

Homework #6 MME 529

Integers Mod n \mathbb{Z}_n and \mathbb{Z}_p

1. in \mathbb{Z}_{11} which numbers have square roots? What are they?
2. In \mathbb{Z}_p show that $x^2 \equiv (p-x)^2 \pmod{p}$.
How does this help with square roots?
Give two examples to illustrate.
3. Solve $17x \equiv 5 \pmod{29}$. Show all steps. (by hand)
4. If we have $ax \equiv ay \pmod{n}$ can we always cancel the a out? What do you think?
5. Simplify $889345234 \pmod{25}$ without doing out the long division.
6. Predict with algebra which members of \mathbb{Z}_{15} will have multiplicative inverse.
7. Solve $x^2 - 2x + 2 \equiv 0 \pmod{13}$. Show all steps. Check your answers.
8. Suppose for sake of discussion we are in \mathbb{Z}_{13} . Show that $a = 2$ is a **generator** for \mathbb{Z}_{13} in the sense that: every member of \mathbb{Z}_{13} is a power of 2 (except 0, of course). For example $9 \equiv 2^8 \pmod{13}$ (kinda wrecks your notion of even numbers, doesn't it?)
What happens if you try to use $a = 5$ as a generator?
Can you find another generator for \mathbb{Z}_{13} ?
9. A **bank routing number** appears in the lower left of all of your checks. Its purpose is to see the check is routed to the correct bank. It is 9 digits.

To increase the chances of detecting an error, the numbers as a group must satisfy an algebraic criteria using mod 10 arithmetic. Specifically

if **ABCDEFGHI** is the routing number then

$7A + 3B + 9C + 7D + 3E + 9F + 7G + 3H + 9I \pmod{10}$ must be congruent to 0

- a) show that **211872946** passes the criteria
 - b) does my own check routing # of **011000138**?
 - c) examine your own routing number. Just report whether it passed or not.
10. What does the symbol a^{-2} in \mathbb{Z}_n mean, in your opinion?

