Proiect Teoria Sistemelor

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1 Filtru Sallen-Key de tip Notch

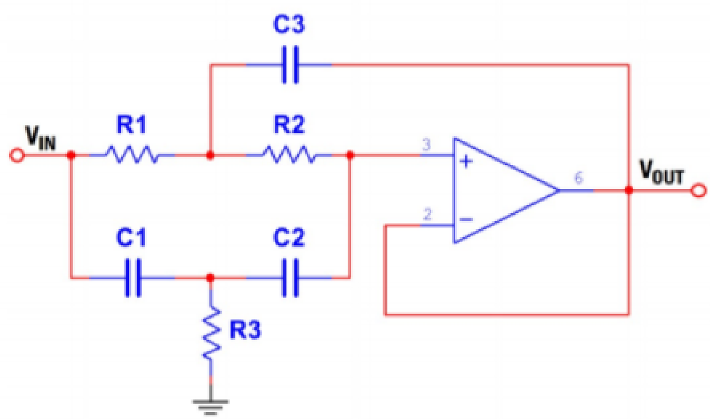


Figure 1: Filtru Sallen-Key de tip Notch

Filtrul ales contine tensiunea de intrare, tensiunea de iesire, 3 rezistente, 3 condensatoare si un amplificator operational. Acesta are in componenta 3 elemente active (cele 3 condensatoare) de unde rezulta ca sistemul este de ordin 3.

$\mathrm{R}_1 = R_2 = 560$

$R_3 = 9k$

$C_1 = C_2 = 33 \cdot 10^{-3}$

$C_3 = 4.7 \cdot 10^{-6}$

$$\mathrm{X}= \begin{pmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \end{pmatrix} = \begin{pmatrix} \mathrm{u}_{C_1} \\ \mathrm{u}_{C_2} \\ \mathrm{u}_{C_3} \end{pmatrix}$$

Realizarea de stare

$$TKI: \begin{cases} i_{R_1} = i_{R_2} + i_{C_3} \\ i_{R_2} + i_{C_2} = 0 \\ i_{C_1} = i_{C_2} + i_{R_3} \end{cases} \tag{1}$$

$$TKII: \begin{cases} u = u_{C_1} + u_{R_3} \\ u_{R_1} + u_{R_2} = u_{C_1} + u_{C_2} \\ u_{C_3} = u_{R_2} \\ y = -u_{C_2} + u_{R_3} \end{cases} \tag{2}$$

y = u - $u_{C_1} - u_{C_2} = -x_1 - x_2 + u$

$u = x_1 + R_3 \cdot i_{R_3} => i_{R_3} = \frac{1}{R_3} \cdot (u - x_1)$

$i_{R_1} \cdot R_1 + i_{R_2} \cdot R_2 = x_1 + x_2 => i_{R_1} = \frac{1}{R_1} \cdot (x_1 + x_2 - x_3)$

$x_3 = i_{R_2} \cdot R_2 => i_{R_2} = \frac{x_3}{R_2}$

$C_3 \cdot \dot{x_3} = i_{R_1} - i_{R_2}$

$\dot{x_3} = \frac{1}{R_1 \cdot C_3} \cdot (x_1 + x_2 - x_3) - \frac{1}{R_2 \cdot C_3} \cdot x_3$

$C_2 \cdot \dot{x_2} = i_{R_2} <=> \dot{x_2} = -\frac{1}{R_2 \cdot C_2} \cdot x_3$

$C_1 \cdot \dot{x_1} = C_2 \cdot \dot{x_2} + i_{R_3}$

$\dot{x_1} = -\frac{1}{C_1 \cdot R_2} \cdot x_3 - \frac{1}{R_3 \cdot C_1} \cdot x_1 + \frac{1}{C_1 \cdot R_3} \cdot u$

$$\begin{cases} \dot{x_1} = -\dfrac{1}{R_3 \cdot C_1} \cdot x_1 + 0 \cdot x_2 - \dfrac{1}{C_1 \cdot R_2} \cdot x_3 + \dfrac{1}{C_1 \cdot R_3} \cdot u \\ \dot{x_2} = 0 \cdot x_1 + 0 \cdot x_2 - \dfrac{1}{R_2 \cdot C_2} \cdot x_3 + 0 \cdot u \\ \dot{x_3} = \dfrac{1}{R_1 \cdot C_3} \cdot x_1 + \dfrac{1}{R_1 \cdot C_3} \cdot x_2 - (\dfrac{1}{R_2 \cdot C_3} + \dfrac{1}{R_1 \cdot C_3}) \cdot x_3 + 0 \cdot u \\ y = -x_1 - x_2 + 0 \cdot x_3 + u \end{cases} \tag{3}$$

$$A = \begin{pmatrix} -\frac{1}{C_1 \cdot R_3} & 0 & -\frac{1}{C_1 \cdot R_2} \\ 0 & 0 & -\frac{1}{R_2 \cdot C_2} \\ \frac{1}{R_1 \cdot C_3} & \frac{1}{R_1 \cdot C_3} & -(\frac{1}{R_1 \cdot C_3} + \frac{1}{R_2 \cdot C_3}) \end{pmatrix} = \begin{pmatrix} -0.0034 & 0 & -0.0541 \\ 0 & 0 & -0.0541 \\ 379.9392 & 379.9392 & -759.8784 \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{1}{R_3 \cdot C_1} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.0034 \\ 0 \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} -1 & -1 & 0 \end{pmatrix}$$

$$D = 1$$

Planul starilor:

Simbolic:

$$\left(\begin{array}{ccc|c} -\frac{1}{C_1 \cdot R_3} & 0 & -\frac{1}{C_1 \cdot R_2} & \frac{1}{R_3 \cdot C_1} \\ 0 & 0 & -\frac{1}{R_2 \cdot C_2} & 0 \\ \frac{1}{R_1 \cdot C_3} & \frac{1}{R_1 \cdot C_3} & -(\frac{1}{R_1 \cdot C_3} + \frac{1}{R_2 \cdot C_3}) & 0 \\ \hline -1 & -1 & 0 & 1 \end{array}\right)$$

Numeric:

$$\left(\begin{array}{ccc|c} -0.0034 & 0 & -0.0541 & 0.0034 \\ 0 & 0 & -0.0541 & 0 \\ 379.9392 & 379.9392 & -759.8784 & 0 \\ \hline -1 & -1 & 0 & 1 \end{array}\right)$$

Relatia intrare-iesire + fdt

$$y = -x_1 - x_2 + u => x_1 + x_2 = u - y; x_1 = u - x_2 - y$$

$$\dot{y} = -\dot{x_1} - \dot{x_2} + \dot{u}$$

$$\dot{y} = -(-\frac{1}{R_3 \cdot C_1} \cdot x_1 - \frac{1}{C_1 \cdot R_2} \cdot x_3 + \frac{1}{C_1 \cdot R_3} \cdot u) - (-\frac{1}{R_2 \cdot C_2} \cdot x_3) + \dot{u}$$

$$\dot{y} = \frac{1}{R_3 \cdot C_1} \cdot x_1 + (\frac{1}{C_1 \cdot R_2} + \frac{1}{R_2 \cdot C_2}) \cdot x_3 - \frac{1}{C_1 \cdot R_3} \cdot u + \dot{u}$$

$$\dot{y} = \frac{1}{R_3 \cdot C_1} \cdot (u - x_2 - y) + (\frac{1}{C_1 \cdot R_2} + \frac{1}{R_2 \cdot C_2}) \cdot x_3 - \frac{1}{C_1 \cdot R_3} \cdot u + \dot{u}$$

$$\ddot{y} = \frac{1}{R_3 \cdot C_1} \cdot (\dot{u} - \dot{y}) + \frac{1}{R_3 \cdot C_1} \cdot (-\frac{1}{R_2 \cdot C_2} \cdot x_3) + (\frac{1}{C_1 \cdot R_2} + \frac{1}{R_2 \cdot C_2}) \cdot (\frac{1}{R_1 \cdot C_3} \cdot (x_1 + x_2) - (\frac{1}{R_2 \cdot C_3} + \frac{1}{R_1 \cdot C_3}) \cdot x_3) - \frac{1}{C_1 \cdot R_3} \cdot \dot{u} + \ddot{u}$$

$\dddot{y} = \frac{1}{R_3 \cdot C_1} \cdot (\dot{u} - \dot{y}) + \frac{1}{R_3 \cdot C_1 \cdot R_2 \cdot C_2} \cdot x_3 + (\frac{1}{C_1 \cdot R_2} + \frac{1}{R_2 \cdot C_2}) \cdot \frac{1}{R_1 \cdot C_3} \cdot (u - y) - (\frac{1}{C_1 \cdot R_2} + \frac{1}{R_2 \cdot C_2}) \cdot (\frac{1}{R_2 \cdot C_3} \cdot \frac{1}{R_1 \cdot C_3}) \cdot x_3 - \frac{1}{C_1 \cdot R_3} \cdot \dot{u} + \ddot{u}$

$\dddot{y} = \frac{1}{R_3 \cdot C_1} \cdot (\dot{u} - \dot{y}) + (\frac{1}{C_1 \cdot R_2} + \frac{1}{R_2 \cdot C_2}) \cdot \frac{1}{R_1 \cdot C_3} \cdot (u - y) - (\frac{1}{R_3 \cdot C_1 \cdot R_2 \cdot C_2} + (\frac{1}{C_1 \cdot R_2} + \frac{1}{R_2 \cdot C_2}) \cdot (\frac{1}{R_2 \cdot C_3} \cdot \frac{1}{R_1 \cdot C_3})) \cdot x_3 - \frac{1}{C_1 \cdot R_3} \cdot \dot{u} + \ddot{u}$

2 Functia de transfer cu ajutorul realizarii de stare

H(s) = C $\cdot (s \cdot I - A)^{-1} \cdot B + D$

$H(s) = \begin{pmatrix} -1 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} s + \frac{1}{C_1 \cdot R_3} & 0 & \frac{1}{C_1 \cdot R_2} \\ 0 & s & \frac{1}{R_2 \cdot C_2} \\ -\frac{1}{R_1 \cdot C_3} & -\frac{1}{R_1 \cdot C_3} & s + \frac{1}{R_1 \cdot C_3} + \frac{1}{R_2 \cdot C_3} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \frac{1}{R_3 \cdot C_1} \\ 0 \\ 0 \end{pmatrix} +$
1

$det(s \cdot I - A) = s \cdot (s + \frac{1}{C_1 \cdot R_3}) \cdot (s + \frac{1}{R_1 \cdot C_3} + \frac{1}{R_2 \cdot C_3}) + \frac{1}{R_1 \cdot R_2 \cdot C_1 \cdot C_3} \cdot s + \frac{1}{R_1 \cdot C_3} \cdot \frac{1}{R_2 \cdot C_2} \cdot (s + \frac{1}{R_3 \cdot C_1}) =$

$= (s^2 + \frac{1}{R_3 \cdot C_1} \cdot s) \cdot (s + \frac{1}{R_1 \cdot C_3} + \frac{1}{R_2 \cdot C_3}) + \frac{1}{R_1 \cdot R_2 \cdot C_1 \cdot C_3} \cdot s + \frac{1}{R_1 \cdot C_3} \cdot \frac{1}{R_2 \cdot C_2} \cdot s + \frac{1}{R_1 \cdot R_2 \cdot R_3 \cdot C_1 \cdot C_2 \cdot C_3} =$

$= s^3 + s^2 \cdot (\frac{1}{R_1 \cdot C_3} + \frac{1}{R_2 \cdot C_3} + \frac{1}{R_3 \cdot C_1}) + s \cdot (\frac{1}{R_1 \cdot R_3 \cdot C_1 \cdot C_3} + \frac{1}{R_2 \cdot R_3 \cdot C_1 \cdot C_3} + \frac{1}{R_1 \cdot R_2 \cdot C_1 \cdot C_3} + \frac{1}{R_1 \cdot R_2 \cdot C_2 \cdot C_3}) + \frac{1}{R_1 \cdot R_2 \cdot R_3 \cdot C_1 \cdot C_2 \cdot C_3}$

$$adj(s \cdot I - A) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}^T$$

$a_{11} = s^2 + (\frac{1}{R_1 \cdot C_3} + \frac{1}{R_2 \cdot C_3}) \cdot s + \frac{1}{R_1 \cdot R_2 \cdot C_2 \cdot C_3} =$

$= s^2 + \frac{R_1 + R_2}{R_1 \cdot R_2 \cdot C_3} \cdot s + \frac{1}{R_1 \cdot R_2 \cdot C_2 \cdot C_3}$

$a_{12} = -\frac{1}{R_1 \cdot R_2 \cdot C_2 \cdot C_3}$

$a_{13} = \frac{1}{R_1 \cdot C_3} \cdot s$

$a_{21} = \frac{1}{R_1 \cdot R_2 \cdot C_1 \cdot C_3}$

$a_{22} = s^2 + (\frac{2}{R_3 \cdot C_1} + \frac{1}{R_2 \cdot C_3}) \cdot s + \frac{1}{R_1 \cdot R_3 \cdot C_1 \cdot C_3} + \frac{1}{R_2 \cdot R_3 \cdot C_1 \cdot C_3}$

$a_{23} = \frac{1}{R_1 \cdot C_3} \cdot s + \frac{1}{R_1 \cdot R_3 \cdot C_1 \cdot C_3}$

$a_{31} = -\frac{1}{R_2 \cdot C_1} \cdot s$

$a_{32} = -\frac{1}{R_2 \cdot C_2} \cdot s - \frac{1}{R_2 \cdot R_3 \cdot C_1 \cdot C_2}$

$a_{33} = s^2 + \frac{1}{R_3 \cdot C_1} \cdot s$

$$adj(s \cdot I - A) = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$$

$$H(s) = \begin{pmatrix} -1 & -1 & 0 \end{pmatrix} \cdot \frac{1}{det(s \cdot I - A)} \cdot adj(s \cdot I - A) \cdot \begin{pmatrix} \frac{1}{R_3 \cdot C_1} \\ 0 \\ 0 \end{pmatrix} + 1$$

$$H(s) = \frac{-\frac{1}{R_3 \cdot C_1} \cdot s^2 - \frac{R_1 + R_2}{R_1 \cdot R_2 \cdot R_3 \cdot C_1 \cdot C_3} \cdot s - \frac{1}{R_1 \cdot R_2 \cdot C_2 \cdot C_3} + \frac{1}{R_1 \cdot R_2 \cdot C_2 \cdot C_3}}{s^3 + s^2 \cdot (\frac{1}{R_1 \cdot C_3} + \frac{1}{R_2 \cdot C_3} + \frac{1}{R_3 \cdot C_1}) + s \cdot (\frac{1}{R_1 \cdot R_3 \cdot C_1 \cdot C_3} + \frac{1}{R_2 \cdot R_3 \cdot C_1 \cdot C_3} + \frac{1}{R_1 \cdot R_2 \cdot C_1 \cdot C_3} + \frac{1}{R_1 \cdot R_2 \cdot C_2 \cdot C_3}) + \frac{1}{R_1 \cdot R_2 \cdot R_3 \cdot C_1 \cdot C_2 \cdot C_3}} + 1$$

$$H(s) = \frac{s^3 + 759.9 \cdot s^2 + 41.12 \cdot s + 0.069}{s^3 + 759.9 \cdot s^2 + 43.68 \cdot s + 0.069}$$

3 Singularitatile sistemului

```
z = zero(H)
```

```
z = 3×1
10^2 ×
   -7.598243031646676
   -0.000523768743012
   -0.000017394139187
```

```
p = pole(H)
```

```
p = 3×1
10^2 ×
   -7.598243029248444
   -0.000558523546259
   -0.000016311767842
```

Figure 2: Poli si zerouri

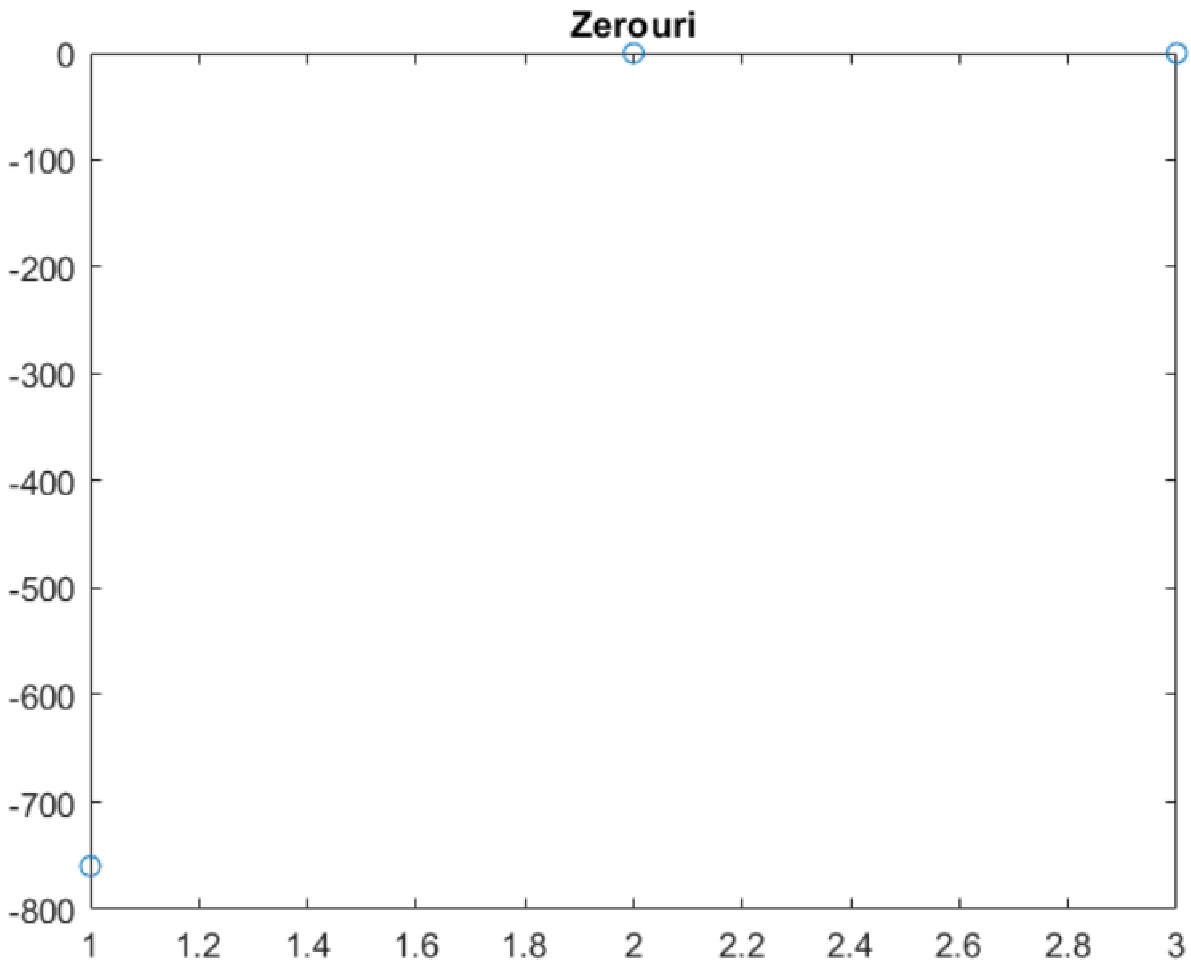


Figure 3: Zerouri

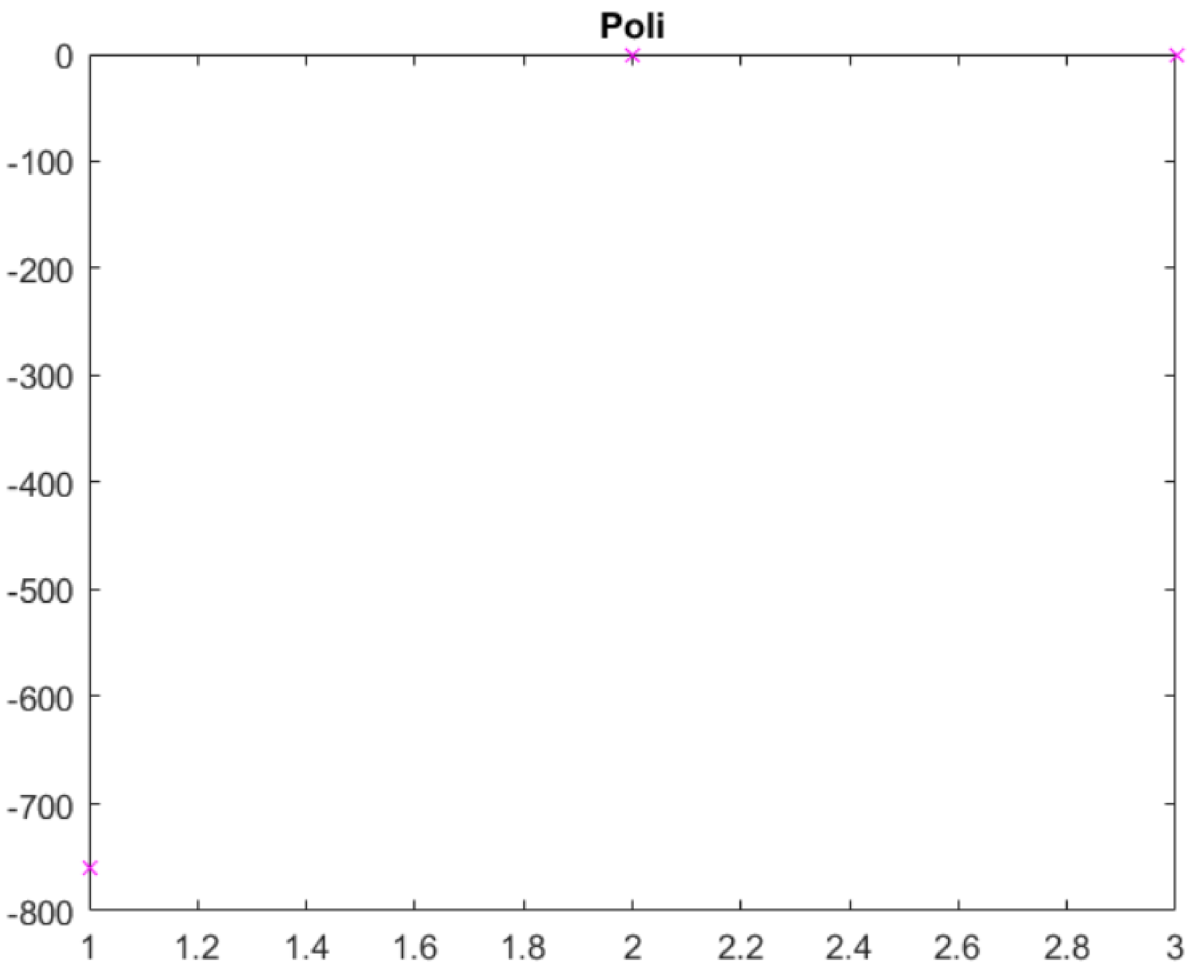


Figure 4: Poli

4 FCC si FCO

$\mathrm{b}_2 = -\frac{1}{R_3 \cdot C_1}$

$b_1 = -\frac{R_1 + R_2}{R_1 \cdot R_2 \cdot R_3 \cdot C_1 \cdot C_3}$

$b_0 = 0$

$a_2 = \frac{1}{R_1 \cdot C_3} + \frac{1}{R_2 \cdot C_3} + \frac{1}{R_3 \cdot C_1}$

$a_1 = \frac{1}{R_1 \cdot R_3 \cdot C_1 \cdot C_3} + \frac{1}{R_2 \cdot R_3 \cdot C_1 \cdot C_3} + \frac{1}{R_1 \cdot R_2 \cdot C_1 \cdot C_3} + \frac{1}{R_1 \cdot R_2 \cdot C_2 \cdot C_3}$

$a_0 = \frac{1}{R_1 \cdot R_2 \cdot R_3 \cdot C_1 \cdot C_2 \cdot C_3}$

FCC: $\left(\begin{array}{ccc|c} -a_2 & -a_1 & -a_0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline b_2 & b_1 & b_0 & 1 \end{array}\right) = \left(\begin{array}{ccc|c} -759.9 & -43.68 & -0.069 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline -0.0033 & -2.5585 & 0 & 1 \end{array}\right)$

FCO: $\left(\begin{array}{ccc|c} -a_2 & 1 & 0 & b_2 \\ -a_1 & 0 & 1 & b_1 \\ -a_0 & 0 & 0 & b_0 \\ \hline 1 & 0 & 0 & 1 \end{array}\right) = \left(\begin{array}{ccc|c} -759.9 & 1 & 0 & -0.0033 \\ -43.68 & 0 & 1 & -2.5585 \\ -0.069 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 1 \end{array}\right)$

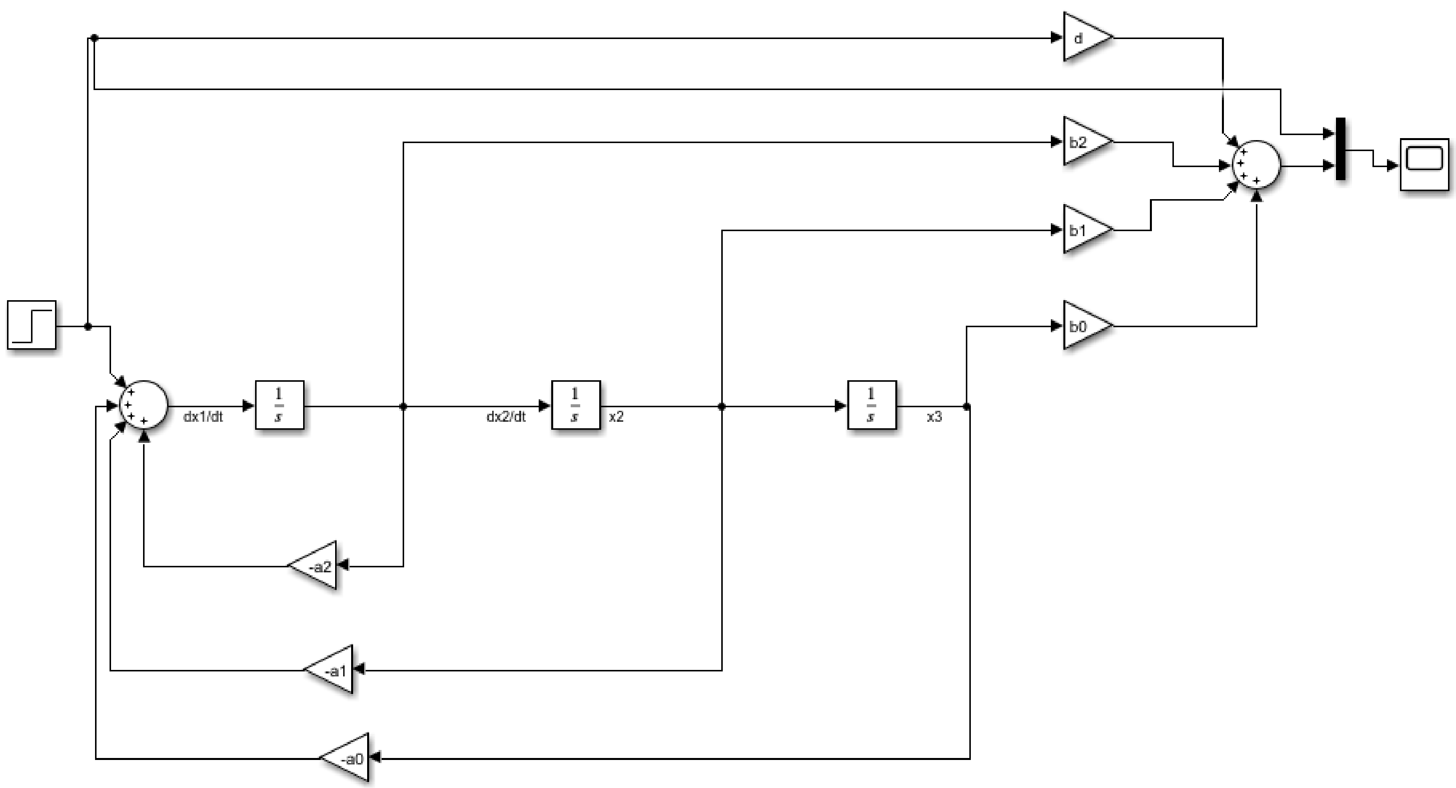


Figure 5: FCC

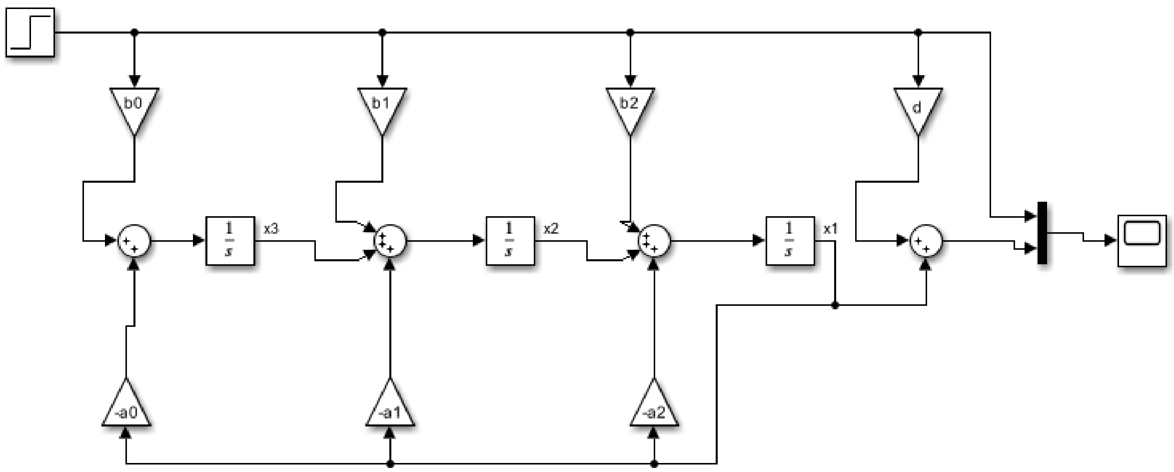


Figure 6: FCO

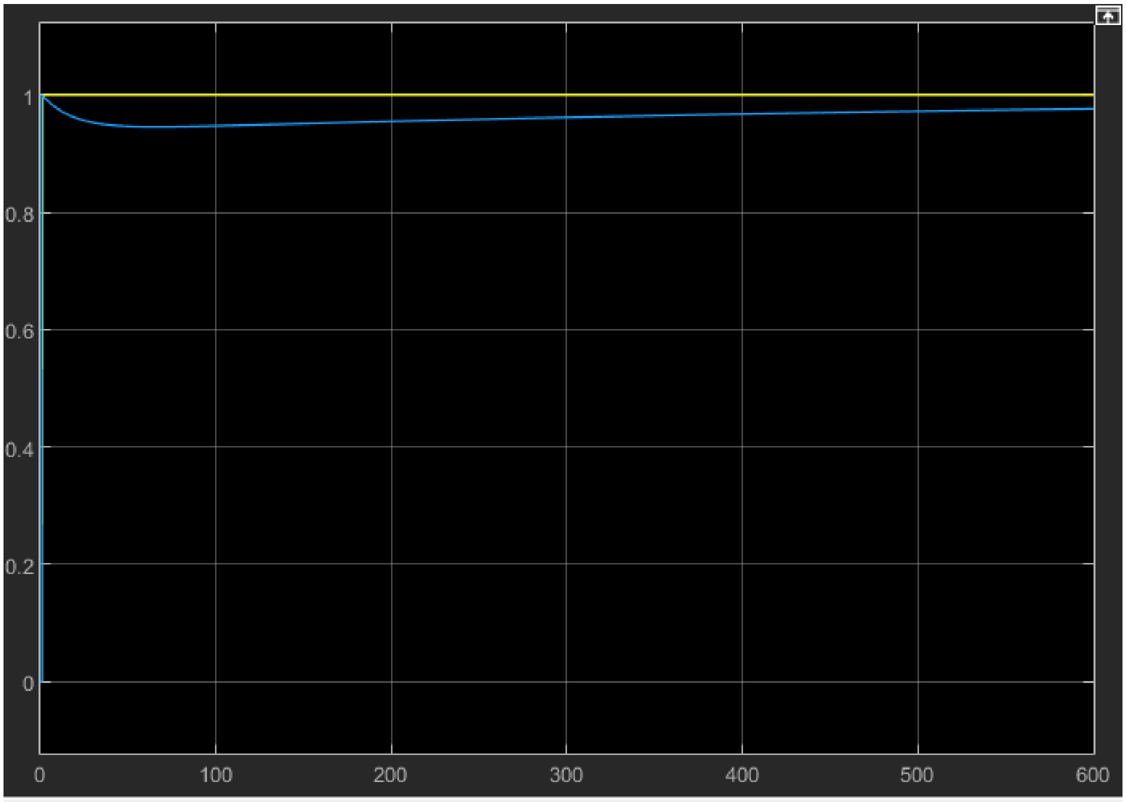


Figure 7: Simulare

5 Forma minimala a functiei de transfer

```
format long
p_Markov = deconv([numarator,zeros(1,6)],numitor);
m_Hankle = [p_Markov(2) p_Markov(3) p_Markov(4);
            p_Markov(3) p_Markov(4) p_Markov(5);
            p_Markov(4) p_Markov(5) p_Markov(6)
            ]
```

```
m_Hankle = 3×3
10^4 ×
   -0.000000336700337   0.000000001133660   0.000013844772796
    0.000000001133660   0.000013844772796  -0.010520416892992
    0.000013844772796  -0.010520416892992   7.993668478111492
```

```
det(m_Hankle)
```

```
ans =
   -8.256042149634151e-05
```

Figure 8: Forma minimala a fdt

Determinantul matricii Hankle este mai mic decat 0 ceea ce demonstreaza faptul ca functia de transfer este scrisa deja in forma minimala.

---

6 Stabilitatea interna

$$\det(\lambda \cdot I_3 - A) = \begin{vmatrix} \lambda + 0.0034 & 0 & 0.0541 \\ 0 & \lambda & 0.0541 \\ -379.9392 & -379.9392 & \lambda + 759.8784 \end{vmatrix} =$$

$= (\lambda^2 + 0.0034 \cdot \lambda) \cdot (\lambda + 759.8784) + 20.5547 \cdot \lambda + 20.5547 \cdot (\lambda + 0.0034) =$
$= \lambda^3 + 0.0034 \cdot \lambda^2 + 2.5835 \cdot \lambda + 2005547 \cdot \lambda + 20.5547 \cdot \lambda + 0.0698 =$
$= \lambda^3 + 759.8818 \cdot \lambda^2 + 43.6956 \cdot \lambda + 0.0698$

λ^3	1	43.6956
λ^2	759.8818	0.0698
λ^1	b_1	b_2
λ^0	c_1	

$$b_1 = -\frac{\begin{vmatrix} 1 & 43.6956 \\ 759.8818 & 0.0698 \end{vmatrix}}{759.8818} = 43.6955$$

$b_2 = 0$

$$c_1 = -\frac{\begin{vmatrix} 759.8818 & 0.0698 \\ 43.6955 & 0 \end{vmatrix}}{43.6955} = 0.0698$$

$1 > 0$
$759.8818 > 0$
$43.6955 > 0$
$0.0698 > 0$
=> nu se schimba semnul => sistemul este stabil intern

Polii sistemului sunt negativi => sistemul este stabil extern

7 Lyapunov

```
A
```

```
A = 3×3
   -0.0034         0   -0.0541
         0         0   -0.0541
  379.9392  379.9392 -759.8784
```

```
A'
```

```
ans = 3×3
   -0.0034         0  379.9392
         0         0  379.9392
   -0.0541   -0.0541 -759.8784
```

```
Q = eye(length(A))
```

```
Q = 3×3
     1     0     0
     0     1     0
     0     0     1
```

Figure 9: Matricile A, A' si Q

```
P = lyap(A',Q)
```

```
P = 3×3
    150.6750 -146.3250    0.0000
   -146.3250  169.4257   -0.0013
      0.0000   -0.0013    0.0007
```

```
eig(P)
```

```
ans = 3×1
      0.0007
     13.4253
    306.6754
```

Figure 10: Matricea P si valorile sale proprii

Valorile proprii ale matricei P sunt strict pozitive, deci sistemul este intern asimptotic stabil.

```
sys = ss(A,B,C,D)
```

```
sys =

  A =
              x1         x2         x3
   x1  -0.003367          0   -0.05411
   x2          0          0   -0.05411
   x3      379.9      379.9     -759.9

  B =
             u1
   x1  0.003367
   x2         0
   x3         0

  C =
       x1  x2  x3
   y1  -1  -1   0

  D =
       u1
   y1   1

Continuous-time state-space model.
```

Figure 11: Sistemul

```
t1 = 0:0.01:4500;
u = zeros(1,length(t1));
[y,t1,x] = lsim(sys,u,t1,[1,2,3]);

V = zeros(1,length(t1));

for i = 1:length(t1)
   V(i)= x(i,:) *P*x(i,:)';
end

plot(t1,V)
```

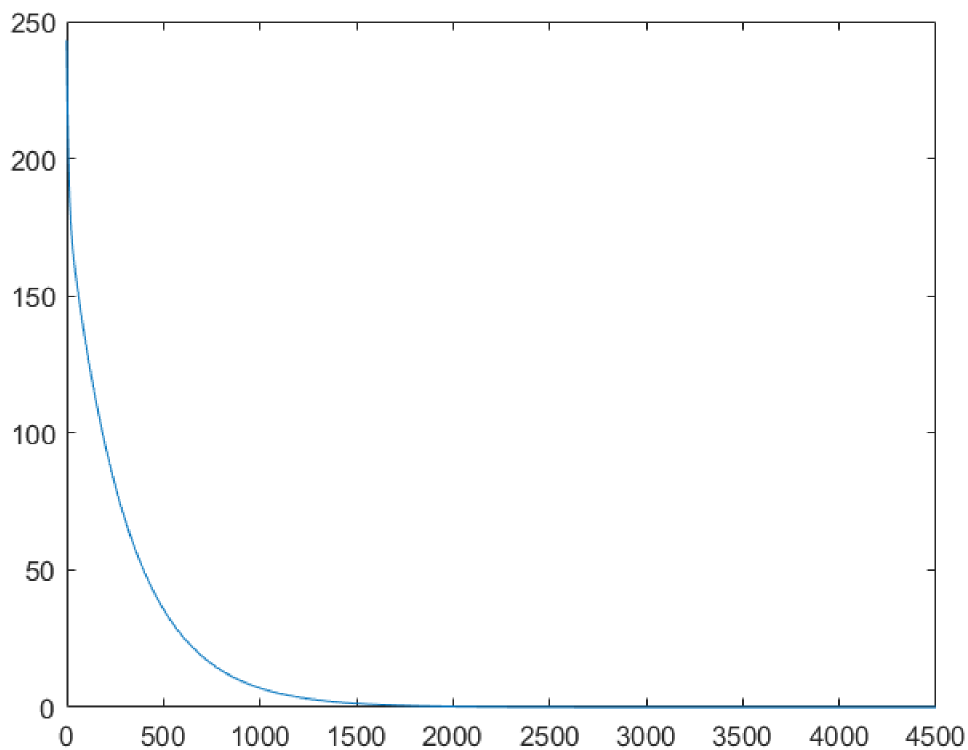


Figure 12: Reprezentarea in timp a energiei

8 Raspunsul sistemului

Raspunsul sistemelor LTI se determina cu ajutorul relatiei:

y(t) = $\mathrm{L}^{-1}\{H(s) \cdot U(s)\}$

unde U(s) reprezinta intrarea data in functie de raspunsul pe care dorim sa-l evidentiem.

$H(s) = \frac{s^3+759.9 \cdot s^2+41.12 \cdot s+0.069}{s^3+759.9 \cdot s^2+43.68 \cdot s+0.069}$

Pentru functia pondere avem U(s) = 1. Cu ajutorul comenzii *residue* din MATLAB a desfacut functia de transfer in fractii simple dupa care am aplicat Laplace invers si am obtinut functia pondere.

$$H(s) = \frac{0.0000002}{s+759.8243} + \frac{-0.0035}{s+0.0559} - \frac{0.0001}{s+0.0016}$$

$$h(t) = \delta(t) + 0.0034 \cdot e^{-760 \cdot t} - 0.0035 \cdot e^{-0.056 \cdot t} + 0.0001 \cdot e^{-0.0016 \cdot t}$$

moduri: $\mathrm{e}^{-760 \cdot t}, e^{-0.056 \cdot t}, e^{-0.0016 \cdot t}$
componenta stationara: $\delta(t)$
componenta tranzitorie: $0.0034 \cdot e^{-760 \cdot t} - 0.0035 \cdot e^{-0.056 \cdot t} + 0.0001 \cdot e^{-0.0016 \cdot t}$

Pentru deducerea raspunsului indicial (sau raspunsul la treapta unitara) se considera $U(s) = \frac{1}{s}$. Apoi se procedeaza ca la pasul anterior.

$$H(s) = \frac{s^3+759.9 \cdot s^2+41.12 \cdot s+0.069}{s^3+759.9 \cdot s^2+43.68 \cdot s+0.069} \cdot \frac{1}{s}$$

$$H(s) = \frac{s^3+759.9 \cdot s^2+41.12 \cdot s+0.069}{s^4+759.9 \cdot s^3+43.68 \cdot s^2+0.069 \cdot s}$$

$$H(s) = \frac{-3.1563 \cdot 10^{-10}}{s+759.8243} + \frac{0.0621}{s+0.0559} - \frac{-0.0621}{s+0.0016} + \frac{0.99}{s}$$

$$y(t) = 1 - 4.43 \cdot 10^{-6} \cdot e^{-760 \cdot t} + 0.0621 \cdot e^{-0.0559 \cdot t} - 0.0621 \cdot e^{-0.0016 \cdot t}$$

moduri: $\mathrm{e}^{-760 \cdot t}, e^{-0.0559 \cdot t}, e^{-0.0016 \cdot t}$
componenta stationara: 1
componenta tranzitorie: $-4.43 \cdot 10^{-6} \cdot e^{-760 \cdot t} + 0.0621 \cdot e^{-0.0559 \cdot t} - 0.0621 \cdot e^{-0.0016 \cdot t}$

Pentru deducerea raspunsului la rampa se considera

$$U(s) = \frac{1}{s^2}.$$

Apoi se procedeaza ca la pasii anteriori.

$$H(s) = \frac{s^3 + 759.9 \cdot s^2 + 41.12 \cdot s + 0.069}{s^3 + 759.9 \cdot s^2 + 43.68 \cdot s + 0.069} \cdot \frac{1}{s^2}$$

$$H(s) = \frac{s^3 + 759.9 \cdot s^2 + 41.12 \cdot s + 0.069}{s^5 + 759.9 \cdot s^4 + 43.68 \cdot s^3 + 0.069 \cdot s^2}$$

$$H(s) = \frac{4.154 \cdot 10^{-13}}{s + 759.8243} + \frac{-1.1118}{s + 0.0559} - \frac{38.0718}{s + 0.0016} + \frac{0.99}{s} + \frac{-36.9599}{s}$$

$$y(t) = t - 37.1 + 5.3 \cdot 10^{-9} \cdot e^{-760 \cdot t} - 1.11 \cdot e^{-0.0559 \cdot t} + 38.2 \cdot e^{-0.0016 \cdot t}$$

moduri: $\mathrm{e}^{-760 \cdot t}, e^{-0.0559 \cdot t}, e^{-0.0016 \cdot t}$
componenta stationara: $t - 37.1$
componenta tranzitorie: $5.3 \cdot 10^{-9} \cdot e^{-760 \cdot t} - 1.11 \cdot e^{-0.0559 \cdot t} + 38.2 \cdot e^{-0.0016 \cdot t}$

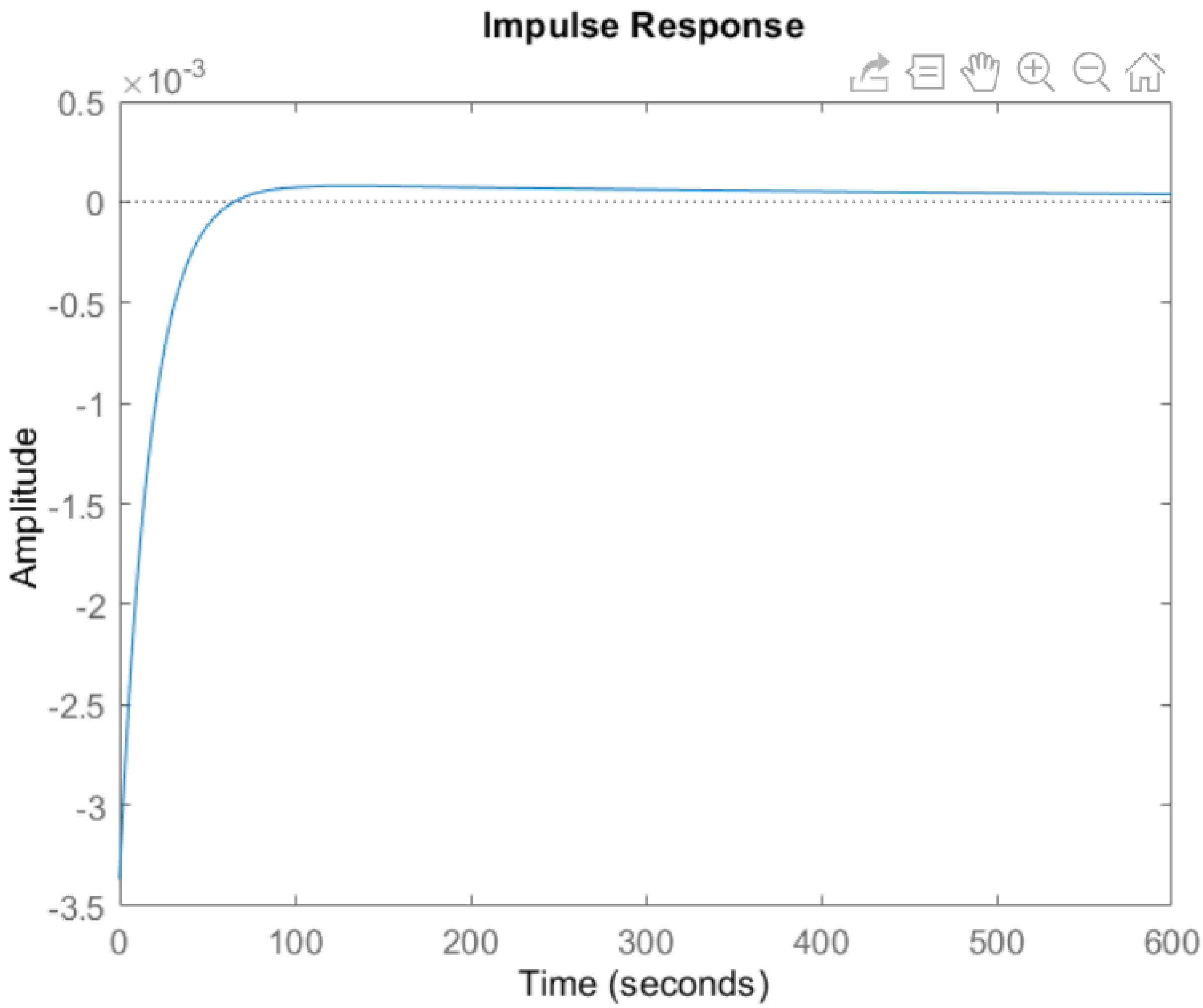


Figure 13: Raspunsul la impuls (functia pondere)

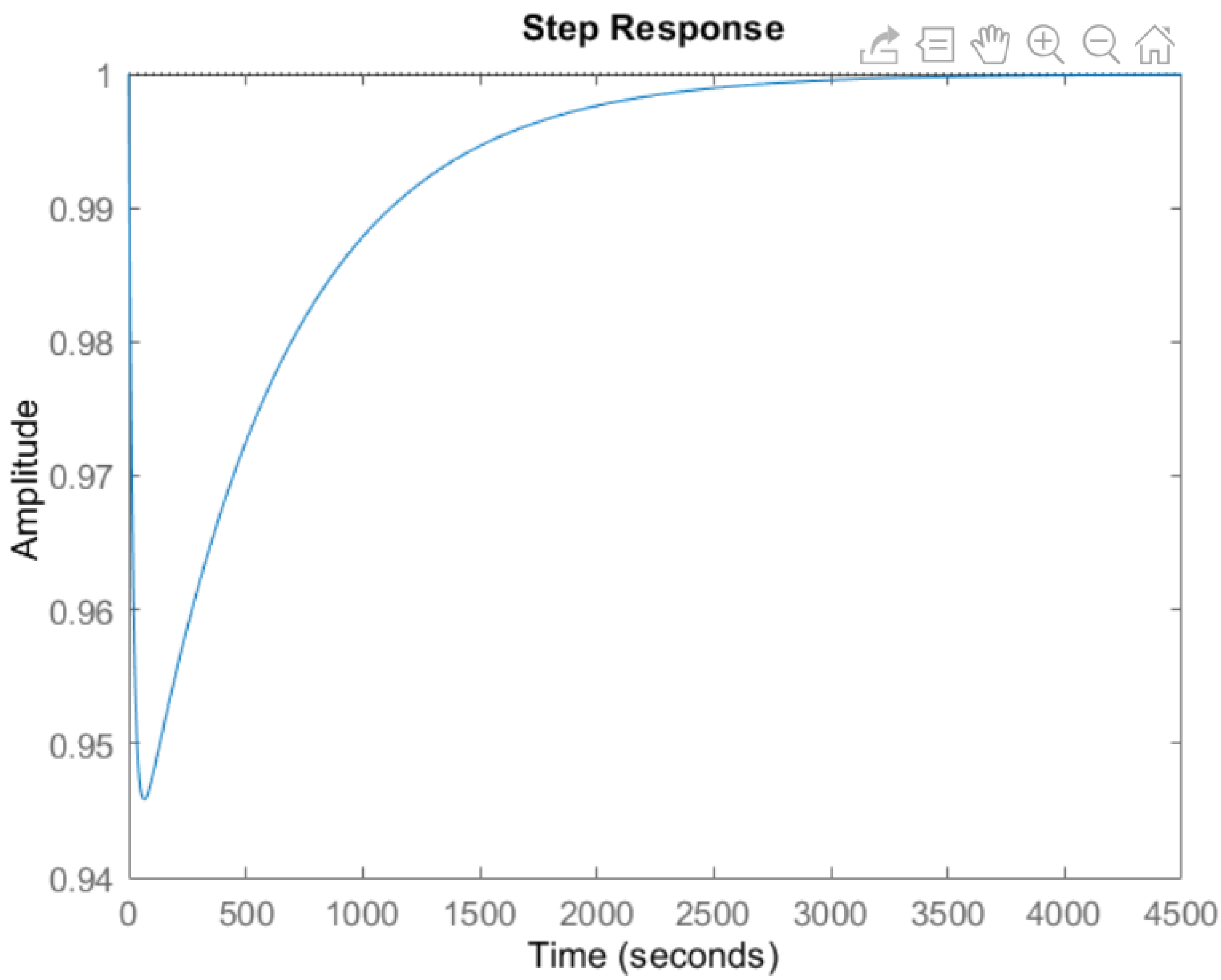


Figure 14: Raspunsul indicial

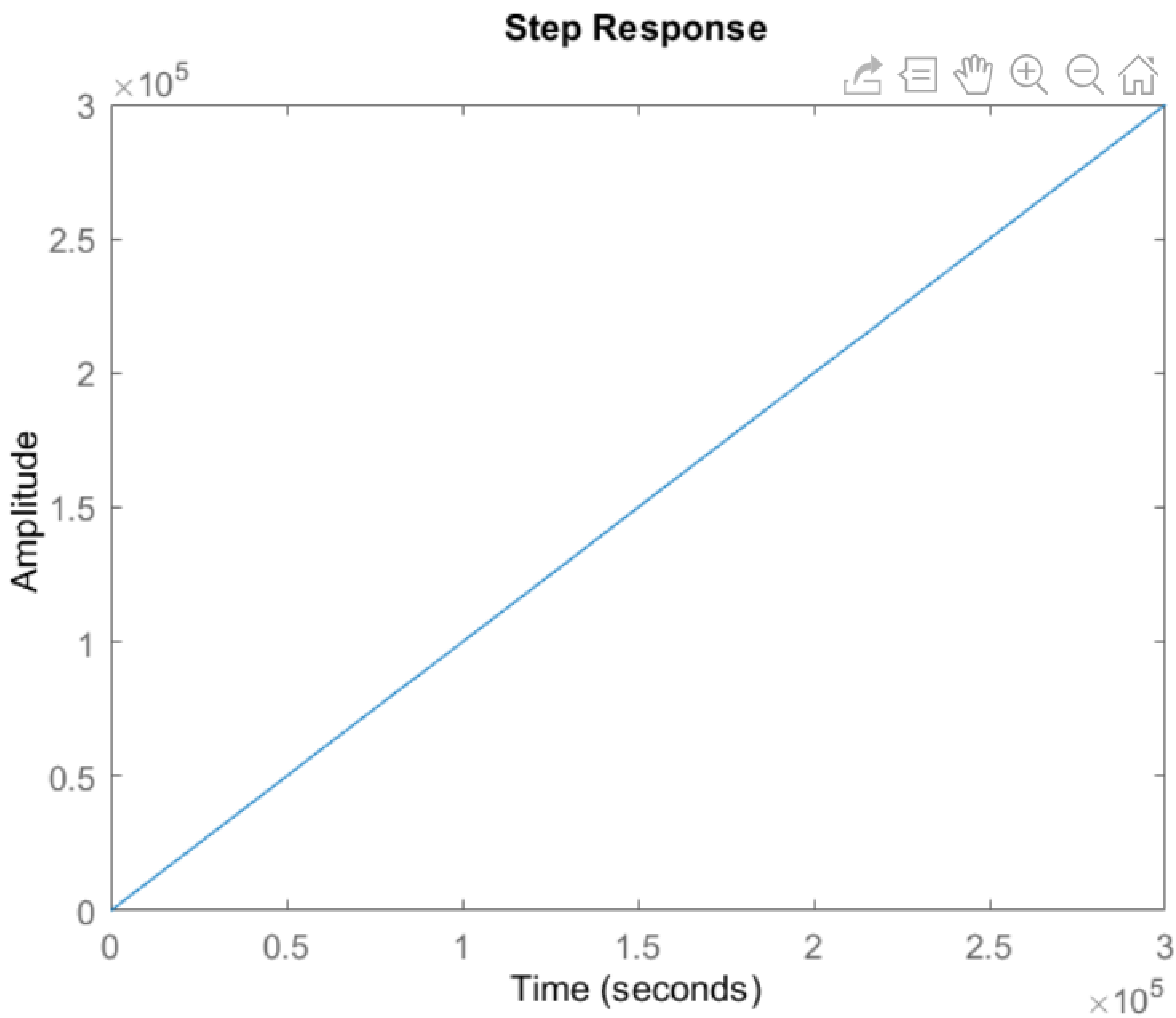


Figure 15: Raspunsul la rampa

9 Performante

Factorul de proportionalitate:
K = H(0)

$$H(0) = \frac{0.069}{0.069} = 1 => K = 1$$

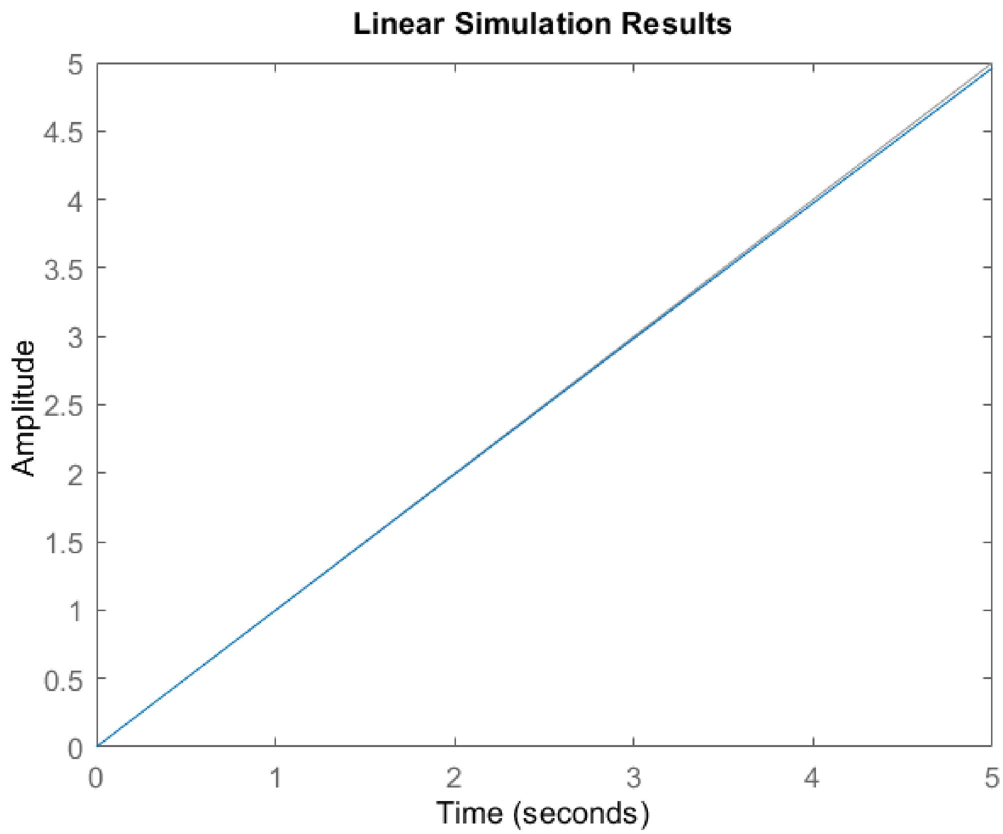
Constante de timp:
$\overset{\circ}{T}_1 = 0.001316093728294$
$\overset{\circ}{T}_2 = 19.0923$
$\overset{\circ}{T}_3 = 574.9$

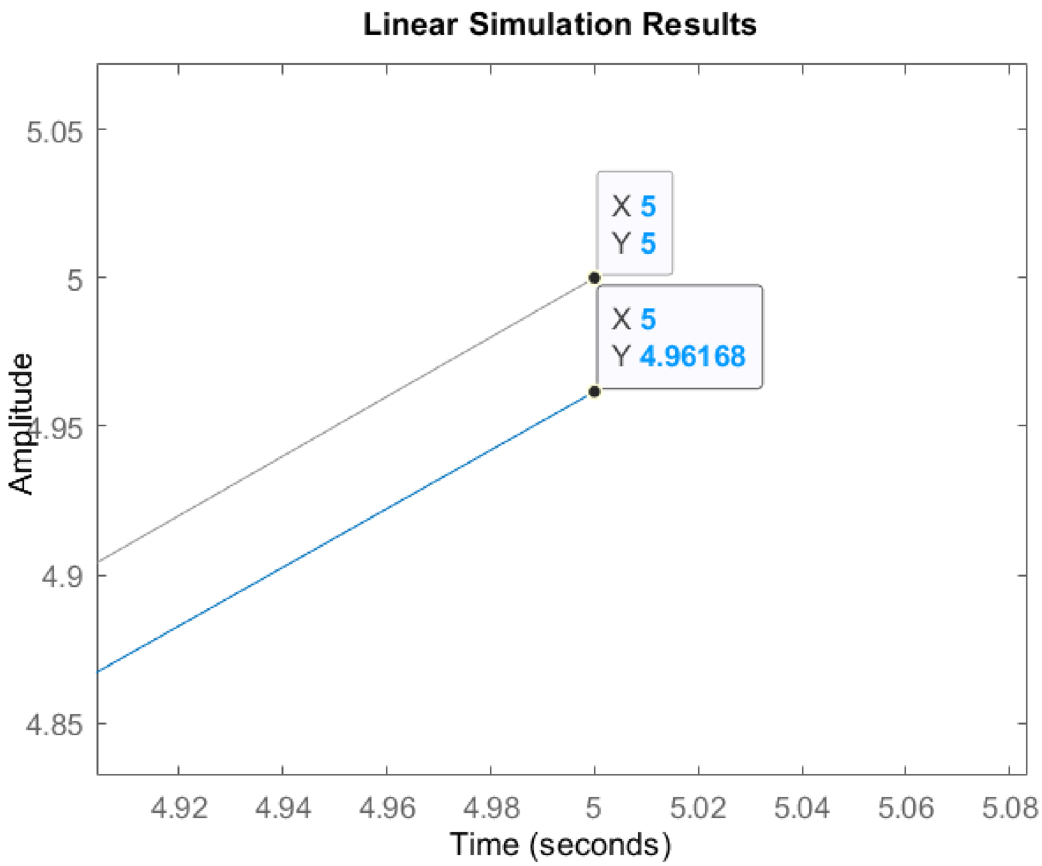
$\hat{T}_1 = 0.001316093728709$
$\hat{T}_2 = 17.9043$
$\hat{T}_3 = 613.05$

Datorita polilor reali, nu se poate calcula pulsatia naturala a oscilatiilor si factorul de amortizare.

Timpul de raspuns:
$t_{r1} = 4 \cdot \hat{T}_1 = 0.0052$
$t_{r2} = 4 \cdot \hat{T}_2 = 71.6172$
$t_{r3} = 4 \cdot \hat{T}_3 = 2452.21$

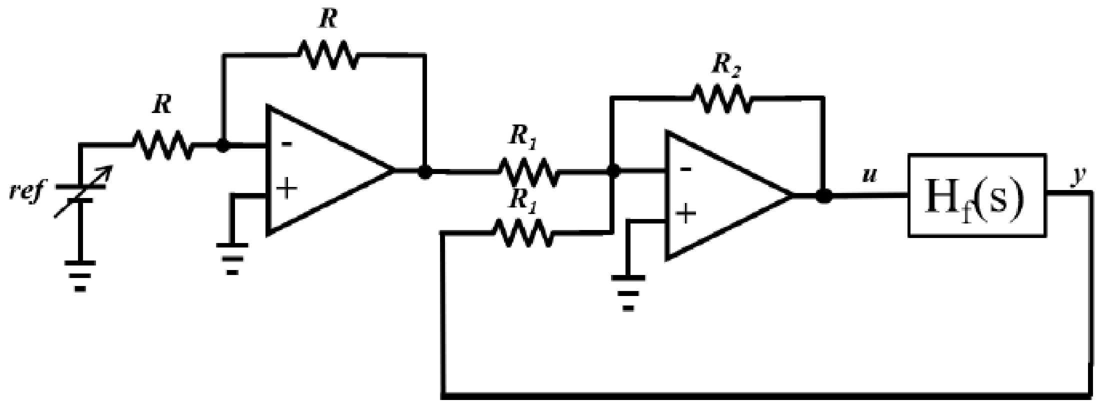
Eroarea la pozitie/viteza:
$\epsilon_{ssp} = 1 - K = 0 =>$ eroarea la viteza este finita
$\epsilon_{ssv} = 0.03832$

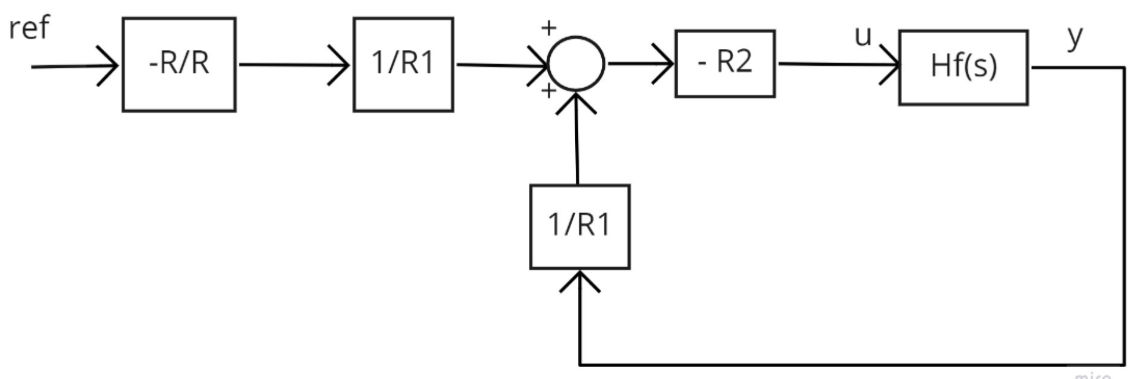

Linear Simulation Results
Amplitude
Time (seconds)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
1
2
3
4
5

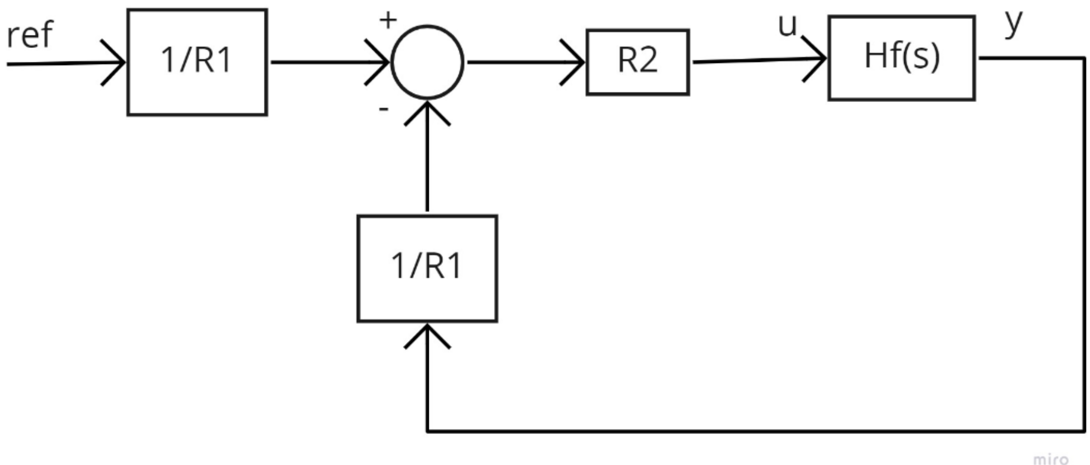


De asemenea, datorita polilor reali nu exista suprareglaj.

10 Sistem de reglare cu regulator proportional







$$H_o = \frac{\frac{R_2}{R_1} \cdot H_f(s)}{1 + \frac{R_2}{R_1} \cdot H_f(s)}$$

$$H_f(s) = \frac{s^3 + 759.9 \cdot s^2 + 41.12 \cdot s + 0.069}{s^3 + 759.9 \cdot s^2 + 43.68 \cdot s + 0.069}$$

$$H_o = \frac{\frac{R_2}{R_1} \cdot \frac{s^3+759.9 \cdot s^2+41.12 \cdot s+0.069}{s^3+759.9 \cdot s^2+43.68 \cdot s+0.069}}{1 + \frac{R_2}{R_1} \cdot \frac{s^3+759.9 \cdot s^2+41.12 \cdot s+0.069}{s^3+759.9 \cdot s^2+43.68 \cdot s+0.069}}$$

Locul radacinilor

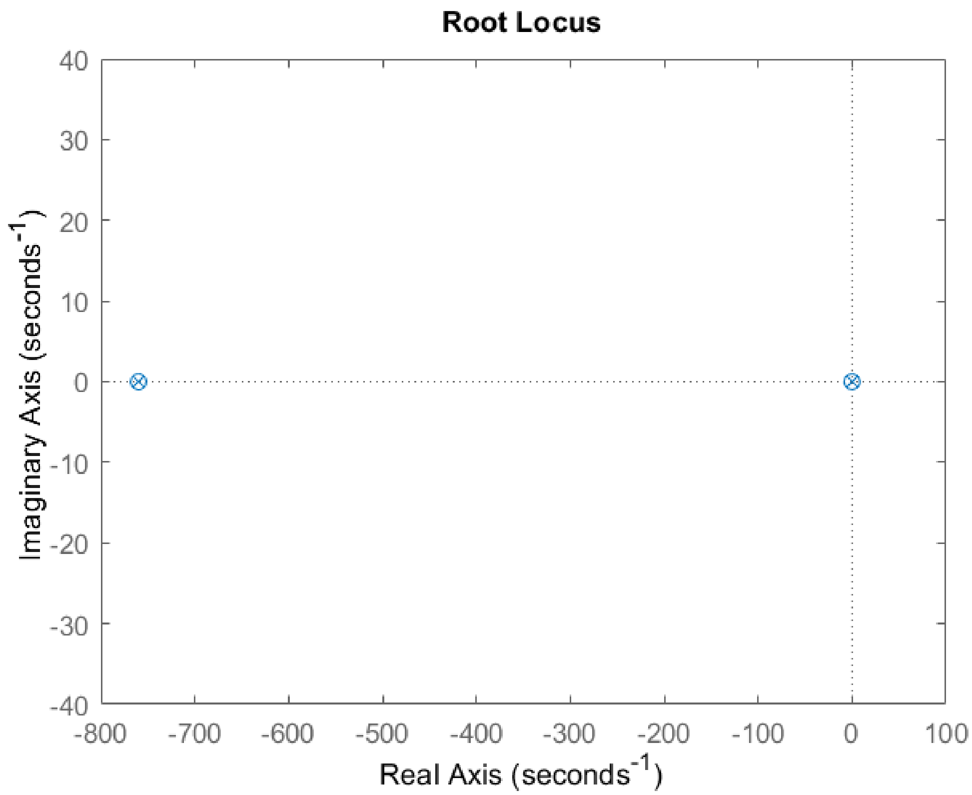
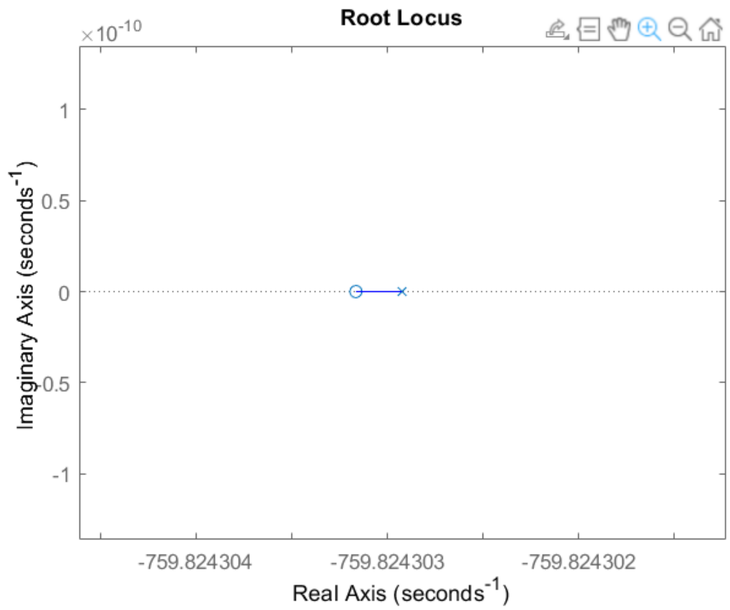
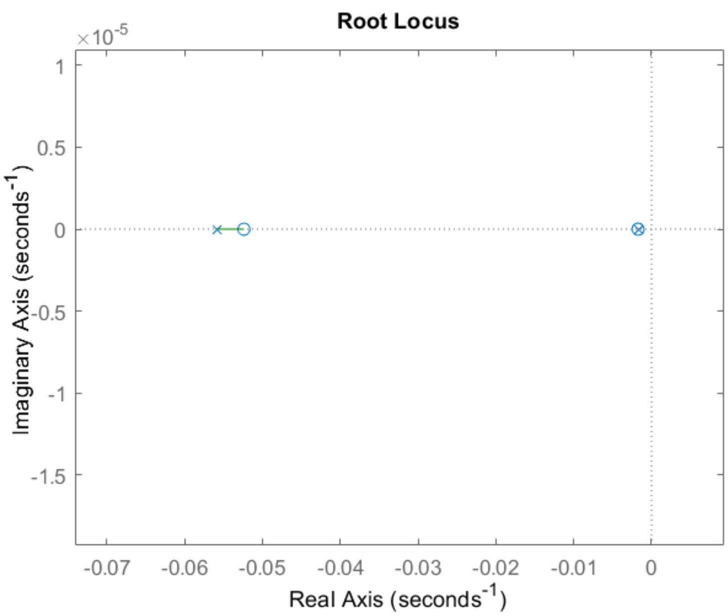
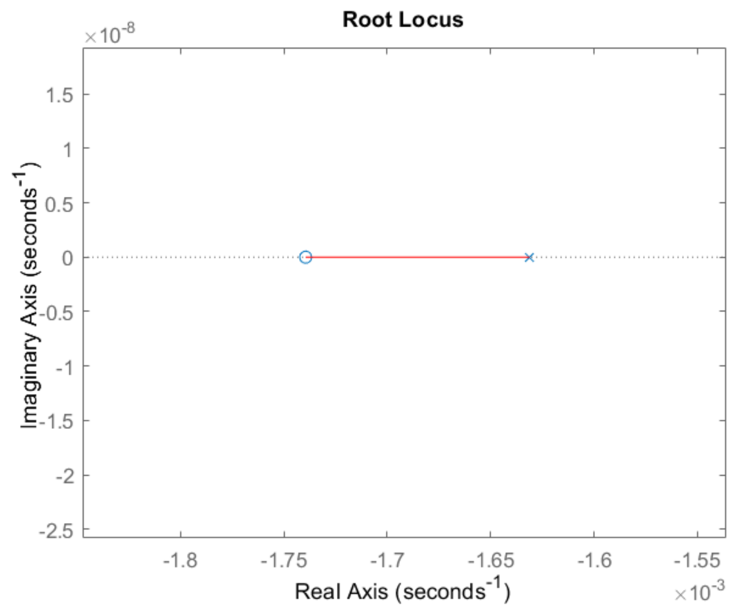


Figure 16: Locul radacinilor


Root Locus
×10^-10
1
0.5
0
-0.5
-1
Imaginary Axis (seconds^-1)
-759.824304
-759.824303
-759.824302
Real Axis (seconds^-1)


Root Locus
×10^-5
1
0.5
0
-0.5
-1
-1.5
Imaginary Axis (seconds^-1)
-0.07
-0.06
-0.05
-0.04
-0.03
-0.02
-0.01
0
Real Axis (seconds^-1)



n = numarul de poli
m = numarul de zerouri
n-m=0, deci nu exista asimptote
$\phi_{\hat{s}_1} = \pi$
$\phi_{\overset{\circ}{s}_1} = 0$
$\phi_{\hat{s}_2} = 0$
$\phi_{\overset{\circ}{s}_2} = \pi$
$\phi_{\hat{s}_3} = \pi$
$\phi_{\overset{\circ}{s}_3} = 0$

Stabilitate: -sistemul este extern stabil pentru orice $\frac{R_2}{R_1} \in (0, \infty)$

Regimuri si moduri:
- regim aperiodic amortizat
- moduri: $e^{\hat{s}_1 t}, e^{\hat{s}_2 t}, e^{\hat{s}_3 t}$

Senzitivitate:
- mica deoarece nu se schimba nici regimul nici stabilitatea sistemului, iar drumul polului dominant este finit.

Sistemul are numai poli reali, deci nu exista suprareglaj.

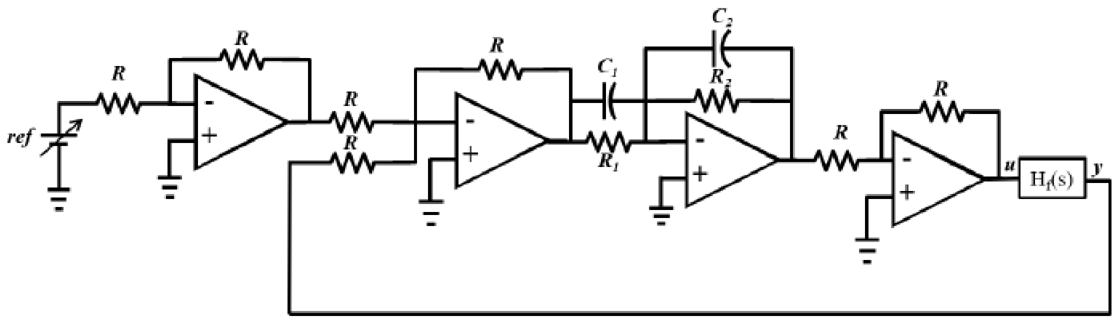
Timp de raspuns minim

$t_{r_{minim}} = 4 \cdot \overset{\circ}{T}_3 = 2299.6$

Din grafic, am extras timpul de raspuns minim ca fiind aproximativ 2340.

Timpul de raspuns este minim atunci cand modulul polului dominant este maxim si anume atunci cand k $->\infty$ si polul ajunge in zerou.

11 Sistem de reglare cu regulator de tip *Lead/Lag*



Determinare $\mathbf{H}_R(s)$

$Z_1(s) = R_1||C_1 = \frac{R_1 \cdot \frac{1}{C_1 \cdot s}}{R_1 + \frac{1}{C_1 \cdot s}} = \frac{R_1}{R_1 \cdot C_1 \cdot s + 1}$

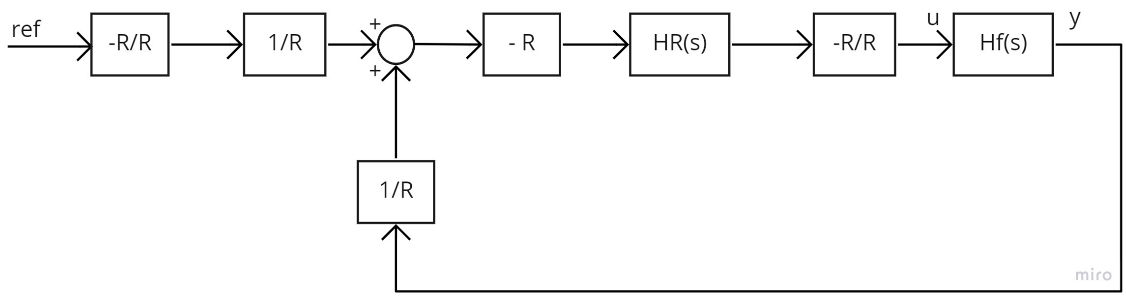
$$Z_2(s) = \frac{R_2}{R_2 \cdot C_2 \cdot s + 1}$$

$$H_R(s) = \frac{-\frac{R_2}{R_2 \cdot C_2 \cdot s + 1}}{\frac{R_1}{R_1 \cdot C_1 \cdot s + 1}} = -\frac{R_2}{R_1} \cdot \frac{R_1 \cdot C_1 \cdot s + 1}{R_2 \cdot C_2 \cdot s + 1}$$

$$K = -\frac{R_2}{R_1}$$

$$T_1 = R_1 \cdot C_1$$

$$T_2 = R_2 \cdot C_2$$



$$H_f(s) = \frac{s^3 + 759.9 \cdot s^2 + 41.12 \cdot s + 0.069}{s^3 + 759.9 \cdot s^2 + 43.68 \cdot s + 0.069}$$

$$H_o = K \cdot \frac{\frac{T_1 \cdot s+1}{T_2 \cdot s+1} \cdot H_f(s)}{1 + K \cdot \frac{T_1 \cdot s+1}{T_2 \cdot s+1} \cdot H_f(s)}$$

$$H_o = \frac{K \cdot (T_1 \cdot s + 1)(s^3 + 759.9 \cdot s^2 + 41.12 \cdot s + 0.069)}{(T_2 \cdot s + 1) \cdot (s^3 + 759.9 \cdot s^2 + 43.68 \cdot s + 0.069) + K \cdot (T_1 \cdot s + 1) \cdot (s^3 + 759.9 \cdot s^2 + 41.12 \cdot s + 0.069)}$$

$$H_o = \frac{K \cdot (T_1 \cdot s + 1)(s^3 + 759.9 \cdot s^2 + 41.12 \cdot s + 0.069)}{1 + T_1 \cdot \frac{K \cdot s \cdot (s^3+759.9 \cdot s^2+41.12 \cdot s+0.069)}{(T_2 \cdot s+1) \cdot (s^3+759.9 \cdot s^2+43.68 \cdot s+0.069)+K \cdot (s^3+759.9 \cdot s^2+41.12 \cdot s+0.069)}}$$

Pentru K=1 si $\mathrm{T}_2 = 1$ avem:

$$H'_{des} = \frac{s \cdot (s^3+759.9 \cdot s^2+41.12 \cdot s+0.069)}{(s+1) \cdot (s^3+759.9 \cdot s^2+43.68 \cdot s+0.069)+s^3+759.9 \cdot s^2+41.12 \cdot s+0.069}$$

```
Hdes =

      s^4 + 759.9 s^3 + 41.12 s^2 + 0.069 s
  ---------------------------------------------
  s^4 + 761.9 s^3 + 1564 s^2 + 84.87 s + 0.138

Continuous-time transfer function.
```

```
pole(Hdes)
```

```
ans = 4×1
10^2 ×
    -7.598422461501188
    -0.020020088593982
    -0.000540671280354
    -0.000016778624478
```

```
zero(Hdes)
```

```
ans = 4×1
10^2 ×
                     0
    -7.598458838824237
    -0.000523825658373
    -0.000017335517391
```

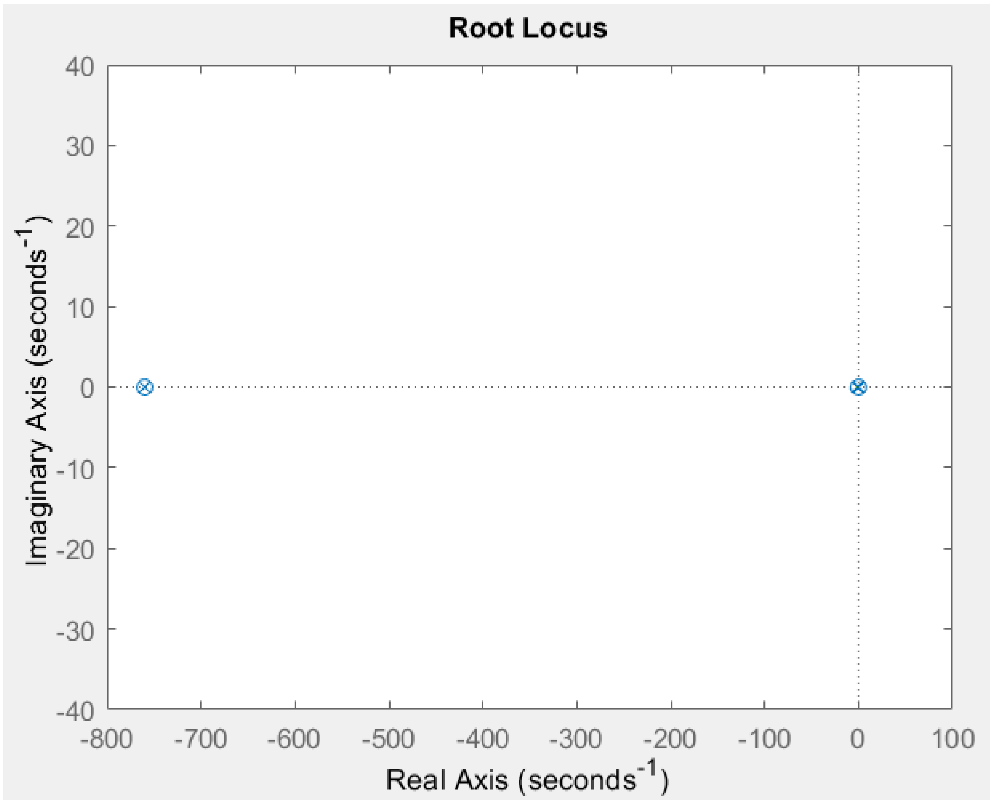
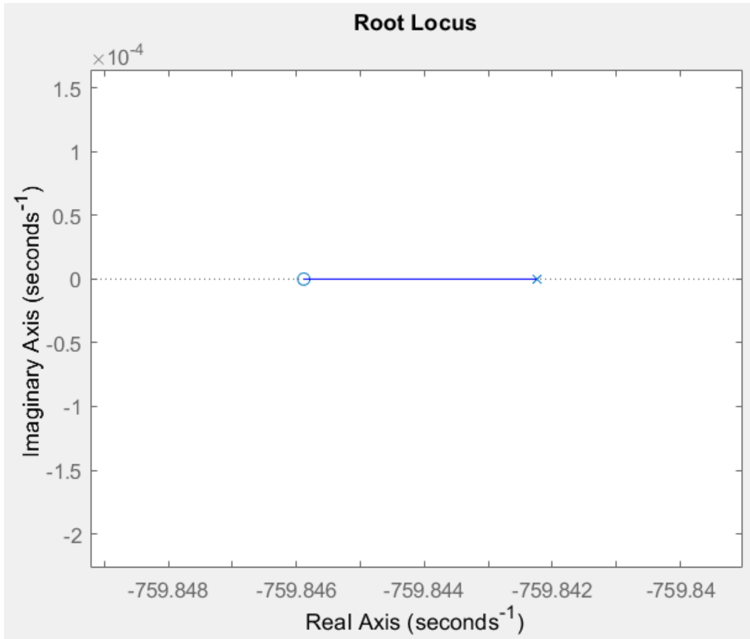
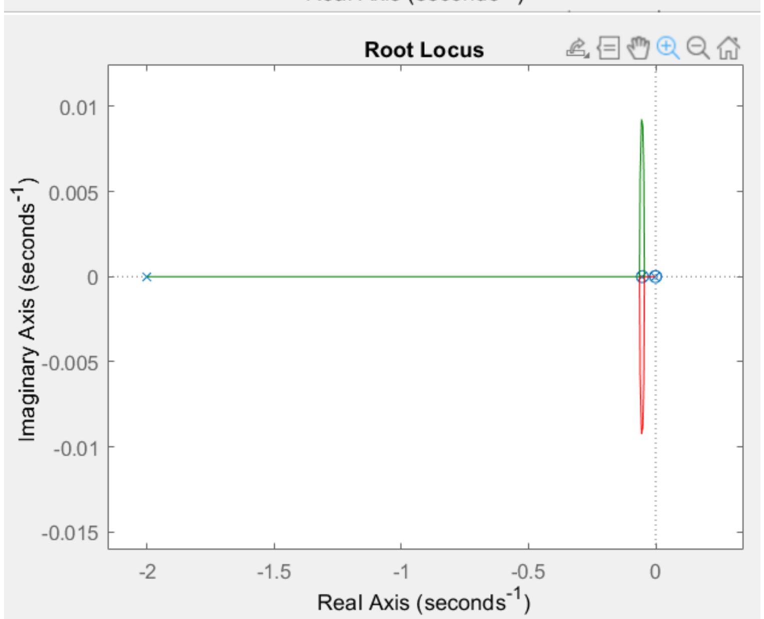
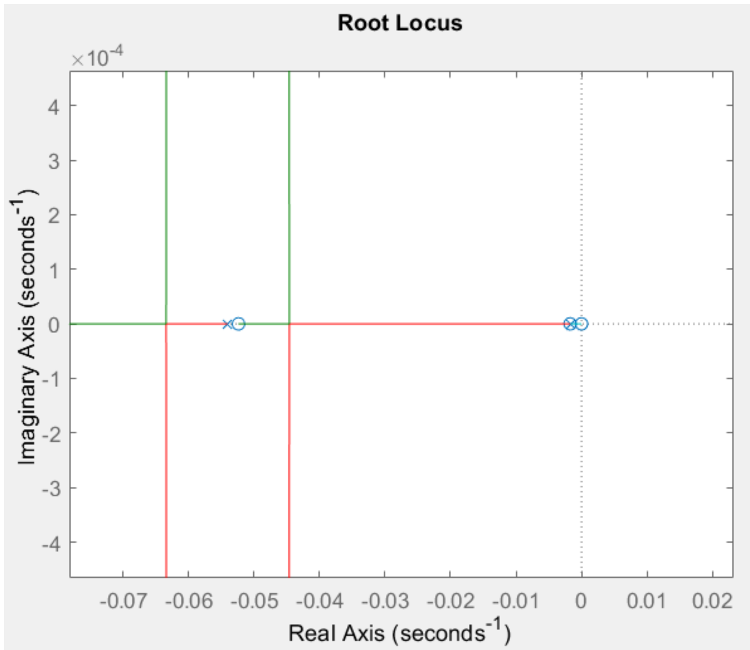
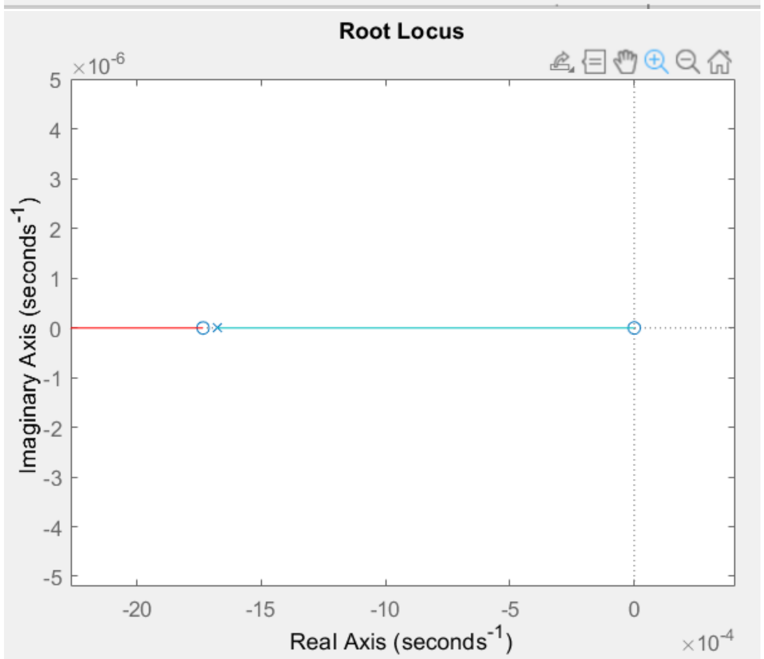


Figure 17: Locul radacinilor


Root Locus
×10^-4
1.5
1
0.5
0
-0.5
-1
-1.5
-2
Imaginary Axis (seconds^-1)
-759.848
-759.846
-759.844
-759.842
-759.84
Real Axis (seconds^-1)


Root Locus
0.01
0.005
0
-0.005
-0.01
-0.015
Imaginary Axis (seconds^-1)
-2
-1.5
-1
-0.5
0
Real Axis (seconds^-1)





n=4
m=4
n-m=0 = Nu exista asimptote
$\phi_{\hat{s}_1} = \pi$

$\phi_{\overset{\circ}{s}_1} = 0$
$\phi_{\hat{s}_2} = 0$
$\phi_{\overset{\circ}{s}_2} = \pi$
$\phi_{\hat{s}_3} = \pi$
$\phi_{\overset{\circ}{s}_3} = 0$
$\phi_{\hat{s}_4} = 0$
$\phi_{\overset{\circ}{s}_4} = \pi$
Din grafic am dedus:
$k_{despr} = 25.9$
$k_{apr} = 53.5$

Stabilitate:
- sistemul este stabil extern pentru orice $k \in (0, \infty)$

Regimuri si moduri:

• $k \in (0, k_{despr})$: regim aperiodic amortizat;
  moduri: $e^{\hat{s}_1 t}, e^{\hat{s}_2 t}, e^{\hat{s}_3 t}, e^{\hat{s}_4 t}$

• $k = k_{despr}$: regim aperiodic amortizat;
  moduri: $e^{\hat{s}_1 t}, e^{\hat{s}_2 t}, te^{\hat{s}_2 t}, e^{\hat{s}_4 t}$

• $k \in (k_{despr}, k_{apr})$: regim aperiodic amortizat;
  moduri: $e^{\hat{s}_1 t}, e^{Re\{\hat{s}_2\}t} \cdot \sin(Im\{\hat{s}_2\}t), e^{\hat{s}_4 t}$

• $k = k_{apr}$: regim aperiodic amortizat;
  moduri: $e^{\hat{s}_1 t}, e^{\hat{s}_2 t}, te^{\hat{s}_2 t}, e^{\hat{s}_4 t}$

• $k \in (k_{apr}, \infty)$: regim aperiodic amortizat;
  moduri: $e^{\hat{s}_1 t}, e^{\hat{s}_2 t}, e^{\hat{s}_3 t}, e^{\hat{s}_4 t}$

Senzitivitate:
- mica, deoarece nu se schima regimurile, nici stabilitatea sistemului, iar drumul polului dominant este finit.

Sistemul are numai poli reali, deci nu exista pulsatii.