Homework 8 (Due: Wednesday, 4/8)

This assignment is due on **Wednesday, April 8**, by 11:59 PM. Your assignment should be well-organized, typed (or neatly written and scanned) and saved as a .pdf for submission on Canvas. You must show all of your work to receive full credit. For problems requiring the use of MATLAB code, remember to also submit your .m-files on Canvas as a part of your completed assignment. Your code should be appropriately commented to receive full credit.

Problems

$\boxed{1}$ (20 points) Given two random variables X and Y, prove the following.

(a) The covariance of X and Y can be equivalently written as

$$\mathrm{Cov}(X, Y) = E[(X - \bar{x})(Y - \bar{y})]$$

or as

$$\mathrm{Cov}(X, Y) = E[XY] - E[X]E[Y]$$

where $\bar{x} = E[X]$ and $\bar{y} = E[Y]$.

(b) If X and Y are independent, then X and Y are uncorrelated.

(c) $\mathrm{Var}(sX) = s^2\ \mathrm{Var}(X)$ for some scalar s

(d) $\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\ \mathrm{Cov}(X, Y)$

$\boxed{2}$ (10 points) Consider a vector-valued random variable

$$A = X\mathbf{e}_1 + Y\mathbf{e}_2$$

where $\mathbf{e}_1$ and $\mathbf{e}_2$ are the orthogonal Cartesian unit vectors, and X and Y are real-valued random variables with

$$X, Y \sim \mathcal{N}(0, \sigma^2).$$

The random variable

$$R = ||A||_2$$

is then distributed according to the *Rayleigh distribution*,

$$R \sim \mathrm{Rayleigh}(\sigma^2).$$

Derive the analytic expression of the Rayleigh distribution, and write a MATLAB program that generates points from the Rayleigh distribution. Make a plot of the distribution and a histogram of the points you generated.

Note: For any of the above problems for which you use MATLAB to help you solve, you must submit your code/.m-files as part of your work. Your code must run in order to receive full credit. If you include any plots, make sure that each has a title, axis labels, and readable font size, and include the final version of your plots as well as the code used to generate them.