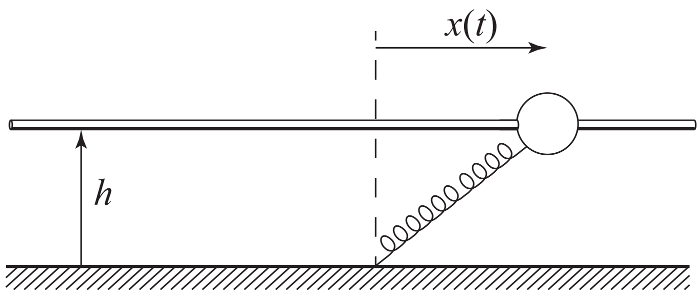

Figure 1

Here is the differential equation

$$\dot{x} = -\frac{k}{b}\left(\sqrt{x^2+h^2} - \ell_0\right)\frac{x}{\sqrt{x^2+h^2}}$$

where k is the spring constant, b the damping constant, h the height of the wire (see Fig. 1) and ℓ_0 is the rest length of the spring.

1a) Non-dimensionalize this equation to get

$$\frac{d\hat{x}}{d\hat{t}} = \frac{\hat{x}}{\sqrt{\hat{x}^2+\alpha^2}} - \hat{x} \tag{1}$$

(you will need to determine what $\hat{x}$, $\hat{t}$ and α are in terms of the variables, x and t, and the parameters, h, b, ℓ_0, and k).

1b) Draw a bifurcation diagram for the system.

1c) Show that, near the bifurcation, Eq. 1 can be written in the normal form for a supercritical pitchfork bifurcation ($\dot{x} = ax - x^3$).

Suppose that you add gravity to the system, and gravity acts along the wire. The governing equation is then

$$\dot{x} = -\frac{k}{b}\left(\sqrt{x^2+h^2} - \ell_0\right)\frac{x}{\sqrt{x^2+h^2}} - \frac{mg}{b}$$

where g is the gravitational constant and m the mass of the bead.

2a) Non-dimensionalize this equation to get

$$\frac{d\hat{x}}{d\hat{t}} = \frac{\hat{x}}{\sqrt{\hat{x}^2 + \alpha^2}} - \hat{x} - \beta \tag{2}$$

(you will need to determine what $\hat{x}$, $\hat{t}$, α and β are in terms of the variables, x and t, and the parameters, h, b, ℓ_0, m, g and k).

2b) Find the fixed points of this equation as a function of α for $\beta = 0$, $\beta = 0.1$, and $\beta = 0.2$. You may wish to do this numerically, using Matlab's **fzero** function, which finds the roots of a scalar function. To learn how to use it, type "help fzero" into Matlab's command line.

2c) Plot a stability diagram for this system. It might be useful to use Matlab's **fsolve** function, which finds the roots of a vector function. To learn how to use it, type "help fsolve" into Matlab's command line.