MA 514 Homework 3

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Exercise 20.2

We can say that L is lower triangular and also $l_{ij} = 0$ for $i - j > p$. We can say that U is upper triangular and also that $u_{ij} = 0$ for $j - i > p$. To see this, note that since A satisfies the conditions of exercise 20.1, A has an LU factorization $A = LU$. When $i - j > p$, because U is upper triangular we have

$$a_{ij} = \sum_{k=1}^{j} l_{ik}u_{kj} = 0 \quad \text{for } i = 1, ..., m-p-1 \ .$$

Therefore, $l_{ij} = 0$ for $i - j > p$. Because L is lower triangular, when $j - i > p$ we have

$$a_{ij} = \sum_{k=1}^{i} l_{ik}u_{kj} = 0, \quad \text{for } i = 1, ..., m-p-1 \ .$$

Therefore, $u_{ij} = 0$ for $j - i > p$.

Exercise 23.1

Let A be a nonsingular square matrix and let $A = QR$ and $A^*A = U^*U$ be QR and Cholesky factorizations, respectively, with the usual normalizations $r_{jj}, u_{jj} > 0$. Then it is true that $R = U$. To see this, first we note that since A is nonsingular, A has a unique QR factorization and $A^*A = (QR)^*QR = R^*R$. Also, because A is nonsingular, $Ax \neq 0$ for every $x \neq 0$. Then $x^*A^*Ax = ||Ax||_2^2 > 0$, which shows that A^*A is positive definite. And because $(A^*A)^* = A^*(A^*)^* = A^*A$, A^*A is also hermitian. So by Theorem 23.1, the Cholesky factorization $A^*A = U^*U$ of A^*A is unique. But then

$A^*A = R^*R = U^*U$ and both factorizations must be unique, we conclude that $R = U$.