Frequency Responses & Filters

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(6th Week Pre-Experiment Lab Report)

I. RESEARCH QUESTIONS

a. Observing voltage variations occurring in the transient and steady states of RC and RL circuits to understand the characteristics of resistors, capacitors, and inductors (RC and RL are each circuits created by capacitors and inductors, and the way that they act in transient and steady states occur due to the way that they store energy in their respective fields[1]).

b. Observing underdamping, critical damping, and overdamping in an RLC circuit to gain understanding of the characteristics of the RLC circuit.

c. Understanding how the characteristics of resistors, capacitors, and inductors vary with frequency.

d. Through measuring frequency response curves using RC and RLC circuits, understanding phase difference and resonance phenomena.

e. Understanding the characteristics of passive filters using RLC circuits.

II. THEORY

A. RLC Circuits and Frequency Responses

1. *RC Circuit Analysis*

An RC circuit is a electrical circuit that contains the resistor (R) and capacitor (C) components. They are most commonly analyzed using impedances and vector representations, where the former denotes the opposition a circuit offers to the flow of the AC circuit, and the later is a technique used to identify the voltage signals experienced by each component. The voltage signal in the first experiment can expect to behave according to the following differential equation.

$$C\frac{dV}{dt} + \frac{V}{R} = 0$$

The differential equation gives the following solution.

$$V(t) = V_0 e^{-t/RC}$$

Thus, the time constant can be expected to have a value of RC.

[1] Personal notes and records are in red text.

2. *RL Circuit Analysis*

An RL circuit is similar to RLC circuits, containing resistors (R) and a inductor (L). Similarly to the case above, the RL circuit follows the following differential equation.

$$V = IR + L\frac{dI}{dt}$$

3. *RLC Circuit Analysis*

An RLC circuit is a electrical circuit that contains the resistor (R), inductor (L), and capacitor (C) components.

4. *Time Constant*

In general, the time constant is a measure of how quickly a system responds to changes in input. In RC, RL, or even RLC circuits, time constants, often denoted by τ, expresses the certain time taken for the voltage to decrease by a certain amount.

5. *Resonance, Bandwidth, Q-factor*

In electrical circuits, resonance typically occurs in circuits containing both capacitors and inductors. At a certain frequency, called the resonance frequency, the reactance of the capacitor and the inductor cancel each other, resulting in a voltage signal influenced purely by the resistance. Bandwidth, on the other hand, in the context of resonant circuits, are sizes of the frequency range. Lastly, Q-factors are a measure of the efficiency of an oscillating system or resonator. It can be seen as the ratio between the amount of energy stored in the system and the energy dissipated by the system. In RLC circuits, it is the ratio between the reactance and resistance.

6. *Bode Plot: Voltage Gain, and Phase*

Bode plots are visualisations of the magnitude or amplitude of systems' signals dependent on the frequency.

Voltage gains, on the other hand, are measures of how much voltage is amplified by an electrical circuit. They are often expressed in decibels (dB).

Phases are measures of how much two signals align with each other. Phase is measured in degrees or radians, which are expressions of the fractions of the cycles that one wave leads the other.

B. Passive Filter

In electronic circuits, **low-pass filters** allow signals below a certain cut-off frequency to be eliminated. On the other hand, **high-pass filters** allow signals above a certain cut-off frequency to be eliminated. **Band-pass filters** allow signals around a certain range of values to be recorded, other signals in other values to be eliminated. **Band-rejection filters** do the exact opposite, with the filter eliminating a certain band of values.

Also called **T-section filters**, T filters consist of resistors and capacitors arranged in the shape of the letter "T", being able to be designed as any of the filters described above.

III. METHODOLOGY

A. RLC Circuits and Frequency Responses

1. Construct the circuit depicted in Figure 1-1.

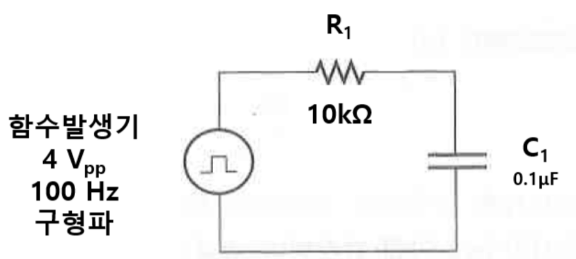

FIG 1-1. Series RC circuit.

2. Using an oscilloscope, observe the voltage variation across the capacitor terminals. Measure the half-cycle period of the capacitor's voltage signal, calculating the time constant of the RC circuit, and compare it with the theoretical value. It is important to note that the output resistance of the function generator should also be taken into account during calculation (the output resistance has a resistance of its own, and the peak-to-peak value of the AC signal is decreased due to this resistance).

3. Observe how the voltage signal across the capacitor terminals changes when a sinusoidal wave with a much larger period than the time constant is inputted. Conversely, observe how the voltage signal across the resistor terminals changes when a sinusoidal wave with a much smaller period than the time constant is inputted. Through this experiment, one can understand differentiation and integration circuits using RC circuits.

4. Replace the resistor with a 1 kΩ, resistor and the capacitor with a 1 μF capacitor. Run the Bode Analyzer on the NI ELVIS to measure the Bode plot. Set the input signal to the signal from the function generator (NI ELVIS) and set the output signal to the voltage signal across the capacitor terminals. Set the frequency range from 10 Hz to 100 kHz and set the per decade to 100. Measure the frequency and phase at which the voltage gain becomes $1/\sqrt{2}$ (or measure the point where it reaches -3 dB on a log scale). This measured frequency is the cutoff frequency or the corner frequency. Compare the measured cutoff frequency with the theoretical value.

5. In the circuit depicted in Figure 1-1, replace the capacitor with a 100 mH inductor and set the frequency of the function generator to 5 kHz.

6. Utilize an oscilloscope to observe the voltage variation across the inductor terminals. Experimentally determine the time constant of both the RC and RL circuits through observation and compare them with their theoretical values.

7. Repeat experiments 3 and 4 (after replacing the inductor with a 22mH inductor).

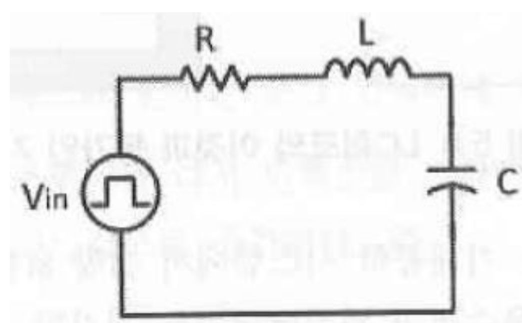

FIG 1-2. Series RLC circuit.

8. Construct the circuit depicted in Figure 1-2. Use a 5 kΩvariable resistor, a 22 mH inductor, and a 0.1 µF capacitor. Set the function generator to output a sine wave with a frequency of 100 Hz and an amplitude of 4 V_{PP}.

9. While varying the variable resistor, observe underdamping, critical damping, and overdamping of the RLC circuit. It is recommended to refer to a dynamics textbook for theoretical understanding before conducting the experiment. Additionally, observe the resonant frequency and compare it with theoretical values.

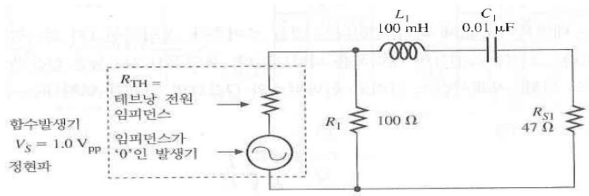

FIG 1-3. series RLC circuit.

B. RLC Circuits and Resonance

1. Construct the circuit depicted in Figure 2-1. If the function generator has an output impedance of 50 Ω, resistor R_1 is not required. This is because it reduces the Thevenin driving impedance R_{TH} of the function generator, thereby reducing the total equivalent series resistance of the circuit.

2. Execute the Bode Analyzer on the NI ELVIS to measure the Bode plot. Set the input signal to the signal from the function generator and set the output signal to the voltage signal across resistor RS1. Set the frequency range from 10 Hz to 100 kHz and set the per decade to 100. Measure the frequency and phase where the voltage gain becomes $1/\sqrt{2}$ and where the voltage gain is maximized. The frequency where the voltage gain becomes $1/\sqrt{2}$ can be used to determine the bandwidth and Q-factor, while the frequency where the voltage gain is maximized is the resonant frequency. Compare the experimentally calculated values with theoretical values.

3. Construct the circuit depicted in Figure 2-1 and repeat steps 1 to 3 identically. However, note that the output signal should be set to the voltage signal across the capacitor terminals.

C. Passive Filter - T Filter

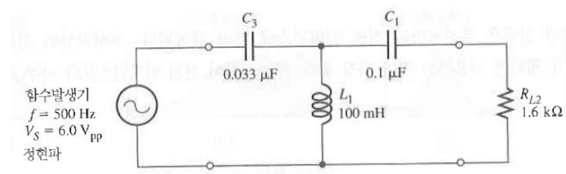

FIG 3-1. T filter circuit.

1. Construct the circuit depicted in Figure 3-1. Set the voltage signal across the load resistor RL2 as the output signal. Execute the Bode Analyzer on the NI ELVIS to observe the Bode plot.

[1] Halliday, D., Resnick, R., and Krane, K. Physics, volume 2. *Wiley, 2nd Edition* (2010), pp. 845–853.
[2] The Sogang University Physics Department. Experimental physics 1 manual. *"Frequency Responce, Filters"*.
[3] Wikipedia. Bode plot. `https://en.wikipedia.org/wiki/Bode_plot`.
[4] Wikipedia. Q factor. `https://en.wikipedia.org/wiki/Q_factor`.
[5] Wikipedia. Rlc circuit. `https://en.wikipedia.org/wiki/RLC_circuit`.