Homework 9 (Due: Wednesday, 4/15)

This assignment is due on **Wednesday, April 15**, by 11:59 PM. Your assignment should be well-organized, typed (or neatly written and scanned) and saved as a .pdf for submission on Canvas. You must show all of your work to receive full credit. For problems requiring the use of MATLAB code, remember to also submit your .m-files on Canvas as a part of your completed assignment. Your code should be appropriately commented to receive full credit.

Problems

1 (20 points) This problem is a prelude to statistical estimators, which we will discuss from the Bayesian perspective as we move forward in class. Consider the Rayleigh distribution

$$X \sim \text{Rayleigh}(\sigma^2)$$

with pdf given by

$$\pi(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2} \right), \quad x \geq 0$$

as derived in Homework 8.

(a) Unlike the normal distribution, the maximizer of $\pi(x)$ for the Rayleigh distribution does not coincide with its expected value. Verify this by computing the maximizer of $\pi(x)$ and the expected value of the Rayleigh distribution, and generate a plot to show your results graphically. In computing the maximizer, it may help to note that

$$\arg\max_x \ \pi(x) = \arg\min_x \ V(x), \quad V(x) = -\log(\pi(x)).$$

(b) Assume we have N independent and identically distributed (i.i.d.) random variables $X_1, \ldots, X_N$, such that

$$X_j \sim \pi(x \mid \theta), \quad j = 1, \ldots, N,$$

where the pdf of X_j depends on the value of a parameter θ.

In frequentist statistics, the *Maximum Likelihood Estimator* (MLE) of the parameter θ is the value of θ that maximizes the probability of the outcomes x_j, $j = 1, \ldots, N$; i.e.,

$$\theta_{\text{ML}} = \arg\max_\theta \ \pi(x_1, \ldots, x_N \mid \theta) = \arg\max_\theta \ \prod_{j=1}^{N} \pi(x_j \mid \theta)$$

provided such a maximizer exists.

If $X_1, \ldots, X_N$ are i.i.d. with

$$X_j \sim \text{Rayleigh}(\sigma^2), \quad j = 1, \ldots, N,$$

compute the MLE of $\theta = \sigma^2$.

2 (10 points) Provide a brief update on your final project, describing the progress you've made over the past few weeks. What have you done thus far? Have you thought about some of the considerations mentioned in the comments on your Final Project Proposals in Canvas? (If you haven't already replied, please outline your replies to my comments here!)

Please keep in mind that we will have synchronous final project presentations via Zoom during our class time 3:00-4:50 PM on Monday, May 11, and Wednesday, May 13 – more details to follow!

Note: For any of the above problems for which you use MATLAB to help you solve, you must submit your code/.m-files as part of your work. Your code must run in order to receive full credit. If you include any plots, make sure that each has a title, axis labels, and readable font size, and include the final version of your plots as well as the code used to generate them.