Homework 2 (Due: Wednesday, 1/29)

This assignment is due on **Wednesday, January 29**, by 11:59 PM. Your assignment should be well-organized, typed (or neatly written and scanned) and saved as a .pdf for submission on Canvas. You must show all of your work to receive full credit. For problems requiring the use of MATLAB code, remember to also submit your .m-files on Canvas as a part of your completed assignment. Your code should be appropriately commented to receive full credit.

Problems

$\boxed{1}$ (25 points) Consider the vertical seismic profiling problem, where a downward-propagating seismic wavefront is generated by a source on the surface and the waves are sensed using seismometers in a borehole (see Example 1.3 in Aster *et al.*, 2019).

The observed travel time t at depth z can be modeled as

$$t(z) = \int_0^\infty s(\xi) H(z - \xi) d\xi \tag{1}$$

where $s(z)$ denotes the vertical slowness (reciprocal of velocity) and the kernel H is the Heaviside step function, which is equal to 1 for nonnegative arguments and 0 for negative arguments. Assume we have $n = 100$ equally spaced seismic sensors located at depths of $z = 0.2, 0.4, \ldots, 20$ m, and we want to estimate n corresponding equal length seismic slowness values for 0.2 m intervals having midpoints at $z - 0.1$ m.

(a) Calculate the appropriate system matrix G for discretizing the integral equation (1) using the midpoint rule.

(b) For a seismic velocity model having a linear depth gradient specified by

$$v = v_0 + kz \tag{2}$$

where the velocity at $z = 0$ is $v_0 = 1$ km/s and the gradient is $k = 40$ m/s per m, calculate the true slowness values, $\mathbf{s}_{\text{true}}$, at the midpoints of the n intervals. Additionally, integrate the corresponding slowness function for (2) using (1) to calculate a noiseless synthetic data vector, $\mathbf{y}$, of predicted seismic travel times at the sensor depths.

(c) Solve for the slowness, $\mathbf{s}$, as a function of depth using your G matrix from part (a) and analytically calculated noiseless travel times from part (b) by using the MATLAB backslash operator (see MATLAB help for \ ). Compare your results graphically with $\mathbf{s}_{\text{true}}$.

(d) Generate a noisy travel time vector where independent normally distributed noise with a standard deviation of 0.05 ms is added to the elements of $\mathbf{y}$. Resolve the system for $\mathbf{s}$ and again compare your results graphically with $\mathbf{s}_{\text{true}}$. How has the result changed?

(e) Repeat the problem using $n = 4$ sensor depths and corresponding equal length slowness intervals. Is the recovery of the true solution improved? Discuss, considering the condition number of your G matrices.

Note: For any of the above problems for which you use MATLAB to help you solve, you must submit your code/.m-files as part of your work. Your code must run in order to receive full credit. If you include any plots, make sure that each has a title, axis labels, and readable font size, and include the final version of your plots as well as the code used to generate them.