The Hall Effect and Helmholtz Coils

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(2nd Week Post-Experiment Lab Report)

I. EXPERIMENT DATA

A. Hall Effect Experiment

semiconductor type	dimension	length (cm) [a]
N-type Semiconductor	horizontal length (l)	0.80
	vertical length (w)	0.40
	thickness (t)	0.05
P-type Semiconductor	horizontal length (l)	0.65
	vertical length (w)	0.45
	thickness (t)	0.05

[a] Vernier calipers, ± 0.005.

TABLE I. Sample dimensions

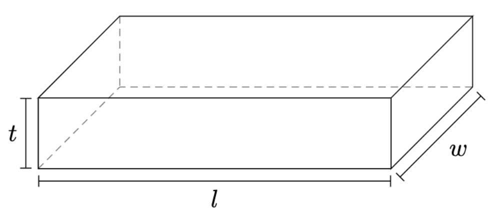

FIG. 1. Sample dimensions.

current (A) [a]	magnetic field [b] strength (mT)	conversion factor (mT/A)
0.26	0.82	3.15
0.50	1.54	3.08
0.75	2.33	3.11
1.00	3.09	3.09
1.25	3.80	3.04
1.50	4.71	3.14

[a] Constant power source, ± 0.005.
[b] Gaussmeter, ± 0.005.

TABLE II. The relationship between the electromagnet's current and magnetic field, ± 0.005.

The first table shows the dimensions of each sample (N-type and P-type semiconductors), where the figure is intended to aid visualisation. The voltage is measured across the width w, the magnetic field is sent through the dimension t, and the current is sent across the length l creating an axial electric field within the sample.

Given the data, a simple graphing of the points in *Python* using the *Matplotlib library* reveals the linear relationship that the current through the electromagnet has with the magnetic field strength.

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}} \tag{1}$$

Additionally, we can quantitatively measure the correlation by using Pearson's correlation coefficient (eq. 1). For this dataset in particular, the regression turns out to be $r = 0.99946$, showing a highly linearly correlated relationship. The average of the conversion factor can be calculated to be 3.10 mT/A, overall justifying our use of this number in the future to deduce the magnetic field strength in milli-teslas of the electro-magnetic from the current value in amperes and vice-versa.

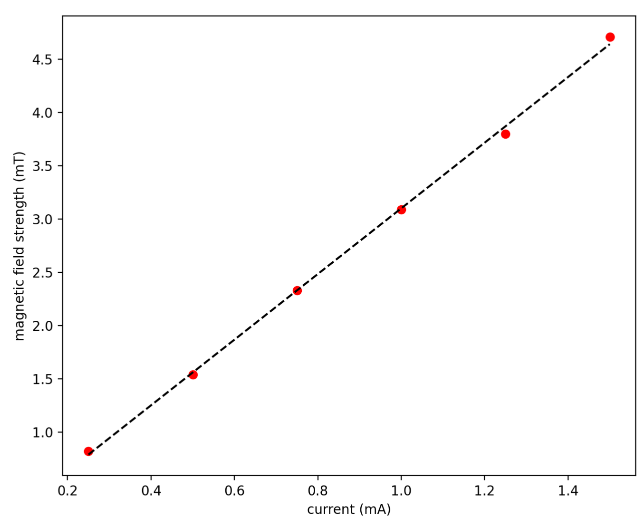

FIG. 2. Current versus magnetic field strength graph.

In measuring the sample's voltage by current (with an absence of a magnetic field), the plotting method, being the more time-efficient method, was used during data collection to check linearity and as the linear relationship was consistent throughout all currents as shown in figure 3, and the current was arbitrary chosen as 4.00 mA.

N-type (4 mA)					
electro-magnet current (A)	electro-magnet field strength (mT)	sample voltage (mV)	sample voltage difference (mV)	charge-carrier density (10^{19} m^{-3})	Hall coefficient (m^{3}V^{-1}s^{-1})
0.13	0.40	187.4	0.5	4.0	0.16
0.27	0.80	185.6	-1.3	3.1	-0.20
0.40	1.20	184.7	-2.2	2.7	-0.23
0.53	1.60	183.3	-3.6	2.2	-0.28
0.67	2.00	182.2	-4.7	2.1	-0.29
0.80	2.40	180.5	-6.4	1.9	-0.33
0.93	2.80	179.5	-7.4	2.0	-0.33
1.07	3.20	178.2	-8.6	1.8	-0.34
1.33	4.00	175.4	-11.5	1.7	-0.36

TABLE III. N-type

P-type (4 mA)					
electro-magnet [a] current (A)	electro-magnet [b] field strength (mT)	sample voltage [c] (mV)	sample voltage difference (mV)	charge-carrier density (10^{19} m^{-3})	Hall coefficient (m^{3}V^{-1}s^{-1})
0.13	0.40	-2.7	0.9	2.2	0.28
0.27	0.80	-1.9	1.7	2.4	0.27
0.40	1.20	-0.9	2.7	2.2	0.28
0.53	1.60	-0.0	3.6	2.2	0.28
0.67	2.00	0.7	4.3	2.3	0.27
0.80	2.40	1.9	5.5	2.2	0.29
0.93	2.80	2.8	6.4	2.2	0.29
1.07	3.20	3.8	7.4	2.2	0.29
1.33	4.00	5.5	9.1	2.2	0.28

[a] Constant power source, ±0.005.
[b] Hall probe, ±0.005.
[c] Hall effect experiment setup, ±0.05.

TABLE IV. P-type.

sample current [a] (mA)	N-type voltage (mV)	P-type voltage (mV)
0.80	38.8	-0.07
1.60	76.2	-0.15
2.40	112.7	-2.20
3.20	148.8	-2.50
4.00	186.9	-3.60

[a] ±0.005, Hall effect experiment setup.

TABLE V. Current-voltage relationships for samples with no magnetic field.

$$n = \frac{iB}{et\Delta V_H} \tag{2}$$

With this current kept constant throughout the sample, the change in voltage across the sample as the magnetic field strength is varied was recorded. the magnetic-field was varied through 10 different values. The electro-magnet's current in amperes, the electro-magnet's field strength in milli-telsas, and the recorded voltage across the sample for both N-type and P-type semi-conductors can be seen above in tables 4 and 5. The electro-magnet's current and the electro-magnet's field strength was simultaneously measured to be recorded, and the sample voltage was constantly measured throughout the variations applied.

The charge carrier density was first obtained using the equation above (eq. 2) from the fixed (sample current), independent (electro-magnet field strength), and dependent (sample voltage) variables. The thickness of the sample was obtained the initial measurements as recorded above and the value 1.602×10^{-19} C was used for the elementary constant e. After the charge-carrier density n was sought, the Hall coefficient was deduced afterwards from the following formula.

sample voltage (mV)	sample voltage difference (mV)	theoretical magnetic field strength (mT)	actual magnetic [a] field strength (mT)	percentage error
185.0	1.9	0.9	1.20	24.4 %
181.0	5.9	2.8	2.40	17.4 %
176.2	10.7	5.1	4.00	27.8 %

[a] ±0.005, Gauss meter.

TABLE VI. Measurements of an arbitrary magnetic field strength with the N-type semi-conductor.

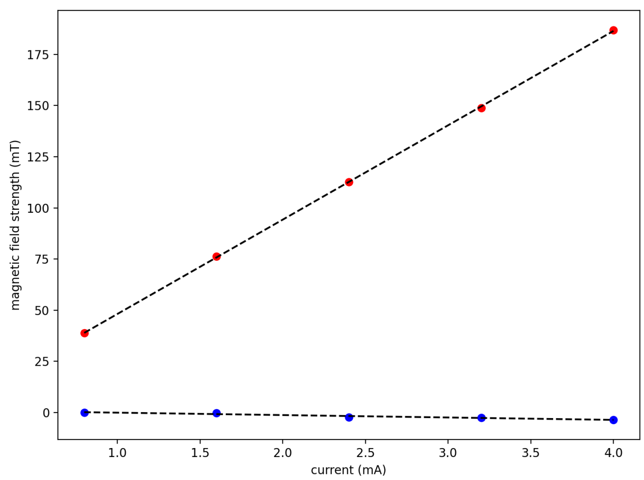

FIG. 3. Sample current versus sample voltage plot.

$$R_H = \frac{1}{ne} \tag{3}$$

The last and final experiment revolving around the Hall effect required voltage measurements to be made for an arbitrary magnetic field. The N-type semiconductor in particular was used for this experiment, and the sample voltage difference, the average hall carrier-carrier density from table 3, and the averge Hall coefficient from table 3 was used to calculate the exact theoretical magnetic field strength. The actual magnetic field strength, measured by the Gauss meter, was compared to this value via percentage error calculations.

$$B \text{ \% error} = \frac{|B_{\text{theoretical}} - B_{\text{actual}}|}{B_{\text{theoretical}}} \tag{4}$$

The error will be further evaluated later on.

B. Helmholtz Coil Experiment

For the Helmholtz coil experiment, the line that connects the centers of the two ends was used as the central axis, and the distance from the center of the two coils was used as the origin.

distance from center (cm) [a]	magnetic field strength (mT) [b]
-4.00	0.07
-3.00	0.13
-2.00	0.17
-1.00	0.16
0.00	0.18
1.00	0.17
2.00	0.16
3.00	0.15
4.00	0.08

[a] Wooden ruler, ±0.005.
[b] Gauss meter, ±0.005.

TABLE VII. Magnetic field strength by distance.

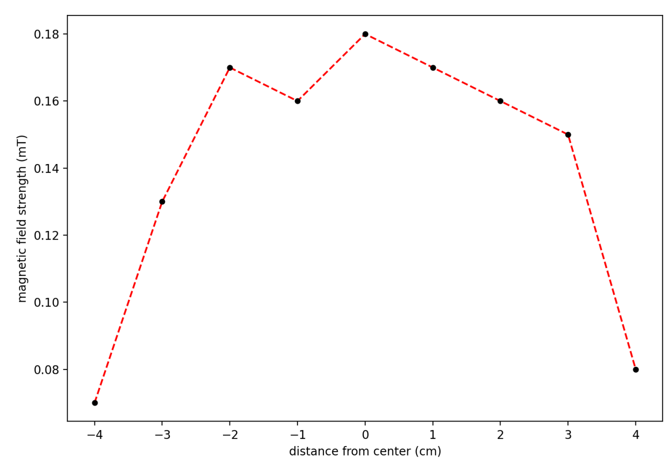

FIG. 4. Distance from the center versus magnetic field strength of solenoid graph (experimental).

II. ANALYSIS AND RESULTS

A. Goals and Recapituation of Experiments

As written in the pre-report, the goals of this experiment was the following, encompassing both the Hall effect experiment and the Helmholtz coil experiment:

a. Understanding the Hall effect and ways of measuring it.

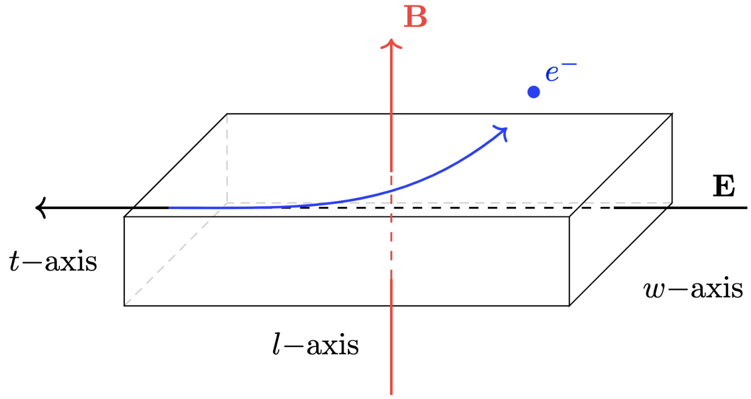

FIG. 5. The path of an electron (the charge-carrier) for a N-type semi-conductor.

b. Measuring a sample's charge carriers' sign (positive or negative) and charge density, and understanding N-type and P-type semiconductors in the process.

c. Understanding how one could utilise the Hall effect to measure (strength of) a field, and discussing the method's limitations.

d. Understanding the functions of Helmholtz coils, acknowledging real-world applications.

Consequently, in total, there were 6 different sets of measurements made for the Hall effect experiment in particular:

1. The dimensions (horizontal length, vertical length, and thickness) of the two semiconductors (N-type and P-type) (see I).

2. The relationship between the electromagnet's current and magnetic field for 6 different current values ranging from 0.26 to 1.50 (see II).

3. The voltage that occurs in each semiconductors due to the fixed current selected in the measurement set above (see V).

4. The electro-magnet current, electro-magnet field strength, the sample voltage, sample voltage difference, the charge-carrier density, and Hall coefficient for 10 different magnetic field strength variations through the two samples (see III and IV).

5. The voltage recorded throughout the sample for three arbitrary magnetic field strength values, measured and theoretically deduced from the sample voltage difference from the Hall effect formula (see VI).

On the other hand, there was 1 set of measurements made for the Helmholtz coil experiment:

6. Magnetic field strength for different distances between the two constituents of the Helmholtz coil (see VII).

B. Evaluation and Error Assessment

1. *Hall Effect Experimment*

The first measurement set was fairly simple, preformed with vernier calipers with high precision. It was intended to be used in further calculations made later on, involving the actual calculations of the Hall voltage and Hall coefficient, thus intended to support the first goal from above. There was an uncertainty of ± 0.005, as the caliper was capable of measuring numbers down to the closest tenth of a millimeter. The figure was then created with LaTeX's *TikZ package* to help visualize the 3-dimensional setup that can be easily misunderstood.

The second measurement set's purpose was in identifying that the current through the electromagnet and the magnetic field strength had a linear relationship, and obtaining the conversion factor that would allow future calculations to be made directly with reference of the current of the electromagnet rather than through using a separate magnetic field measuring device. The theoretical errors were calculated in par with the conventional error calculation methods for digital devices (errors for digital devices are suppose to be half the highest precision decimal point). The information on six different current values (the maximum was set as 1.50 A as the manual instructed) were plotted via the Python *Matplotlib package* (See 2, and in addition, Pearson's correlation coefficient was obtained with equation (1), together showing great correlation of the two physical quantities. The errors in the current and the magnetic field was again obtained from conventional error analysis methods, being ± 0.005 for both the constant power source measuring current in amperes and the Gauss meter measuring the magnetic field strength in milli-teslas.

The third measurement's purpose lied in the fact that semi-conductors with current flowing in it automatically gain voltage across its thickness (t), which would hinder the measurement of the proper Hall voltage, which would should only be the voltage dependent on the magnetic field sent across the vacuum containing the semi-conductor. Following the manual, the voltage produced by different current values were measured, plotted (See 3), and evaluated. The evaluation was on whether the current was adequate for the Hall measurement. The current should have produced a consistent voltage value across the samples, and this was evaluated through looking at the local linearity of magnetic field values around particular intervals of current values. If a certain local interval showed high linearity, it would guarantee important circuit characteristics such as accurate signal amplification, reduction of distortion, compatibility among standard components. These together would help in credible calculations to be done later on in the next measurement step. The graph, again plotted by Python, showed linearity throughout all values, and the value of 4 A was used as it induced field strength values big enough for large sample voltages well above the smallest detectable decimal point to be sufficiently detected.

The fourth measurement(s) were where the most important and relevant data collection and calculations were made. The N-type and P-type semi-conductors were both put between electro-magnets with the constant current of 4 mA flowing within them, and the magnetic field strength was varied through 9 different values of the magnetic field, each differing by approximately 0.40 mT. The corresponding current of the constant power source that powered the electromagnet was also measured for more information, and the ultimate sample voltage was measured with the Hall effect experiment setup. Again, the critical step was to subtract the default voltage measured in the previous step (the voltage across the samples when 4 mA was sent through them) to this voltage, and accurately finding the voltage *difference* induced uniquely due to the magnetic field. This sample voltage, shown in the 4th column of the table, was what was used in equation (2) to get the charge-carrier density and ultimately the Hall coefficient. The overall process helped in accomplishing the second goal and third goals from above, supporting the understanding the Hall effect and the two types of semi-conductors. As seen in 5, for the N-type semiconductor, the charge carrier had a negative charge, and as the Lorentz force was applied, the electron was accelerated and displaced towards the positive w-axis, in the top side of the page in reference to the figure. As the electron had a negative charge, the accumulation them in the back side of the cube the on the wl-plane induced a lower voltage and thus a negative voltage difference. For the P-type semi-conductor, the exact opposite would of happened, with an accumulation of the "holes" (the *absence* of electrons) on the top part would have created a relatively positive charge, hence inducing a positive voltage difference.

the overall results hugely give insight into what N and P-type semi-conductors are and what behaviour they display. With the charge carriers being different, the Lorentz force changes the inner structure of these semi-conductors differently, and to conduct proper applications, we need to hugely differentiate how they are oriented in reference to electric and magnetic fields. The values of the charge-carrier density and the Hall coefficient have no proper benchmark that we can use to compare the values and calculate errors as different materials hugely differ and such values.

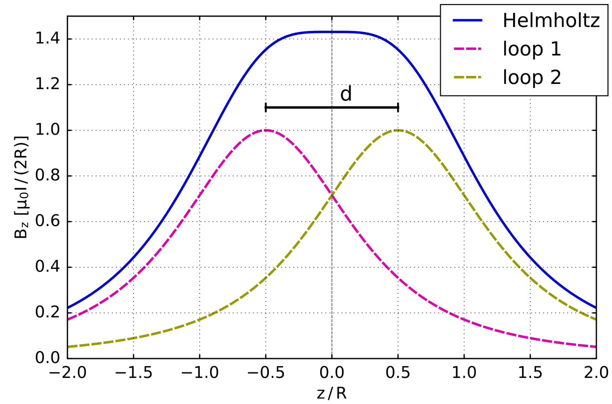

FIG. 6. Distance from the center versus magnetic field strength of solenoid graph (theoretical) [7].

The Last measurement, the sixth measurement, was the last measurements revolving around the Hall-effect. Arbitrary magnetic fields were produced around the N-type semi-conductor, to reverse the steps taken in the calculations of the charge-carrier density and the Hall coefficient and determine the magnetic field from the understanding of the inner workings of the semi-conductor. The actual values were taken from the current, with the conversion factor obtained in measurement set 2. The result of the reverse experiment was highly effective, with low error values, ranging from 17.4% to 24.4%.

2. Helmholtz Coil Experiment

The Helmholtz coil experiment involved one singular measurement set, where the magnetic field strength between the two coils were measured. Theoretically, the magnetic field strength should have shown a hill like shape with the highest value exactly at the middle of the two coils. In the special case considered in this experiment, the derivation is relatively straight forward, directly from the Bio-Savart Law. For one coil:

$$B(x) = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}} = \xi(x)\frac{\mu_0 I}{2R} \tag{5}$$

The influence of two coils with n turns can thus be expected to be

$$B(x) = \frac{\mu_0 n I}{2R}\left[\xi(x-R/2)+\xi(x+R/2)\right]$$

$$= \frac{\mu_0 n I}{2R}\Big(\left[1+(x/R-1/2)^2\right]^{-3/2} \tag{6}$$

$$+\left[1+(x/R+1/2)^2\right]^{-3/2}\Big)$$

After expanded as a Taylor series,

$$\begin{gathered}(x/R)(96/\sqrt{5}/125-(x/R)^3(512\sqrt{5})/625 \\ +\mathcal{O}((x/R)^5)\end{gathered} \tag{7}$$

given $x \ll R$. A visual representation can be seen below in figure 6. Compared with the experimental data, visually displayed in 4, the evident similarity of the curves can be seen, suggesting a well preformed experiment. There does exist a slight lean towards the right in addition to the a slight dip of the value relative to the trend for the distance -1 cm, most likely a error of some sort. The probable reason for this error, along with some other possible sources of error in the experiments mentioned above, other than the ones already mentioned, can be listed like the following:

Parallax error occurs in analog measurements of all sorts, and the analog tools used (vernier calipers, wooden ruler) could have induced this particular type of error. The certain type of error can be fairly simply avoided by using digital instruments.

Improper calibration the experiments preformed required fairly many calibrations to be done. The Gauss probe in particular required calibration for proper measurement of the magnetic field in the last experiment, and a faulty calibration for the measurement done in -1 cm is a very probable reason for the dip occurred in 4.

Faulty wiring of the circuits are also a highly probable, inducing irregular signals and currents. The Gauss meter and the measurements done with the Hall experiment setup all highly varied my time, measurements increasing and decreasing frequently. The low values of physical quantities would have more so increased the probability that these errors would have affected the results, with even the smallest spikes capable of influencing the result. A leeway for this kind of error would have been higher magnetic fields and currents to be used, with less frequent changes of the values.

Voltage drops are always a factor that highly disturbs electronic experiments, and it is very likely that voltage drops would have affected this experiment. The resistance of the cables and possible groundings in the middle of the experiments would have caused frequent and instantaneous changes in voltage and other measurements influenced by it.

To finish, there are multiple applications of the results of these experiments and the series of measurements made. Semi-conductors in general are used in many different devices with their utility in allowing and blocking currents. With the better understanding of the inner-workings of the semi-conductor samples, we can use these materials in devices with caution of how they will behave within magnetic fields like those applied in the experiment.

In the case of the Helmholtz coils, as mentioned in [4], applications include the reversing of the effect of the Earth's magnetic field. In (magnetic) field-sensitive experiments, the coils that creates a field vector towards a certain direction can be used against the direction of the field vector created by Earth's magnetic field, were the superposition of these fields will result in a net field of zero. They can also be used to do the opposite: increase a point's magnetic field. In cases where moving charged particles are intended to gather in a certain point (for example in doping processes), magnetic field can be created through the Helmholtz coils with a consistent, calculable, and variable spread and intensity, unlike the simple uses of magnets.

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[3] Hu, C. Modern semiconductor devices for integrated circuits. *Prentice Hall,* (2010), pp. 1–6.

[4] The Sogang University Physics Department. Experimental physics 1 manual. *"Hall Effect, Helmholtz Coil"*.

[5] Wikipedia. Charge carrier density. `https://en.wikipedia.org/wiki/Charge_carrier_density`.

[6] Wikipedia. Hall effect. `https://en.wikipedia.org/wiki/Hall_effect`.

[7] Wikipedia. Helmholtz coil. `https://en.wikipedia.org/wiki/Helmholtz_coil`.

[8] Wikipedia. Magnetic field. `https://en.wikipedia.org/wiki/Magnetic_field`.