1. Show that the equation $x^2+y^2=z^3$ has infinitely many solutions for x, y, z positive integers.

Let $x=n(n^2-3)$ and $y=3n^2-1$ for $n \geq 2$.
Then,

$$\begin{aligned} x^2+y^2 &= n^6-6n^4+9n^2+9n^4-6n^2+1 \\ &= n^6+3n^4+3n^2+1 \\ &= (n^2+1)^3 \\ &= z^3 \text{ for } z := n^2+1. \end{aligned}$$

2. Prove that nonnegative integer solutions of the equation $x^2+2y^2=z^2$ with $\gcd(x,y,z)=1$ are given by:

$$x=\pm(2s^2-t^2), \quad y=2st, \quad z=2s^2+t^2, \quad s,t \geq 0.$$

$$x^2+2y^2 = 4s^4-4s^2t^2+t^4+8s^2t^2 = 4s^4+4s^2t^2+t^4 = (2s^2+t^2)^2 = z^2.$$

3. In a pythagorean triple $x^2+y^2=z^2$, prove that x and y cannot both be perfect squares.

Pf (contradiction): Suppose $x=k^2$ & $y=j^2$ for $K, j \in \mathbb{Z}$.

By Fermat's Last Theorem, $z^2=(k^2)^2+(j)^2$ has no solution's. Conclude that x, y cannot be perfect squares.

11. Prove that nonnegative integer solutions to $1/x^2 + 1/y^2 = 1/z^2$ are given by:
$x=2st(s^2+t^2)$, $y=s^4-t^4$, $z=2st(s^2-t^2)$.

$$1/x^2+1/y^2 = 1/[4s^2t^2(s^2+t^2)^2] + 1/[s^4-t^4]^2$$

$$= \frac{(s^2-t^2)^2+4s^2t^2}{4s^2t^2(s^2+t^2)^2(s^2-t^2)^2}$$

$$= \frac{s^4-2s^2t^2+t^4+4s^2t^2}{4s^2t^2(s^2+t^2)^2(s^2-t^2)^2}$$

$$= \frac{(s^2+t^2)^2}{4s^2t^2(s^2+t^2)^2(s^2-t^2)^2}$$

$$= \frac{1}{[2st(s^2-t^2)]^2}$$

$$= 1/z^2 . \quad \checkmark$$