Homework 7 (Due: Wednesday, 4/1)

This assignment is due on **Wednesday, April 1**, by 11:59 PM. Your assignment should be well-organized, typed (or neatly written and scanned) and saved as a .pdf for submission on Canvas. You must show all of your work to receive full credit. For problems requiring the use of MATLAB code, remember to also submit your .m-files on Canvas as a part of your completed assignment. Your code should be appropriately commented to receive full credit.

Problems

1. (20 points)   Consider the `gravity` test problem from Regularization Tools, which models a one-dimensional gravity surveying problem (see the 'Regularization Tools' manual for more details). This test problem was used as an example demonstrating a "too-good-to-be-true" reconstruction that can occur when committing an *inverse crime*; i.e., when the model used to solve the inverse problem is the same exact model used to generate the data, such that there is no model/data mismatch.

   In the file `MA590_InverseCrimes_ex.m` (posted on Canvas), the command

   $$\texttt{[G,y\_true,x\_true] = gravity(64,3,0,1,0.05);}$$

   is used to generate synthetic data `y_true` using a short source `x_true` defined by the piecewise function

   $$x = x(t) = \begin{cases} 2\,, & 0 \le t < \frac{1}{3} \\ 1\,, & \frac{1}{3} \le t \le 1 \\ 0\,, & \text{else} \end{cases} \;.$$

   The file `gravity_mod.m` is then used to generate data from a longer source, extended to be nonzero on the interval $[-0.5, 1.5]$. An inverse crime can be avoided by using the operator matrix G (modeling the short source) to solve the inverse problem with the long source data – we will refer to this as the model/data mismatch problem.

   (a) Implement Tikhonov regularization to solve the model/data mismatch problem; i.e., using the source short operator matrix G to solve the inverse problem given the long source data. Does regularization affect your results? Discuss your results, including relevant plots to support your findings, and state your "optimal" choice of the regularization parameter α for this problem.

   (b) Corrupt the synthetic data by adding normally distributed noise with zero mean and standard deviation 0.1. Discuss how this affects your results in solving the inverse problem with both the crime data (i.e. short source data) and mismatch data (i.e. long source data). In each case, compute both the "naive solution" and the Tikhonov regularized solution, and discuss your findings. Include relevant plots to aid in your discussion.

(c) Change the value of the depth parameter `d` from 0.05 to 0.1. How does this affect the condition number of the operator matrix `G`? Repeat parts (a) and (b), discussing how the conditioning of the operator matrix plays a role in the results for the different approaches considered.

Note: For any of the above problems for which you use MATLAB to help you solve, you must submit your code/.m-files as part of your work. Your code must run in order to receive full credit. If you include any plots, make sure that each has a title, axis labels, and readable font size, and include the final version of your plots as well as the code used to generate them.